"A OR NOT-A" — that is the question

At the end of June’04 the following note concerning the philosopher and logician Gotthard Günther could be found within Wikipedia, the free internet encyclopedia[1]:

Gotthard Günther (in the USA as well: Gotthard Guenther), June 15, 1900 - November 29, 1984, has been a German philosopher.

Work and Vita

His work based upon Georg Wilhelm Friedrich Hegel, Martin Heidegger and Oswald Spengler. He developed a trans-Aristotelian logical approach (omitting the tertium non datur). Günther’s transclassical logic was the attempt to combine improved results of modern dialectic with formal logic. His focus on the philosophical problem of the “Du” (“You”/”Thou”) was trailblazing. He contributed as well to the fields of cybernetics and to both natural and social sciences, especially to sociology...

We have cited this Wikipedia contribution because there is at least one point which has to be discussed, namely the rejection of the law of excluded middle (LEM) – the so-called tertium non datur (TND) –, which according to the author(s) of the Wikipedia article has been omitted by Gotthard Günther in his design of a "trans-Aristotelian" logic. We will not discuss the statement that Günther’s work is based upon Hegel, Heidegger and Spengler, because this statement is not completely wrong but it is a somewhat shortened view, a kind of oversimplification.

However, the interpretation of Günther’s polycontextural logic (PCL)[2] as an elimination of the TND is not only a misunderstanding, it is totally wrong. Since the PCL is an extension of classical logic and mathematics it contains necessarily all laws of classical logic and mathematics. What has changed are the conditions under which these laws hold. Since the interpretation of the law of excluded middle (LEM) as well as the law of contradiction (LC) are of importance for all theories of inconsistent or paraconsistent logical systems[3] we will focus our discussion only on this particular point of the Wikipedia article from June’04.

In the German version of the Wikipedia contribution there is a hyperlink to the term tertium non datur (TND) which does not exist in the English contribution for reasons that are unknown to the present authors. However Wikipedia also offers an explanation for the TND in English[4], viz.:

The law of excluded middle (tertium non datur in Latin) states that for any proposition p, it is true that (p or not-p).

Joe is bald

then the inclusive disjunction Joe is bald, or Joe is not bald is true.

This is not quite the same as the principle of bivalence, which states that P must be either true or false. It also differs from law of non-contradiction, which states (P and not-P) is false. The law of excluded middle only says that the total (p or not-p) is true, but does not comment on what truth values P itself may take. This leaves the possibility open that certain systems of logic may reject bivalence (by allowing more than 2 truth values) but accept the law of excluded middle, by accepting that (p or not-p) is always true, even when P itself is neither true nor false. The distinction is far less important in traditional logic, however, where bivalence is accepted. Some nontraditional logics, most notably intuitionistic logic, are not bivalent, and in such logics the law of excluded middle does not necessarily hold. The page bivalence and related laws discusses this issue in greater detail.

The law of excluded middle can be misapplied, leading to the logical fallacy of the excluded middle, also known as a false dilemma.

Note: For reasons of a better overview all hyperlinks have been omitted.
On one side, the author of this contribution states that "the law of excluded middle only says that the total (p or not-p) is true" and on the other side he claims that the inclusive disjunction 'Joe is bald, or Joe is not bald' "is not quite the same as the principle of bivalence, which states that p must be either true or false" where he defines the principle of bivalence "that for any proposition p, either p is true or false". From a traditional logical point of view these statements are not very convincing, because if we consider "p or non-p" as true, i.e.,

\[ p \lor \neg p \equiv 1 \]  \hspace{1cm} (1)

the principle of bivalence is fulfilled. In order to see what is hidden behind such foggy statements it is worth to have a short look behind the scenes of so-called 'alternative logics' in which certain 'classical' laws are abandoned or replaced. 'Alternative' or 'non-consistent' logics "have become rather fashionable in logical circles; even receiving a kind of blessing from one of the Church Fathers (cf. Quine \[5\])", as Van Benthem calls it in *What is Dialectical Logic ?*[6]. In ref.[7] Peter Suber gives a birds-eye view of the problems concerning non-consistent logical concepts with respect to the classical 'Laws of Thought'. These are the 'Law of Identity', the 'Law of Contradiction', the 'Law of the Excluded Third' and the 'Law of Ground'. What happens if these laws of thought should fall, has been described by Oliver Reiser\[8\] as "the most profound modification in human intellectual life will occur, compared to which the Copernican and Einsteinian revolutions are but sham battles". And as V.J.McGill – one of the forgotten American philosophers who was professionally injured during the McCarthy era\[9\] – emphasized "if our reasoning is correct, that the present philosophers and logicians who are ready to restrict the law of excluded middle must be prepared to restrict the law of contradiction as well, or to work out a new system which evades the paradox".\[10\] In other words: both logical principles, the law of contradiction and the law of excluded middle, are conditionally equivalent because each can be deduced from the other one, provided the classical rules of inference, deMorgan’s law and the 'Law of Double Negation', i.e. \(p \equiv \neg \neg p\), are accepted to hold.

