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Formal Logic, Totality and The Super-additive Principle

If the title of this paper combines Formal Logic and Totality (Ganzheit) it is resisting a general trend which is still strong in present scientific activities. The most comprehensive theory of Totality which we possess is contained in Hegel´s logic. But every student of this thinker knows how emphatically Hegel denounces formalization. According to him the structure of all totalities is "dialectic". Formal logic is based on a strict dichotomy of form and content (matter). But dialectics fuses the two in the superadditive principle of synthesis which combines thesis and antithesis in a way in which the contradiction between the two is not only retained but elevated to a higher level. The general consensus still is that the retention of contradiction – which is indeed demanded by all systems to which we ascribe the character of totalities – obviates all attempts of formalization. This belief is now more than two thousand years old and it is hard to shake.

However, a re-evaluation of the theory of dialectics and its super-additive principle, where the whole is more than the sum of its parts, has recently become a pressing necessity. Among the new scientific disciplines which have sprung up in recent times Cybernetics seems to have the widest interdisciplinary spread. The topics it deals with range from mathematics (information theory) and physics (quantum mechanics) over biology (bionics) to the theory of consciousness, of culture and of human history.[1] It Is hardly necessary to point out that the problem of the structure of totalities turns Up various aspects within-the scope of Cybernetics. Nevertheless a basic investigation into the formal logical texture of totalities is still missing. The ancient prejudice that such inquiry loads us straight out of the realm of formal, codifiable procedures of logic is still too strong.

Some progress has been made just the same. In a very relevant paper on biologic "coalitions"[2] H. von Foerster has pointed out that such phenomena are characterized by what he calls, a super-additive nonlinear principle of composition where some measure Φ of the whole is more than the sum of the measures of its parts:

$$\Phi(x+y) > \Phi(x) + \Phi(y)$$

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reprinted in: Gotthard Günther, *Beiträge zur Grundlegung einer operationsfähigen Dialektik*, Band 1, Meiner Verlag, Hamburg, 1976, p.329-351.

1 In this respect attention is drawn especially to the Russian efforts in this field. Cf. A. I. Berg, *Kiberneticu na Sluzhbu Kommunizmu*, Vol. I, Moscow/Lenongrad 1961. (Engl. Translation. *Cybernetics At the Service of Communism*, Publ. Joint Publ. Research Service, Washington D.C. JPRS 14592). Also: *Filosofskye voprosy Kibernatiki* (*Philosophical Problems of Cybernetics*, JPHS 11503),

2 Heinz von Foerster, *Bio-Logic*, in: Biological Prototypes and Synthetic Systems) I; Plenum Press, Now York 1962.

H. von Foerster´s argument cannot be repeated in detail. It will be sufficient to say that by applying the concept of "logical strength" (Carnap, Bar-Hillel) according to which a truth function increases its strength with the number of negative values it applies the author shows that a "coalition" of two statements A and B signifies such a super-additive principle:

A	B	≡	&
1	1	1	1
0	1	0	0
1	0	0	0
0	0	1	0

(I)

Table (I) shows on the left side the value constellations (0 for negative and 1 for positive) of the statements A and B. It is obvious that the logical strength of each is ½. On the right side we have first the equivalence relation (≡) of A and B which gives us their average strength as a result of what may be called a normal adjunction. This average strength is, of course, again 1/2. The last value sequence represents conjunction (&), in von Foorster´s words a "coalition", and the logical strength of the value sequence is in this case 3/4 since, compared with the equivalence relation the last value of the sequence has turned from 1 to 2 which adds one quarter the strength of the function.

