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Cybernetic Ontology and Transjunctional Operations

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INTRODUCTION

This essay presents some thoughts on an ontology of cybernetics. There is a very simple translation of the term "ontology". It is the theory of What There Is (Quine). But if this is the case, one rightly expects the discipline to represent a set of statements about "everything". This is just another way of saying that ontology provides us with such general and basic concepts that all aspects of Being or Reality are covered. Consequently all scientific disciplines find their guiding principles and operational maxims grounded in ontology and legitimized by it. Ontology decides whether our logical systems are empty plays with symbols or formal descriptions of what "really" is.

The following investigation arrives at the result that our present (classic) ontology does not cover "everything". It excludes certain phenomena of Being from scientific investigation declaring them to be of irrational or metaphysical nature. The ontologic situation of cybernetics, however, is characterized by the fact that the very aspect of Being that the ontologic tradition excludes from scientific treatment is the thematic core and center of this new discipline. Since it is impossible to deny the existence of novel methods and positive results produced by cybernetic research, we have no choice but to develop a new system of ontology together with a corresponding theory of logic The logical methods that are used *faute de mieux* in cybernetics belong to the old ontological tradition and are not powerful enough to analyze the fresh aspects of Reality that are beginning to emerge from a theory of automata.

The first section of this essay deals with classic ontology. The second is devoted to some perspectives of a trans-classic ontology. Sections three and four try to develop a new theory of logic capable of meeting the demands of cybernetics better than the two- or many-valued systems currently in use. In the first two sections the philosophical view-point dominates. In the last two, technical problems of logic are accentuated.

The author strongly suspects that a majority of readers will hold the opinion that it would have been amply sufficient to restrict the investigation to Section 3 and 4 and to forget about the ontologic prelude of Section 1 and 2. The consensus that basic "metaphysical" reflections about logic have little or no practical value at all is widely spread. There is even some justification for this belief and it may be safely said that, as far as our two-valued traditional logic is concerned, the cyberneticist will gain nothing by submitting his logical procedures to a renewed scrutiny of its fundamental presuppositions. This logic is in its basic features now more than two thousand years old. A long historical process has worked its ontology into the very marrow of our

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bones, so to speak. We use this ontology with reasonable precision without being in the least aware of doing so.

There seems to be no reason why this happy and comfortable state should not continue. Einstein's widely quoted exclamation: "Der liebe Gott spielt nicht mit Würfeln"+) is a poignant expression of the deep-seated belief in classic ontology. And everything might be very well, indeed, except for the advent of transclassic calculi which demanded an ontologic interpretation. From then on, the logician was faced with an alternative. He could either try to interpret his new procedures in terms of the Aristotelian ontology or he could assume that a many-valued system is incompatible with the classic foundations of logic. This second part of the alternative involves, of course, a much greater risk. So it is understandable that Jan Łukasiewicz looked for ontological support in Aristotle's *Organon* when he introduced a third value into logic. It is important to know that he succeeded to a certain degree and that he was able to find a philosophic interpretation for a calculus with three values, and for another one with a denumerably infinite number of values. This happened between 1920 and 1930. It is quite significant, however, that after about ten years of research he was forced to admit that he could not find any ontologic significance for calculi between three and an infinite number of values. Since then hardly any progress has been made in this direction. Four- five- and other finite n-value systems have been used with practical applications but without any genuine insight into their basic ontologic significance. C.I. Lewis's sceptical statement with regard to many-valued systems, that "the attempt to include all modes of classification, and all resultant principles would produce, not a canon, but chaos" still stands unchallenged [1a]. For the first time the unity of logic is endangered! To preserve it, competent logicians have suggested that formal logic should be restricted to two values.

We are going to show that this suggestion is untenable. But so is the assumption that many-valued theories should be restricted to interpretation in terms of classic ontology. There is no doubt that this can be done within certain narrow limits and valuable results have been obtained with such procedures. Jack D. Cowan´s *Many-valued Logics and Reliable Automata* is a recent and notable example of this method[1b]. We should be very clear about the fact that the interpretation of many-valued systems on the basis of Aristotelian ontology is by no means "false". It is quite legitimate. In fact a vigorous continuation along this line is absolutely necessary.

However, there is another aspect to the question of the relation between a formal logic and its ontology. Is it *possible* to *exploit the immense capacities of many-valued* systems if we use them only to analyze what the classic tradition calls Reality ? This author confesses that the present use of many-valued logic reminds him of a man who might spend a fortune on a Ferrari racer in order that his wife should have transportation to the super-market.

An ontologic analysis of many-valued structures shows that only a tiny, almost infinitesimal, part of them coincides with the concept of Being or Reality that we have inherited from the Greeks. If we intend to use the full range of logical possibilities now

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+) Transl.: "God does not play with dice."

[1a] C.I. Lewis; Alternative Systems of Logic, *The Monist*, XLII 4, P. 507 (1932).

[1b] J.D. Cowan: "Many-Valued Logics and Reliable Automata," *Principles of Self- Organization*, ed. H. Von Foerster, G. W. Zopf, Pergamon Press, London, PP. 135-179 (1962).

available to us but still cling to ancient ontological concepts, the result will indeed not be a canon but logical chaos. The basic conceptual foundations with which a logic meets Reality are established as far as two-valued theories are concerned. But with regard to many-valuedness we have not even started to lay the proper foundation. An ontology is nothing but a very general prescription of how to use a logic in an existing world. It tells us how much of this world is approachable by exact scientific procedures. It is the aim of this essay to show which specific data of Reality that the classic ontology judged to be "irrational" or "transcendent" are within the grip of cybernetics if a certain type of many-valued logic is applied. For this very reason we claim that a careful analysis of the ontologic foundation of cybernetics is an eminently practical undertaking. The cyberneticist may find it useful to learn about a new way to interpret transclassic systems of logic. He should therefore not begrudge us the time and the effort to get acquainted with the contents of Sections 1 and 2.

This is a first attempt to outline an ontology for cybernetic logic. The author is aware of its considerable shortcomings. Among other things it is too abbreviated. But time was short and did not permit a more detailed analysis. The author hopes to make up for it in the second volume of his *Idee und Grundriss einer nicht-Aristotelischen Logik* which is in preparation.

The present essay deals only with one phenomenon, which will be called subjective self-reflection. Some of its elementary features are already recognizable in very primitive, inanimate systems. Nevertheless we shall focus our attention on its highest and richest representation, the self-awareness of Man. It may seem more reasonable to start with the simple manifestations of self-reflection in elementary models of self-organizing systems. Alas, this is not possible for a formal logic which claims general ontological validity for all structures of self-reflection. What will be valid for the self-awareness of man will also be valid for systems of lower reflective organization. But not vice versa. It is not possible to develop a new ontological theory of logic by starting at the bottom. Aristotle did not do so. The general principles of his theory of thinking which stood us in good stead till the advent of cybernetics were developed at the very outset of the evolution of Western science. Aristotle started with an answer to the primordial question: what is, "logically speaking", objective Being? We try to follow a great example if we pose and try to answer the question: what is "logically speaking" subjective self-awareness?

1. REMARKS ON CLASSIC ONTOLOGY

Philosophy has played a negligible part in the development of modern science since the times of Newton and Leibniz. The reasons are rather obvious and have frequently been stated. Descartes, Pascal, and Leibniz created the mathematics of their period out of the spirit of metaphysical problems. And Newton´s great work *Philosophiae Naturalis Principia Mathematica* not only carried the word philosophy in its title, but fully deserved this label because the transcendental problem of the relation between motion and time played a decisive part in the development of his theory of "fluxions". But then the ways of philosophy and exact science (including mathematics) begin to part. Kant´s philosophical speculations about the mutual relations of space and geometry on the one hand, and time and arithmetic on the other were actually refuted by Euler and d´Alembert even before they were stated in the Critique of Pure Reason[1]. For Hegel the mathematical type of thinking had nothing to do with philosophy. And Schopenhauer´s ideas about the exact sciences of his time show a complete lack of understanding of the very essence of mathematical or experimental reasoning. Since then the regrettable alienation between philosophy and science has progressed even further. What might be the most profound metaphysical investigation of our own time, the ontological thought analysis by Martin Heidegger, remains intrinsically incomprehensible to the exact scientist or mathematician. It is not the fault of either side. This alienation has unfortunately provoked indifference, contempt, or even outspoken enmity against philosophy in the scientific camp. Perhaps the strongest and most radical expression of the present discord between philosophy and science is represented by the following statement of a well known thinker in the scientific camp: "Es gibt keine Philosophie als Theorie, als System eigener Sätze neben denen der Wissenschaft". (There is no philosophy as theory, as a system of statements *sui generis* apart from those of science.[2]

It seems a rather hopeless task under the circumstances to recommend some philosophical considerations from the field of ontology to the present-day scientist. Yet the attempt has to be made; the radical developments that have taken place within Science during the last decades, have made us suspect that certain fundamental philosophical concepts and presuppositions on which all our scientific efforts are (more or less unconsciously) based are in dire need of a thorough reexamination. The recent arrival of the youngest member of the scientific family, cybernetics, has made this suspicion almost a certainty[3]. Moreover, there is a special reason why the ontologist is interested in this situation. Formal (symbolic) logic, which has so often served as the arbiter in scientific controversies, is at present unable to help: its explosive expansion since about the middle of the last century has made the security of its own foundations dubious. Today it is still impossible to evaluate the effects which such discoveries as those that have come to us from Kurt Gödel and others will have on the future development of this discipline. The ontological basis of logic itself is in question, proof of it is the impossibility of resolving the claims of Intuitionism against Formalism and Platonism at this juncture[4].

There is no escape! When the formal logical foundations of science and mathematics become doubtful, the issue automatically reverts back to the ontological sector of philosophy. But even now the ontologist hardly dares offer his services: he knows only too well how unwelcome his reflections are, even under the present mental tribulations. The shout of logical positivism that the metaphysician is a fictioneer still reverberates loudly in the Hall of Science. But lately events have taken an ironic twist. The scientists themselves have invaded ontology. W. Heisenberg did so some time ago with a very

[1] Oswald Spengler: *Der Untergang des Abendlandes*, Vol. I, C. H. Beck, München, p. 163 ss (1923).

[2] Rudolf Carnap: Die alte und die neue Logik, *Erkenntnis* I, p. 2 3 (1930).

[3] G. Günther: *Das Bewusstsein der Maschinen*, AGIS Verlag, Krefeld-Baden Baden (1957) for the necessity of new philosophic foundations of cybernetics. (Note_vgo: a third edition appeared in 2002)

[4] Wolfgang Stegmüller: *Metaphysik, Wissenschaft, Skepsis*, Humboldt-Verlag, Frankfurt a. Main-Wien, pp. 232-241_(1954).

valuable essay *Kausalgesetz und Quantenmechanik*[5]. E. Schrödinger gave in his Tarner Lectures[6] a very competent exposition of the ontologic relations between consciousness and world. As far as cybernetics is concerned one has only to mention W. S. McCulloch, whose articles offer us quite concentrated doses of metaphysics[7] and Norbert Wiener´s essay on *Newtonian and Bergsonian Time* [8] which in our opinion refutes certain basic aspects of traditional metaphysics.

Since cybernetics is much younger than quantum mechanics and, ontologically speaking, less developed, the new ontological situation naturally is delineated most sharply in the statements of Heisenberg and Schrödinger. In the above-mentioned essay Heisenberg offers the following reflections: Kant introduces in his *Critique of Pure Reason* the law of causality as an *a priori* principle by demonstrating that without this principle we could never form the concept of an objective world that exists independently of the subjective thought-processes that take place within our consciousness. Kant poses precisely this question: what "mechanism" in our mind enables us to distinguish between a sequence of events that occurs exclusively in our psyche – for instance a sequence in a dream – and a sequence that takes place in the external world independent of our observation? It is evident, so the *Critique of Pure Reason* points out, that we need a formal criterion to make the desired distinction; for we are aware of objective reality, as well as of our dreams and fantasies, only as content of our consciousness[9]. Nevertheless, we obstinately believe that some of these impressions have their origin in a world outside the mind and others have not. The source of this conviction, Kant declares, is the category of causality, which makes us look at a specific series of impressions as a rigid temporal succession that our mind is powerless to alter or stop. And what our consciousness cannot modify and control must necessarily have an existence outside and independent of it. The law of causality appears thus as a criterion to distinguish between subject and object, between consciousness and world. If we look at our impressions without interpreting them as causally linked to each other, they can be understood only as "a play of imaginations with no reference to an object"[10].

Heisenberg quotes the relevant passage (where Kant demonstrates that causality is our mental mechanism for the distinction between Subjectivity and external Reality) and admits that, if we use this interpretation, we have obtained a genuine a priori principle[11]. As such it is, of course, irrefutable – for the very simple reason that this *a priori* principle does not make the slightest assumption about the factual contents of the external Reality. It only states that if we want to think of a Reality that exists independently of the subject who is aware of it, we cannot do so without using the category of causality. To put it differently: if we want to establish an absolutely objective natural science which completely describes Reality without reference to the

[5] W. Heisenberg: Kausalgesetz und Quantenmechanik, *Erkenntnis* II, 2/3, pp. 172-182 (1931).

[6] E. Schrödinger: *Mind and Matter*, University Press, Cambridge (1959).

[7] W. S. McCulloch: Towards Some Circuitry of Ethical Robots, *Act. Biotheoret*. XI, p. 147 (1955); "Of I and It" (not published).

[8] N. Wiener: Newtonian and Bergsonian Time, *Cybernetics*, Wiley, New York, pp. 40-56 (1948).

[9] Kant: *Critique of Pure Reason*, B, pp. 232-256.

[10] "… ein Spiel der Vorstellungen…, das sich auf kein Object bezöge." See above Note 9, B, p. 239.

[11] W. Heisenberg: loc. cit. p. 176, "Es ist evident, dass eine solche Auffassung des Kausalgesetzes als a priori Postulat nicht widerlegt werden kann, da es über die Erfahrung nichs aussagt."

subjective origins of our scientific terms and concepts then everything must be understood in terms of causality. Laplace´s famous Spirit would face in his differential equations a world devoid of any subjectivity whatsoever. This relation between subject and object depicts the classic ideal of scientific knowledge.

This ideal, however, Heisenberg points out, cannot be pursued since the advent of quantum mechanics. A radically objective system of physics, with a dichotomy of Reality into "thing" and "thought" is now impossible: "the radically isolated object has, on principle, no describable properties"[12].

If Heisenberg´s claim remains valid, and there is overwhelming evidence that it will, an entirely new type of logic must be developed. However, the term *New Logic* has been grossly misused since the Cartesian Johannes Clauberg (1622-1665) first spoke of *Logica Vetus et Nova* [13]; it will therefore be necessary to state what should be understood if such an expression is used. *A system of logic is a formalization of an ontology* [14]! If there seems to be a need for a new logic a new concept of ontology must be formed and *vice versa* In the present situation, outstanding representatives of the physical sciences express viewpoints which are *de facto* statements from a new ontology. A new concept of logic is consequently called for. But since such a new concept can only be developed in contrast to our classic tradition and theory of thought, it will be useful to offer a brief sketch of the reciprocity of traditional logic and ontology.

The correspondence theory of logical and ontological structures dates back at least to the dialogues of Plato, the Aristotelian Organon, and the logic of the Stoics. During this epoch the question was raised (and answered): what are the formal and ontological requisites for making verifiable and generally valid statements about the objective world ? It was found that such statements are possible only if we assume that the laws of Nature (Being) and the laws of Thought are essentially identical but differ in their formal aspects. This formal difference between a mathematical law in physics and the corresponding law in logic is due to the fact that, in the first case a description of the external world is intended, while in the second case the mirror image of this world, as it is repeated in our thought processes, is the motive and semantic theme of our representation.

Thus the set of natural laws (objectivity) and the inverse set of the rules and structures of logic together form an enantiomorphic system of rationality. The two subsets of this system constitute a symmetrical exchange relation which is as simple as our familiar

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[12] W. Heisenberg, loc. cit.: "Wenn Kant gezeigt hat, dass für eine objektive Naturwissenschaft das Kausalpostulat die Voraussetzung sei, so ist dem entgegenzuhalten, dass eben eine in dem Sinne "objektive" Physik, d.h. eine ganz scharfe Trennung der Welt in Subjekt und Objekt, nicht mehr möglich ist … der völlig isolierte Gegenstand hat prinzipiell keine beschreibbaren Eigenschaften mehr."

[13] First published by Schalbruch, Amsterdam, 1691. It also seems that Clauberg first suggested the term "ontology." cf. J. E. Erdmann, "Grundriss der Geschichte der Philosophie" II, Whilhelm Hertz, Berlin, p. 33 (1878). About the misuse of the term "new logic" cf. G. Günther, "Die Aristotelische Logik des Seins und die nicht-Aristotelische Logik der Reflexion," Zeitschr. f. philos. Forsch. XX, 3; part I, pp. 360-381 (1958).

[14] This has been analyzed in detail in G. Günther: Metaphysik, Logik und die Theorie der Reflexion, *Arch. d. Philos*., VII, 1/2; pp. 1-4.4 (1957).

distinction between left and right. This exchange relation is defined by our traditional operator of two-valued negation. Any datum of experience is either positive or negative, objective or subjective, and no third term (*tertium non datur*) is allowed. The disjunction is exclusive and total. The classic tradition, in a time-honored expression, speaks of the metaphysical identity of Thought and Being. In the realm of the ultimate, absolute Reality, Thought and Being are the same. They can be distinguished only on a relative empirical level where they appear as opposites. But our ontologic tradition insists that even in this opposition they express the same meaning and represent only two different aspects of the same "subject-matter" as our language profoundly says. However, it should never be forgotten that these two empirical aspects of Reality constitute a strict exchange relation of two sets or subsystems of a universal enantiomorphic structure which is, as such, indifferent to the distinction between subject and object (Cusanus´ *coincidentia oppositorum*).

However, this system of classic (two-valued) ontology, successful as it has proved for the development of Western science, suffers from an enormous drawback. The symmetrical exchange relation and the resulting ontological equivalence of subject and object governs only the mutual relations between the two subsets as inverse totalities. It is not applicable to any individual member of either set. In other words, the context of terms that describe the structure of our external objective world permits not the slightest penetration by concepts that refer to the epistemologic subject of cognizance that comprehends and is aware of objects. We may either discourse about objective reality (i.e. nature) in ontological terms or we may refer to the perceiving subject in logo-logical concepts, but we are absolutely not permitted to mix the two. If we ignore this prohibition we invariably get lost in a jungle of contradictions and paradoxes. The very fact that we nowadays possess an accurate science and base on it a vast technology is due to an ontologic tradition which was reasonably strict in adhering to the principle of dichotomy between matter and form and between subject and object.

The two-valued character of our logical tradition from the time of the Greeks up to the present day[15] testifies to the fact that our logic is a faithful attempt to formalize the ontology of the ultimate parity of form and matter, or subject and object as it was expressed in the ancient maxim of the metaphysical identity of Thought and Being. As long as our logical endeavors are orientated to this ontology we have no right to speak of a new logic, despite the enormous amount of detail that has been added to the older system in the course of the past century. But our logic still insists that it is meaningful to conceive the idea of a thought-object being fully identical with itself and therefore capable of isolation. The assumed metaphysical parity of Thought and Being permits a consistent system of formalization (logic) only if we regard these two primordial components of Reality as a symmetrical exchange relation. But such a relation isolates the two components completely from each other. Mind and Matter belong to different metaphysical dimensions; they do not mix. There is no such division between the energetic and the material state of the Universe. The Einstein equation $E = mc^2$ states

[15] A striking example how little our traditional logic has deviated from its two-valued structure is J. M. Bochenski´s *Formal Logic*, Karl Alber, Freiburg, München, which was published in 1956. Research in many-valued logic was started by E. L. Post and J. Łukasiewicz in 1920. But Bochenski´s 640 page volume which was published 36 years later reserves only a little more than two and one half pages for this topic!

that energy may be converted into mass and *vice versa*. But there is no analogous formula for the conversion of thought into matter or meaning into energy. We know as an empirical fact that our brain is a physical system where certain largely unknown – but physical – events take place. These represent to the observer a combination of electrical and chemical data[16] producing a mysterious phenomenon which we might call meaning, consciousness, or self-awareness. In view of this fact we must either retreat into theology and speak of a supernatural soul which only resides in this body as a guest, or assume that matter, energy and mind are elements of a transitive relation. In other words there should be a conversion formula which holds between energy and mind, and which is a strict analogy to the Einstein equation. From the view-point of our classic, two-valued logic (with its rigid dichotomy between subjectivity and objective events) the search for such a formula would seem hardly less than insanity. The common denominator between Mind and Matter is metaphysical and not physical according to a spiritual tradition of mankind that dates back several millenia. The very structure of our logic implies this metaphysical belief.

