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MANY-VALUED DESIGNATIONS AND A HIERARCHY OF FIRST ORDER ONTOLOGIES

Remarks concerning a philosophical interpretation of many-valued systems

In two-valued logic the distinction between affirmation and negation happens to coincide with the dichotomy between designation and non-designation. It will be shown that this is not the case for many-valued systems. It will be further shown that the distinction between affirmation/negation on the one hand and designation/non-designation on the other leads to the concept of a hierarchy of first order ontologies [1], each with a corresponding sequence of systems of logic.

We shall define an 'ontology' as a structural system in which the distinction between designating and non-designating values is inapplicable, and which is determined by nothing else but the number of values available. In an ontology all values designate. However, if values permit a division between designation and non-designation, the system in question may be considered a 'logic'.

It is obvious that a one-valued system must be considered an ontology. Our familiar two-valued system of classic origin on the other hand qualifies as a logic. In it the relation between affirmation and negation is represented by Table I,

Table I

x	N(x)
1	2
2	1

where the first integer may denote affirmation and the second integer negation. Negation may thus be interpreted as a symmetrical exchange relation of two values, where the initial application of the negation to any value generates a successor-value m+1 and the once iterated application reproduces its immediate predecessor (m).

Assume we have a many-valued logic with m values i (i = 1, 2, 3, ..., m) which are generated by a successor operation i' = i + 1 (1 < i < m) where the successor of m shall be, by definition, the initial value of the system, say m' = 1. Starting from the two-valued exchange procedure in Table I, one might extend this modus operandi into many-valued systems. By doing so we observe that in any m-valued logic only m–1 independent negations N; exist.

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1 We understand by "first order ontology" a theory of Being (ὄντως ὄν) in contrast to the plurality of second order ontologies referring to the plurality of classes of existing objects.

Table II

x	$N_i(x)$
1	1
2	2
:	:
i – 1	i – 1
i	i + 1
i + 1	i
i + 2	i + 2
:	:
m	m

Table II may convey the impression that there are m independent negations because

$$N_m(m) = 1.$$

But N_m is not an independent negation because it can always be replaced by a suitable combination of the preceding negators. If we speak of independent negations we mean negational procedures which produce by initial applications only values which have not previously occurred in the system. Only the reversal of an independent negation (its first iteration) may obtain a value already available in the system. However, if $N_m(m)$ then both the initial negation and its first iteration produce values which have occurred in exchange operations $N_{m-n}(m - n)$ where $n < m$. It is useful for the logician to refuse $N_m(m)$ the character as a bona fide negator because by doing so one obtains a criterion for the distinction between positive and negative values. *A value is considered positive if it cannot be generated by an independent successor operation*. Any value generated by such an operation is considered to be negative. It follows that, no matter how many values a system has, only one value will be positive. Thus, all m-valued systems where $m > 2$ shall be considered asymmetrical with regard to the distinction between positiveness and negation.

Turning to the problem of designation and reminding ourselves that the difference between affirmation and negation coincides in two-valued logic with the dichotomy between designation and non-designation, we may obtain a deeper understanding of what a non-designative value is, if we consider the well-known isomorphism of two-valued logic[2] which is associated with its symmetry of affirmation and negation and the fact that the negative value is always *the* non-designative value. This isomorphism implies that, semantically speaking, the second (negative) value does not represent anything new which is not already designated by the first value. In other words, the non-designative value is a "repeater-value". It merely iterates (reflects) what is already available through the designating value. It follows that, if we intend to designate something different from what a one-valued ontology presents, it cannot be done by the second value. Ontological differences mean, in formal logical terms, different degrees of richness (complexity) of structure. But such structural difference can only be generated by a *difference in the number of values employed*. Thus if we

2 Cf. Reinhold Baer: "Hegel und die Mathematik"; Transactions of the II. Hegel Congress, October 18-21, 1931, Publ. Mohr, Tübingen.

intend to designate something apart from what is the ontological concern of a two-valued logic, we would require a plurality of values for any designational intents – and *non-designation would have to repeat this very plurality* in order to obtain a logic for such a many-valued ontology.

What would then be an adequate definition of the distinction between designation and non-designation values in a given m-valued system? Keeping in mind the concept of the non-designative value as "repeater-value", non-designation is, in many-valued systems, represented by the *excess* of values remaining after collecting complete designational systems of ascending valuedness. For it is precisely this excess of values that cannot but repeat one of the previously collected systems of designation.