The argument used by von Foerster has the great merit of showing that a super-additive principle of logical strength is already extant in classic formal logic (and so is its opposite of super-subtractivity in disjunction). But the history of traditional logic has shown that the form in which super-additivity manifests itself in simple conjunctive relations does not suffice to develop all the peculiar characteristics of totalities which we find displayed in systems of reasonably high complexity. This is why the history of formal Aristotelian logic is accompanied by an equally long history of dialectic (non-formal) logic. The latter was supposed to take up the logical problems, where formal logic, due to its specific limitation, had to drop them.[3]

It will pay to investigate the basic shortcoming of traditional formal logic. To put it in a nutshell: it excludes the subject of thought from the logical picture of the Universe.[4] Thus this picture is entirely "objective" in the full double meaning of the term. It goes without saying that the mental image of the Universe, thus obtained, does not describe it as a totality. A very important structural element is missing in this logical imagery: the indubitable power of the Universe to form subsystems which act as centers of objective reflection as well as of self-reflection. But since this property is excluded it stands to reason that the totalities of lower order which we encounter in biology, psychology, social sciences or history are also outside the scope of traditional logic. They are parts

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3 The terms "traditional", "classic" and "Aristotelian" shall be used in this assay as applied and interpreted in: Gotthard Günther, *Idee und Grundriss einer nicht-Aristotelischen Logik*, Hamburg (Meiner) 1959.

4 An excellent description of this epistemological situation is given in E. Schrödinger, *Mind and Matter*, University Press, Cambridge 1959. See esp. p.51.

of the Universe and available for their description are only the very same logical elements and procedures which are applicable to the objective world in its entirety. This means they cannot be described as totalities either.

It will help to understand the epistemological situation of our traditional formal logic (including modern mathematical logic!) if we draw a diagram:

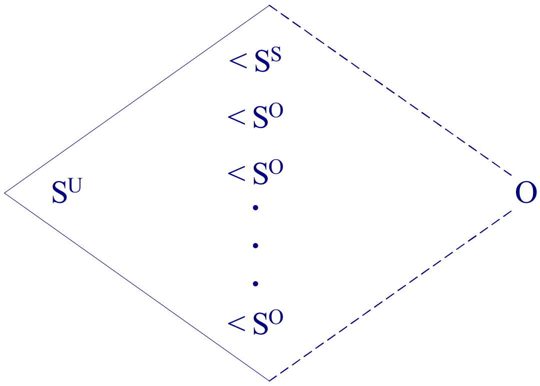

Fig_1

In this figure O means, of course, the objective world as reflected in the consciousness of a subject S··· . But since subjectivity is a phenomenon shared by an indefinite number of relatively independent centers of self-reflection and, moreover, only one of them may, for the purpose of developing a theory of thinking, be regarded as *the* subject who thinks whereas the others are thought of, we have to distinguish three different meanings of S···. We show this by writing: S^U, S^S and S^O. With S^U we indicate what in traditional logic is usually called universal subjectivity (Kant´s "Bewusstein überhaupt"). When we write S^S (or subjective subject) we refer to what is in a given process of thinking the actual subject of the mental event. All the other potential subjects of thought are, of course, relative to the designated one (S^S) objective subjects, i.e. possible objects of the reflection of S^S. In our figure they are indicated by S^O.

The classic theory of thinking as expressed in all our present systems of logic assumes that subject (S^U) and object (O) represent logically speaking an absolute dichotomy: what is not object is necessarily subject and what is not subject is correspondingly object. It is assumed that looking at the world all subjects form a closed phalanx confronting the object and since they are all – in some unexplained manner – parts of the universal subject (S^U) they will have a common basis of thought. Because if S^S agrees in its reflection of O with S^U then the resulting judgments of S^S will be binding for all S^O. It follows that the general (metaphysical) dichotomy between S^U and O is reflected in a second order dichotomy which separates S^U and S^S from S^O and O.