But if Heisenberg´s statements about the mathematical inseparability of subject and object in a quantum-mechanical description of the physical world are correct, then it becomes impossible to subscribe further to our traditional ontology and its consequences in formal logic. However, the mental step implied is enormous, and should not be taken on the testimony of a single witness no matter how great his scientific reputation. We shall, therefore, turn our attention to Erwin Schrödinger´s more elaborate discussion of the problem.

In the main, Schrödinger´s ideas take the same epistemological trend as those of Heisenberg. He discusses in detail the principle of objectivation which interprets objects as ontologically isolated identities. This has led to great successes. But the price we have paid for it is indicated by the fact that "we have not yet succeeded in elaborating a fairly understandable outlook on the world without retiring, our own mind, the producer of the world picture, from it …"[17]. The principle of radical objectivation was undoubtedly necessary for the past period of scientific research and it will remain so for certain borderline cases.

However, since the advent of quantum mechanics in physics, of meta-theory in logic and mathematics, and, last but not least, since the emergence of cybernetics the scientific situation has changed so radically that a new appraisal of this principle is overdue. Schrödinger draws our attention to the fact that as long as our thinking objectivates without hindrance and inhibitions it "… has cut itself off from all adequate understanding of the subject of Cognizance, of the mind"[18]. And he continues: "But I do believe that this is precisely the point where our present way of thinking does need to be amended … That will not be easy, we must beware of blunders … We do not wish to lose the logical precision that our scientific thought has reached …"[18]. This is not a passing thought in the Tarner Lectures. On another page we find a similar statement where Schrödinger again admonishes us to give up "… the time-hallowed discrimination

[16] John von Neumann: *The Computer and the Brain*, Yale University Press, New Haven, p. 42 (1958).

[17] E. Schrödinger, see Note 6, p. 66.

[18] E. Schrödinger, see Note 6, p. 54s, cf. also p. 38.

between subject and object. Though we have to accept it in everyday life for ´practical reference´ we ought, so I believe, to abandon it in philosophical thought"[19].

Unfortunately, that seems to be easier said than done. Schrödinger himself draws our attention to a very peculiar relation between subject and object when he remarks: "the reason why our sentient, percipient, and thinking ego is met nowhere within our world picture can easily be indicated in seven words: because it is itself that world picture. It is identical with the whole and therefore cannot be contained in it as a part of it"[20]. Yet common sense and daily experience tell us that our thinking ego is a content of this world which science describes as an utterly subjectless context of existence. The Tarner Lectures call this an "antinomy" and refer to it with the following remarks: "The thing that bewilders us is the curious double role that the conscious mind acquires. On the one hand it is the stage, and the only stage on which this whole world-process takes place, or the vessel and container that contains it all and outside which there is nothing. On the other hand we gather the impression, maybe the deceptive impression, that within this world-bustle the conscious mind is tied up with certain very peculiar organs (brains) … On the one hand, mind is the artist who has produced the whole; in the accomplished work, however, it is but an insignificant accessory that might be absent without detracting from the total effect"[21].

If Schrödinger states that the phenomenon of consciousness or self-awareness has no legitimate place in our world picture because it is itself this very picture, he says in effect, that to be a subject means to be a mirror for an object. But since no subjects are to be found in this world this mirror must be an object too. The conclusion is unavoidable that if we use the term "subject" we actually mean a special class of objects which have the mysterious quality that they can reflect any other object in such a way that not only the object but the process of reflection is mirrored. Fichte significantly called the subject (ego) an "image of an image" and in another context "the image of a capacity" (to have images)[22]. So there is nothing but objects and "images". And insofar as a subject "exists" it does so only as an object. Qua subject it simply isn´t there. In fact it is nowhere. No wonder classic ontology delivered a startling dictum through the person of William James who published, in 1904, an essay: "Does Consciousness Exist"?[23] He first notes that Kant in the *Critique of Pure Reason* weakened the philosophic concept of "soul". He replaced it with his concept of the transcendental ego which in its turn attenuated itself to the "thoroughly ghostly condition" of a *Bewusstsein-überhaupt* (general consciousness) "of which in itself absolutely nothing can be said[24]. James´ careful analysis finally leads to the assertion that consciousness does not exist! "That entity is fictitious, while thoughts in the

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[19] E. Schrödinger, see Note 6, p. 51.
[20] E. Schrödinger, see Note 6, p. 52.
[21] E. Schrödinger, see Note 6, p. 64s.
[22] J. G. Fichte: *Die Bestimmung des Menschen*, W. W. II, ed. J. H. Fichte, Mayer und Willer, GmbH, Leipzig, p. 245 (1844); N. W. W. I, p. 428.
[23] W. James: Does Consciousness Exist? *Journal of Philosophy, Psychology and Scientific Methods*, 1, 18, Sept. 1 (1904).
[24] Quoted from *Essays in Radical Empiricism*, ed. R. B. Perry, Longmans, Green and Co., New, York, p. 2 (1947).

concrete are fully real. But thoughts in the concrete are made of the same stuff as things are"[25].

This conclusion may sound somewhat melodramatic, but it does not come as a surprise to the student of the history of Western science. He knows that all scientific endeavors of the past are based on the ontological proposition that every law that contributes to a verifiable description of Reality must be resolvable into statements about objects and objective events, because the terms that our cognitive mind forms as categories of mental comprehension are at the same time ontic properties of things and their modes of physical existence[26]. This "metaphysical" identity of Thought and Being is, according to Aristotle, the fundamental prerequisite of any science that deserves the name. And we cannot deny that the faithful adherence to this ancient tradition has stood us in good stead.

However, this basic epistemologic attitude, which still dominates our thinking, entails, a fatal weakness. All our scientific terms – as they are developed on this Aristotelian ontological basis – retain a semantic ambiguity. They can, in their entirety, either be taken as a description of the Universe as the absolute Object or as the absolute Subject. In other words: there is nothing in our present theories of thinking to enable us to distinguish logically between a genuine object like a stone and a subject or center of consciousness that appears to us to be a pseudo-object if we locate it in the body of all animal or human and call it all ego. This is the relevant meaning in Schrödinger´s remark that the mysterious entity we are accustomed to call a subject is nothing but our world picture taken as a totality.

It is interesting to note that it has occurred to neither Heisenberg nor Schrödinger that this situation makes their suggested inclusion of subjectivity into our scientific world picture quite impossible. Our classic system of (two-valued) concepts represents an enantiomorphic structure of rationality where the object exhaustively mirrors the subject and *vice versa*. This system offers two and only two ways to provide us with an ontological description of the relation between subject and object. This relation may either be interpreted as a conjunction or as a disjunction. But these two interpretations are inextricably compounded. If we consider the relation between subject and object with regard to the totality of the world and define it as conjunctive, then both form a disjunction relative to any arbitrarily chosen part of the world. But if we take the opposite view and presume that their ultimate ontological relation is disjunctive, then their relation inside the world must necessarily be conjunctive. This is the law of duality of two-valued logic stated by the two DeMorgan expressions:

$$p \wedge q \equiv \sim(\sim p \vee \sim q)$$

$$p \vee q \equiv \sim(\sim p \wedge \sim q)$$

Since it does not matter from which angle we look at the situation we shall take our orientation in the following arguments mostly from the conjunctive viewpoint.

[25] Loc. cit. p. 37.

[26] W. Windelband: *A History of Philosophy*, Macmillan, New York, p. 134 (1893). ´The general, the Idea, is, as the true Being, the cause of occurrence and change … On the other hand, the general. is in thought the ground by means of which and from which the particular is proved.´

If we assume that subject and object are the inverse unit elements of an enantiomorph system, then it is possible to make empirically conjunctive statements about subjects and objects in a context where all terms are uniformly designated. We do that in our discourse daily and think nothing of it. But, of course, everything we say about subjects is expressed in terms that designate objects. We cannot help it because there are no other terms available owing to the collaboration between the principle of objectivation and two-valued logic. We are so accustomed to this epistomological deficiency in our language that we make automatically and unconsciously the necessary allowances when we receive information of this sort. If somebody told his friend to pick up his wife at the steps of the Lincoln Memorial and he reported afterwards: I could not pick "her" up because I located only her body standing on the steps, that would be considered a very stupid joke. However, in a strict ontological sense the friend would have been right. Subjectivity cannot be located in this manner. And what could have been picked up was merely an "it", not a "she".

But if Reality is actually the conjunction of the inverse components of subject and object, and we insist on a precise scientific language which does not permit the liberties of everyday speech, we arrive by logical necessity at a duality of interpretations for our system of objective terms. H. Reichenbach has drawn our attention to the fact that this is what has actually happened in quantum mechanics. The Schrödinger wave equation guarantees logically a "strict duality of wave and corpuscle interpretation for free particles"[27]. This is the only way to obtain an "exhaustive" description of Reality in purely objective terms. The contraposition of subject and object is transposed into Bohr´s rule of complementarity. The two quantum mechanical concepts of corpuscle and wave still designate objective reality. But the degree of objectivation that is represented by them is much lower than for corresponding terms of classic physics. What dilutes their ontological significance is their complementary contraposition[28]. The degree of objectivity that was formerly represented by a single concept is now distributed over two. This *property of distribution is the disguise under which the subjective component of our quantum mechanical terms conceals itself.*

Since we will later demonstrate that this element of distribution is the general logical criterion for determining whether a given theoretical system contains smaller or larger traces of subjectivity in its terms, it may be useful to explain a little further how it shows up in Bohr´s rule of complementarity. The so-called Copenhagen Interpretation of quantum theory starts from the fact that any experiment in physics must be described by using the two-valued classic terms of physical science. These terms cannot be replaced as an epistemological basis of our thinking because our consciousness assumes a two-valued structure whenever it contacts objective facts. Our classic theories of nature use these terms exclusively because they strive for that scientific "idealization in which we can speak about parts of the world without reference to ourselves"[29]. Quantum mechanics on the other hand maintains that this radical dichotomy between subject and object is a purely formal concept. Subject and object constitute a clear-cut

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[27] H. Reichenbach: *Philosophic Foundation of Quantum Mechanics*, University of California Press, Berkeley and Los Angeles, pp. 71 and 33 (1946).

[28] C.F. von Weizsäcker: *Komplementarität und Logik*, Die Naturwissenschaften, XXXXII, 19, p. 522 (1955). ´Die Komplementaritiät der Begriffe (schränkt) ihre Objektivierbarkeit ein.´

[29] W. Heisenberg: *Physics and Philosophy*, ed. R. N. Anshen, Harper & Brothers, New York, p. 55 (1958).

division of Reality only as long as we conceive the objective world as a self-contained totality and put it as such in contrast to subjectivity in general. But as soon as we want to observe part of the world the symmetrical character of our formal system of logic is affected and special provisions have to be taken to preserve it.

Heisenberg has described the epistemological imbalance of terms in quantum mechanics by making the statement that modern physics "starts from the division of the world into the ´object´ and the rest of the world". But dichotomy implies "already a reference to ourselves and insofar our description (of the world) is not completely objective"[30]. It is important that we are fully aware of the ontologic consequences of this statement. If the dichotomy radically separates object and subject so that the first represents all of the world and the second only our description of it, then this description would be completely objective. Our set of descriptive terms and the corresponding set of objective properties of the external world would represent a structural equivalence and not an implicative relation. There would be no Reflexionsgefälle (gradient of reflection) between the subject and the object. But the division which Heisenberg proclaims is not such a simple one. He places the object on one side and the "rest of the world" on the other. But the rest of the world means a conjunction of object and subject! This is exactly his point.

But if we accept this second dichotomy, and there is no reason why we should not, we will have to remember that in any description of objective Reality the two terms "object" and "subject" are inversely equivalent. This means: Heisenberg´s dichotomy is only acceptable if it is supplemented by a corresponding dimension which separates the subject from "the rest of the world". In this way we arrive at three ontologic dichotomies as the following table demonstrates:

object (O^O)	subject (S^S)
object (O^O)	object (O^S) < subject (S^S)
object (O^O) < subject (S^O)	subject (S^S)

The indices refer to the "als" (as if) category of transcendental logic. Something is thought of as having reference only to itself or as referring to something else. The distinction corresponds roughly to that of world in itself (an sich) and "world" as content of our awareness, and to that of consciousness as inner subjective awareness and consciousness as objective event in the external world. Heisenberg´s dichotomy implies that distinction, but it seems that he is not aware of what his "rest of the world" means. The division above the horizontal line refers to the "absolute" dichotomy of the classic tradition of logic which believed in the ideal of a radically objective description of Reality. The Copenhagen interpretation of quantum mechanics is represented directly below and further down its necessary corollary. If we represent the possible formal relations between O and S in symbolic form we obtain

$$O^O \equiv S^S \qquad (1)$$

$$O^O \supset (O^S < S^S) \qquad (2)$$

$$(O^O \wedge S^O) \subset S^S \qquad (3)$$

[30] W. Heisenberg: loc. cit. p. 56.

Formula (1) is always true if O^O and S^S have the same value and it is always false if their values differ. Formula (2) is invalid if and only if O^S is true and the conjunction of O^S and S^S does not hold. In Formula (3) this situation is reversed. This time the implication is not valid if S^S is true and again the conjunction does not hold. It is obvious that if Formula (1) holds then Formula (2) cannot stand alone. It must be complemented by Formula (3). Otherwise the value symmetry which the Copenhagen Interpretation expressly demands is destroyed. It is significant that a two-valued calculus of logic (as applied in quantum mechanics) cannot assign different values to S^S and S^O or to O^O and O^S. In other words: although the Copenhagen Interpretation acknowledges epistemological differences between S^S and S^O or between O^O and O^S, from the view-point of a formal classic calculus the indices are redundant.

This co-validity of the Formulas (2) and (3) points at two distinct phenomena of distribution of terms in quantum mechanics. There must be one type of distribution concerning the O^O-range describing the object) and another one in the S^S-range (developing the logical theory). We have already taken notice of Bohr´s rule of complementarity in this context and observed that the duality of corpuscle and wave indicates a distribution of subjectivity over two sets of objective terms. The second feature of distributivity shall be mentioned three paragraphs below. Whatever the epistemologic frame of a scientific discipline, the thinking that is done in it is nothing else but the mapping of a set of conceptual terms onto a field of objective data. The simplest case is represented by Formula (1). Here the set of S-terms corresponds one-to-one with the set of O-terms. But in order to give this two-valued system ontological significance either "S" or "O" must be declared as designated value. If we choose "O" we are entitled to state that our formulas provide us with an abstract picture of the objective world. But the subject as the onlooker, who has this image, remains an unknown x because "S" was not the designated value. In other words, the procedure of designation implies that the ontological character of either "S" or "O" must remain unknown. If "O" is the designated value, then we assume a mysterious "soul" that perceives a real world and knows about it in genuine objective terms. If, however, the designation favors "S" as for instance Fichte´s and Hegel´s logic does, then the resulting philosophy seems to know all about the subject but the genuine object, the thing-in-itself, disappears. Kant still admits its existence in the *Critique of Pure Reason* but emphasizes that we will never know anything about it. His successors Fichte and Hegel are not even satisfied with that. They demonstrate rather convincingly that the very concept of an isolated object-in-itself is a logical contradiction. That means we cannot even make meaningful statements which assert the radical objective existence of such things.

We have gone in such detail about this ontological issue because it is of overriding importance to understand why a two-valued theory of thought can never describe an order of Reality in which subject qua subject and object qua object co-exist. A logic in the usual sense of the word cannot be applied at all unless we designate a value. But as soon as we have done this we are committed. We cannot have it both ways. If we use our logic to describe the object, then the context of our terms is never applicable to the subject. But if our theory aims at describing the relations between our mental (subjective) concepts, then we do not obtain a picture of the objective world, only of its reflected image, with typical properties of reflection that the objects do not possess.

The peculiar epistemological structure of quantum mechanics stems from the fact that it uses a logic in which subject and object permit only an inverse transmission of terms but it applies it to a dimension of Reality where subjective and objective properties are inextricably mixed. The result is, as we have pointed out, a distribution either of objective terms over the range of subjectivity or an inverse distribution of subjective concepts over the field of objects. Our Formulas (2) and (3) indicate these reciprocal situations. The practical effect of this unusual situation can be described as follows: As long as no factor of distribution enters the picture, the case in classic physics, we use *two and only two* distinctly different values to describe *one single* object that is fully and unquestionably identical with itself. But as soon as we allow for distribution two things happen. On the objective side it becomes impossible to retain the concept of an object that has an indivisible identity with itself. Instead of it we obtain two pseudo-objects which complement themselves as mutually exclusive pictures of the objective component of Reality. This is the duality of the corpuscle-wave concept which mirrors the classic contraposition of the two logical values "positive" and "negative". In pre-quantum-mechanical physics only one value designates the object. Consequently it is sharply focussed and single. But from the very moment the physicist claims that it is impossible to separate non-ambiguously in his observational data the share of the subject and the object, *both* values have to be used for the description of what he sees. Hence the splitting of the identity of the object in its two images as corpuscle and wave. So much for the object and the rule of complementarity.

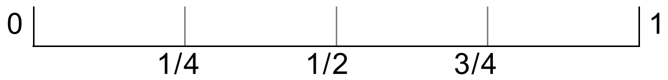
But in any science we can think of a comprehending subject facing a certain context of the world. If this context is changed, it must necessarily modify the conditions of thought under which the relevant context can be understood. This reciprocity is expressed in our Formulas (2) and (3). It means, as far as quantum mechanics is concerned, that the principle of distribution manifests itself not only in our description of objects and objectivity in general but also in the epistemological conditions that determine the logic of our scientific thought processes. We remember that on the classic level of epistemology we had two distinct logical values (true-false) on the subjective side facing one single self-identical object in the external world. Now the identity of this very same object is distributed over two complementary concepts of objectivity. But the argument applies both ways. The reciprocity of Formulas (2) and (3) implies that the principle of distribution should equally hold on the side of thought, affecting the rigid contraposition of our two values. And this is what happens indeed. "True" (T) and "false" (F) are distributed over each other; instead of the clear distinction between them which is expressed in Table 1:

Table I

F	T
1	0
0	1

we are forced to adopt a sliding scale of "mixed" values:

Table II



The result is that we can describe the properties of observed " objects" only in terms of probability functions. Not only external existence manifests itself in complementary forms. There is subjective complementarity too. "The *Knowledge* of the position of a particle is complementary to the *Knowledge* of its velocity or momentum"[31].

To prevent a misinterpretation of the term "subjective" as used by Heisenberg, by Schrödinger or the present author, it should be emphasized that it never means dependency on the *arbitrariness* of any subject, not even the impassioned scientific observer. Heisenberg has clearly stated: "The probability function combines objective and subjective elements. It contains statements about possibilities, or better, tendencies … but … these statements are completely objective: they do not depend on any observer"[32]. The expression "subjective" if used in quantum mechanics with regard to the corpuscle-wave duality and the probability of functions, *can never mean anything but that the logic applied uses its two values in a distributed state.*

With these remarks we conclude our presentation of the part played by subjectivity in modern physics. However, the definition of subjectivity as a phenomenon of value-distribution in logic and as ambiguity in the concept of the object (particle plus wave) that emerged from our arguments is not sufficient for the purpose of cybernetics. We have seen how the introduction of the subject into our scientific frame of reference changes the ontology of the object. But a parallel ontology of the subject has not yet been introduced. Its discussion will be our next concern.

2. TRANS-CLASSIC ASPECTS OF ONTOLOGY

The reasons why the logical properties of subjectivity disclosed in quantum mechanics do not by themselves satisfy the requirements of cybernetics can be stated in simple terms. Physical science is – quite rightly so – only interested in the description of genuine objects and of objective events. Subjectivity enters the picture only in a negative manner, as a lack of certainty and as a duality of terms weakening their power to designate objectivity. The subject as such, as a center of reflections with self-reference, is not the topic of any science with the methodological aim to explore this whole world the way it is given to us as the objective content of our consciousness. Even if the ideal of objectivity seems to be rather tarnished nowadays it still remains a regulative principle of scientific conduct.