Before we continue to discuss the meaning, the interpretation of the LEM, it might be useful to collect the four (not three!) so-called 'Laws of Thought':

1. **The Law of Identity (LI):**

   The law of identity – that \(x = x\) – constitutes the foundation of logic and mathematics, and by extension of a coherent and meaningful philosophy. The law recognizes, quite simply, that something is itself – that it is what it is, with all the implications of being what it is. It institutionalizes the prevailing theory of natural sciences stating that every member of a class, say class \(C\), has the same nature as every other member of that class. If \(x\) and \(y\) are two objects or elements of a class \(C\), they are identical if every attribute of \(x\) is also an attribute of \(y\) and vice versa, i.e.,

\[(x \equiv y) =_{def} \forall P [P(x) \leftrightarrow P(y)]\]  \hspace{1cm} (2)

where "\(\equiv\)" symbolizes equality in the meta-language, "\(\leftrightarrow\)" symbolizes equality on the level of the object language, and "\(\forall P\)" is the universal quantifier on the predicate \(P(\ldots)\) which reads as "\(\ldots\) is \(P\)".

As obvious as the law of identity appears, its consequences are not, neither from a philosophical nor from a formal logical point of view. A very good discussion on these points is given by Gotthard Günther in *Idee und Grundriss*[11].
2. **The Law of Contradiction (LC):**

The law of non-contradiction states that each member of a class is not only identical to every other member in that class, or that \( x = x \), as the law of identity states, but also that the nature of a class cannot be what it is not, say \( x \) and non-\( x \). Symbolically the law of non-contradiction states that \( x \) is not non-\( x \), i.e.

\[
\sim (x \land \sim x) \quad (3)
\]

If this law will be accepted, then (3) is true or, i.e., \( \sim (x \land \sim x) \equiv 1 \)

3. **The Law of Excluded Middle or Third (LEM):**

This law states that not only is \( x \equiv x \), and that \( x \) is not non-\( x \), it adds that \( x \) is \( x \) and nothing in between or if there is the statement \( p \) either \( p \) or \( \sim p \) must be true, there is no third or middle true proposition between them, viz.

\[
p \lor \sim p \quad (4)
\]

If the this law will be accepted, then (4) becomes logical true as in eq.(1), i.e., \( p \lor \sim p \equiv 1 \).

4. **The Law of Sufficient Ground (LSG):**

Everything that exists must have a necessary and sufficient reason for existence – and that reason can be discovered and communicated to others. This conception was formulated in 1646 by Leibniz, the great German logician, mathematician and philosopher, as "the principle of sufficient reason".

These principles are known as the 'Laws of Thought' because they have been considered in the past to be some of the most fundamental principles of logic. Notice that although they are called 'Laws of Thought', it does not mean that people actually follow them all the time. In fact people violate these principles when they think, such as when they are being inconsistent. These laws can be summarized as follows:

A statement is either true or false. It is precisely one of both (law of identity). It cannot be both true and false (law of contradiction) and it cannot accept another value between true and false, i.e. a third does not exist (law of excluded middle), and it must have a sufficient ground for its existence.

Although the law of identity is basic for all natural sciences as well as for mathematics and logic, this law is not explicitly used within axiomatic deductive derivation of the propositional calculus.[12] This law is simply taken as granted.[13]

The law of sufficient ground is even not known to most mathematicians and logicians and therefore no attempt of its foundation is known within the frame of classical mathematics or logic.

The law of excluded middle and the law of contradiction can be deduced on the basis of the mathematical axiomatic deduction process and as a consequence the 'Laws of Thought' as given above in their semantic form lost much of their importance and have been pushed more or less into the background of our scientific thinking. By the majority of logicians the 'Laws of Thought' are considered as needless. However, this is a big mistake because these laws represent the foundation, the meaning, the semantics of the logical calculus, while the mathematical (algebraic) axioms stand for the syntax of the theoretical building.[12]

Gotthard Günther’s book *Idee und Grundriss* (cf. ref. [11]) is certainly one of the most excellent discussions concerning the logical importance and philosophical interpretation of the 'Laws of Thought'. Since this book has not yet been translated, we first will give
some citations of Herbert Francis Bradley[14], an English philosopher, quoted by Günther in Idee und Grundriss. The importance of Bradley’s œuvre for the development of non-Aristotelian, non-consistent or paraconsistent logical systems has not been discovered neither by the Anglo-American nor by the German scientific community (if we neglect Günther’s Idee und Grundriss). In some way Bradley bridges the German philosophical idealism and the Anglo-American pragmatism and it is this bridging, this dialectical interplay, which seems to be the intellectual obstacle for a reception not only of Bradley’s but also of Günther’s work.

In Writings on Logic and Metaphysics[15] James W. Allard characterizes the idea of the third chapter of Bradley’s Principles of Logic as follows:

"The third chapter of The Principles of Logic, 'The Negative Judgement', is an application of Bradley’s general view of the nature of judgement to the specific case of negative judgements. Bradley applies this general view in the preliminary form in which he states it in 'The General Nature of Judgement' rather than in the considerably deepened form of 'The Categorical and Hypothetical Forms of Judgement'. His concern is to explain how negative judgements contain an ideal content which is predicated of reality.