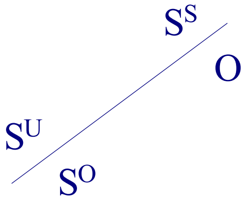

Fig_2

But since what is in the mental eye of the subjective subject (the self) only an objective subject (a thou) may in its turn become the thinker, Figure 2 is not complete, because S^U and S^O may also be dichotomically separated from S^S and O. Thus we obtain

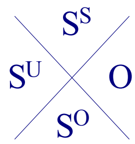

Fig_3

The pattern obtained in Figure 3 yields if we replace S and O by P and N (P for positive and N for negative), the well known table of two-valued negation:

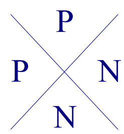

Fig_4

The distinction between S^U and S^S resp. S^O which has disappeared in Figure 4 re-occurs later as difference between partial and total negation and reflects itself in the qualificational equivalences:

$$
\begin{aligned}
(x)f(x) &\equiv \sim(Ex)\sim f(x) \\
\sim(x)f(x) &\equiv (Ex)\sim f(x) \\
(x)\sim f(x) &\equiv \sim(Ex)f(x) \\
\sim(x)\sim f(x) &\equiv (Ex)f(x)
\end{aligned}
$$

which may be derived from the laws of the famous square of opposition.

In other words: the founding relation of all classic thought and its ultimate basis on which everything is built is an *exchange relation of absolute symmetry* between total affirmation and total negation.[5] Its most famous expression is Hegel´s terse remark at the beginning of his Dialectic logic: "Das reine Sein und das reine Nichts ist … dasselbe"[6]. A formalized equivalent of it is:

$$A \equiv \sim(\sim A)$$

which holds only in a two-valued system of logic where each value is the mirror image of the other.

There can be no doubt that the operational basis of classic logic is an exchange relation between subject and object or between a mapping process and that which is mapped. However, if we have a second look at Figure 2 or 3 we will notice that the complete symmetry of the exchange relation between S··· is guaranteed only by the introduction of the concept of a universal subject (S^U) which according to the metaphysical tradition of classic logic (e.g. Nicolaus Cusanus) is, ontologically speaking, identical with O.

5 Hence the isomorphism of classic logic. Cf. Reinhold Baer, *Hegel und die Mathematik*, Verhandlungen des zweiten Hegel Kongresses vom, 18.-21. Okt. Publ. Tübingen 1932.

6 Hegel III (Meiner 1923) p. 67.

The modern scientist who tries to discover the formulas in which the code of the Universe is written is usually not aware of the basic ontologic assumptions which govern his mode of thinking. But they show up in his results just the same. Because if S^U is ultimately identical with O then his world picture will contain no traces of bona fide subjectivity – as Schrödinger has pointed out correctly. And if S··· and O represent an exchange relation of enantiomorphic equivalence then the basic laws of Nature must obey the principle of reflection-symmetry (parity). Whenever a phenomenon shows up which seems to display the structural features of non-parity there will be cogent reasons for a turn to more general principles of reasoning which explain the event again in terms of reflection-symmetry. These reasons will not only be strong, nay, they will be invincible as long as we stick to the ontologic tradition of classic logic and its principle of reflection-symmetry.

We are here not concerned with the fate of parity in the future development of physics but it must be pointed out that the concept of Totality should be ruled out as logically analyzable if parity reigns supreme in our theory of thinking. We have given the main reason above: if the relation between thought and its object is basically understood as a symmetric exchange relation the phenomenon of subjectivity disappears. But a "totality" in which everything is reduced to objectivity can never be total because something is missing.

A totality is, in Hegel´s terminology:

1) an iterated self-reflection of
2) a non-iterated self-reflection, and
3) a hetero-reflection.

If we permit, for the description of this structure, only logical operations which lead to reflection-symmetry then 1) is eliminated, and 2) and 3) turn out to be indistinguishable and logically identical … because 1) is nothing else but the capacity of keeping 2) and 3) apart.

However, if the concept of the universal subject, i.e. of 'Bewusstein überhaupt' (Kant), is eliminated the logical constraint to reduce everything to ultimate parity relations disappears. We will still have reflection-symmetry between S^S and S^O but not longer between S··· and O in general. In other words: it will turn out that the founding relation between subject and object or between Thought and Being is not a symmetrical exchange relation but something else. This is the point where the transition is made from formal classic logic of Aristotelian type to a theory of trans-classic, non-Aristotelian Rationality.