Under the circumstances it might seem doubtful whether subjective consciousness could become the topic of a serious scientific treatment. It is true that we possess a very profound epistemological theory of self-consciousness, but it was developed by metaphysicians in India as well as in the Western World. Its terminology is suspect and, in its traditional form, almost useless for scientific purposes. On the other hand, present day cybernetics is so enamored of its imposing arsenal of hardware and of a

[31] W. Heisenberg: loc. cit. p. 49. The italics are ours.
[32] W. Heisenberg: loc. cit. p. 53.

terminology attuned to the radically objective character of physical models that there seems little chance these two shall ever meet.

Yet they must be brought together. When computer theorists pose such questions as: can machines have memory? do they think? are they able to learn? can they make decisions? do they possess creativity? we can see that subjectivity enters into cybernetics from the very beginning in a much stronger fashion than into physics. Nobody has ever seriously asked whether electrons think or whether they are gifted with the power of mental creativity. Classic, as well as modern, physics are not interested in the fact that our universe contains several groups of systems with such a high capacity for self-organization that they produce a mysterious quality called consciousness or self-awareness. It is quite different with cybernetics. This novel theory potentially encompasses every scientific discipline that, by its very nature, is obliged to recognize the actual existence of a plurality of centers of self-awareness which we commonly call consciousness. In his *Design for a Brain* Ross Ashby[33] has given a very clear exposition of the methodological situation that confronts us in cybernetics. He points out that the (originally subjective) category of "learning" can be defined in a way that has no necessary dependence on consciousness. But he significantly adds that the "observation, showing that consciousness is sometimes not necessary, gives us no right to deduce that consciousness does not exist. The truth is quite otherwise, for the fact of the existence of consciousness is prior to all other facts. If I perceive – am aware of – a chair, I may later be persuaded, by other evidence, that the appearance was produced only by a trick of lighting; I may be persuaded that it occurred in a dream, or even that it was an hallucination; but there is no evidence in existence that could persuade me that my awareness itself was mistaken – that I had not really been aware at all. This knowledge of personal awareness, therefore, is prior to all other forms of knowledge"[36]. From this it follows clearly, as Ashby has pointed out in another context, that "cybernetics has its own foundations"[34]. It should be noted that the concept of consciousness is not built into the foundations of physics – despite its empirical admixture of subjective elements. However, if Ashby is right (and we believe strongly that he is) that the existence of consciousness is prior to all other facts in cybernetics, then the ontological foundations of any cybernetic theory must differ essentially from those of physics. In the latter discipline we shall continue to search, despite all modern developments, for the basic laws of materiality. Materiality is what we mean if we imply that there is an outside world beyond the confines of our or any consciousness. It does not matter at all how diaphanous this idea of materiality has become during the last decades. There is some possibility it might even fade into the concept of a "self-field", the ultimate speculation of modern physics[35]; but even such a field would be an objective order of Reality. Objectivity has always meant and will always mean materiality. Ontologically speaking it makes not the. slightest difference whether we define materiality as that which we can see or touch, or whether we interpret it as a "hypostatized" field of self-interaction. It still remains the very same

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[33] W. R. Ashby*: Design for a Brain*, Wiley, New York, p. 11 (1952).
[36] An approximate idea of it in G. Günther: Das Bewusstsein der Maschinen, see Note 3.
[34] W. R. Ashby: *An Introduction to Cybernetics*, Wiley, New York, p. 1 (1956).
[35] A.W. Stern: *Space, Field and Ether in Contemporary Physics*, Science, 116, pp. 493-496 (1962). See also O. L. Reiser: Matter, Anti-Matter and Cosmic Symmetry, Philosophy of Science XXIV 3, pp. 271-274 (1957).

objective "It" as the trivial objects of our daily life. The concept of consciousness does not enter into this picture at all. In fact it has been irrelevant for the entire development of Western science from the Greeks till this present century.

For cybernetics, on the other hand, the fact of self-awareness is fundamental. It follows that Man is about to enter a new epoch in his scientific history[34]. The transition from the physical sciences to that new group of disciplines which are originating under the general label cybernetics is so basic that the magnitude of this mental revolution is not yet fully grasped even by the cyberneticists themselves. We shall try to give an approximate idea of its size by starting from some principal statements made by Ashby. He remarks in his *Introduction to Cybernetics,* under the very appropriate heading "What is New?", that "the truths of cybernetics are not conditional on their being derived from some other branch of science." Accordingly, "it depends in no essential way on the laws of physics or on the properties of matter ... The materiality is irrelevant, and so is the holding or not of the ordinary laws of physics"[37].

This leads to surprising conclusions. It will be useful, however, before stating them to give the working definition of cybernetics that Ashby offers under the same heading. He interprets this novel science as "the study of systems that are open to energy but closed to information and control"[38]. From a purely logical viewpoint this definition is somewhat preliminary and redundant, for the concept of control can to some degree be subsumed under information. However, it will serve, together with Ashby´s other remarks, as a good starting point for a general definition which might satisfy the ontologist. Since the distinction between "open to energy" and "closed to information" implies the irrelevancy of the material aspects of a cybernetic system one might describe cybernetics from the ontological angle as the study of a specific type of systems that must be described in terms presuming but not designating the materiality of the system. However, this definition also can only be provisional. It suffers from the fact that the designating character of cybernetic terms is only negatively circumscribed. Especially since we do not know how these specific types of systems should be defined in logical terms which do refer to its susceptibility to information. But we have already learned something of considerable importance: in our universe there exists a class of physical systems which have a non-material aspect. This aspect can be scientifically investigated! It can be treated experimentally, and we may build a new type of technology on it.

The transition of our thinking to this new outlook has come to us so gradually and partially disguised in the cloak of trustworthy traditional patterns of thinking that very few contemporary thinkers realize how radical the change has been and how many innovations it will induce in the future. The idea that we encounter in our universe phenomena that seem not only to have a nonmaterial aspect but in whom this aspect alone describes their essence is one of the oldest of mankind. We have ancient, cryptic words for it like Life and Soul. But these non-material manifestations of Reality were always considered the domain of religion and theology, beyond the reach of scientific treatment. Only in the nineteenth century did this outlook begin to change, when the influence of Kant, Fichte and Hegel made itself felt in the new scientific theories. Kant

[37] Loc. c it., p. 1.
[38] Loc. c it., p. 4.

had deprived the concept of soul of all metaphysical substantiality, declaring it to be a regulative principle of thought. Following in his steps, Fichte and Hegel developed the first full-fledged logic of consciousness: the secularization of the concepts of Life and Soul had entered its first phase. A significant new term was coined during this period: *Geisteswissenschaft.* The word "Geist" is untranslatable, and since 1871 we find it in English dictionaries as an adopted foreign word. It is interesting to notice that if we divest the word of all specific nuances with which the German tradition has impregnated it and penetrate to its logical core then it means nothing but an aspect of objective Reality that must be described in terms which are indifferent to the materiality of the objective context that is under discussion. But the idea of Life or Soul as a metaphysical essence that resides temporarily or even permanently as an alien in our empirical reality died hard. In natural science it survived for some time in the theory of vitalism. In philosophy it continues to plague us in many disguises like, for instance, the division between the humanities and science or the modern varieties of irrationalisms.

It seems to us that cybernetics is taking up the heritage of those ancient metaphysical traditions if it deals with that sector of Reality where the question of the material character of the observed phenomenon has become irrelevant. However, the range of the phenomena that belong to this category is enormous. It encompasses the whole scope of the Universe. To it belong all inanimate systems that show even the slightest degree of capacity for self-organization. It includes as a second group all organic systems from the simplest unicellular through the whole sequence up to man. And it encircles with its terms all historical institutions that have ever been or potentially could be produced by mankind. Nobody will find it difficult to see that the mental amplitude of our cybernetic theories surpasses any other scientific discipline that has been conceived since the times of Plato and Aristotle. One might say that cybernetics stands between the whole array of our individual sciences on the one side and philosophy on the other.

This exceptional position of cybernetics has not yet been fully realized by the scholars working in this field. And therefore, no serious need has been felt to provide this novel mental undertaking with logical foundations of its own. But foundations are necessary nonetheless. It should be evident that if cybernetics is of such scope that it comprises not only natural systems of both varieties, inanimate or animate but also historical institutions as self-organizing units, then the theoretical foundations of such isolated disciplines as physics, chemistry, biology, and sociology are ridiculously insufficient. And so are our present day mathematics, which are not yet prepared for a mathematical theory of consciousness and self-awareness. But if consciousness is a basic prerequisite for the behavior of certain self-organizing systems of animal type, we shall make little progress in the cybernetic analysis of animal or human behavior until we possess a mathematical method for the treatment of the still mysterious phenomenon of self-awareness. On the other hand it has been impossible, up to now, to develop the required procedures because the underlying logical concepts are still missing. The logic which science has used so far is minutely tailored to the needs of the classical concept of intellectual pursuit with its methodological ideals of excluding subjectivity from the formation of all theoretical terms and of being radically objective. It stands to reason that this attitude is worse than useless when the behavior of a system is due to its possessing self-awareness. And self-awareness is subjectivity, a phenomenon that can only be described in terms irrelevant to the materiality of the object it is related to.

The demand for a suitable new logic should be recognized in cybernetics more than anywhere else. And this investigation has imposed upon itself the task of furnishing the basic concepts for a formal theory of self-reflection that might satisfy the comprehensive demands of cybernetics. Since all previous sciences have derived their fundamental ideas and theoretical procedures from philosophy, as the history of human knowledge amply demonstrates, we shall turn again to this great source of primordial concepts to see what more can be learned from it.

The previous relations between philosophy and empirical science can be described briefly by stating that philosophy in its ancient pre-Greek form was the only universal "science" that encompassed all material as well as non-material aspects of Reality, and that investigated the prototypal relations between these aspects. What begins approximately with the Greeks is a process of mental comminution. Certain parts of knowledge undergo a subtle change. Philosophy itself remains unaffected, but these parts detach themselves from it and become independent bodies of knowledge. Geometry is an early example of the breaking off. After a slow start this process has continued without major interruption through the last two millennia. It persisted delectably during the Middle Ages, and after the Renaissance it accelerated rapidly. In the last century it has assumed such fantastic proportions and has split our knowledge into such a gigantic labyrinth of single sciences that no human brain is still capable of understanding its general context and meaning. It is only natural that the cry for a Unified Science was heard long before the arrival of cybernetics. But, although famous names like Łukasiewicz, Bohr, Russell, and Carnap were connected with it, one is forced to admit that the undertaking has not succeeded. It was predicated on the assumption that philosophy had dissolved itself in this process of comminution and that nothing was left. Its successor was supposed to be a special discipline among others, to be called "the logical analysis of scientific language"[39].

There were good reasons for believing that the attrition of the former substance of philosophy was complete. What seemed to be the last metaphysical stronghold of old-style philosophical thinking – the theory of, infinite actuality – had been conquered by George Cantor[40] during the last two decades of the nineteenth century. His theory of transfinite sets (*Lehre vom Transfiniten*) appeared to be a purely mathematical discipline; when, later on, paradoxes developed from it, no mathematician went back to the metaphysical origins of set theory. The solution of the difficulties was considered a merely technical affair of symbolic logic even if it meant resorting to such desperate

[39] R. Carnap: *Logische Syntax der Sprache*, Verlag Julius Springer, Wien, p. 111 (1934). ´Philosophie wird durch Wissenschaftslogik, d.h. logische Analyse der Begriffe and Sätze der Wissenschaft ersetzt´.

[40] G. Cantor: Über die verschiedenen Standpunkte in Bezug auf das aktuale Unendliche, *Zeitschr. f. Philos. und philos. Kritik*, 88; pp. 240-265 (1886) - "Mitteilungen zur Lehre vom Transfiniten," I, *Ibid*. 91, pp. 81-125 and 252-270; continued as "Mitteilungen..." II, 92, pp. 240-265 (1888) - "Beitrage zur Begründung der transfiniten Mengenlehre I." Math. Ann. 46, pp. 481-512. Continued as "Beitrage... II," Ibid, 49, pp. 207-246 (1897). Also see E. Cassirer, *Substanzbegriff und Funktionsbegriff*, Bruno Cassirer, Berlin, pp. 80-87 (1910). ´Cantor hat, indem er in seinengrundlegenden Untersuchungen das System der transfiniten Zahlen schuf, zugleich alle die scholastischen Gegensätze des Potentiell- und Aktuell-Unendlichen, des Infiniten und Indefiniten wiederum heraufbeschworen. Hier scheinen wir somit endgültig von der Frage nach der reinen Erkenntnisbedeutung der Begriffe zu den Problemen des absoluten Seins und seiner Beschaffenheit hinüber gedrängt zu werden.'

measures as the restrictions that Brouwer, Heyting and other intuitionists wanted to impose on mathematics. At any rate after Cantor´s initial steps, there was no turning back; and although the Transfinite is not yet fully conquered, it has irrevocably been claimed as a mathematical problem and has thus lost its dignity as a metaphysical archetype. But does that really mean that the last bulwark of classic metaphysics has crumbled? Our answer is emphatically No. But since the proof of the pudding is in the eating we intend to demonstrate that there remains at least one genuine transcendental problem of the classic tradition awaiting its exact scientific treatment and subsequent solution. Incidentally, our claim of "at least one problem" does not exclude the possibility that there may be an infinite number of them; we strongly believe this to be the case. Metaphysics is by its very nature an *inexhaustible* source of transcendental categories offered for transformation into exact scientific concepts. In fact, that seems to us to be the intrinsic difference between philosophy and the positive scientific disciplines that have emerged and separated from it. The latter are in principle exhaustible and can be completed. The former can not! And this is our only guarantee that the well of human creativity will never dry up.

The problem that remains is covered – but not defined – by such questions as: what is life? What is consciousness? What is subjectivity? and finally: what is history? It seems strange to name in one breath such divergent and apparently heterogeneous topics.

Life which is assumed to be treated fairly well by Biology and History belongs to the humanities. Here metaphysics, which has fallen in such disrepute among scientists, proves its practical usefulness.

To the philosopher it has always been clear that such heterogeneous phenomena as Life and History have this in common: they both represent self-reflective systems. In other words, they display a subjectivity of their own. However, the very fact that this has been recognized at a very early date has hampered the scientific treatment of the phenomenon of subjectivity. It is a curious situation. The overwhelming number of metaphysicians in East and West agree that Reality as such can only be understood in analogy (*analogia entis)* to a self-reflecting subject. Spinoza even chose for ultimate Reality a term that indicated its self-reflective structure: *natura naturans.* But the very fact that this category seemed to point at the metaphysical secret of all Existence made the sober scientist shy away from it. He was always familiar with the concept of ordinary physical reflection. There he had no difficulty in regarding the world as a reflection (content) of his consciousness. But self-reflection is different. From its lowest forms as the spark of Life in the primitive organism to its highest manifestations in Man it denoted always a metaphysical essence, the primordial stuff that is the very core of Reality. The prejudice voiced by Spinoza that only an *intellectus infinitus* may understand self-reflection still dominates our scientific thinking. There is a silent consensus that it is impossible to develop a strict formalism for self-reflection.

Of course, as long as self-reflection, the essence of life, consciousness and subjectivity, is considered to be something mystical and supernatural it would be hopeless to look for an exact formal logic that describes its structure. It would be even more absurd to expect a mathematical treatment of it. How would one compute the divine breath that penetrated the deadness of mere matter on the day of creation? The answer to this question is so much a foregone conclusion that we cannot help but suspect that there is a gross misunderstanding involved. Even if cybernetics should ever succeed in designing systems that must be recognized as perfect behavioral equivalents of life or conscious subjectivity it would be arrant nonsense to say: this computer is alive or is conscious. Physics has learned long ago that it does not investigate what Is. It deals only with phenomena and not with what lies behind them. The same attitude should govern cybernetics. The question is not what life, consciousness, or self-reflection ultimately is, but: can we repeat in machines the behavioral traits of all those self-reflective systems that our universe has produced in its natural evolution? It is not impossible that the computer theorist might succeed completely. But even then, consciousness in a machine and consciousness in a human body would only be phenomenally identical. Ontologically speaking they would be as far apart as any two things can be. The reason is obvious: the natural product originated in a cosmic evolution lasting several billions of years and, unless we assume a divine spirit in the beginning no personal self-consciousness directed the production. The cybernetic system, however, would be produced in a radically abbreviated time scale and the development would be guided by other systems (humans) with a highly developed self-awareness. And finally the physical resources, as well as the methods of manufacture, would hardly bear any resemblance to the conditions under which Nature did its work.

Thus, even if there existed an absolute behavioral equivalence between the manifestations of self-awareness in a human body and in some other physical system designed by the methods of cybernetics, we would not know in the least what a human (or animal) personal ego actually is. In other words: the metaphysical concept of a "soul" does not enter into the theory of automata at all. *Ergo,* this novel undertaking is not hampered by any sort of metaphysical restriction. *It nowhere competes with meta*physics[41].

If this distinction is kept in mind, the possibility of developing automata which display all characteristics of self-reflection depends entirely on finding a formal logical criterion for self-consciousness or subjectivity which would be amenable to treatment in a calculus, and consequently in mathematics. Such a criterion is still unknown to science and would forever remain so if terms such as life, subject, and consciousness denoted only something supernatural. Without detracting from their possible metaphysical implications we shall show now that this is not the case. Our demonstration will be specifically associated with the concepts of subject and subjectivity because they have, by their logical connotations, played a greater part in epistemology than other related terms[42].

What strikes even the superficial reader of philosophical texts is that the term "subject" is used in two almost diametrically opposed senses. The texts talk about an absolute subject and an epistemological subject of our individual thought processes. The absolute subject represents ultimate Reality or Being that reflects itself. It is the fountain of Truth. It is supposed to be the origin of all cosmic order and harmony. And it is totally

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[41] W. Sluckin: *Minds and Machines*, Pelican Books, pp. 231 (1954) confronts cybernetics directly with metaphysics. Unfortunately, this is done very inexpertly, as is shown by the use of such self-contradictory terms as 'psychological metaphysics.'

[42] Kant's *Critique of Pure Reason* is perhaps an exception. Here, the term 'consciousness' plays a dominant role, but his successors, especially Hegel, turn again to 'subject.'

indifferent to the distinction between form and matter[43]. Clearly, no logic or computer theory can define this meaning of the term in any technically usable way. Even Cantor´s theory of the transfinite would fail. But the very same philosophic tradition talks about the subject and subjectivity in quite a different view when it refers to the finite empirical subject. Whereas the infinite subject represents the highest Good, finite subjects have no reality of their own. They are the source of all falsity and delusion. They represent disorder and boundless arbitrariness. Their very existence is based on the distinction between form and matter. As pure subjects they are nothing but empty form. Therefore they cannot reflect themselves in their true nature as subjects[44]. They only reflect objects, and consequently if they try to think of themselves they do so only in terms of objectivity, with a consequent semantic falsification of their self-reflective thoughts. And if human history resembles a "slaughter house", as Hegel remarks[45], this is so because this type of subject has never learned and cannot learn anything from history.

This is not exactly an impressive record. Certainly the subject empirical has nothing of the majesty and unapproachability of the subject absolute. There seems to be no reason why the former should not be imitated. Maybe in the process of doing so we might learn how to improve upon the natural product, which is by no means perfect. If it still sounds utopian to design automata, which display the behavioral traits of life, consciousness and subjectivity (and even ethical personality if Warren McCulloch[46] is right), our present disability is due to the fact that we have not yet developed a logic, and a corresponding mathematical procedure, which can demonstrate that these terms, and others related to them, have a precise rational and computable core. What gives them a mystical and irrational flavor is our previous incapacity to connect them with categories which belong to a strict formalism. That a datum of experience is way beyond the present scope of logic and mathematics does not necessarily give it metaphysical dignity.