After summarizing his view in §1, Bradley devotes the rest of this chapter to a defence of two theses that support it. The first thesis, defended in §§2-6, is that negative judgements stand on a different level of reflection from affirmative judgements. By this Bradley means that negative judgements portray what they deny as something that has a truth-value. This is analogous to the way in which negation is represented as a truth-functional operator in sentential logic. Bradley explains this claim in §§2-3 and elaborates it in §§4-6 by contrasting it with three claims about negative judgements that he rejects.

The second thesis, defended in §§7, 12, and 15-20, is that negative judgements presuppose a positive ground. This is a denial of any view that takes negative judgements to be true or false irrespective of their relation to any other judgements. It further specifies that at least one of these other judgements must be affirmative. Bradley explains this claim briefly in §7 and applies it to different kinds of negative judgements in §12. In §§15-19 he attempts to clarify it further by arguing that the positive ground that a negative judgement presupposes is, in the terminology of the logical tradition, the contrary, not the contradictory. Bradley concludes the chapter in §20 by suggesting that his view of negative judgements has metaphysical implications. Bradley’s view of negative judgements plays a crucial role in his metaphysics, a fact to which he makes no reference in this chapter. (Its role in his metaphysics is explained in 'The General Nature of Reality', [121-2], reprinted below.)."

The following citations from chapter 3 and 5 can be found in full length as pdf-file in ref.[14a]:

**F.H. Bradley: The Principles of Logic** - Chapter III : The Negative Judgement

§ 2:  
... It is not merely as we shall see lower down (§7), that negation presupposes a positive ground. It stands at a different level of reflection....

§ 3:  
... Thus in the scale of reflection negation stands higher than mere affirmation.

§ 7:

Every negation must have a ground, and this ground is positive. It is that quality x in the subject which is incompatible with the suggested idea. A is not B because A is such that, if it were B, it would cease to be itself. Its quality would be altered if it accepted B; and it is by virtue of this quality, which B would destroy, that A maintains itself and rejects the suggestion. In other words its quality x and B are discrepant. And we can not deny B without affirming in A the pre-existence of this discrepant quality.

§19:

... Contradiction is thus a ‘subjective’ process, which rests on an unnamed discrepant quality. It can not claim 'objective reality'; and since its base is undetermined, it is hopelessly involved in ambiguity....
§20:  
... I think most of us know that one can not affirm without also in effect denying something ...

And in chapter 5, we find:

F. H. Bradley: The Principles of Logic - Chapter V: The Principle of Identity, Contradiction, Excluded Middle, and Double Negation

§1  
...For identity without difference is nothing at all. It takes two to make the same, and the least we can have is some change of event in a self-same thing, or the return to that thing from some suggested difference....

§12.  
We have to avoid, in dealing with Contradiction, the same mistake that we found had obscured the nature of Identity. We there were told to produce tautologies, and here we are by certain persons forbidden to produce anything else. "A is not not-A" may be taken to mean that A can be nothing but what is simply A. This is, once again, the erroneous assertion of mere abstract identity without any difference...

§19  
...Excluded Middle .... In it we affirm (i) that any subject A, when the relation to any quality is suggested, is determined at once with respect to that predicate within the area of position and negation, and by no relation which is incompatible with both. And (ii) we assert that, within this area, the subject is qualified as one single member. And then we proceed to our "either-or".

§20  
... the axiom of Excluded Middle goes beyond the limits of disjunctive judgment. It contains a further principle, since it asserts a common quality of all possible existence. It says, Every real has got a character which determines it in judgment with reference to every possible predicate. That character furnishes the ground of some judgment in respect of every suggested relation to every object. Or, to put the same more generally still, Every element of the Cosmos possesses a quality, which can determine it logically in relation to every other element.

§29  
... I can not say "It is false that A is not b", unless I already possess the positive knowledge that A is b. And the reason of my incapacity is that no other knowledge is a sufficient ground.

Before we discuss Bradley’s statements, we will introduce some arguments which are given by Günther in Idee und Grundriss concerning the meaning of the LEM and the 'Law of Double Negation', i.e. $p \equiv \neg \neg p$.

In Idee und Grundriss Günther refers several times to Elements of Symbolic Logic by Hans Reichenbach\[16\]. Reichenbach introduces four different possible versions for the LEM\[17\]:

a) the version of propositional calculus:  
\[ p \lor \neg p \]

b) the version of predicate calculus with universal quantifier:  
\[ \forall x [P(x) \lor \neg P(x)] \quad (5) \]

c) the totally bound version of second order predicate calculus:  
\[ \forall P \forall x [P(x) \lor \neg P(x)] \]

d) the version of predicate calculus with exclusive disjunction:  
\[ \forall x [P(x) \oplus \neg P(x)] \]

Reichenbach, however, does not give any explanation concerning the content, the meaning of the different possible versions. In other words, the reader has to decide which version he would prefer. In the literature normally only (5a) is discussed as the formal representation of the LEM (cf. ref\[17\]).