We begin by re-drawing Figure 1 omitting S^U and having the phalanx of the S^O replaced by a single S with the index O. We indicate the relations between S^S , S^O and O by arrows of four different shapes. According to the logical character of the relation an arrow will either be double-pointed or it will have one shaft or be double-shafted having either continuous or dotted lines. Figure 5 will then show the following configuration:

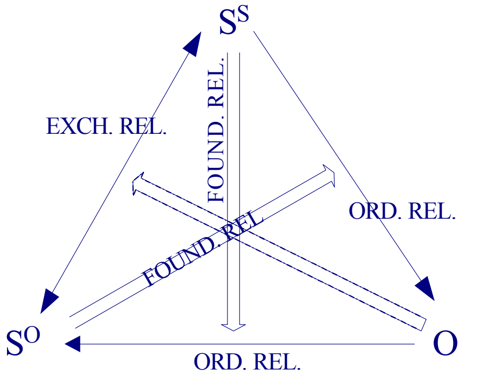

Fig_5

If S^S designates a thinking subject and O its object in general (i.e. the Universe) the relation between S^S and O is undoubtedly an *ordered* one because O must be considered the content of the reflective process of S^S. On the other hand, seen from the view-point of S^S any other subject (the Thou) is an observed subject and it is observed as having its place *in* the Universe. But if S^S is (part of) the content of the Universe we obtain again an ordered relation, this time between O and S^O. There remains the direct relation between S^S and S^O. This is obviously of a different type. S^O is not only the passive object of the reflective process of S^S. It is in its turn itself an active subject which may view the first subject (and everything else) from its vantage-point. In other words S^O may assume the role of S^S thus relegating the original subject, the Self, to the position of the Thou. And there is neither on earth nor in heaven the slightest indication that we should prefer one subjective vantage-point for viewing the Universe to another. In short, the relation between S^S and S^O is not an ordered relation. It is a completely symmetrical *exchange* relation, like "left" and "right". An ordered relation between different centers of subjective reflection cones into play only if we re-introduce the concept of a universal subject which contains all human "souls" as computing sub-centers.[7] Of the two relations we have so far considered, the exchange relation is symmetrical and the ordered relation represents non-symmetry.

There is, however, one more relation to be considered which combines in a peculiar way the aspects of symmetry and non-symmetry. In the previous two cases the members or arguments of the relation could be considered as unanalyzed units. Or to talk in terms of our diagram, the relations hold between

$$S^S \rightarrow O$$
$$O \rightarrow S^O$$
$$S^O \leftrightarrow S^S$$

as the corners of our triangle. What we still have to consider is the relation any of the three terms S^S, O and S^O may assume to the relation which holds between the other two

7 The other case, that the computing mechanisms of animals, plants or artifacts may be logically regarded as subsystems in a theory which describes the epistemological structure of human consciousness is not considered here. Considerable work has been done with regard to this problem in Cybernetics, but not on a purely *logical* level The interest of application to *physical* systems is always dominant. Cf. W. Ross Ashby, *An Introduction to Cybernetics*, New York 1956.

terms. From a purely combinational view-point three possibilities exist for a demanded relation … r^F … they are:

$$S^S\, r^F\, (O \rightarrow S^O)$$

$$O\, r^F\, (S^O \leftrightarrow S^S)$$

$$S^O r^F\, (S^S \rightarrow O)$$

From these we shall, for the time being at least, eliminate the second. It tells us nothing new. It describes only the situation we are familiar with from classic (two-valued) logic where all subjects S··· form, what we called earlier in this paper a "closed phalanx" excluding the object from themselves and thus obtaining an "objective" aspect of the Universe. Consequently $Or^F(S^O \leftrightarrow S^S)$ only informs us that if O develops its mirror image in S··· it will do so in dichotomic terms of positive and negative forming a strict exchange relation since S··· will be either S^S or S^O and a Tertium will always be excluded.[8] We have pointed out before that such an exclusion principle obviates our conceiving totalities in terms of traditional logic. Since the relation $Or^F(S^O \leftrightarrow S^S)$ is known to logic since the times of Aristotle and has its own specific properties we distinguish its graphic representation from the other two by having drawn it with dotted lines.