But what is an individual subject, and what is general subjectivity as the medium that connects different egos ? The ground is much better prepared for a fruitful answer than most scientists realize. So far we have only listed two contributions. We possess the knowledge provided by quantum mechanics, that the introduction of subjectivity into our physical picture of the external world generates a peculiar phenomenon of distribution. And we are indebted to Ross Ashby for the insight that cybernetic systems must be described in terms not designating the materiality of the system. But there is one more relevant contribution. It was made by Heinz von Foerster, and from the viewpoint of a future logic of cybernetics it is in fact the most significant one.

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[43] This motive of indifference was especially stressed by Schelling. Cf. System der Philosophie *W. W.* 111, pp. 1- 108 (1801). See also Fichte's trenchant criticism of it. *N. W. W.* III, pp. 371-389.

[44] E. Schrödinger: *What is Life?* University Press, Cambridge., pp. 90-92 (1955) significantly speaks of 'deception' with regard to the plurality of individual finite subjects. He even invokes the Veda concept of mãya.

[45] G. W. F. Hegel: Philosophie der Geschichte, Einleitung, W. W. XI, ed. H. Glockner, Fr. Frommanns Verlag, Stuttgart, p. 49 (1949) uses the term 'Schlachtbank.' Cf. also ibid., p. 31.

[46] An unavoidable conclusion from his essay "Toward some Circuitry of Ethical Robots." See Note 7.

It originated from von Foerster´s evaluation of Schrödinger´s thesis (in his monograph What is Life?) that orderly events can be produced according to two basic principles: "order-from-order" and "order-from-disorder", principles which establish two types of natural law, the dynamical and the statistical[47]. Von Foerster makes the profound observation that there is one more principle which should not be confused with Schrödinger´s order-from-disorder. He called it, order-from-noise, and announced it in his contribution to the Conference of Self-Organizing Systems in 1960[48]. He demonstrated his idea by a simple mental experiment. Cubes with surfaces magnetized perpendicular to the surface are put into a box under conditions which permit them to float under friction. All these cubes are characterized by opposite polarity of the two pairs of those three sides which join in two opposite corners. Now let undirected energy (noise) be fed into the box by the simple expedient of shaking it. If we open the box after some time an incredibly ordered structure will emerge, "which, I fancy". says von Foerster, "may pass the grade to be displayed in exhibition of surrealistic art." No order was fed into the box, just "noise"! But inside the box a principle of selection (the polarities) governed the events. "Only those components of the noise were selected which contributed to the increase of order in the system"[49].

The exemplification of the principle may be trivial to the physicist but it delights the logician, for it demonstrates the difference between order-from-disorder and order-from-noise so clearly that a logical theory can be based on it. To do so, we should return once more to Schrödinger and his two principles. The distinction he makes cannot withstand the scrutiny of the logician. He gives all example of the order-from-order principle and describes how it represents the dynamical type of law. Later, however, he takes a second look at his example and admits that it depends on our own attitude whether we assign the motion of a clock to the dynamical or to the statistical type of event. His final conclusion is "that the second attitude, which does not neglect them (statistics), is the more fundamental one"[50]. On the other hand we have to admit that lie has made an excellent case for his thesis that the "real clue to the understanding of life"[51] is the order-from-order principle. But according to his own admission this is not really a basic principle; order-from-disorder is more fundamental. If we want to develop a formal logic for self-organizing systems we cannot be satisfied with a principle which turns out to be a derivative from some other which is more general. Moreover, one gets the impression that he does not take his order-from-order principle, as exemplified by a clock quite seriously because, according to his own words, "it has to be taken with a very big grain of salt." What makes it dubious is that Max Planck's interpretation of this principle (which was adopted by Schrödinger) is a straightforward physical concept! But are we supposed to forget now that we agreed with Ross Ashby that cybernetic laws do not belong in the same class as physical laws!

It seems to us that the key to the problem is to be found in von Foerster´s principle of order-from-noise. We are going to show that it is as fundamental as the order-from-

[47] Loc. cit., p. 82.

[48] Heinz von Foerster: On Self-Organizing Systems and Their Environments, In *Self-Organizing Systems*, ed. M. C. Yovits and S. Cameron, Pergamon Press, London, pp. 31-50 (1960).

[49] Loc. cit., p. 45.

[50] Loc. cit., p. 83.

[51] Loc. cit., p. 83.

disorder concept because it involves certain new logical operations which have not yet been recognized in formal logic and which we would like to name "transjunctions."

Since the Planck-Schrödinger principle of order-from-order is not basic we shall have only two fundamental concepts: order-from-disorder and order-from-noise. This requires two comments. First: we will need a logical criterion to distinguish in a calculus between disorder and noise in the specific sense which is implied by von Foerster´s new principle. Second: we will have to reconcile the order-from-noise idea with the fact that self-organizing systems feed on negative entropy. Taking first things first we like to draw the attention of the reader to the fact that Schrödinger´s term "disorder" has already its equivalent in formal logic. He calls his disorder "statistical". But statistical laws are handled by a logic of probability. Thus probability is the logical equivalent of disorder. On the other hand it is quite obvious that the feeding of noise into von Foerster´s box did not create a logical probability situation, or more disorder. We know that exactly the opposite took place. But still we must admit that disorder and noise are closely related and the old recipe for a logical definition is *genus proximum et differentia specifica*. Consequently we ask what is, from a logical point of view, the *genus proximum* or common denominator for disorder and noise ? This question was already discussed in part I of this paper. It was shown that a probability logic resulted from a distribution of the two available values over the range of their "distance" such that if 0 = false and 1 = true these two values are spread over the range of all denumerable fractions between 0 and 1.

What noise has, logically speaking, in common with disorder is that it produces a distribution. But what is distributed must be something else. Certainly not logical values, since their spreading produces only probability. A closer look at von Foerster´s model will give us a hint. His box contains cubes with magnetized surfaces. It is trivial to state that these cubes are in some state of distribution in the box before we start shaking it. It is also trivial to note that our shaking results in a different state of distribution. So far we may admit that noise also manifests itself as an agent of distribution. But now let us look at our Cubes with the magnetized surfaces. Each individual cube may be regarded by us as a tiny logical system, the two values being north pole on the outside or north pole on the inside. *Et tertium non datur.* It goes without saying that our two values exist in their system in a non-distributed state. When the shaking begins a distribution does take place and it concerns our little two-valued systems – but not their individual values! What has happened when von Foerster's surrealistic architecture finally emerges is that without any change in their internal value structure the individual systems which represent this rigid two-valuedness have been rescued from their haphazard initial position of disorder and redistributed in a fashion such that they form a system of sorts which is composed of as many two-valued systems as there are magnetized cubes in our box.

It will be useful to have another look at the state of the box before the shaking started. The cubes were at that time in some unspecified state of disorder. But the cubes themselves represented units of order. Consequently the initial situation that existed inside the box must be described as a conjunction of order and disorder. This gives us one more hint as to the significance of von Foerster´s noise influx. The noise is something which is capable of instigating a process that absorbs lower forms of order and thereby converts a corresponding degree of disorder into a system of higher order. In other words: it is a synthesis of the order-from-order and the order-from-disorder ideas. Having discarded Schrödinger´s simple order-from-order concept we obtain now two basic principles:

Schrödinger: order-from-disorder
von Foerster: order-from-(order-plus-disorder)

In both cases the logical equivalent of disorder is a distribution of logical terms. But what is distributed is different. Schrödinger´s principle refers to the distribution of individual values. von Foerster´s concept refers to the distribution of value-systems. In the first case the internal structure of the logical system which suffers the distribution is changed: a theory of formal certainties is transformed into a theory of probabilities. In the second case nothing of this sort happens: The distribution does not concern the elements which constitute a given system but the system itself as an inviolate entity.

This gives us two entirely different meanings of distribution and consequently of disorder. von Foerster´s distinction of disorder and noise is a profound one and opens up much deeper perspectives than his unassuming demonstration with the magnetized cubes suggests at first sight. Of course everything depends now on the question whether we will be able to define a logical operator that would represent a distribution not of values but of closed value-systems. It will not be necessary to discuss value-distribution. The corresponding logic of probability is well established and we could not add anything of special relevance. It suffices to point out that our traditional two-valued logic takes care of the ordinary order-from-order concept as well as of Schrödinger's order-from-disorder principle. A simple logical demonstration of order-from-order would occur if the *Principia Mathematica* were rewritten in terms of Sheffer´s stroke function. The undertaking might have some merits but we confess we cannot find the prospect exciting. The order-from-disorder principle enters classical logic in its more important part. It is the predicate calculus that introduces probability and makes it basically ineliminable since we know that the "objective" verifiability of the argument of a function f(x) will ultimately depend on statistical terms.

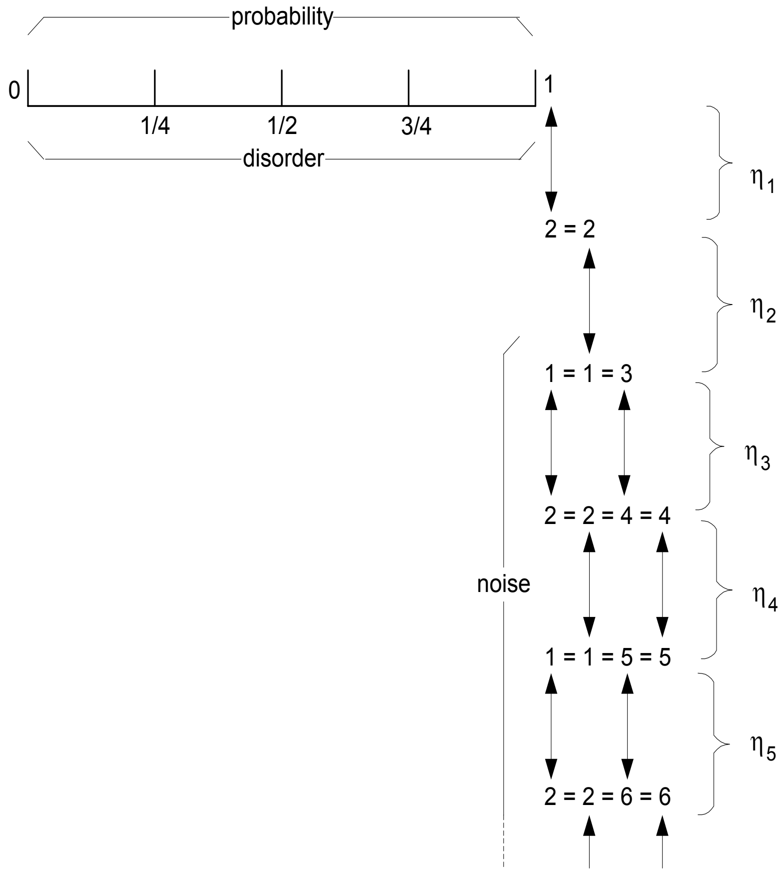
As far as the second meaning of "distribution" is concerned, which we culled from von Foerster´s order-from-noise, no recognized model exists. It is up to us to give a formal demonstration of it and to introduce the new logical operation a "transjunction" which is responsible for a logic of distributed systems. The next and the last section of our investigation will give an outline of a transjunctional formalism.

3. LOGIC WITH TRANSJUNCTIONS

If we want to distribute not logical values but systems of values our next question should be: what permits values to form a system? This system-producing factor obviously must be that which allows distribution. The demanded factor is by no means unknown; in fact its indication is rather trivial: what enables our two traditional values to form a logic is the *existence of the unary operator that we call negation* (η). Table I in Part 1 shows that a negation is nothing but a simple exchange relation between two values. This exchange relation is not in the least different from the familiar relation between the terms left and right. But if such an exchange relation establishes the basis for the formation of a logical system, then the distribution which von Foerster's principle, order-from-noise, is supposed to produce is actually a spreading of exchange relations. This extension, of course, can only be made by the introduction of additional values. In other words, Von Foerster's principle is logically definable only if we introduce a many-valued calculus.

For our further demonstration we shall, for convenience, use the set of positive integers as members of such an exchange relation, and we shall refer to them – if they are so related – as "logical values". We also introduce the term "successor" as it is known from Peano's group of axioms for such a numerical sequence, and we stipulate that each integer or value forms an exchange relation with its successor. By doing so we obtain a linear sequence for potential classic systems of logic; or to be more precise, we locate the very same two-valued system of logic in a linear sequence of "places". We further stipulate that the designation "classic" should apply only to systems that are established by an exchange relation between a value and its – successor. For the time being we ignore possible exchange relation which may be formed by any two values where one is not the immediate successor of the other. It goes without saying that such a linear sequence of exchange relations does not yet represent a many-valued calculus, let alone the idea of a new trans-classic system of logic. Our foregoing remarks are only intended to prepare the way for a scheme that shall illustrate our two concepts of distribution and their relation to Schrödinger's "disorder" and to von Foerster's "noise". We indicate distribution of values by an ordinary straight line. Along this line negation is indicated by 0. For the distribution of systems, negation is represented by any positive integer m where m > 1. Both distribution patterns have in common the value "1". In its horizontal relation "1" should be interpreted as true. In its vertical reference as positive or irreflexive.

Table III



The vertical column of value-systems is written twice. On the left, only the integers "1" and "2" are used. On the right, "1" is identified in its second occurrence with "3", from there on subsequent odd numbers are always used whenever the positive value turns up; even numbers indicate the reoccurrence of places for the negative value. This parallel arrangement helps to point out that "3", "4", "5"… do not represent values in their own right but are, for the time being at least, solely chosen for the task of identifying the place where a specific classic value is located (if it is part of a system which has suffered distribution). It is important to signify this by a unique number for each place because the very same system, and with it its values, acquires different functional properties in different stages of distribution. Furthermore, this method or a similar one is required if we want a notational opportunity to introduce a discrete series of η-operators. But it should not be forgotten that the sole object of distribution is the same classic system, 0 - 1, which provides, us with the logical frame for a theory of probability (as indicated in the horizontal part of our diagram).

The reader is reminded that Table III serves only as an illustration of what is meant if we distinguish two different forms of distribution in logic. We have not yet shown how a new theory of calculi for system distribution may originate from von Foerster's principle order-from-(order-plus-disorder). So far we only know that a type of distribution that does not produce probability might be effected by a sequence of negational operators ($\eta_{1 \ldots k}$) such that any m-valued position might be reached by the use of the operators $\eta_1 \ldots \eta_{m-1}$ as the matrix below Table IV shows:

Table IV

	η_i
i	i + 1
i + 1	i

If we state that a many-valued system is a distributive order for the classic two-valued system we shall have to qualify this proposition. The avowed purpose of our undertaking is, of course, to make Von Foerster´s "noise" logically treatable. In other words, the traditional system of logic will appear in our trans-classic order in a form in which it possesses values that transcend its structural frame and therefore represent "noise" from the viewpoint of a strictly dichotomous theory of thought. We shall see later oil that this gives the value concept a double meaning in higher systems of logic. This ambiguity reduces its importance considerably. It will be seen in due course that what is really distributed in trans-classic structures of logic are not so much value-systems as a new logical unit which serves as basis for systematic value constellations. But the interpretation of many-valuedness as system-distribution will serve us to It should only be remembered that the concept of value Permits only a very one-sided evaluation of trans-classic logic.

We shall now present our approach to the problem of system distribution and show that this yields a new type of logic which might be the answer to some problems of cybernetics. This theory will permit a positive operational definition of "subject" and introduces a new logical unit which complements the value concept. We take our start from the familiar table of the 16 two-valued, binary truth functions and demonstrate our departure by using as an example inclusive disjunction, as shown in Table V:

Table V *)

p	q	p ∨ q
1	1	1
1	2	1
2	1	1
2	2	2

Now we remind ourselves that we intend to develop a logic capable of defining subjectivity in logical contraposition, to everything that designates mere objects and objectivity. If we examine Table V from this viewpoint, it occurs to us that the variables "p", as well as "q", represent objective data. In the usual interpretation of the propositional calculus they are identified as unanalyzed statements. But statements are clearly objects and carry an objective meaning. The same must be said – although in a lesser sense – of the values that are attached to the variables: they too have, in this two-valued context, an objective meaning. They designate whether something is or is not. In our special case the values determine two mutually exclusive properties that a statement might have. There might be some doubt about the symbol "∨" which is supposed to denote disjunction. One might argue that this is a subjective concept and as such not really designating objectivity. But one might also say that it refers to a psychological act performed by our brain and in this case "∨" should be classed with the other symbols contained in our table. In fact, we shall do so because we wish to be cautious and because we intend to eliminate from Table V everything that may semantically refer to the objective context and meaning of Reality.

It seems there is nothing left to represent the subject in this context: we seem to have obliterated the whole table. But this is not quite so, for something else is offered by Table V; it also represents, apart from variables, values and operations, three abstract patterns of *possible* value occupancy. These and only these we shall retain. To claim that these empty patterns by themselves designate objective data and have a concrete semantical meaning relative to an objective world would be rather difficult. So we shall accept patterns of possible value occupancy as the basic elements of a new logic which should be capable of defining subjectivity. We obtain more patterns of this type if we extend our procedure of getting rid of symbols with reference to objectivity to all 16 truth-functions of classic logic. In order to distinguish these patterns we shall use the two symbols ∗ and □ which, we stipulate, shall have no *logical meaning*. They only indicate that if a meaningful logical sign occupies a ∗ place in a given pattern it cannot also occupy a place which is marked by □ and *vice versa*. Using these two marks we obtain, from two-valued logic, eight abstract patterns:

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*) Note (evgo) :
in classical logic the disjunction is given as in the table, where " 0 " symbolizesthe negation and " 1 " the affirmation, respectively. Günther uses for the affirmation the value " 1 " and for the negation the values " 2 ", " 3 ", and so on.

		classic	Günther
a	b	a ∨ b	a ∨ b
0	0	0	2
0	1	1	1
1	0	1	1
1	1	1	1

Table VIa

1	2	3	4	5	6	7	8
*	*	*	*	*	*	*	*
*	*	□	□	*	□	*	□
*	□	*	□	*	*	□	□
□	□	□	□	*	*	*	*

Since each mark – for the time being – holds the place for two values, these patterns yield, if so used, our familiar 16 two-valued truth-functions. We have numbered the patterns for easy identification; no other significance is attached to the numbers.

It is obvious, however, that Table VIa does not represent all possible abstract patterns for occupancy by meaningful logical symbols. And since the patterns by themselves are completely indifferent to the question whether there are enough such symbols to fill additional patterns there is no objection to introducing two more meaningless marks in order to give us an opportunity to complete the table of. all possible four-place patterns. (If we intend to regard these patterns – without prejudice to value occupancy – as the basic elements or units of a new system of logic we cannot afford to select arbitrarily just eight out of a larger number).

In order to complete our table we shall use the additional marks ▲ and •, to which also no logical significance is attached, in order to indicate possible value-occupancy by more than two values. We then obtain the rest of the patterns as shown in Table VIb:

Table VIb

9	10	11	12	13	14	15
*	*	*	*	*	*	*
▲	*	▲	□	▲	□	▲
*	▲	□	▲	▲	▲	•
□	□	□	□	□	*	□

Thus a table displaying all possible patterns has precisely 15 entries, a number which can be derived from Stirling´s numbers of the second kind.[+)] It will be noted that some rule of placing the marks has been followed: for instance, starting the columns always with *. This is more or less a matter of convenience and we might as well, write the pattern No. 14 with, e.g., the following order of marks: * ▲ □ *. This is for the time being quite irrelevant. We are at this moment only concerned with the abstract patterns of potential value-occupancy and from this view-point both arrangements, * □ ▲ * and * ▲ □ *, represent the same pattern. The case, of course, is different when we replace the

+) See note [69] and Formula (28) on page 50 meaningless marks by actual values with specific logical significance. The simplest case is pattern No. 5: * * * * . But even this pattern can assume an infinite number of meanings. In two-valued logic it has just two aspects of theoretical relevance expressed by the value sequences T T T T and F F F F for true and false. These aspects would grow to three in a three-valued logic and to infinity if we permitted the number of values to increase beyond any limit.

However, no matter what the actual value-occupancy of a pattern may be, the identity of the abstract pattern or structure, and therefore the continuity of meaning, would always be retained. This indicates that the fifteen patterns of the Tables VIa and VIb, although composed of signs without logical significance, represent some sort of meaningful order. Their full meaning still escapes us, but this much may be said now: no matter how comprehensive the logical systems we construct and no matter how many values we care to introduce, these patterns and *nothing else* will be the eternally recurring structural units of trans-classic systems. Our values may change but these fifteen units will persist.