Günther does not discuss the relational statement (5c) explicitly, a relation which Reichenbach designates as "the tertium non datur written completely in bound variables".
Instead Günther introduces the complementary version of (5c) using the existential quantification for a universe of discourse with only one object x, i.e.,

$$\exists x \ [P(x) \lor \neg P(x)] \text{universe with only one object } x$$  \hspace{1cm} (6)

The first order relation (6) represents the LEM using the existential quantifier in the limit case of a universe with only one object. Its complement results from a qualified universal quantification which leads to a second order predicate logical formula such as given by Reichenbach:

$$\forall P \forall x [P(x) \lor \neg P(x)]$$  \hspace{1cm} (5c)

In order to understand the meaning of (6) and (5c) we will discuss an everyday example. First we will use relation (6), where the existence of a universe of discourse with only one object x (say a tomato) will be considered with an attribute P(.), e.g., P(.) ≡ "… is red":

$$\exists x \ [P(x) \lor \neg P(x)] \text{universe with only one object } x :=$$  \hspace{1cm} (6)

"There exists (exactly) one object x (a unique tomato) which is red or which is not-red"

This statement cannot be denied without inconsistency, i.e., it is always true. There is nothing left to think about – there is no process of reflection left – because this universe possesses only one and only one object x which is (independent of the kind of object) either red or not-red – and that’s it.

On the other hand, if we consider all possible objects x of a universe of discourse (e.g., all tomatoes of the world) and if we again use the attribute "… is red" then (5b) – the first order predicate expression – yields the following statement:

$$\forall x \ [P(x) \lor \neg P(x)] :=$$  \hspace{1cm} (5b’)

"All objects x (e.g., all tomatoes) are red or not-red"

This declaration about x is not necessarily always true, it can be denied without inconsistency because the attribute "… is red" stands for an attribute which is not the characteristic property of a special object x – such as our tomatoes – but it also stands for many other objects which also could be red. Since we neither can exclude the existence of tomatoes which are never red but yellow or which are characterized by their special taste, or smell, or shape etc. as a distinguishing quality, the statement (5b’) leaves a rest of reflection focused on the object(s) x. In other words, relation (5b’) does not completely interpret the object(s) x and therefore requires some further information or some restrictions on a basic set of objects; restrictions which cannot be given by a machine, but by a thinking human being. This is a completely different situation as depicted by relation (6).

If we use relation (5c) instead of the first order predicate expression (5b), then we have a qualified quantification which now yields a so-called second order predicate expression where the quantification of x has to be accomplished on all properties of x. This means on all properties of x which are known to us and on all properties of x which are still unknown to us. Again this is a limit case which describes 'The Essence' of x – the object-in-general or the object-as-such. Therefore the second order predicate

$$\forall P \forall x [P(x) \lor \neg P(x)]$$  \hspace{1cm} (5c)

is always true, it cannot be denied without inconsistency, because now the object(s) x is (are) completely interpreted, there is nothing left to think about. The object is or it is not — "to be or not to be" —, or since (5c) indicates the plural of x one possibly could state: "the idea, the concept of an object is or it is not".
From a structural point of view relation (5c) and (6) can be considered as complementary logical interpretations of the law of excluded middle. Both relations are always true, i.e., they cannot be denied without inconsistency. Both of them are limit cases, which are unrealistic for the daily life but nevertheless they are necessary as a "thought experiment" in order to extend the Aristotelian logic towards a non-Aristotelian logic, a step which already has been taken by Gotthard Günther by his creation of the polycontextural theory.

In the language of Günther relation (5c) and (6) represent a so-called universal contexture, a term which will be explained shortly. At this stage we have to point the reader's interest again to Bradley's (more than hundred years old) statements given above and to Günther's (more than fifty years old) *Idee und Grundriss* where all these points have been discussed in detail. *Idee und Grundriss* still represents one of the most brilliant outline of the European philosophy analyzed from logical point of view.

Günther discusses explicitly – as we already have mentioned – the case of relation (6) which corresponds to a universe where there is no difference between the essence (the concept) of an object and the object in itself, or in metaphysical words between 'Sein und Seiendem'. In other words, the universe of (6) with only one object can be described without the need of quantifiers and without the need of any predication at all, i.e.,

$$\forall x P(x) \equiv \exists x P(x) \text{ resp. } \sim \forall x P(x) \equiv \sim \exists x P(x) \text{ (for an universe with only one object x)}$$

(7a)

and

$$P(x) \equiv \forall x P(x) \text{ resp. } \sim P(x) \equiv \sim \forall x P(x) \text{ (for an universe with only one object x)}$$

(7b)

and

$$P(x) = \text{def } p \text{ resp. } \sim P(x) = \text{def } \sim p \text{ (for an universe with only one object x)}$$

(7c)

For the complementary limit case, a universe which is charcterized by objects where all attributes are known, i.e., with $\forall P \forall x P(x)$, relation (7) is also fullfilled. Both cases (6) as well as (5c) are distinguished by the fact that the 'Laws of Thought' strictly hold, i.e. the 'Law of Identity' (cf. eq.(2)), the 'Law of Contradiction', the 'Law of Excluded Middle' as well as the 'Law of Double Negation',i.e., $\sim \sim p \equiv p$. And as Günther tells us that relation (7) describes – within the terminology of classical metaphysics – the so-called *coincidentia oppositorum*, an existence where all reality would be invested in "god" or a *summum bonum*, or more pragmatic in a single point of designation. This is a world where all differences between the concept of an object and the object-in-itself, or between the meta- and object language coincide (cf. ref. [19]).