However, the other two relations of the type …r^F… have so far not obtained a legitimate place in formal logic. They define the way in which an individual consciousness (as a logical subject) may establish its position confronting the world. Formally speaking it is the relation any of the two realizations of S···, namely S^S or S^O, may have toward the connection of the other S··· and O. We call this the *founding* relation (r^F) because by it, and only by it, a self-reflective subject separates itself from the whole Universe which thus becomes the potential contents of the consciousness of a Self gifted with awareness. In contrast to it the classic relation $Or^F(S^O \leftrightarrow S^S)$ is still a founding relation – but not for consciousness. Not a self-reflective subject but only the *content* of the consciousness of a potential subject is established by it.

In Figure_5 the founding relations for subjectivity are indicated by the double-shafted arrows which issue from S^S and S^O and hit the center of the opposite side of the triangle. These arrows illustrate in diagrammatic form the relations between consciousness as a self-reflective activity and the world in general. The world is always *both* O (bona fide objectivity) and S^O subjects viewed as part of the objective world … where S··· is always excluded only as S^S. This last statement seems to be contradicted by our figure because the arrow issuing from S^O seems to point to a world which includes S^S and O. This is the unavoidable fault of a still picture. An adequate representation would demand a moving picture in which the double shafted arrow would oscillate between S^S and S^O. One should not forget: what is in our diagram S^O may at any time assume the role of S^S, thus relegating the latter to the logical position of S^O. Let us repeat that S^S and S^O constitute the exchange relation between subjectivity as the Self and the other subject which appears to the Self as the Thou. For any given logical position only one

8 See Figure I

of the two double-shafted arrows represents actualization of a center of self-reflection. Since such actualization requires all *three* components S^S , S^O and O it is impossible if we have located the center in S^S to find it also in S^O. But it has no fixed status in S^S and it may be shifted to S^O. Fichte calls this "die Duplizität im Ich" because, as he puts it, such a center of self-reflection can neither be fully identified as an existing entity (als seiend) nor as a structural principle of active organization (als Prinzip). This is the Duplicity of the Self.[9]

What we have so far ignored in our contraposition of S^S and S^O is the fact, well known to all of us, that no Ego, or Self exists in solipsistic splendour and that this universe of ours permits the coexistence of an indefinite number of centers of self-reflection who all claim to be thinking Egos comprising the total realm of Being as potential contents of their awareness. It is obvious, therefore, that the exchange relation is an *exclusive* disjunction on a level of reflection which is identified with the logical position of S^S.

But an impartial observer, S^{S^S} , who assumes his place neither in S^S or S^O but "outside" of Figure 5 will come to a different conclusion. He will still concede the existence of a disjunctive relation between two subjects but to him this disjunction must be inclusive. He is forced to admit that two concurring S··· may both be S^S although relative to him both will be S^O as long as he is claiming the exalted position of an S^S of higher reflexive capacity.

But this claim also extracts from the "outside" observer, S^S an interesting admission. He will state that, seen from his vantage point, the inclusive disjunction does not only hold in the case of:[10]

$$S^S r^F(O \rightarrow S^O) \ .\vee.\ S^O r^F(S^S \rightarrow O) \qquad (1)$$

but also in the other two cases:[11]

$$S^O r^F(S^S \rightarrow O) \ .\vee.\ O r^F(S^S \leftrightarrow S^O) \qquad (2)$$

$$S^S r^F(O \rightarrow S^O) \ .\vee.\ O r^F(S^S \leftrightarrow S^O) \qquad (3)$$

provided, of course, that he uses a two-valued logic. But in doing so he realizes by self-reflection that he has committed a momentous logical mistake. Since in classic logic only two values are available for the determination of the distinction between subject and object, it is impossible to describe the *triadic* relation between the subjective subject; the objective subject and the object.