In order to stress the logical significance of these patterns, and to point out that they, and not their actual value occupancies, represent invariants in any logic we shall give them a special name. These patterns will be called "morphograms", since each of them represents an individual structure or Gestalt (μορφή). And if we regard a logic not from the viewpoint of values but of morphograms we shall refer to it as a "morphogrammatic" system.

If we look from this angle at classic logic we see that we should more properly speak of it as a system of values. As a morphogrammatic order it is incomplete, for only the eight patterns of Table IVa are utilized. It is, therefore, impossible to say that its logical units are *the* morphograms. The tradition rightly considers the classic system as a value theory. The values are its formal units. The actually employed morphograms assume only a secondary role in this context. In more comprehensive systems the situation is reversed. The reliance on the value concept makes the interpretation of trans-classic calculi so difficult that many logicians refuse to recognize them as the potential base of a new logic[53]. They claim that the two-valued system (with the theories of probability and modality) represents the only genuine formal theory of thinking.

We shall now look at the situation from the morphogrammatic stand-point. As a system of morphograms the classic logic is incomplete. It employs only those eight patterns that are, if occupied by the two classic values, logical equivalents of the objective component of Reality. This is quite as it should be. This theory was developed for the very purpose of describing the world in radically objective terms with all subjective traits rigidly excluded. The subject was traditionally considered the metaphysical source of all arbitrariness, error, and fraud: objects never lie but the subject may. As long as this prejudice was cultivated it was, of course, absurd to try to give a formal logical definition of what is meant if we use words like "subject" or "subjectivity." On the other hand, if we look at the problem without any of the traditional prejudice and rid

[53] B. von Freytag - Löringhoff: Über das System der Modi des Syllogismus, *Zeitschr. f. Philos. Forsch*. IV, pp. 235-256 (1949). Same author*: Logik; Ihr System und ihr Verhältnis zur Logistik*, Kohlhammer, Stuttgart (1955). Also H. A. Schmidt: Mathematische Gesetze der Logik I, Springer, Berlin - Göttingen - Heidelberg, p. 124s (1960).

ourselves of the associations of irrationality that commonly accompany these two terms we shall find that a very precise logical meaning can be connected with them. Since Table VIb is excluded from a logic that describes the objective character of the world it can, if interpreted in a morphogrammatic logic, not refer to objectivity. It can consequently only refer to the part that the subject plays in a logic which does not suffer under the restrictions which an old ontological tradition has imposed on our theories of rational thought.

However, there is some grain of truth in the tradition. If we use a term borrowed from information theory we might say that a formal logic is required to be a "noiseless" system. The introduction of subjectivity into it would make it very noisy. Since this cannot be tolerated in classic logic, but is demanded in cybernetics, we are required to develop a more comprehensive theory which is not hampered by the morphogrammatic restrictions of two-valued logic. Subjectivity is a logical theme beyond the boundaries of our traditional ontological concept of Reality. We repeat again: the tradition equates Reality and objectivity and excludes the subject from it. This has led, during the long history of metaphysics, to the identification of subjectivity or consciousness with the concept of a transcendental soul which has arrived from Beyond and is but a guest in this Universe. But there is also a different concept represented by primitive religion and pointedly worded by an American Indian tribe, the Algonquins. They define a subject as "that which has cast itself adrift." With these ideas in mind we shall try to interpret Table VIb .

Since it will make our task easier, we repeat the Tables VIa and VIb but this time not as abstract morphograms. We shall present them as occupied by values. Since we will have to introduce four values, "1" and "2" shall represent the traditional values; and, since we only discuss four-place sequences for the time being, we stipulate that they may retain their full ontologic significance. "3" and "4" will be the additional values which the filling out of Table VIII requires. The value sequences thus obtained may be referred to as the "standard forms" of the morphograms. This, however, is a mere convention since any other choice of values would represent the patterns equally well.

Table VII

[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
1	1	1	1	1	1	1	1
1	1	2	2	1	2	1	2
1	2	1	2	1	1	2	2
2	2	2	2	1	1	1	1

A logic which is two-valued and uses only these eight morphograms is severely restricted in its value occupancy. There is just one non-standard form which is obtained by traditional negation.

We add now the standard forms of the additional morphograms in Table VIII:

Table VIII

[9]	[10]	[11]	[12]	[13]	[14]	[15]
1	1	1	1	1	1	1
3	1	3	2	3	2	3
1	3	2	3	3	3	4
2	2	2	2	2	1	2

If a logic uses the morphograms of Table VIII, with [15] excluded, a three-"valued" system is required. The number of nonstandard value occupancies increases then to five. But only a four-"valued" logic is morphogrammatically complete. It becomes so by adding pattern [15]. Twenty-three non-standard value occupancies are available in this case. If more value-occupancies are desired, systems with more values have to be chosen. And there is, of course, no limit how far we want to go.

But this raises the question: what is meant if we use the term "value" in systems which employ Table VIII? The answer will lead us straight to the problem how subjectivity may be defined in a system of formal logic. To make our point we will take the standard forms of the morphograms [1], [4] and [13] and consider them as functions resulting from – the traditional variables "p" and "q" as is done for [1] and [2] in the truth-tables or in matrices of the propositional calculus. We now only add [13] and put all of them together, for demonstration purpose, in another Table IX. As classic values we shall use "P" and "N" for "positive" and "negative" and for the additional value required by morphogram [13] the number "3 " as in the preceding Table IX:

Table IX

p	q	[1]	[4]	•	[13]
P	P	P	P	•	P
P	N	P	N	•	3
N	P	P	N	•	3
N	N	N	N	•	N

The additional dotted line shall indicate that [13] does not properly belong to this Table. In this arrangement "p" and "q" are supposed to represent any objective system that offers an (exhaustive) choice between two values. We notice that [1] and [4] have something in common. Where two values are proffered, as is the case in the second and third position of the value-sequence, the two classic functions *accept the choice.* Between them they take what is available in terms of values. They differ only insofar as the function which is carried by morphogram [1] prefers the lower value and the one represented by [4] picks the higher one. It is obvious that the function carried by [13] is not of this type. Where there is a choice of values offered by "p" and "q" the *very choice is rejected.* This is the only formal logical meaning any additional value beyond "P" and "N" can have. Any value that does not accept the proffered choice is a rejection value: it transcends the objective (two-valued) system in which it occurs. In analogy to disjunction and conjunction we shall therefore call a morphogram which requires more than two values for its filling a "transjunctional" pattern; an operation performed with it a "transjunction."

It stands to reason that the rejection of a value choice does not have to be total (but undifferentiated) as in [13]. There are also the possibilities arising from partial rejection: the morphograms [9] to [12] represent them in all their variations. And there is also a radical rejection [15] which differentiates the total refusal to accept the alternative of two values. Finally we have to acknowledge that equivalence too may have its transjunctional extension. It should be noted that from the morphogrammatic point of view the transjunctional equivalence cannot assume total form, for if we wrote in [14] the value sequence 1 3 3 1 we would only repeat, with different value occupancy, the morphogram [8].

So far we have interpreted the value occupancies which were effected by "3" and "4" in Table VIII from a purely formal standpoint. We characterized them as rejections of a pair of alternative values. But such abstract characterization does not provide us with an ontological interpretation of these value sequences. In other words, we also want to know the semantic meaning of the transjunctional morphograms. A clue was given in this direction when we referred to the Algonquian definition of a soul as that which has cast itself adrift. This means something that does not anymore belong to the ordered context of things that surround us and that make up the physical reality of our Universe. On the other hand, since the dawn of History, whoever used a term like "subject" (or some equivalent of it) was capable of conceiving anything else but a purely negative thought. He tried to I conceive a mysterious x that defied description in terms of any predicate that was applicable to some objective content of the Universe. We find the classic expression of this ontological attitude in one of the oldest religious texts, in the Brihadārnyaka-Upanishad, where it is tersely said that the ālman (the soul) can only be described by the terms *"neti neli"*. Translated from the Sanskrit it means: not this and not that. The sentences preceding the neti-term in the Sanskrit text make it quite clear that from any duality of (contradictory) terms neither is applicable[54]. But this is exactly what morphogram [13] indicates. Where there is a choice of two alternative values both are rejected. It is impossible for us to connect any other formal logical meaning with terms like "subject", "subjectivity" or "consciousness" but rejection of an alternative that is total as the (exclusive) disjunction between true and false. For this very reason the morphograms [9] - [15] express as logical structures what we intend to say if we make statements which include references to the non-objective side of Reality.

It should be clearly understood that the issue for the cyberneticist is not whether there is an occult essence in the Universe which is called "subjectivity" and whether our definitions and methods conform to it or whether such metaphysical quale does not exist. The situation is exactly the reverse. Our logic does not depend on the fact that there are such more or less mysterious phenomena as subjects and subjective processes in the Universe, the secret properties of which we have first to discover so that afterwards we can talk about them and form categories and concepts for their empirical description. This is hopeless! Subjectivity can only be experienced by personal introspection. But the latter is not communicable in scientific terms and will never be. The procedure we propose to employ is not interested at all in what our private insight

[54] Brihadaranyaka - Upanishad, IV, 2, 4 and IV, 5, 15.

might tell us about our innermost subjective life – this is the business of artists and theologians – it only stipulates the acceptance of the morphograms [9] - [15] in the logic of cybernetics.

In the future it will be unavoidable to talk about subjective functions in cybernetic theory. This will be the case when we discuss systems that have an actual center of reflection or which at least behave in a way that such conclusion is forced upon us. Under the circumstances it will be of paramount importance to have a general agreement about what we mean if we refer to the subjectivity or the subjective functions of a given system. We propose as basis for a general consensus the following statement: *if a cyberneticist states that an observed system shows the behavioral traits of subjectivity he does so with the strict understanding that he means only that the observed events show partly or wholly the logical structure of transjunction*. There is nothing vague and arbitrary in this use of the term "subjectivity." It implies clearly that we are not interested in what a subject metaphysically is – even if – this question might have some meaning – but what definitions we intend to use if we try to discourse about subjectivity in a communicable scientific manner.

However, since Table VIII presents a certain richness of transjunctional structure (when compared with the simple duality of disjunction and conjunction), some explanatory remarks are in order. The variety of morphograms refers to the fact that we cannot talk about the subjective component of Reality unless we distinguish three different states of it. It may be

a) a property of something else
b) a personal identity structure, called a subject
c) a self-reference of (b).

Everybody is familiar with these three aspects of subjectivity. The first is commonly called a thought; the second, an "objective" subject or person; the last, self-awareness or self-consciousness. These three distinctions correspond to the three varieties of rejection of a two-valued alternative which Table IV demonstrates:

a) partial rejection : morphograms [9] - [12] and [14]
b) total, undifferentiated, rejection : morphogram [13]
c) total, differentiated, rejection : morphogram [15]

A thought is always a thought of something. This always implies a partial refusal of identification of (subjective) form and (objective) content. This fact has been noted time and again in the history of philosophic logic, but the theory of logical calculi has so far neglected to make use of it. Any content of a thought is, as such, strictly objective; it consequently obeys the laws of two-valued logic. It follows that for the content the classic alternative of two mutually exclusive values has to be accepted. On the other hand, the form of a thought, relative to its content, is always subjective. It therefore rejects the alternative. In conformity with this situation the morphograms [9] - [12] and [14] always carry, in the second and third rows of Table IV, both an acceptance and a rejection value. Together, they represent all possible modes of acceptance and rejection.

A personal identity structure or subject is logically characterized by the fact that not even a partial identification with anything objective (two-valued) is tolerated. The subject, qua subject, is in total contraposition to the whole of the Universe as its logical and epistemological object. It has "cast itself adrift." Morphogram [13] corresponds to this situation. On the other hand it is obvious that the actual refusal of identification with anything objective that is implied by [13] does not provide us with a logical pattern which would denote the potential capacity of self-awareness of subjectivity. The last discussed morphogram indicates awareness of something (which may be its objective content) but no reflection of its state of being aware. The abstract pattern of this situation is furnished by morphogram [15] which incorporates four different values. The two center values have in common that they reject the alternative of "1" and "2" But in one case the rejection is effective in a three-valued system. In the other the rejection has an iterated character. This function designates self-consciousness and the latter is, indeed, an iteration of consciousness. The morphograms [1] – [8] require for their application only a two-valued system of logic. For the patterns [9] - [13] and [14] a three-valued order is necessary. But [15] cannot be used unless a four-valued logic is accepted as basis for a theory about all subjective components of Reality.

By introducing the morphograms [9] - [15] into his logic the cyberneticist becomes able to speak in a finite and non-ambiguous way about subjectivity in self-organizing, and therefore self-reflecting systems. Warren S. McCulloch has stated that if somebody can "specify in a finite and unambiguous way what a brain does with information, then we can design a machine to do it"[55]. The above described logical situation does not yet meet McCulloch's demand, but we think it indicates at least the formal logical structures any sort of consciousness and self-consciousness must use in order to become aware of and use information that infiltrates the brain. By referring to the morphograms we are in a position to state in a finite, non-ambiguous, and computable way what we mean if we say a system has subjective properties or represents a subject or has self-awareness. The precise meaning of such a statement is simple *that the behavioral properties of the system in question display a logical structure that includes rejection values*. And the individual morphograms which come into play will indicate precisely which of the three described varieties of subjective behavior we are referring to.

The introduction of the fifteen morphograms as the basic logical units of a trans-classic system of logic has far-reaching consequences. Such units would have hardly more than decorative significance unless there exists a specific operator able to handle them and to transform one morphogram directly into another. Negation is not capable of doing this as long as we adhere to the classic concept of negation. It is traditionally a reversible exchange relation between two values. It follows that by negating values we only change the value occupancy of a morphogram, not the morphogram itself; no matter how many negations are used, the abstract pattern of value occupancy remains always the same[56].

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[55] W. S. McCulloch: Mysterium Iniquitatis of Sinful Man Aspiring into the Place of God. *Scientific Monthly*, 80: No. 1, pp. 35-39 (1955).

[56] The situation would, of course, be different if we introduced negators like

N	
1	3
2	3
3	1

or

N´	
1	2
2	1
3	1

footnote continues on the next page

However, there is another way to look at the matter. Kant and his successors in the field of transcendental logic: Fichte, Hegel, Schelling, discovered it. Its significance for a formal calculus of logic has so far not been understood. This was partly the fault of its initiators because they insisted that it could not be formalized. These philosophers introduced an operation into their systems of metaphysical logic which they called: "setzen". Although the term is untranslatable – it could at best be rendered as "objectivate" – its meaning is quite clear. Every concept we use, so goes the theory, has to be treated as an objective reflection of itself. Only as such does it acquire significance. The principle of identity cannot be stated as "A" but as "A = A" (Leibniz)[57]. In order to emphasize the point that any concept we use behaves as a mirror image of itself Fichte introduces an interesting notation[58]. He does not write A = A like Leibniz but $\frac{B}{B}$ and $\frac{B}{S}$, where the horizontal line is meant to indicate the plane of reflection. For an iterated reflection[59] he extends his notation to $\frac{B}{\frac{B}{B}}$. He further produces formulas of reflection[60] like $I = \frac{S}{O}\,\infty$, where "I" stands for identity, "S" for subject and "O" for object. But he gives no formation rules. The attempt was let down by the ineptness of the technique he used, but it showed very clearly that Fichte was groping for a specific calculus of reflection. Hegel later added the idea that not only terms but also the operation commonly called "negation" should be treated as a reflection of itself. His *Logik* is an attempt to implement this program. We shall use these ideas of Leibniz, Fichte, and Hegel and show that they point the way to a general logical operator for reflection which satisfies the demand for a formal transformation of one morphogram into another regardless of their value occupancy.

For the time being we shall retain Fichte´s notation; but instead of the letters A and B we shall use our nondescript marks: ∗, □, ▲, and • because we intend to generalize the concept of self-reflection to the point where it includes our morphograms. By placing the appropriate marks above the plane of reflection and their mirror images below we obtain the following arrangement of morphogrammatic patterns. (We shall, however, not use Fichte´s notation for iterated reflection: $\frac{A}{\frac{A}{A}}$ since a formal logic takes care of this phenomenon with other methods). See "Fichte-Table" X for shapes. The one-place reflection (a star and its mirror-image) is easily recognized as the classic identity principle which Leibniz wrote A = A. This star represents the only morphogram which

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Although these negations have been used by Łukasiewicz and Reichenbach we cannot recognize them as basic principles of negation. Their application was dictated by material viewpoints. In Łukasiewicz' case the aim. was to gain an intuitive understanding of three-valued logic. Reichenbach was moved by specific demands of quantum-mechanics.

[57] G. Leibniz: *Nouveaux Essays sur Pentendement humain*, IV, 2, Section 1. See also Fichte's remarks on Schelling's transcendental idealism. N.W.W., Ed., J. H. Fichte, III, pp. 368-389.

[58] *N.N.W.*, ed. J. H. Fichte, 1, p. 160 ss. 'B' stands for 'Bild' and 'S' for 'Sein'.

[59] *N.N.W.*, ed. J. H. Fichte, 1, p. 419.

[60] *N.N.W.*, ed. J. H. Fichte, 111, p. 381.

could be ascribed to a so-called one-valued logic. The fifteen examples of four-place reflection are provided by the morphograms of a two-valued logic. If we were dealing with a three-valued logic our table would have to show nine-place reflections. Generally: for any m-valued system the reflection would have m^2 places.

It is worth mentioning that a generalized concept of reflection that plays an important part in Fichte´s and Hegel´s logic interprets negation as a specific form of reflection. If we wrote negation

$$\frac{\begin{matrix} * \\ \square \end{matrix}}{\begin{matrix} \square \\ * \end{matrix}}$$

instead of using the conventional table form one can easily see why the process of negation was interpreted in this manner. However, we do not want to delve into this aspect of reflection. It is sufficient to say that reflection in a larger sense may utilize any number of places. In this more general theory all Stirling numbers play their proper parts. Be that as it may, this investigation considers only morphogrammatic reflections of m-valued systems with m^2 places. If $m > 2$ it will be advisable not to speak of morphograms alone but also morphogrammatic compounds. The distinction is essential. With increasing m the number of morphogrammatic compounds increases too. But the number of morphograms as basic units of formal logic remains the same no matter how large m is. The hierarchy of all m-valued orders represents a "quindecimal" system of morphogrammatic reflection.

Table_X

	1	2	3	5	6	8	9	10	13	14	15
	*	*	*	*	*	*	*	*	*	*	*
	*	*	□	*	□	□	▲	*	▲	▲	□
	*	□	*	*	*	□	*	▲	▲	●	▲
*	□	□	□	*	*	*	□	□	□	□	*
*	□	□	□	*	*	*	□	□	□	□	*
	*	□	*	*	*	□	*	▲	▲	●	▲
	*	*	□	*	□	□	▲	*	▲	▲	□
	*	*	*	*	*	*	*	*	*	*	*
	4	2	3	5	7	8	12	11	13	14	15

Fichte´s notation of a horizontal line as a symbol of reflection is not very practical. We shall replace it by the sign "$\mathfrak{R}$" which we will call a reflector. A reflector is an operator that produces the reflection of a given morphogrammatic pattern; be that a single morphogram, a morphogrammatic compound or a morphogrammatic sub-unit of such a compound. This means that $\mathfrak{R}$, if so indicated, may operate one, two, three or any number of morphograms which make up a larger compound.

Since, however, morphograms do not occur as empty structural patterns in logic, but are always occupied by values, the symbol η ... for negation will, of course, be retained. If applied it will always carry the appropriate suffix indicating the specific values which are operated. If there is only one suffix and the suffix is an integer it is indicated that the negation represents an exchange relation between two values which are not separated by a third. All other cases will be treated as composites of such elementary exchange relations. Their composition will be indicated by adding to η the suffixes of the negations which contributed to the given constellation of values. Our sequence of elementary tables looks as follows:

	η_1
1	2
2	1

	η_2
2	3
3	2

	η_3
3	4
4	3

if $1 \leq i < m$ negation is defined

$$\eta_1 \, (1,2,\ldots i,\ i+1\ldots m) \rightarrow (1,2,\ldots\ i+1,\ i,\ \ldots\ m)$$

for all m-valued systems. Thus the table of negations of a three-valued logic is represented by Table XI.