Another result which is demonstrated by relation (7) is given by the fact that the propositional calculus is exclusively determined by "non-analyzed", so-called atomic sentences, that can be processed automatically by machines. This is the reason why for any automatic resolution process the quantified predicated expressions have to be transformed into their propositional logical – their Skolemized form –, a process which cannot be reversed. This point is important since most of non-consistent logical concepts are based on propositional logic and precisely this is their weakness, because all expressions have to be interpreted again and therefore they are in principle useless for any attempt to model or to implement algorithms with the ability of automatic reasoning or deciding (see for example refs.[3][7]).

The unforgotten 'law of ground'

As we have seen above, (5c) and (6) define a universal logical domain where the four 'Laws of Thought' strictly hold. Such a logical domain has been designated by Gotthard Günther as 'universal contexture'. Such a universal contexture is defined by the law of
excluded middle, the tertium non datur which according to relations (6), (5c) and (7) now can be formulated again in its usual way as,

$$ p \lor \sim p $$

where the mark in (4') indicates that the LEM has to be considered in connection with the discussion concerning the universe of discourse related to the logical expressions (6) and (5c).

For any designation (acceptance, affirmation) or non-designation (rejection, negation) of the LEM in (4') a 'point of designation' – a logical place – has to exist from which the designation or non-designation of the total situation as given by the LEM occurs. Such a logical place (or ground) has to be indicated by a number – a third value. This value (place value) cannot be located between true and false, i.e., between 1 and 0 (law of excluded middle) but beyond 1 and 0, i.e., beyond true and false. As it was already mentioned above based on a monocontextural view the 'point of designation' has been called the coincidentia oppositorum which corresponds to the existence of a summum bonum.[21] In other words, only a summum bonum is able to decide whether or not to accept or to reject such a situation as given by (4') on the basis of discussion associated with the meaning of (5c), (6) and (7).

So far we reached a point which in principal should have be known for more than 300 years if Leibniz is considered as the inventor of the law of (sufficient) ground.[22]

From an engineering point of view a single ground or a summum bonum represents a logical concept which is not very helpful for any attempt to design and construct, for example, decision-making machines.(cf. ref.[21]) This is a point which was very well known to the philosopher and logician Gotthard Günther[23] – an expert of the German idealism – who neither fits into the category of idealism nor into the category of the Anglo-American pragmatism.

For any technical application we have to go back to the situation described by expression (5b'):

$$ \forall x [P(x) \lor \sim P(x)] := (5b'') $$

"All objects x (e.g., all tomatoes) are red or not-red"

If we define a logical domain – a contexture – only for the considered characteristic attribute "red" or "not-red", then according to the rules of analytic definition, the genus proximum of the considered attribute is "colour"[24] which would represent the apex in the pyramid of terms given by the diairesis (Platonic division) – which then fixes the quantified expression (5b'') again to the LEM with no reflection process left. The apex can be considered as logical place of an indexed contexture or as a 'point of designation' placed beyond true and false. As we already emphasized, within a contexture, i.e. intra-contextural, all logical rules strictly hold.[25]

The idea behind polycontexturality, however, is not accomplished by introducing a single contexture as the name 'polycontexturality' already enunciates and this makes the difference to Bradley's idea of a positive "ground". In the theory of polycontexturality there are not only many grounds (or logical places which define the contextures) but the grounds, or logical places, or contextures are mediated. The grounds (logical places, contextures) have to be considered as an interplay, as an exchange between what is grounded and the ground, i.e., the ground turns to the grounded and the grounded mutates to the ground and so forth. Or in other words, the contextures are not simply isolated but
they are mediated by new logical operators. These operators act on the (indexed) contextures, i.e., these are inter-contextural operations between contextures or there corresponding place values. Within a contexture these operators are meaningless, i.e., intra-contexturally they are not defined.

From a polycontextural point of view the predicate logical expression (5c) yields an 'universal contexture' which encloses many contextures. At the bottom of such a tree-like structure as it is defined by a universal contexture (an example for such a structure is the well known Platonic pyramid of concepts) there are so-called 'elementary contextures' and above of these elementary contextures so-called 'compound contextures' may be involved (for more details see for example, ref.[19a]).

It is important to envision that all contextures are characterized by indices – so-called place values. In the beginning Günther uses natural numbers in order to label and to operate on different contextures ('logic of place values'). Later (in the Sixties of the last century) he introduced so-called qualitative numbers which are defined by the keno- and morphogrammatic and the theory of polycontextural numbers.[26] Without such an indexing and the new operators (poly-negations, transjunction, etc.), which also have been introduced by Gotthard Günther (and which have been enhanced later by Rudolf Kaehr and others), the contextures would not be mediated with each other.