The common fallacy committed by logicians who reason along traditional lines is that if subject and object are different it is sufficient to assign different values to them. But since the structure of classic negation represents a symmetric exchange relation and since there can be no preference to assign a definite value to S··· or to O, it is

9 Cf. Joh. G. Fichte: *Die Tatsachen des Bewusstseins*, Posthumous Works (ed. J. H. Fichte, vol. I) p. 573. Fichte also mentions a second duplicity of the contents of consciousness.

10 … rF…and …→… will both be interpreted as material implications and … ↔ …as negated equivalence or exclusive alternation.

11 For the transition of O to S^O note Hegel´s remark: "…im Lebendigen schlägt das *Objekt* in das *Subjekt* um …", Hegel (ed. Glockner) X, p. 263.

impossible to distinguish the subject from the object by saying, for instance, that the positive value ultimately designates the object (because we describe the Universe in affirmative statements) and that the negative value refers to the subject. Although there can be no doubt that the existence of negational processes is a symptomatic index for the presence of subjectivity in the Universe, it is not one or the other value which points to the subject but their mutual relation which displays "Reflexionsidentität" in contrast to the one-valued, stable and irreflexive identity which is incorporated in the bona fide object.

Nevertheless, it is indeed possible to determine the distinction between subject and object by logical values. Not by assigning *another* value to the subject but by engaging *two* values for the designation of *one* identity. And since we can think at least of one more theme beyond a) object, b) subject namely "reality" as the ultra-conscious context c) in which object and subject cooperate we would have to allot three values for the identity theory of c). In the case that we may be able to conceive something of even higher logical order, the difference between it and everything else would be determined by a tetradic structure of values.

The following table (II) will illustrate this relation between object designation, logical theme, value differential and n-valued logical system:

theme					
object	non-object		value-differential	log. value-system	(II)
1	1	hierarchy of themes	0	2	
1	2		1	3	
1	3		2	4	
1	4		3	5	
…	…		…	…	

Since the object, completely isolated from the subject, is designated, by one and only one value, the object column only repeats this number. In classic logic the numerical difference between the values for the object and those which designate the subject – or anything else for that matter – is zero. The third column therefore starts with 0. This informs us that the only way to think of a subject or any system gifted with self-reflection, is to conceive it as an object – which means *without* self-reflection. In other words: the first theoretical approximation to the problem of subjectivity is offered in a three-valued logic. Here again one value designates the object, but two are left over for everything which is not an object. The numerical difference between the values assignable to the object and non-object is now 1. Something can now be said (in terms of logical structure) about the non-object which would differ from all statements about bona fide objects.[12]

12 The reader´s attention should be drawn to the significant fact, that the numbers in the centre column of Table II are the numbers of rejection values in ascending value systems. Cf. G. Günther: *Cybernetic Ontology and Transjunctional Operations*, Self-Organizing Systems 1962 (ed. Yovits, Jacobi, Goldstein) p. 313-392 (Washington 1962).

Our ideal observer who contemplates the relations between subject and object as illustrated in the triangle of Fig. 5 must ultimately arrive at the conclusion that table (II) is applicable in his case. *He cannot differentiate between himself and the triangle unless he assigns to himself a logical value which does not occur in the triangle*. But what is sauce for the goose is sauce for the gander. Our observer expects that S^S in the triangle is capable of differentiating between itself and O. Consequently he has to concede that S^S in contradistinction to O possesses an additional value. Since O is described in a two-valued system, the description of the triangle requires a three-valued logic. Finally this description is the content of the consciousness of our ultimate observer who must consequently reason with four-valued structures.