Table XI

	η_1	η_2	$\eta_{2.1}$	$\eta_{1.2}$	$\eta_{1.2.1}$ or $\eta_{2.1.2}$
1	2	1	2	3	3
2	1	3	3	1	2
3	3	2	1	2	1

$\eta_{2.1}$ is defined by

$$\eta_{2.1} =_{\mathrm{Def}} \eta_1 \cdot \eta_2$$

In words: operate η_1 on the result of the operator η_2. Since the order of the suffixes is somewhat awkward and $\eta_{2.1}$ produces the mirror-image of η_2 we may as well use the reflector "$\Re$" and write $\eta_{2.R}$ and $\eta_{1.R}$ It is worthwhile to note that these negations are not commutative:

$$\eta_{1.R} \neq \eta_{2.R}$$

If the whole standard sequence of values is reversed we omit all numerical suffixes and add only ... R. Thus we may write on the basis of Table_XI:

$$\eta_R =_{\mathrm{Def}} \eta_{1.2.1} = \eta_{2.1.2}$$

This notation may be advantageous if we have a long row of suffixes for η... . The reflector "$\Re$" may be, according to Table_X, added to non-negated (standard) value sequences or to negations as it is convenient.

In order to indicate (in the case of Table_XI and also in the .case of tables of negation with a large number of values) that "$\Re$" applies to constellations of, individual values and not of morphogrammatic structures, the operator of reflection will always be written in index form after "η". If the original order of values is that of the normal sequence of integers the negational reflexion "η_R" shall have no index unless it is not certain to which value system the operation applies. If we want to point out, for instance, that "η_R" does not signify the sequence 3-2-1 but 5-4-3-2-1, we add the number of values as subscript to $\mathfrak{R}$: η_{R5}. However, this will not be necessary if the morphogrammatic compounds carry the index of the value-system to which they belong. If "$\mathfrak{R}$" operates on a morphogram, it is placed before it.

The reflective properties of the morphograms can now be written with a provisional notation (if we assume that they have standard form):

$\mathfrak{R}[1]$	=	$\eta_1[4]$	$\mathfrak{R}[4]$	=	$\eta_1[1]$
$\mathfrak{R}[9]$	=	$\eta_1[12]$	$\mathfrak{R}[12]$	=	$\eta_1[9]$
$\mathfrak{R}[10]$	=	$\eta_1[11]$	$\mathfrak{R}[11]$	=	$\eta_1[10]$
$\mathfrak{R}[6]$	=	$[7]$	$\mathfrak{R}[7]$	=	$[6]$
$\mathfrak{R}[2]$	=	$\eta_1[2]$	$\mathfrak{R}[3]$	=	$\eta_1[3]$
$\mathfrak{R}[13]$	=	$\eta_1[13]$	$\mathfrak{R}[15]$	=	$\eta_{1.3}[15]$
$\mathfrak{R}[14]$	=	$\eta_1[14]$			
$\mathfrak{R}[5]$	=	$[5]$	$\mathfrak{R}[8]$	=	$[8]$

We notice that the reflection-operator $\mathfrak{R}$ affects different morphograms in different ways. The first group of our "formulae" shows that the law of duality holds not only for disjunction and conjunction but also for all forms of partial transjunction. The second group, which consists of only one line, shows the reflective symmetry between the conditional and its inverse. From the third group we learn that for morphograms [2], [3], [13], [14] and [15] the $\mathfrak{R}$-operator is equivalent to various forms of negation. And the last group shows that due to their symmetrical structure neither morphogram [5] nor [8] is affected by the operator of reflection.

These limitations of the $\mathfrak{R}$-operator show clearly that, even if we could use transjunction in a two-valued logic, which we cannot, the classic formalism does not provide us with a satisfactory theory of reflection. In a physical universe which is adequately described by a two-valued logic some phenomena show reflective properties and others do not. But this situation is unacceptable for a logical theory which is to include the subject. Fichte has pointed out repeatedly that subjectivity of the subject means nothing but perfect transparency ("Durchsichtigkeit")[61]. This does not mean, of course, that a subject or consciousness is, at all times and in every respect, completely transparent to itself: there are opaque spots in our subjectivity, as everybody knows from his own experiences. There was no need for Fichte to point that out, for Kant had already established what was meant by this term. One of the most important passages in the *Critique of Pure Reason* reads (in translation): "That: *I think* (I am aware of) must be capable of accompanying all my representations ..."[62]. In other words, the point is not that the self-transparency of the subject must be present in every moment and with regard to every content of the reflexive mechanism but that it is on principle always

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[61] *N.N.W.*, ed. J. H. Fichte, II, p. 43; *Was ist die Ichheit am Ich ?* Es ist die absolute Durchsichtigkeit.

[62] B 131 'Das *Ich denke* muss alle meine Vorstellungen begleiten können...'

capable of doing so. It is impossible for any subject to be aware of something, and to be at the same time constitutionally incapable of acknowledging it as its own.

This is in fact a maxim that has been incorporated in our scientific concepts for a long time, though couched in a different terminology. Physicists would reject something to be physically real if that something could never be observed, either directly or indirectly and could never be the possible object for any sort of thought. A "subjective" awareness which faced and reflected a "world" which contained such mythical objects would indeed be partly opaque. A subject is an all or nothing proposition. In other words: a partly opaque subjectivity could not exist at all. To understand this fully, one has to remember the distinction between the operation of reflection and what is reflected. It corresponds roughly to the difference between consciousness and what one is conscious of (commonly called its content). There are, of course, always gaps and discontinuities in the content of our consciousness. The reflexive mechanism of our body registers at any given moment a practically unlimited number of impressions from the external world we are actually not aware of. That means that any consciousness is, with regard to its content, highly fragmentary and discontinuous. But what cannot be fragmentary and full of gaps is the process of reflection itself. A simple example may make this clear. If we say: "one, two, three, four …" we are dimly aware of a nervous activity which we call "counting". This is at the very moment the actual content of our reflection. And nobody will deny that this content may be discontinuous and fragmentary in an indefinitely large number of ways. We may stop counting and we may resume again. A small child trying to learn it may skip numbers. Our attention may be diverted while our lips continue to articulate numerical terms or we may finally give up from sheer exhaustion. But no same person would seriously assert that the law of conscious reflection which manifested itself in this activity could be fragmentary or break down all of a sudden. The law which we applied was the principle of numerical induction; and although nobody has ever counted up to 10^{1000}, or ever will, we know perfectly well that it would be the height of absurdity to assume that our law might stop being valid at the quoted number and start working again at 10^{10000}. We know this with absolute certainty because we are aware of the fact that the principle of induction is nothing but an expression of the reflective procedure our consciousness employs in order to become aware of a sequence of numbers. The breaking down of the law even for one single number out of an infinity would mean there is no numerical consciousness at all! This is what we intended to say with the statement that a system of self-reflection cannot be partially opaque: its transparency is complete. And when Fichte uses this term he always means that consciousness has a knowledge of itself that it does not have to acquire empirically. It possesses it by dint of its own nature of "total reflection" (Hegel).

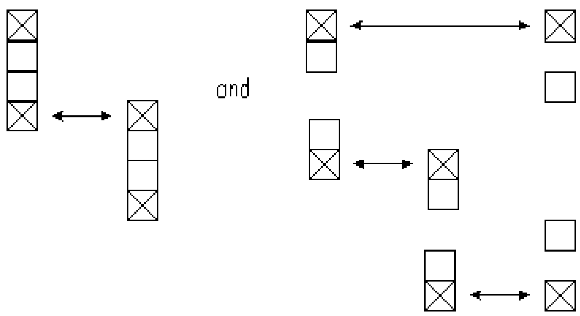
These considerations should make clear why a logical system that displays only partial reflexivity is an insufficient theoretical basis for a theory of consciousness. Even if we add the transjunctional morphograms to the classic array we discover that the reflections produced by the $\mathfrak{R}$-operator on four-place patterns are fragmentary. If we are restricted to four places it is non-sensical to assume that morphogram [13] could be a reflection of [5]. But a theory of total reflection would demand this very thing! On the other hand, such a demand can be met if we proceed from the single morphograms that the traditional logic uses to compounds of morphogrammatical structures.

There are still many competent thinkers who object to the proposal of a trans-classic logic (which would include the traditional two-valued theory) as a new organ of philosophy as well as of science, so the step into this novel realm should not be taken lightly. On the other hand we are forced to make it. The classic system is morphogrammtically incomplete; even if we could add the missing patterns (treating the additional values as merely some trans-logical "noise" of irrational origin and as indices of probability) the situation would not improve. As a system of reflection the revised theory would still be incomplete. The operator "$\mathfrak{R}$" is not capable of deploying its possibilities with individual morphograms.

4. MORPHOGRAMMATIC COMPOUNDS IN M-VALUED SYSTEMS

In order to establish logical continuity in compounds of morphograms, the individual patterns have to be joined in such a way that all joinable places are actually connected with each other. These places are the top and bottom value occupancies of each morphogram. If we look at the two arrangements:

Table XII



we see that a compound of only two patterns does not produce a system of morphograms. Both patterns have joinable places, indicated by x, which are not joined. The compound on the right side, however, represents a system. All joinable places of value occupancy are connected. It should also be noted that the pseudo-compound on the left side offers only seven places of value occupancy. This is too much for two values and not enough for three.

It seems at first to be trivial to point out that the value occupancies in the joinable places must always be identical, but we shall see later that this has in fact far-reaching consequences for the theory of the $\mathfrak{R}$-operator. The m o r p h o g r a m m a t i c arrangement on the right side provides the nine places for value-occupation which are required in a three-valued logic. But whereas the traditional theories of many-valuedness, such as those of Post, Łukasiewicz, Wajsberg, and Slupecki, consider the sequence of values as continuous, we arrange them in smaller or larger compounds of morphogrammatic units. As our nineplace pattern shows, it is not necessary that the values which fill and represent a morphogram form continuous four-place sequences. In fact this is impossible. No more than two values belonging to the same pattern can ever be direct neighbors. On the other hand there is no limit to how far they can be apart.

This too has weighty consequences for a general theory of reflection. The fact that we may connect individual morphograms only as allowed by their actual value-occupancy imposes, of course, certain limits on the construction of morphogrammatic compounds. The rules for it cannot be given within the frame of the present discussion. Instead we shall give a demonstration of how the $\mathfrak{R}$-operator handles values, and changes value occupancies, for a given array of morphograms. As a model we shall use a table of several value sequences belonging to a three-valued logic. We select our value-sequences with the stipulation that they shall represent only compounds of the morphograms [1] and [4]. This limits us to exactly eight sequences:

Table XIII

[4,4,4]	[1,4,4]	[4,1,4]	[4,4,1]	[1,1,4]	[1,4,1]	[4,1,1]	[1,1,1]
1	1	1	1	1	1	1	1
2	1	2	2	1	1	2	1
3	3	3	1	3	1	1	1
2	1	2	2	1	1	2	1
2	2	2	2	2	2	2	2
3	3	2	3	2	3	2	2
3	3	3	1	3	1	1	1
3	3	2	3	2	3	2	2
3	3	3	3	3	3	3	3

We shall now apply the operator for total reflection ($\mathfrak{R}$ without index) to the first sequence, which contains in all three positions the morphogram [4]. In order to demonstrate the effect that this operation has on the value-occupancy of all three patterns we will separate them in the intermediate stage:

Table XIV

$\mathfrak{R}$[4,4,4]		η_R		[1,1,1]
1	3		3	1
2	3			1
3			3	1
2	3			1
2	2	2		2
3		2		2
3			3	1
3		2		2
3		1	1	3

This table shows drastically that the $\mathfrak{R}$-operator is completely indifferent to the actual value-occupancy of the four-place pattern it transforms. It just changes morphograms into each other and implements these transformations with the values that are demanded by the value-occupancy of the key positions where the morphograms are joined together. This happens in Table XIV, at the first, fifth and ninth places in the column. Since the key values of the third morphogram exchange their places in the first and last position of the column, the original values "1" and "3" are retained. This, however, is not possible in the case of the first and second morphograms. Here the key values are now "3" and "2" and then "2" and "1". These key values and the structure of the morphogram determine the other value occupancies. Since this treatment of values is rather unusual we shall demonstrate this issue of value-occupancy also for the $\Re$-operation of a single morphogram within a compound of three morphogrammatic patterns. We choose for the demonstration the first morphogram of [4,4,4] which has the standard form 1222:

Table XV

$\Re^1$[4,4,4]	η_1			[1,4,4]
1	2		2	1
2	2			1
3			3	3
2	2			1
2	1	1		2
3		3		3
3			3	3
3		3		3
3		3	3	3

Although the operator changes only the first morphogram [4] to [1], the value-occupancy of the other patterns is also altered. The first values of the second and third pattern are exchanged. By again exchanging all classic values ("1" and "2") with the help of the negation "η_1" we obtain the standard version of [1,4,4]. An explanation is due of how an $\Re$-operations applied to one or several morphograms within a larger compound. First, we produce the mirror-image of the morphogram that is affected by the $\Re$-operator. If the operator changes two or more morphogrammatic patterns, their combined value-sequence must be put down in reverse order. By doing so, possible intervals that are produced by values from other patterns must be observed. These intervals are then filled with the values that occur in the original sequence wherever there is such an interval. Thus after having reversed the sequence 1222 in Table XV the third, sixth, seventh, eighth and ninth place is filled with the corresponding values of [4,4,4]. The following Table XVI gives an example of the application of $\Re$ to two morphograms. This time we choose the patterns 1222 and 1333 of [4,4,4]:

Table XVI

$\Re^{1.3}$ [4,4,4]	η_R		[4,1,1]
1	3		1
2		2	1
3	3		3
2		2	1
2	2		2
3	2		3
3	3		3
3	2		3
3	1		3

In order to-illustrate how the $\Re$-operator works with two patterns the morphograms in the center of Table XVI have not been separated. First the value-sequence that is affected by $\Re^{1.3}$ is written in reversed order. This leaves us with two intervals. In the second column the values which [4,4,4] provides are written for the open places. The appropriate negation η_R then returns the value-sequence to its standard form for [4,1,1].

By operating [1,1,1] in a corresponding way we obtain the following definitions for several value-sequences of Table XIII. From Table XIV we derive:

$$[1,1,1] =_{Def} \eta_R\ \Re[4,4,4] \qquad (4)$$

from Tables XV and XVI

$$[1,4,4] =_{Def} \eta_R\ \Re^{1}[4,4,4] \qquad (5)$$

$$[4,1,1] =_{Def} \eta_R\ \Re^{1.3}[4,4,4] \qquad (6)$$

And using [1,1,1] as definitorial basis we further obtain:

$$[1,4,1] =_{Def} \eta_R\ \Re^{2}[1,1,1] \qquad (7)$$

$$[4,1,4] =_{Def} \eta_R\ \Re^{2.3}[1,1,1] \qquad (8)$$

It is important to note that Table XIII contains two more morphogrammatic compounds which cannot be defined in this simple manner. [4,4,1] as well as [1,1,4] have specific properties which set them apart from the other value-sequences. It will be interesting to compare the Formulas (4), (5), (6), (7), and (8) with corresponding formulas, that use only negations and no $\Re$-operations. We obtain then DeMorgan-type relations that look as follows:

$$p[1,1,1]q =_{Def} \eta_R\ (\eta_R\ p[4,4,4]\ \eta_R\ q) \qquad (9)$$

$$p[1,4,4]q =_{Def} \eta_1\ (\eta_1\ p[4,4,4]\ \eta_1\ q) \qquad (10)$$

$$p[4,1,4]q =_{Def} \eta_2\ (\eta_2\ p[4,4,4]\ \eta_2\ q) \qquad (11)$$

and with [1,1,1] as definiens:

$$p[4,1,1]q =_{Def} \eta_1\ (\eta_1\ p[4,4,4]\ \eta_1\ q) \qquad (12)$$

$$p[1,4,1]q =_{Def} \eta_2\ (\eta_2\ p[4,4,4]\ \eta_2\ q) \qquad (13)$$

Again [4,4,1] and [1,1,4] remain undefined. If we want a definition for them and still rely, apart from negation, only on [4,4,4] and [1,1,1] as definitorial basis we are forced to resort to the following cumbersome sequence of symbols:

$$p[4,4,1]q =_{Def} \eta_1\ (\eta_1\ p[1,1,1]\ \eta_1\ q)\ [4,4,4]\ \eta_2\ (\eta_2\ p[1,1,1]\ \eta_2\ q\ ) \qquad (14)$$

$$p[1,1,4]q =_{Def} \eta_1\ (\eta_1\ p[4,4,4]\ \eta_1\ q)\ [1,1,1]\ \eta_2\ (\eta_2\ p[4,4,4]\ \eta_2\ q\ ) \qquad (15)$$

It is, of course, possible to shorten Formulas (14) and (15) if we do not restrict ourselves to the use of [4,4,4] and [1,1,1]. However, there might be reasons when this restriction is desirable. The introduction of transjunction [13,13,13] provides us with such a motive. In two-valued logic disjunction may be defined by the use of negation and conjunction and the latter by the inverse procedure with disjunction. It would be important to have a corollary to DeMorgan´s law that would establish an analog basic relation between conjunction and disjunction on one side and total transjunction in a three-valued system on the other. But if we do this with negational operations we arrive at the following involved formula:

$$\begin{aligned} p[13,13,13]q =_{Def} &< \eta_1 (\eta_1\, p[4,4,4]\, \eta_1\, q)\ [1,1,1]\ \eta_2 (\eta_2\, p[4,4,4]\, \eta_2\, q) > \\ &\eta_1 (\eta_1\, p[4,4,4]\, \eta_1\, q)\ [1,1,1]\ \eta_2 (\eta_2\, p[4,4,4]\, \eta_2\, q) \\ &\eta_{2.1} < \eta_1 (\eta_1\, p[1,1,1]\, \eta_1\, q)\ [4,4,4]\ \eta_2 (\eta_2\, p[1,1,1]\, \eta_2\, q) > \end{aligned} \tag{16}$$

By using the Formulas (14) and (15) we may, of course, reduce the awkward Formula (16) to the very simple formula:

$$[13,13,13] = ([1,1,4])\ [1,1,4]\ (\eta_{2.1}\ [4,4,1]) \tag{17}$$

and

$$[13,13,13] = ([4,4,1])\ [4,4,1]\ (\eta_{1.2}\ [1,1,4]) \tag{18}$$

But this is not exactly what we want. Here a new morphogrammatic distinction becomes important. Only two of the value-sequences of Table XIII represent one morphogram. They are [4,4,4] and [1,1,1]. We shall call sequences in which the same morphogrammatic pattern is repeated in all "places" of the system a monoform value-sequence. If more than one morphogram is used to cover all "places" we shall speak of a polyform structure. The polyform sequences [1,4,4], [4,1,4]) [4,4,1], [1,1,4], [1,4,1] and [4,1,1] are all we know so far. We see now that in Formulas (17) and (18) the monoform structure of [13,13,13] is equated with two polyform expressions. The relation is, in fact, interesting in many respects; but it is not what we want. We search for a corollary to DeMorgan´s law for our function [13,13,13].

Since all basic morphograms of the Tables VI and VIa must be classified as monoform it means that the DeMorgan law expresses a relation that is established with the exclusive use of monoform value-sequences. If we assume this morphogrammatic viewpoint Formulas (17) and (18) do not qualify as corollaries. Formula (16) does, but in such an awkward manner that we cannot. feel very happy about it. And since it is impossible to blame [4,4,4] and [1,1,1] for the length of the formula the blame must fall upon the η-operator.