The following table depicts some basic operations of the classical propositional logic as well as of Günther’s ‘logic of place values’. It is important to note that the conjunction (K), disjunction (D) and negation (N) have to be considered as inter-contextural, i.e., between different contextures and not intra-contextural (i.e., within) a contexture.

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(a) Three basic logical operations: conjunction, disjunction, and negation.
   a) conjunction of the standard logic with 0 (false) and 1 (true)
   b) conjunction of Günther’s many-placed logic with place values 1 and 2
   c) disjunction of the standard logic with 0 (false) and 1 (true)
   d) disjunction of Günther’s many-placed logic with place values 1 and 2
   e) negation of Günther’s many-placed logic with place values 1, 2, and 3 with the (non-standard) non-classical negations N1, N2, N3, N2.1, N1.2 and N1.2.1 = N2.1.2

Although natural numbers are quite helpful for some "paper and pencil" operations they are completely inappropriate for an implementation of computer models because natural numbers would not allow any modelling of heterarchical structured processualities (for more details see ref.[27]).

If we come back to the citations 3 and 4 we can summarize shortly, that an interpretation of Bradley’s work in front of the background of Gotthard Günther’s œuvre and his interpretation of Hegel gives a completely different view as compared to the interpretation given in the philosophical literature.[28] If, for example, Bradley states "that negation presupposes a positive ground [and that] it stands at a different level of reflection" then this can never be fully understood or interpreted on the basis of a positive-lingusitic
frame such as it is given by all classical standard and non-standard logical concepts including the paraconsistent models. It only can be understood if we accept the model of multi-negational operations –, the model of negative languages as it was introduced into science by Gotthard Günther.[29] In order to demonstrate this on a very simple example, we will consider the classical 'Law of Double Negation', i.e., \( \neg \neg p \equiv p \).

In its usual notation \( \neg \neg p \equiv p \) this law is indeed ambiguous at least for those thinkers who accept the existence of dialectical principles. From a polycontextural point of view, however, negation can be interpreted in two different ways: intra-contextural and inter-contextural. The intra-contextural negation corresponds to the well known law \( \neg \neg p \equiv p \) which strictly holds intra-contexturally (see also ref.[25]). In its inter-contextural interpretation (see table_1) a total situation (a whole contexture) as it is given, for example, by the LEM in (5b") is rejected. In the particular case of (5b") it is the colour of the objects \( x \) which will not be designated, i.e. which will be rejected – for reasons whatsoever, may be the price of \( x \) is more important so the corresponding contextures will be designated in order to make a decision.

It is not surprising that all discussions on dialectical logic, which rest on a thinking in classic logical categories, are somewhat frustrating. This is reflected by the so-called 'positivism debate' and/or 'Methodenstreit' in the late Sixties.[30] A good example of the extremely poor argumentation based on classical logical thinking is given by Karl Popper’s lecture *What is dialectic?* from 1937, which was reprinted in 1963[31], and which was part of the so-called 'positivism debate'. Günther also presented a contribution[32] to the 'positivism debate'. However Günther’s contribution never was quoted by Habermas in *Zur Logik der Sozialwissenschaften*[33], a publication which generally is considered as the résumé of this debate. Even in the latest edition[34] Günther’s contributions to the foundation of dialectical logic – collected in *Beiträge zur Grundlegung einer operationsfähigen Dialektik*[35] – were still ignored by Habermas.

In order to give the reader an opportunity to form his own judgement about the level of the "intellectual event" in Germany in the late Sixties, we digitized several contributions of the 'positivism debate' – the best at first:

Listing of the new text-files_2004-1 in vordenker.de

- All new text files of Gotthard Günther are listed in the new bibliography and have been marked by "summer edition 2004" in the last column.

In addition the following text files are presented in the 'Sommer-Edition 2004':

   http://www.vordenker.de/ggphilosophy/kaehr_skizze_36-120.pdf


   http://www.vordenker.de/ggphilosophy/reiser_non-aristotelian-logic.pdf

4. V.J. McGill: Concerning the Laws of Contradiction and Excluded Middle, 1939.
   http://www.vordenker.de/ggphilosophy/mcgill_contradiction-excl-middle.pdf

   http://www.vordenker.de/ggphilosophy/bradley_principles-logic_chp-3-5.pdf


   http://www.vordenker.de/ggphilosophy/adorno_logik-sozialwiss.pdf

   http://www.vordenker.de/ggphilosophy/popper_logik-sozialwiss.pdf

   http://www.vordenker.de/ggphilosophy/habermas_logik-sozialwiss.pdf

10. J. Habermas: Analytische Wissenschaftstheorie und Dialektik
    http://www.vordenker.de/ggphilosophy/habermas_analyt-wissenth.pdf

    http://www.vordenker.de/buehl/buehl_ende-zweiwert-soziol.pdf

    http://www.vordenker.de/ggphilosophy/popper_was-ist-dialektik.pdf

    http://www.vordenker.de/ggphilosophy/popper_what-is-dialectic.pdf

    http://www.vordenker.de/ggphilosophy/schmitz_rezens-idee-grundr.pdf

    http://www.vordenker.de/ggphilosophy/schmitz_zeiterfahrung.pdf

    http://www.vordenker.de/ggphilosophy/schmitz_hegels-logik.pdf
Notes


2. Considering the total work of Gotthard Günther, we will not use the name "trans-Aristotelian logic". Instead we will speak about polycontextural logic or more generally about polycontextural theory which comprises polycontextural logic as well as the keno- and morphogrammatic which also has been introduced into science by Gotthard Günther and in the following by Rudolf Kaehr and others.