However, as soon as our observer realizes that the founding relations in Fig. 5 obey the laws of a three-valued logic, he realizes that not all the inclusive disjunctions which he established in the formulas 1), 2) and 3) are analytic formulas and generally valid. He will find that only 1) still holds and that the disjunctive relation in S··· between S^S and S^O is indeed basic and invariant to a transition into a higher-valued system. With regard to 3) he will discover that its general validity has completely disappeared. Formula 2), on the other hand, has assumed a peculiar equivocality. Since a three-valued logic operates with five negational states[12] – where two-valued logic uses just one – an exchange-relation may be interpreted in five different ways. In the case of three of them formula 2) will be as valid as 3); i.e. for all possible states of the system of Fig 5. In the case of two others formula 3) will be invalid if the system O, S^O, S^S assumes the following values: classic negation for O and the irreflexive value for S^O as well as S^S. This is a most significant result!

Unfortunately the scope of this paper precludes an interpretation and discussion of such details no matter how important they are. This investigation intends only to show that the concept of Totality or Ganzheit is closely linked to the problem of subjectivity and trans-classic logic and that it is based on three basic structural relations:

an *exchange* relation between logical positions
an *ordered* relation between logical positions
a *founding* relation which holds between the member of a relation and a relation itself.

It may be said that the hierarchy of logical themes as indicated in table (II) represents an hierarchy of implicational power. All themes have in common that they are self-implications; they imply themselves. However the first theme (objective existence) implies only itself and nothing else. In this respect it differs from any succeeding theme which implies itself as well as all subordinated themes. For this reason it is proper to call the initial theme "irreflexive" and all the following "reflexive". Irreflexivity means that something we think of is only an implicate but not an implicand for something else. On the other hand if we refer logically to reflexivity we mean that our (pseudo-)object of thought is an implicand relative to a lower order and as well an implicate relative to a theme that follows it in the hierarchy of table (II).

We are now able to establish the fundamental law that governs the connections between *exchange*-, *ordered*- and *founding*-relation. We discover first in classic two-valued logic that affirmation and negation form an *ordered* relation. The positive value implies itself and only itself. The negative value implies itself *and* the positive. In other words: affirmation is never anything but implicate and negation is always implication. This is why we speak here of an *ordered* relation between the implicate and the implicand. The name of this relation in classic two-valued logic is – inference.

It is now necessary to remember that the possibility of coexistence of two independent subjects (I and Thou) in the Universe is based on an *exchange* relation between equipollent centers of reflection. Moreover, these subjects are all capable of being implicands. More objects do not operate inferentially. That means they do not imply anything else.

If we now consider the *founding* relation in which a subject constitutes itself as diametrically posed relative to all objects and the *total* objective concept of the Universe we will discover that this relation represents an interesting synthesis of *exchange* and *order*. The *founding* relation is in itself an *exchange* relation in so far as the linking subject (S^S) may assume the logical position of the other subject which is thought of (S^O). S^O may in its turn assume the rank of S^S. Any two centers of subjective reflection of the same order mutually imply each other. But such an exchange does not operate between S··· and O. As we pointed out before: the bona fide object cannot infer the subject and by doing so usurp the role of a subject. If it could it would imply that subjects are irreflexive entities which for a subject is a contradictio in adjecto.

It follows that the relation between implicate and implicand has two different aspects: between two subjects this relation assumes the role of a symmetrical *exchange*. Between subject and object it appears however as an *ordered* relation. The *founding* relation is therefore also an ordered relation. Or to put it differently: the founding relation is a combination of *exchange* and *order*. What is the implicand (S^S) may become the implicate *not relative to* O but to our impartial observer S^{S^S}. We might say that the *founding* relation is a concatenation of sequences of *exchange* and sequences of *ordered* relations.

The diagram of Fig._6 will illustrate what we mean:

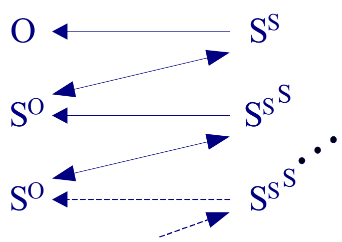

Fig_6

Fig._6 indicates a sequence of single-pointed and a second sequence of double-pointed arrows such that a single-pointed arrow always alternates with a double-pointed one. A concrete example of what the figure illustrates is the father-son relation. This is first an ordered relation. But the son can also become a father. In this sense father-son is also an exchange relation. But the son does not acquire the status of father relative to his own father but relative to the grandson of his father. In abstract terms: what is member (or argument) of the ordered relation $O \leftarrow S^S$, namely S^S, may become an argument of an exchange relation not relative to O but relative to S^{S^S} which implies this *exchange* $S^S \leftrightarrow S^O$.