One cannot help but wonder under the circumstances whether trans-classic systems of logic are basically also orders of value-assertion and value-negation. The Formula (16) leaves one with the impression that negation is somehow too weak an operator within these new realms. For this very reason we introduce the $\mathfrak{R}$-operator. A many-valued system, interpreted as a morphogrammatic logic, is basically not a negational order but a system of reflection. This has never been clearly recognized by previous investigations in this field. The very meritorious researches of Lukasiewicz, Wajsberg, Slupecki and others still lean on the ontology of the Axistotelian terms of δυνατὸν εἶναι (potentiality), ἐνδεχὸ μενον εἶναν (contingency) and ἀναγκγῖον εἶναι (necessity) as elaborated in "De Interpretatione". This is an ontology of objective Being but not of objective-subjective Reflection. But for any ontology of the object the natural way to handle values is to assert or negate them. Using Fichte´s symbolism (see Table X) we noticed that negation is equivalent to reflection for inverse value constellations like 1, 2 and 2, 1 or 1, 2, 3 and 3, 2, 1. It is true that Aristotle hints at a third value in the famous ninth chapter[63] of "De Interpretatione", but this value seems to coincide with Fichte´s horizontal line. Very significant also is that considerable difficulties exist to complement the "third value" of Aristotle with a fourth. And it becomes almost impossible to interpret this ontology with five, six, or seven individual values. This was clearly recognized by Łukasiewicz. As early as 1930 he made the following statement: "Es war mir von vornherein klar, dass unter allen mehrwertigen Systemen nur zwei eine philosophische Bedeutung beanspruchen können: das dreiwertige und das unendlichwertige System[64]. This is undoubtedly true if the extension of traditional logic into trans-classic regions is based on "De Interpretatione". Aristotle´s "third value" can only be understood as the indifference (Schelling) between "true" and "false". Another way to put it is to say that the decision between the two values remains suspended because of the specific properties of the designated ontological situation. Aristotle is concerned with propositions in the future tense. He argues that it is still undetermined whether there will be a sea-battle tomorrow … or not. But although neither side of the alternative can be said to be true or false the disjunction itself: "Either this battle will be or it will not be" is accepted as true regardless of the future tense. And there will, or course, come a moment when the datum in question moves from the modal realm of possibility (δυνατὸν εἰναι) into that of reality or non-reality. Consequently the decision between the two values is suspended only because of the time element involved. It is now very easy to take the step from this third suspension value to a logic of probability. Since we have to assume that the interval between the δυνατὸν εἰναι and the ontological state of ἐνδεχὸ μενον εἰναν may be very long (and to all practical intents and purposes even infinite) the suspension may remain forever; the time for a final decision may never come. We have then to choose between probability values, of which there must be at least a denumerable infinity. A fourth, fifth, or sixth value between this third value of indifference and the infinity of probability data makes very little or no philosophic sense. One cannot help but agree with Łukasiewicz´s statement that finite m-valued systems where m > 3 have no philosophic significance.

Of course, it might be argued that Aristotle´s third "value" introduces reflection into formal logic … in a manner of speaking. Deciding to suspend the decision between two values is a sort of subjective reflection. This has already been admitted, and we discussed this type of subjectivity when we mentioned the part that is played by reflection in quantum mechanics. But we also cited Heisenberg´s comment that the probability functions are "completely objective" with regard to their semantic significance[65]. And this is what Aristotle is concerned about. His envisaged value of suspension designates exclusively possible or actual states of *objective* existence. His philosophical theme is – in his own words – τὸ ὀν = Being as an object. This ὀν turns up as the verb ϵἰναι in the modal terms which we quoted in the preceding paragraph. It is what the subject – faces, but never the subject itself! Obviously a logic which takes

[63] Cf. Aristotle *De Interpretatione*, DC, 19 9. It seems to us that the καὶ μᾶλλον μὲν ἀληθῆ τὴν ἑτέραν indicates degrees of truth of falsity. In other, words: a probability logic where two - and only two - ontological values are distributed over an interval between them.

[64] J. Łukasiewicz: Philosophische Bemerkungen zu mehrwertigen Systemen des Aussagenkalküls. *Comptes Rendues des Séances de la Société des Sciences et des Lettres de Varsovie*, XXIII, class III, p. 72 (1930).

[65] W. Heisenberg: *Physics and Philosophy*. See Note 29, p. 53.

its bearings from the objective side of Reality is not very well equipped to deal with subjectivity as such and as a state of being in contraposition to any thinkable object.

The defenders of the classic position in logic may, of course, say that the ultimate Reality behind the Aristotelian ὀν and ϵἰναι namely the τὸ τἰ ἡν ϵἰναι is the absolute indifference of Object and Subject. But this is the viewpoint of a mystic. It cannot be the basis of a logic of cybernetics. This much may, however, be admitted: the minimum of reflection which is involved in the description of the external world as a bona fide object is indeed capable of defining subjectivity. In other words: it is possible to define the subjective function of transjunction [13,13,13] in terms of negation combined with conjunction and disjunction. We did so when we produced the Formula (16). It was based on the system {[4], η_1, η_2}. However, it took logic a long time to recognize the following point. It is not sufficient that we are *able* to describe something in formal terms: it is equally important how we describe it. This is one of the basic tenets of the transcendental logic of Kant, Fichte, Hegel, and Schelling. These thinkers were fully aware of the fact and pointed out that it is, of course, permissible to describe a subject exclusively in terms of objective existence and that there is no limit to such a description (for no subjective phenomenon can be demonstrated which could not be submitted to such a treatment). The procedure is in itself irreproachable. But by doing so, as Fichte and his successors point out, we have described a subject as an object. If we intended to do so, nothing more can be said. But if we intended to describe the subject qua subject we have failed! We have interpreted something in terms of *being* although we wanted to know something in terms of *reflection*. In order to avoid this mistake we introduced the $\mathfrak{R}$-operation. This gives us an opportunity to express the DeMorgan law in a double fashion. First it can be presented with the help of η. In this form it demonstrates structural relations of objective existence. But the same law may also be expressed with the $\mathfrak{R}$-operator. In this case we define it as a law of reflection. We still owe the reader this second definition. We shall produce it after a demonstration of the capacities of the $\mathfrak{R}$-operator in morphogrammatic compounds.

It is obvious that the concept of subjectivity in formal logic, as represented by the $\mathfrak{R}$-operation, has nothing to do with distribution of values. The logical unit of many-valued systems is the morphogram. η-operations cannot directly transform one morphogram into another because they deal with values and not with abstract patterns incorporated in more or less irrelevant values. But the new $\mathfrak{R}$-operator demands, in its turn, distribution of morphograms. We observed that if "$\mathfrak{R}$" is applied to single morphograms the result is sometimes nothing, sometimes a negation, and only in a few cases a second morphogram. But the few morphogrammatic compounds which we demonstrated in the Table XIII contained only the patterns [1] and [4] which are amenable to $\mathfrak{R}$-transformation even in their isolated state. We shall now show that in a morphogrammatic compound a given pattern can be transformed into any other pattern. If we look, for instance, at Table XIV we observe that after operation by $\mathfrak{R}$ (total reflection) the second morphogram, represented by the value sequence 2333, becomes the reflection of the first 1222. But 2333 appears, of course, as its mirror-image 3332 in this operation. One morphogram has been transformed into another but both belong to the same Table VII. We have not yet demonstrated that an $\mathfrak{R}$-operation may also t r a n s f o r m a non-transjunctional pattern into one with transjunction. If we want to establish a DeMorgan relation between disjunction and conjunction on one side and transjunction on the other we require exactly this sort of operation.

When we produced [1,4,4] and [4,1,1] with the help of $\mathfrak{R}^1$ and $\mathfrak{R}^{1.3}$ from conjunction (see Tables XV and XVI) we omitted to use $\mathfrak{R}^2$, $\mathfrak{R}^3$, $\mathfrak{R}^{1.2}$ and $\mathfrak{R}^{2.3}$ on [4,4,4]; and later we did not apply $\mathfrak{R}^1$, $\mathfrak{R}^3$, $\mathfrak{R}^{1.2}$ and $\mathfrak{R}^{1.3}$ in our definitions based on [1,1,1]. We will now apply these not yet used $\mathfrak{R}$-operators on conjunction and disjunction. The next two tables show the results:

Table XVII

[4,4,4]	$\mathfrak{R}^2$	$\mathfrak{R}^3$	$\mathfrak{R}^{1.2}$	$\mathfrak{R}^{2.3}$
1	1	3	3	3
2	2	2	3	3
3	3	3	3	3
2	2	2	3	3
2	3	2	2	2
3	3	3	2	3
3	3	3	3	3
3	3	3	2	3
3	2	1	1	1
[4,4,4]	[13,1,13]	[4,13,1]	[1,1,1]	[1,13,1]

and

Table XVIII

[1,1,1]	$\mathfrak{R}^1$	$\mathfrak{R}^3$	$\mathfrak{R}^{1.2}$	$\mathfrak{R}^{1.3}$
1	2	3	3	3
1	1	1	2	1
1	1	1	1	1
1	1	1	2	1
2	1	2	2	2
2	2	2	1	1
1	1	1	1	1
2	2	2	1	1
3	3	1	1	1
[1,1,1]	[4,13,13]	[13,1,4]	[4,4,4]	[13,4,4]

First it should be noted (see also Table XIV) that:

$$[1,1,1] = \eta_R\ \mathfrak{R}[4,4,4] = \eta_R\ \mathfrak{R}^{1.2}[4,4,4] \tag{19}$$

$$[4,4,4] = \eta_R\ \mathfrak{R}[1,1,1] = \eta_R\ \mathfrak{R}^{1.2}[1,1,1] \tag{20}$$

This operational identity of $\mathfrak{R}$ and $\mathfrak{R}^{1.2}$ is by no means general. The following example will show that $\mathfrak{R}$ and $\mathfrak{R}^{1.2}$ do not always produce identical results:

$$\mathfrak{R}[4,2,12] = \eta_R\ [2,1,9] \tag{21}$$

$$\mathfrak{R}^{1.2}[4,2,12] = \eta_R\ [2,1,1] \tag{22}$$

On the other hand:

$$\mathfrak{R}[4,2,12] = \mathfrak{R}^{2.3}\ [4,2,12] \tag{23}$$

A discussion of the occasional operational identity of total 'R with one of its sub-operators (although interesting in itself) goes beyond the scope of this investigation. However, we are very much concerned with the other $\mathfrak{R}$-operations of Table XVII and XVIII because they show us examples of transformations of classic morphograms into transjunctional patterns. The value-sequences thus obtained are polyform but with their help it is now easy to give a formulation of the DeMorgan law for transjunction using $\mathfrak{R}$-operators. Instead of Formula (16) we may now write:

$$[13,13,13] =_{\mathrm{Def}} \eta_2 < (\mathfrak{R}^2[4,4,4])\ [1,1,1]\ (\eta_{1.2}[1,1,1]) > \tag{24}$$

This expression satisfies our stipulation that only the monoform sequences of conjunction and disjunction may be used. The considerable reduction in negational operations that Formula (24) represents when compared with Formula (16) shows that the reflectional element contained in η is not adequate to cope with a logic of reflection.

We may approximate the classic law of DeMorgan even further. Instead,of using both, conjunction and disjunction, to express the value-sequence of transjunction we may confine ourselves to one of the two. If we choose disjunction we obtain the desired formula by a simple substitution which gives us the new definition:

$$[13,13,13] =_{\mathrm{Def}} \eta_2 < (\eta_R\mathfrak{R}^2\mathfrak{R}[1,1,1])\ [1,1,1]\ (\eta_{1.2}\mathfrak{R}^1[1,1,1]) > \tag{25}$$

By an analog procedure we can define transjunction with the exclusive use of conjunction.

$$[13,13,13] =_{\mathrm{Def}} \eta_1 < (\eta_R\mathfrak{R}^1\mathfrak{R}[4,4,4])\ [4,4,4]\ (\eta_{2.1}\mathfrak{R}^2[4,4,4]) > \tag{26}$$

It stands to reason that no transformation of a classic morphogram into morphogram [15] can be accomplished with nineplace value-sequences. But this situation is easily remedied by progressing to a system which requires four values. The procedure then is analogous.

The Aristotelian ontology which advances *à la* Łukasiewicz from a hypothetical third value of logical indifference between "true" and "false" directly to an infinity of probabilities would make the introduction of an individual fourth value very difficult from the interpretational viewpoint. In a theory of objective existence the fourth value seems to represent a redundancy. It has no status of its own to keep it apart from the subsequent values. In the theory of morphograms it is different: there value four has a special significance insofar as a three-valued system is, morphogrammatically speaking, still incomplete. And in the first philosophical theory of consciousness which really deserves the name[66] – the Transzendentale Elementarlehre in the *Critique of Pure Reason* – Kant provides a table of categories[67] which, so he points out, represent the basic logical structure of the mind. These categories are subsumed under four primordial motives of consciousness which he calls:

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[66] M.Bense: Bewusstseinstheorie, Grundlagenstudien, II, 3, P. 65 (1961).

'Beuwsstseinstheorie im Sinne einer philosophischen Theorie, also einer Theorie, deren Aussagen erkenntnistheoretisch und ontologisch hinreichend allgemein formuliert sind, so dass sie von einer speziellen Fachwissenschaft unabhängig bleiben, aber für jede verbindlich sind, *gibt es erst seit Kan*t.'

[67] B 106; See also B 95.

quantity
quality
relation
modality

This would require, so far as a formal logical theory of consciousness is concerned, a system with four values. That means a structural order which is morphogrammatically complete. Thus the fourth value has a specific significance. But this significance could not mean anything to Aristotle because his philosophical theme is objective Being, and not its subjective reflection as awareness and self-consciousness.

This should take care of the fourth value. However, we have to admit that it does not solve the problem of the ontological identification of a fifth, sixth or any subsequent value. And unless we resign ourselves to their interpretation as probabilities we have to admit that the task of identifying a potential infinity of values with regard to their individual semantic significance, other than modality or probability, is hopeless. This is a further motive for giving up the value theory and for resorting to the morphogrammatic interpretation of trans-classic systems of logic. It is justifiable to call these systems non-Aristotelian because the concept of the morphogram means a departure from the way a trans-classic logic has to be developed if such development is guided by Aristotle´s speculations in "De Interpretatione".

The non-Aristotelian viewpoint considers logical systems which transcend the scope of the two-valued traditional theory as vehicles of the distribution of systems. And since each individual morphogram indicates the place of a two-valued logic, ion, which is, of course, disturbed by the "noise" of transjunction, we might as well say that a many-valued logic is a place-value order of morphograms and of compounds of morphogrammatic patterns. This relegates the concept of value in these higher systems to a subsidiary role. The use of value, and therefore the use of negation, is still necessary because it is impossible to construct compounds of morphograms in a logical sense without value-occupancy. But it is not the value but the morphogram which determines the semantic s i g n i f i c a n c e of the non-Aristotelian theory of thought. The classic concept of ratiocination is incomplete only from the morphogrammatic viewpoint. And it is this new aspect which introduces the idea and the operations of transjunction. The concept of a value of rejection is incompatible with the metaphysics of Aristotle. His hypothetical third value from the ninth chapter of "De Interpretatione" is anything but a rejection of the alternative of the two values on which his theory of thought is based.

If we interpret many-valued systems as place-value orders of morphograms and morphogrammatic compounds we should say something about the formal composition of these arrangements, which grow rapidly in complexity if more values are introduced. The two-valued system is not only morphogrammatically incomplete, as we have frequently noted: it is also not a compound of morphograms. Only one morphogram may be used at a time and in a single operation as far as the definition of such operations as conjunction, disjunction, conditional and so on is concerned. A three-valued system is morphogrammatically richer although still incomplete, as we know, but it also represents morphogrammatic compound structures. A four-valued system is finally both. It is complete as to the number of morphograms and it is also an order of compounds. It is important not to confuse the hierarchy of value-systems with the hierarchy of morphogrammatic compounds. A three-valued system using three connected morphograms incorporates just 3 "four-place" sub-systems which are basically "two-valued" *but open for transjunction.* A four-valued system represents 6 "two-valued" logics, 4 "three-valued" systems and 1 "four-valued" formal order. The number of two-valued subsystems for any m-valued order is

$$\frac{m^2 - m}{2}$$

Moreover, any m-valued logic has m sub-systems of the value-order m-1. Generally it can be said that the number of s-valued sub-systems that are formed by an m-valued logic is

$$\binom{m}{s}$$

when $s \leq m$. The following Table XIX gives the values for $\binom{m}{s}$ where s ranges from 2 to 7:

Table XIX

m	$\binom{m}{2}$	$\binom{m}{3}$	$\binom{m}{4}$	$\binom{m}{5}$	$\binom{m}{6}$	$\binom{m}{7}$
2	1					
3	3	1				
4	6	4	1			
5	10	10	5	1		
6	15	20	15	6	1	
7	21	35	35	21	7	1
...	...	...	...	...	...	...
...	...	...	...	...	...	...

According to our table a five-valued logic would include as subsystems 10 "two-valued" logics, the same number of "three-valued" systems, and 5 "four-valued" logics. We have put the value-designation in quotation marks because they all permit rejection values to enter their order. A "two-valued" subsystem in a "three-valued" logic is determined by 3 values. This awkwardness shows the inadequacy of the value concept when applied to higher systems of logic. It is more adequate to say that a three-valued logic is a compound of 3 morphograms.

Table XIX is nothing but a fragment of the well-known table of binomial coefficients[68] adopted for our purpose. An interesting fact that can be obtained from Table XIX is that the sum of the numbers of all sub-systems of s^{th} order for a given m-valued logic is always equal to the number of sub-systems of s + 1 order in a logic with m + 1 values. It is implied that each logic contains itself as sub-system.

[68] The author is indebted to Professor H. von Foerster for having drawn his attention to this fact.

In the described sense we may interpret all m-valued systems of logic, classic as well as trans-classic, as place-value systems of sub-logics with the order indices 1,2,…,m-1. It is by no means superfluous or trivial that we include the two-valued logic. The very fact that the traditional logic, in its capacity of a place-value structure, contains only itself as subsystem points to the specific and restricted role which reflection plays in the Aristotelian formalism. In order to become a useful theory of reflection a logic has to encompass other sub-systems besides itself.

More important than the interpretation of all logics as place-value systems of suborders that are made up of values is the morphogrammatic orientation which looks at a given logic as a set of morphograms and morphogrammatic compounds. In the classic logic these two concepts coincide. There are no compounds in the proper sense unless we say that each morphogram represents its own compound. In any m-valued system where $m > 2$ they differ. It stands to reason that the number of morphograms which make up a compound is always identical with the number of first order systems which are incorporated in a given logic. In one (and the most important) respect, however, there is no difference between the Aristotelian and the many-valued logic: the number of morphograms and morphogrammatic compounds is always smaller than the number of value-sequences or functions. A two-valued system has eight morphograms which are represented by 16 functions of four places. A three-valued logic possess $3^9 = 19683$ nine-place value-sequences. The number of morphograms that are represented in it is, as we know, 14 and the system is therefore not yet morphogrammatically complete. However, as far as unique morphogrammatic compounds are concerned this system contains 1 compound represented by one value, 255 compounds incorporated by two values, and 3025 compounds where the structure requires three values for systematic representation. In the classic system all morphograms claim double value occupancy. In the three-valued system we find the following correlation between values and morphogrammatic compounds:

Table XX

compounds	value-occupancy
1	3
255	6
3025	6

The more comprehensive the logical systems become, the higher is the rate of value-occupancy, or the smaller becomes the number of unique morphogrammatic structures compared with the number of value-sequences that represent them in a given logic. The author´s attention was drawn by H. von Foerster to the fact that the number of ways $\mu(m)$ in which m values can be put into n different places can be defined with the aid of S(n,k), the Stirling numbers of the second kind[69], the first few values of which are given in Table XXI. It can be shown that

[69] J. Riordan: *Introduction to Combinatorial Analysis*, Wiley, New York (1958); See p. 32 ss, Table p. 48.

$$\mu(m) = \sum_{i=1}^{m} S(m^2, i) \tag{27}$$

Thus, if we wish to know the number of morphograms, or morphogrammatic compounds, the answer will be given by Formula (27).