3. See for example:


7. See for example: Peter Suber, *Non-Contradiction and Excluded Middle*, in: <http://www.earlham.edu/~peters/courses/logsys/pnc-pem.htm>


9. The Honor Roll – American Philosophers during the McCarthy era: <http://www.mail-archive.com/marxism-thaxis@lists.econ.utah.edu/msg00679.html>


Within the text we will use a shortened form for this title, namely *Idee und Grundriß*

12. For the propositional calculus several axiomatic systems have been developed, in order to demonstrate its consistency, completeness, and independency. The most common axiomatic systems are those developed by Gottlob Frege (1848-1925), David Hilbert (1862-1943) and Jan Łukasiewicz (1878-1956).
example: Axiomatic system by David Hilbert.

axioms
A1: \((a \lor a) \rightarrow a\)
A2: \(a \rightarrow (a \lor b)\)
A3: \((a \lor b) \rightarrow (b \lor a)\)
A4: \((a \rightarrow b) \rightarrow ((c \lor a) \rightarrow (c \lor b))\)

In addition to the four axioms two rules and one definition are necessary:

rules
R1: If B results from a deduced expression (or axiom) A, by substitution of a variable in A on each place of existence by any other expression, then one can pass from A to B.
(For example: On the basis of R1 one can substitute in A4 c by its negation \(\neg c\))
R2: From the concluded expressions (or axioms) A and \(A \rightarrow B\) one can pass to B (modus ponens)

definition D1: Instead of \(A \rightarrow B\) the following logical expression can be used: \(\neg A \lor B\)

If the law of identity would not be valid, it would be impossible, for example, to develop any theory on natural numbers.


a) We have digitized chapter 3 (The Negative Judgement) and chapter 5 (Principles of Identity, Contradiction, Excluded Middle, and Double Negation).

A pdf-file of chapter 3 and 5 can be found under www.vordenker.de (Summer Edition, 2004)

Joachim Paul (ed.), URL: <http://www.vordenker.de/ggphilosophy/bradley_principles-logic_chp-3-5.pdf>

b) More information about Bradley: <http://plato.stanford.edu/entries/bradley/>}


Note: \(\forall x\) symbolizes the universal quantifier ("Every object, x, is such that")

\(\forall P\) symbolizes the the qualification of the universal quantifier, i.e., the quantification on the objects x has to performed on all attributes P of x. "All attributes P of x" means on all qualities P of x which are known to us and all which are still not known to us.

\(\oplus\) symbolizes the exclusive OR – the so-called EXOR.

Cf. ref. 11, p. 149.

For classical metaphysics x stands not longer simply for tomatoes but for the existent in the sense of Martin Heidegger and we are speaking about is the being of Being (existent) wich are invested in "god" or more generally in a *summum bonum.*


In the (monocontextural) world of tomatoes or other objects of the daily life the *summum bonum* is the user, the consumer who decides whether he will accept or not accept such an alternative situation. Since our computer models still belong to a monocontextural world, they are not able to make such a decision by their own abilities.

F.H.Bradley belongs to the minority of thinkers who obviously was aware about these problems.
Gotthard Günther, in: Idee und Grundriss... ref. [11], p.131 ff.

"Dem Universalprädikat gegenüber, das den Unterschied von Dasein und Sosein aufhebt, weil es beides in gleicher Weise definiert, gibt es nur noch eine totale Negation, nämlich die der nicht-Identität von Dasein und Sosein; und damit existiert ein nur einzig der obersten Bestimmungsgesichtspunkt von Sein und Nichtsein, demgegenüber das Tertium non datur ohne Einschränkung gilt. Derselbe ist also, wie wir sehen, metaphysisch.

Bei Bradley kommt das dadurch zum Ausdruck, dass er darauf hinweist, dass das Prinzip des Tertium non datur einen gemeinsamen "Grund" für Positivität und Negation impliziert. Aber eben nur impliziert. "This principle ... says ... that there is a ground of relation, though it does not know what the relation is." Der berühmte englische Metaphysiker fährt dann fort: "... on the one hand, Excluded Middle transcends disjunction, since it possesses a self-determining principle which disjunction has not got. On the other hand, in its further development, it is nothing whatever but a case of disjunction and must wait for the sphere of discrepant predicates to be given it as a fact."[The Principles of logic, p. 153] Das selbstdeterminierende Prinzip, von dem Bradley spricht, ist das transzendent-objektive Sein, auf dessen Hintergrund alle Prädikate des Denkens abzubilden sind und das nur die totale Alternative von Positivität und Negativität, von Objektivität und Subjektivität, von Gegenständlichkeit und Denken zulässt. Hier ist das Dritte unbedingt ausgeschlossen. Gegenüber diesem metaphysischen Geltungsbereich aber steht die praktische Anwendung des Drittensatzes im empirischen Denken. Und hier gilt das dritte Axiom der klassischen Theorie des Denkens nur eingeschränkt, nämlich relativ zu dem obersten faktischen Bestimmungsgesichtspunkt, unter dem ein gegebenes Prädikat steht. Im Falle unserer Alternative "rot" oder "nicht-rot" ist jener Bestimmungsgesichtspunkt der Disjunktion "Farbe". Damit sind die zu den Prädikaten "duftend", "dornig" usw. gehörenden Bestimmungsgesichtspunkte ausgeschlossen."