Thus we may say: the *founding*-relation is an *exchange*-relation based on an *ordered*-relation. But since the *exchange*-relations can establish themselves only between *ordered* relations we might also say: the *founding*-relation is an *ordered* relation based on the succession of *exchange*-relations.[13] When we stated that the *founding*-relation establishes subjectivity we referred to the fact that a self-reflecting system must always be:

self-reflection of (self-and hetero-reflection).

As Hegel pointed out in his dialectic logic one and a half centuries ago, the opposition of hetero- and self-reflection is not a parity relation because it requires an iteration of self-reflection in contrast to the non-iterative character of hetero-reflection. It follows as was pointed out above, that one value is sufficient to designate in hetero-reflection but two values are required – apart from the value for object-designation – to separate self-reflection from the object. This is confirmed by the character of the *founding*-relation. Table (VI) clearly shows that it requires a minimum of three values for its own establishment.

But the introduction of a third value generates a new principle of superadditivity. In von Foerster´s case the super-additivity concerned only the increase of the classic negative value in a truth function. In the case of the *founding*-relation an increase in the number of two-valued systems is concerned. All "truth functions" of a three-valued system are compositions of three two-valued systems represented by the values 1+2, 2+3 and 1+3. For each value we might further add, we would obtain a new super-additive increase of (two-valued) systems. We can determine this increase in analogy to von Foerster´s formula $\Phi(x+y) > \Phi(x)+\Phi(y)$ by introducing the expression

$$\Phi(z) = 1/2z(z-1)$$

If z is composed of two terms, a and b, representing the poly-validity of two logical systems we have

$$z = a + b$$

The super-additivity we are looking for is then demonstrated by

$$1/2(a+b)\ (a+b-1) > 1/2a(a-1) + 1/2b(b-1)$$

where clearly the left hand side of this inequality exceeds the right hand side by

$$ab$$

This is nothing other than the cross-term interaction of a and b.

Thus a four-valued system which our impartial observer S^{S^S} would require must consist of 6 two-valued systems of reflection. In the case of a five-valued logic this number would increase to 10 two-valued subsystems.

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13 The author found that a practically identical formal pattern of the relation between symmetrical exchange and order was discussed in an earlier book by Karl Heim, *Das Weltbild der Zukunft*, Berlin 1904, esp. p. 35ff.; Heim calls its pattern "Grundverhältnis". However, it was developed for a very different purpose and it does not assume our initial state O.

The logical prototype of all totalities (Ganzheiten) is the system of consciousness. We know this at least since the advent of the Critique of Pure Reason. But consciousness involves as we have seen a *synthesis*, of the two most basic relations in logic: the symmetrical *exchange* of values and the hierarchal *order* of values. *Exchange* and *order* are combined in a new codifiable principle which we call the *founding* relation. This principle establishes the totality of consciousness but since it is entirely formal it also governs the structural laws of any totality we may conceive as such.

Already in 1950 L. von Bertalanffy wrote in an essay on General Systems Theory "that many concepts which have often been considered as anthropomorphic, metaphysical or vitalistic, are accessible to exact formulation." [14] However, what is still missing in General Systems Theory is the representation of such concepts as exemplifications of a universal formal theory of totalities grounded in the concept of logical value and its operation by affirmation and negation. This paper tries to make a contribution in this direction following the example given by Hegel.

vordenker
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14 Cf. L. v. Bertalanffy, *An Outline of General Systems Theory*, Brit. Journ. f. the Philos. of Science I, 2 (Aug. 1900) p. 134-165.