Table XXI

n / k	1	2	3	4	5	6	7	8	9
1	1								
2	1	1							
3	1	3	1						
4	1	7	6	1					
5	1	15	25	10	1				
6	1	31	90	65	15	1			
7	1	63	301	350	140	21	1		
8	1	127	966	1701	1050	266	28	1	
9	1	255	3025	7770	6951	2646	462	36	1

In a two-valued logic we have, e.g.:

$$\mu(2) = \sum_{i=1}^{2} S(4, i) = 1 + 7 = 8$$

Or, in the case of a three-valued system:

$$\mu(3) = \sum_{i=1}^{3} S(9, i) = 1 + 255 + 3025 = 3281$$

There is, however, another aspect to the theory of the morphogrammatic compounds which we will call their μ-structure. It arises from the formula:

$$\bar{\mu}(m) = \sum_{i=1}^{m^2} S(m^2, i) \tag{28}$$

We require Formula (28) as justification of our statement that a logic of reflection has 15 basic morphogrammatic units. If we assign m the value 2 then we obtain from Formula (28)

$$\bar{\mu}(2) = 15$$

the number of morphograms represented by Tables VIa and VIb. However, Formula (28) has a deeper significance. If we equate m = 3 then

$$\bar{\mu}(3) = 21147$$

Since we know that a three-valued logic has only $3^{3^2} = 19683$ value-sequences it seems to be stark nonsense to ascribe to a trinitarian logic 21147 morphogrammatic compound structures. It is indeed impossible if we assume that 3 is the highest value in the system; or to put it into different words that our logic is only a sub-system of itself. In this case Formula (27) applies. On the other hand, we face a different situation if our trinitarian logic is a sub-system of, let us say, a logic with 9 values. The number of rejection values any two-valued system may have within an m-valued logic is always m-2. If a three-valued logic is only a sub-system of itself only one rejection value is available for each of its two-valued sub-systems. But if the same trinitarian logic is part of a nine-valued structure of reflection our Table IX would grow into Table XXIV.

Table XXIV

p	q	[1]	[4]	[13]	[13]	[13]	[13]	[13]	[13]	[13]
P	P	P	P	P	P	P	P	P	P	P
P	N	P	N	3	4	5	6	7	8	9
N	P	P	P	3	4	5	6	7	8	9
N	N	N	N	N	N	N	N	N	N	N

Table XXIV demonstrates clearly that nothing is or can be added to the morphogrammatic structure of "two-valued" logic. But the case is quite different for the trinitarian system. By being a sub-system of a nine-valued order of reflection it acquires a greater richness of morphogrammatic structure. We give as an example a value-sequence which may occur in a trinitarian system if and only if it is a sub-system of a logic where $m \geq q$:

1 2 3 4 5 6 7 8 9

This is a function with the morphogrammatic order [15,15,15] which a three-valued logic that is only a sub-system of itself could not have. If the trinitarian system is a sub-system of, e.g. a four-valued logic, the increase of morphogrammatic richness would be considerably smaller. But there is a limit for such an increase. It is given by the formula

$$\bar{\mu} - \mu$$

which in the case of a three-valued logic is

$$\begin{array}{r} \bar{\mu}(3) = 21147 \\ -\,\mu(3) = 3281 \\ \hline 17866 \end{array}$$

No doubt the increase in morphogrammatic compound structure is impressive. But for a "three-valued" logic it ends with that number. Generally, no sub-system will increase its morphogrammatic richness if $m > s^2$.

Every logic, if included as a sub-system in a higher order of reflection finally reaches a point of morphogrammatic saturation, provided, of course, that s is finite. For a two-valued logic this stage is reached when the classic system is incorporated in a four-valued order. If, e.g., [15] as a four-place sequence is penetrated by higher values and assumes, let us say, the shape 1792, the original transjunction is monotonously iterated. As far as the classic system is concerned no new logical motive has been added. We all know from our own psychological introspection that our consciousness has a capacity for a theoretically unlimited self-iteration of its concepts. Fichte has drawn our attention to its (negative) logical significance. We have, he says, a concept of something and may iterate it into a:

concept of a concept of a concept..............of something

and so on *ad nauseam*. He and later Hegel point out that after the second step no increase in logical structure can be expected. The endless iteration of our reflection is, to use a term of Hegel, "eine schlechte Unendlichkeit" (a bad infinity). It is important to point out that there are indeed two utterly different ways in which a formal increase of reflection may be obtained: first, by (empty) iteration of a morphogrammatically saturated system and second, by a growth of morphogrammatic structure. It is a serious argument against the reflective power of the infinite hierarchy of two-valued meta-systems that this hierarchy represents an iteration of the first kind.

From a logical point of view it is also important to know that there is a semantic difference between the morphogrammatic structure any m-valued system has as an independent logic and the additional structure it gains by becoming a sub-system of a more comprehensive order of reflection. It will be useful to stress this difference by speaking of morphogrammatic compounds of first and second order. The first is by far the more important – at least as far as the semantic interpretation plays a part.

Despite the rapid growth of the first order compounds their numerical ratio to the value-sequences grows steadily smaller. This gradually enhances the importance of the morphogrammatic structures. The higher the rate of their possible value-occupancy the more flexible they are in their employment for a theory of reflection or subjectivity. In our traditional logic they cannot be used at all in this sense since their value-occupancy means a strict alternative of two values producing a perfect involution. Morphograms indicating transjunction are useless in this situation. It may be said that the concept of Being or of Reality developed on the platform of two-valued logic is entirely irreflexive. This is why Schrödinger´s complaint that it is impossible to discover subjectivity and subjects in our present scientific world-conception is more than justified. If a morphogram changes its value-occupancy, and there is only one other value available, and this value entails perfect negational symmetry (Nicholas of Cusa´s *coincidentia* oppositorum), then nothing is gained by this change – except the insight into the futility of this operation for a theory of reflection.

This helps us to obtain a reliable definition of what we mean when we use the terms "irreflexive", "reflexive" and "self-reflexive"[70]. We shall stipulate that we refer with the first concept to those structures of any system that can be described by a logic which uses only the morphograms [1] to [8]. Thus the value-occupancy is automatically

[70] This is the distinction which occurs in Hegel's Logic as ´Reflexion-in-Anderes´, ´Reflexion-in-sich´ and 'Absolute Reflexion´. Cf. Hegel, ed. Glockner (See Note 45) IV, p. 493 ss; VIII, p. 288.

restricted to two inverse values. In other words, there exists a symmetry between the designating and the non-designating value. A system which is described with the exclusive use of categories derived from a logic with the above morphogrammatic restriction has a most significant property: *it has no environment of its own!* Environment would mean a third value! It also means structural asymmetry**.** If one reads H. von Foerster´s essay *On Self-Organizing Systems and Their Environments* with the eye of a logician then it is not difficult to discover this lack of logical symmetry between what is supposed to be the system itself and its possible environment.

In fact there is only one system known to us which forces us, by logical necessity, to conceive it as having no environment. It is the objective universe as a whole representing the sum total of Reality. This is why our traditional logic applies so perfectly to all of *it* – so long as we are willing to forget about the subject. The very moment we say that we perceive the Universe, it has acquired an enveloping environment: the "space" of perception. And it does not help us in the least if we argue that the dimension of perception is enclosed in the Universe. In the same essay, H. von Foerster correctly points out that it is irrelevant whether the environment is inside or outside the "closed surface" which separates it from that which it "envelops"[71].

Our classic tradition of science assumed that it was possible to treat, even inside the Universe, certain data of observation in total isolation and without regard to an environment. Quantum mechanics has first disabused us of this notion. But having an environment and being affected by it is one thing. A probability logic takes care of this situation. Quite a different thing is a system which *reflects* its environment by organizing itself and producing additional structure. An elementary particle which is affected by the observation does not do so: the closed surface is missing. Logically speaking, the applied values are just diffused and distributed over an area of uncertain character. But such a structure-producing action takes place in von Foerster's experiment with the magnetized cubes. The "noise" which enters the box containing them is reflected in an incredibly ordered structure. We have already pointed out that it is senseless to view this situation with categories which have sprung from a probability logic. In the case of the cubes a phenomenon of distribution is again involved, but it is no longer a distribution of single data, with corresponding individual probability values, but of arrays of data which are capable of forming systems. It is evident that this requires the service of a logic which is capable of distributing systems. The basic unit of such a logic must be something which represents an array of data. This unit is the morphogram.

We have demonstrated that such a logic exists, and we have also shown that the introduction of morphograms with transjunctional structure, [9]-[15], produces a distribution of systems. If we ignore the value-occupancy of our structures we call the distribution of our original four-place morphograms over different positions a morphogrammatic compound. If we look at the same structure from the viewpoint of value-occupancy, we speak about a many-valued logic having a given number of m, m-1, m-2,…,m-n valued sub-systems. Both aspects are essential. The first is necessary because it indicates the structural incompleteness of two-valued logic and it provides us with a new logical unit, the morphogram, which is capable of representing a system and

[71] Loc. cit., p. 31.

at the same time of demanding distribution if we intend to apply more than one of these structural patterns in the, same binary function. The second aspect is essential because the morphograms, to be fully usable in terms of logic, have to be occupied by values; and values are the only means by which their distribution may be accomplished. If a many-valued logic is basically a place-value system of distributed morphograms then such places of distribution must be marked by values. If, for instance, 123223333 represents a function in which the morphogram [4] is distributed over four places then the first position is indicated by the value-sequence 1222. What we decide to call the second place is occupied by 2333 and the third place shows its location by using 1333.

These 3 four-place sequences may be considered mutual "negations". But negation in a many-valued system has, under certain conditions, an entirely different function from the corresponding operation in traditional logic. If we negate 1222 and obtain 2111 in classic logic we have negated the *meaning* of the original sequence. But if we apply the negator η_2, thus changing 1222 to 1333, we insist that the second value-sequence carries exactly *the same meaning* as the first. What the operator did was only to shift the meaning from one given location in a system of reflection to some other place. A change of values in a many-valued order may under given circumstances produce a change of meaning. But it does not necessarily do so. In traditional logic a value has one and only one function. By negating one value it unavoidably *accepts* the other one as the only possible expression of a choice. And by doing so it implicitly accepts the alternative that is offered by the given values. In this sense negation is a function of acceptance in the classic theory and the values "true" and "false" are *acceptance* values. All thinking starts from the primordial fact that there is something to think about. Consciousness is, seen from this angle, nothing but the acceptance of the fact that there is an objective world. And if we think about this objective "there is" we use only the morphograms [1] through [8] which can be arranged in a logic where each value functions as an acceptance value. And here a change of value results always in a change of meaning. A negated conjunction is not a conjunction anymore. It now carries a different meaning. It signifies incompatibility.

As soon, however, as we enter the domain of many-valued logic by making use of the morphograms [9] through [15] all values assume a second function. They may or they may not be acceptance values. And if they are not, then they represent *rejection*. In our standard form of morphogram [13] as shown in Table IV the value "3" represents a rejection. But any value may be considered a rejection value. If a given system provides for its variables, in a specific instance, the values "2", "3", "4" "5" and "6" and the applied function chooses "1" the selected value represents a rejection of the structural context which is circumscribed by the offered values. If the value-sequence [4,4,4] of Table XIV gives us conjunction with the standard and two "negated" value-occupancies the η-operations which determine the relations between these 3 four-place sequences do not change the meaning of [4]. Instead of it they state the fact that conjunction is also valid within two additional structural contexts which originate from the rejection of the $1 \longleftrightarrow 2$ alternative. In other words: [4,4,4] asserts that conjunction is simultaneously valid on three different levels of reflection and that these levels are related to each other via the operations $\eta_{2.1}$ and η_2. In this specific case it is the value "3" which transjugates the meaning from one sub-system to another.

Table XXII

[4]	$\eta_{2.1}[4]$	$\eta_2[4]$
1	2	1
2	3	3
2	3	3
2	3	3

By interpreting transjunction as a logical act of rejection this type of operation acquires a specific cybernetic significance. We have already suggested on a previous page that transjunction isolates a system (by rejecting it). In doing so, it produces the distinction between a closed system and its environment. This is exactly what a two-valued logic can never do. Its very nature of having only two values makes it impossible. One value is not sufficient to define a system. Every description of it absorbs two values! But the very same values which do the job of describing it cannot be used to tell us what it means to have an envelope around it (Wittgenstein). For this very purpose we require a value which *transcends* the scope of the system. However, as we have seen, there is no way to make such a value operable as long as we stick to the classic ontology and the concomitant logic of Being-as-the-irreflexive-It. For this logic only Reality as Totality has a closed surface. In other words: all of the Universe may be considered a system of "retroverted" self-reflection. It is retroverted because the Universe as such has no environment. Or, to put it differently, the environment coincides with the system it "envelops".

On the other hand, when we speak of individual centers of self-reflection in the world and call them subjects we obviously do not refer to retroverted self-reflection. Such individual centers have, as we know very well, a genuine environment (which the Universe has not!) and what they reflect is this very environment. It stands to reason that these systems of self-reflection with centers of their own could not behave as they do unless they are capable of "drawing a line" between themselves and their environment. We repeat that this is something the Universe as a totality cannot do. It leads to the surprising conclusion that *parts of the Universe have a higher reflective power than the whole of it*, as has been recognized for a long time. In Hegel´s logic the phenomenon of reflection is subdivided into three parts: He defines them as:

a) retroverted reflection (Reflexion-in-sich)
b) transverted reflection (Reflexion-in-Anderes)
c) retroverted reflection of retroversion and transversion (Reflexion-in-sich der Reflexion-in-sich und-Anderes)

Section (a) represents the physical system of the external world described by its specific reflective properties. But (b) and (c) signify the additional capacities of reflection which sub-systems of the Universe must possess if they are to be called subjects.

This shows that the early philosophic theory of reflection is still ahead of the present logical state of cybernetics. We talk about self-organizing systems and their environments; but Hegel´s distinction between (a), (b) and (c) shows that this is not enough. A self-reflective system which shows genuine traits of subjective behavior must be capable of distinguishing between two types of environment and be able to react accordingly. First it must reflect an "outside" environment which lies beyond its own adiabatic shell and second it must be capable of treating (b) as an environment to (c). These two environmental meanings are not yet clearly distinguished in present cybernetics although von Foerster´s experiment with the magnetized cubes may give a very rough idea of it. The cubes themselves obviously require two different environments in order to build up their complicated architecture. They could not do so unless they possessed an environment inside the box where they could move freely. If there was no such environment, i.e., if they were locked in their initial position no structure could originate. But it is equally obvious that a second environment is required as place of origin of the "noise." In our example the three orders (a), (b) and (c) are rather haphazardly thrown together. They do not represent a fully organized system of reflection – although there is reflection of a very artful kind – but the arrangement gives at least an approximate idea of what is meant when we say that a system showing subjective traits of behavior must have an inner *and* an outer environment. And it must have the inherent ability to distinguish between the two.

This leads us back to transjunction and to our interpretation of transjunctive values as operations of rejection. We stated that if a system is rejected the value which acts as rejector places itself outside of it. By doing so, it establishes a boundary or a logically closed surface for the rejected system. In other words: it makes a distinction between the system and something else, i.e., an environment. This is achieved by the operations trans-classic values perform on the basis of the morphograms [9] through [14]; but we know: a logic which uses only the patterns [1] through [14] has at its disposal only one rejection value for a given two-valued system. Thus it can only establish one boundary and one environnient for the system it rejects. In other words: the distinction between an inner and an outer environment does not yet exist on the level of a three-valued logic. However, there is still one morphoaram left which becomes usable in a four-valued system. It is morphogram [15] which incorporates two different values of rejection as Table XXIII shows:

Table XXIII

		(15)
1	1	1
1	2	3
2	1	4
2	2	2

If we look at our standard value sequence which represents this operational pattern we notice as trans-classic values "3" as well as "4". Both of them have in common that they reject the alternative 1⟷2. And since value "4" implies the logical power of "3" both share in this operation. In other words: for the reflective level of "3" the operation is total. The system 1⟷2 is now clearly separated from an environment. But "4" has an additional power of rejection. It establishes a second environment within the sphere of rejection itself. This new environment has a weaker boundary. We all know what this means from our personal introspection. Our capacity to reflect upon our own thoughts and thought-processes implies that we are capable to make our own system of reflection the environment of a second order reflection. In other words: systems of reflection and environment may reverse their roles. Expressed in morphogrammatic terms: the pattern remains the same if we write [15] as sequence 1342 or as 1432. But there is a difference with regard to the functional significance for $1 \longleftrightarrow 2$ implied in the exchange of the positions for "3" and "4".

It goes beyond the scope of this investigation to discuss the functional significance of the exchange relation between 1342 and 1432. If we did so it would lead us into very intricate questions about the outer and the inner environment of self-reflective systems. We have confined our theory of transclassic logic to the development of some basic terms of reflection which we derived from von Foerster´s experiment. It served us well as a starting point for our discussion of a logic with transjunctional operations. Transiunction was interpreted as "noise" relative to a two-valued system. We then showed that the only possible logical interpretation of subjectivity is formally equivalent to the order-from-noise principle. Thus we equated noise with subjectivity. However, it seems rather preposterous to say that von Foerster´s experimental arrangement displays a subjectivity of its own. Although the noise that effects changes in the arrangement of the cubes has a general transjunctional (= subjective) character it lacks an essential quality. Von Foerster´s principle does not permit us to distinguish between the different varieties of transjunction. Ergo, it is impossible to define in reflective terms what is inner and what is outer environment, not for us, but for the noise. There is, of course, a crude analogy to the distinction between an inner and an outer environment which every subject (potentially) has. In von Foerster's experiment it is the difference between the environment of the box and the environment of the cubes inside the box. The question may be settled for us, but we are idle spectators in this situation. Our opinions are quite irrelevant. The important issue is: what is inner and outer environment for the noise as the "soul" of this self-organizing system? If the cubes form a strange architecture is this something the noise erects in its external world in the way we build cathedrals, airports or communities ? Or does this architecture belong to the inner (subjective) environment of this organizing principle and do the cubes and their arrangement play the part of the "thoughts" of von Foerster's principle? The structure of the experiment in question is, of course, too undifferentiated to answer these and similar questions. But it is highly instructive to see how many formal characteristics of subjectivity, e.g. distribution of systems, transjunctional organization, inner and outer environment, rejection and self-reflection are incorporated in such a simple arrangement. That these traits display themselves in a very rudimentary form is of much less importance than the fact that they exist at all and can be demonstrated in such primitive experiments.

The issues of an advanced theory of reflection cannot be discussed on such a narrow experimental basis. Least of all the problem: what is inner and what is outer environment of a system that behaves as a fully developed subject of reflection? to obtain a complete answer to this question would be equivalent to the challenge to construct a trans-classic ontology of the subject as detailed as the classic ontology of the object. This is a goal that lies in a distant future.

5. SUMMARY

We are coming to the conclusion of our discussion on ontology and transjunctional logic in cybernetics. Our argument started with the observation that cybernetics requires an ontology and logic which provides us with a basis from which we may include the subject and the general phenomenon of subjectivity into a scientific frame of reference without sacrificing anything of clearness and operational precision. We hope to have shown that this is entirely within the range of our logical capacities. We defined subjectivity as logical distribution and we distinguished between distribution of values and of systems which are formed by groups of values. The basic units of such groups we called "morphograms". From there the concept of a place-value system of morphograms and morphogrammatic compounds originated. This theory brought forth the idea of a set of logical operators called transjunctions. A short analysis of these operators led to the discovery that logical values have two basic functions: they can be considered either as acceptance values or as rejection values. In classic two-valued logic values are only capable of acting as acceptance values. In a morphogrammatic logic with $m > 2$ they also function as rejection values. Herein lies the difference between their objective and subjective significance. In a complete system of logic, referring to the object as well as to the subject, a value must always carry a double semantic meaning, namely being a value *of something* and for a *subject* of reflection. Our final Table XXIV illustrates this inverse relation:

Table XXIV

for	value	of
	acceptance	object
subject	rejection	

The difference in the functional character of the values which occupy the various places of the morphograms and their compounds is far reaching. The acceptance capacity of a value is precisely limited to the values that are offered for acceptance. In other words: *there are no degrees of freedom in this function.* If a value sequence which results from a binary operation is designated as a conjunction, then the higher value must be chosen in a two-valued system. However, it is different with rejection. A system $1 \longleftrightarrow 2$ may be rejected by "3" or "4" or by any higher value we care to select, provided our logic is of an order sufficiently comprehensive to provide the value we intend to use for this operation. Theoretically our choice is infinite. This situation refers to the often observed and widely discussed infinite iterativity of systems with total reflection of the order (c). The subject seems to be bottomless as far as its "self" is concerned. This however is, from the viewpoint of the logician, an unwarranted assumption. We are only permitted to say that a system represents all structural characteristics. of subjectivity if it is complete with regard to the number of basic morphograms and functional representations. As a further provision it requires a logic with two stages of rejection over and above the number of values that are demanded for the description of its physical properties. In this sense a cyberneticist may talk in a definite, communicable and computable manner about the subject.

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