The universal predication not only abrogates the difference between 'existence' and 'suchness' because both are defined by the universal predication it also allows only one total negation, the non-identity of 'existence' and 'suchness'. Thus only one uppermost 'point of designation' between being and non-being exists from which the LEM strictly holds. As we can see this is a metaphysical point.

This is articulated by Bradley if he states that the tertium non datur implies a common "ground" for positivity and negation – but only implies.

"This principle ... says ... that there is a ground of relation, though it does not know what the relation is." The famous English metaphysician then continues: "... on the one hand, Excluded Middle transcends disjunction, since it possesses a self-determining principle which disjunction has not got. On the other hand, in its further development, it is nothing whatever but a case of disjunction and must wait for the sphere of discrepant predicates to be given it as a fact."[The Principles of logic, p. 153] The self-determining principle, as it is called by Bradley, corresponds to the transcendental-objective being – a background on which all predicates of thinking have to be mapped and which only allows the total alternative between positivity and negativity, objectivity and subjectivity, concreteness and thinking. In this case any Third is excluded unrestrictedly. Against this metaphysical range of validity stands the practical application of the law of excluded middle within our empirical thinking. Here, the third axiom of the classical laws of thought can only be applied in a limited way given by a relative umost actual 'point of designation' determined by the given predicate. In the case or the alternative "red" or "not-red" the corresponding 'point of designation' for this disjunction is given by the "colour" and all other 'points of designation' which belong to predicates such as "smell"or "thorny", etc. [of a rose] are excluded.

For the differentia specifica a range of wavelengths or frequencies or some values of the rgb scheme are technically adequate possibilities in order to decribe "red".

<table>
<thead>
<tr>
<th>definiendum</th>
<th>:=</th>
<th>definiens</th>
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<tbody>
<tr>
<td>definiendum</td>
<td>:=</td>
<td>genus proximum</td>
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</table>

examples:

<table>
<thead>
<tr>
<th></th>
<th>:=</th>
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<th>:=</th>
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<tbody>
<tr>
<td>square</td>
<td>quadrangle</td>
<td>&quot;plus&quot;</td>
<td>four sides of equal length</td>
<td></td>
</tr>
<tr>
<td>red</td>
<td>colour</td>
<td>&quot;plus&quot;</td>
<td>vacuum wavelength: 640 nm ≤ λvac ≤ 750 nm</td>
<td></td>
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</tbody>
</table>
If \( p \) symbolizes for example "the rose is red", then \( \neg p \) ("the rose is not non-red") does not necessarily mean that the rose must be red, it also could be yellow or any other colour. In other words, the definition of a contexture by \textit{genus proximum} and \textit{differens specifica} does not necessarily yield an elementary contexture, where the negation is related to a diairetic disjunction of neighboured attibutes of the platomic pyramid. A diairetic disjunction of neighboured attibutes is given for example by the attribute "wizened" and "non-wizened" of the rose. In this case the law of doule negation strictly holds. In case of the colour, we are faced with a so-called compound contexture in which corresponding contextures for the different colours can be defined which are mediated to each other by corresponding operators such as inter-contextural negations and transjunctions etc. (cf. ref.19).


See for example:


See also: Rudolf Kaehr, \textit{Derrida’s Machines – Cloning the Natural}, URL: <http://www.thinkartlab.com/plk/media/DERRIDA/DERRIDA.htm>

Some links concerning 'Positivismusstreit' ('positivism debate') and 'Methodenstreit':

<http://www.wordiq.com/definition/Theodor_Adorno>


<http://en.wikipedia.org/wiki/Methodenstreit>

<http://en.wikipedia.org/wiki/Theodor_Adorno>

<http://de.wikipedia.org/wiki/Methodenstreit>


There exists the rumor that Habermas commented Günther’s contribution with the words: 'you don’t have to read Günther’s article because it was financed by the US Air force.' We don’t know whether this rumor is correct or not. Fact is, that even in 1985 Habermas has ignored not only Günther’s contribution but also the contribution of his colleague Walter L. Bühl \textit{Das Ende der zweiwertigen Soziologi: Zur logischen Struktur der soziologischen Wandlungsstheorien}, in: Soziale Welt Jhrg. XX, 1969, Heft 2, p.162-180. – A pdf-file is available under www.vordenker.de (Summer Edition, 2004) Joachim Paul (ed.), URL: <http://www.vordenker.de/buehl/buehl_ende-zweiwert-soziol.pdf>