The first system of designation would be represented by one-valuedness and if a second value is available it could do nothing but *repeat* this one valuedness and thus attain non-designational character. Thus a two-valued system would be "mono-thematic". (Designation defines an ontological theme.) If we add a third value we are able to define two themes, one based on one- and a second theme based on two-valuedness. However, no repeater-value is available in this case to mirror thematic one-valuedness, and thematic two-valuedness cannot be reflected by one-valuedness anyway. Adding a fourth value would alter this situation. After designating two themes, one value is left over to repeat the one-valuedness of the first theme. If five values are at our disposal we have an excess of two values over the values absorbed by our two themes and this excess mirrors the two-valuedness of the second designational theme. But the addition of another value produces a third ontological theme of three-valuedness, and the value-excess disappears.

Table III provides us with an example of designational value distribution in an m-valued system where m = 10.

Table III

m	collected designational systems	excess of values
10	1 2 3 4	0

A 10-valued system obviously has no such excess of values available for non-designation. According to our definition it is an ontology with four ontological classes of objects referred to by 1-, 2-, 3- and 4-valuedness.

Table IV

m	collected designational systems	excess of values
24	1 2 3 4 5 6	3

(3 ├── repetition ──→ 3)

A 24-valued system, on the other hand, is a logic and not an ontology if we understand by "logic" a system which divides its values into designational and non-designational values. We notice that six ascending modes of designation are available which leaves an excess of three values for non-designation. The question whether it would be permissible to split the excess of values and to argue that in a 24-valued system we face an alternative of having either three-valued designation repeated in non-designational values, or one- and two-valued designation singled out for non-designational repetition will be discussed later.

Generalization of the procedure exemplified in Table III and IV is obtained by observing that the excess of values in an m-valued system is equal to the difference between the sum of consecutive integers 1 $\rightarrow$ k:

$$\sum_{1}^{k} i = \frac{1}{2}k(k+1)$$

$$\text{where} \quad k = \begin{cases} p = \frac{1}{2}\left(\sqrt{1+8m}-1\right), \text{ if } p = \text{integer} \\ \text{next integer} > p, \text{ if } p \text{ is not integer} \end{cases}$$

and m = number of values available [[3]].

Let us now turn to the hierarchy of ontologies, i.e., the structure with no value-excess after the collection of designational systems with ascending valuedness. Table V displays the first four of them, three with their full retinue of logical systems.

Table V

m	des.	non-designation				character of system	interval
1	1	0				ontology (mono-thematic)	I
2	1	1				logic	
3	1	2	0			ontology (dia-thematic)	II
4	1	2	1			logic	
5	1	2	2				
6	1	2	3	0		ontology (poly-thematic)	III
7	1	2	3	1			
8	1	2	3	2		logic	
9	1	2	3	3			
10	1	2	3	4	0	ontology (poly-thematic)	
11	1	2	3	4	1	logic	
:	:	:	:	:	:		

The extreme left column of Table V indicates the total number of values (m) available. The double line descending in staircase fashion divides designation from non-designation. The numbers appearing on both sides do not indicate individual values but give the sum of values required for a specific designative or non-designative purpose. If a zero appears, no value is at our disposal for non-designation. According to its numbers of designated themes, every ontology is followed by a corresponding

[3] The author owes this formula to H. von Foerster.

number of logical systems and, if the value-excess for non-designation is not split up, each logical system "focuses" on one of the designated themes offered by the preceding ontology. The theme thus mirrored by non-designation is underlined in Table V. Each ontology, together with the logical systems which follow it, forms what we shall call a logical interval. The number of values required to start a new interval is computable by

$$\frac{m(m+1)}{2}$$

Thus the m-th interval where m = 7, e.g., is initiated by a system of 28 values displaying a choice of seven ontological themes for logical reflection in the systems with 29 to 35 values.

The fact that an ontology, provided it displays a certain richness of themes, is followed by an ascending order of logical systems suggests a short remark on the familiar distinction between object-language and meta-language. According to the iterative capacity of self-referential systems any object-language may be arbitrarily iterated in meta-languages as often as we wish. But none of these repetitions increases the number of values which are employed in the object-language. This means that the question of the logical structure of the subject which has the marvelous capacity of iteration never enters this endless sequence. There is no meta-language in which it can be discussed because there is no ultimate meta-language which ends the iterative process: in the realm of meta-languages, the thinking subject manifests itself as nothing but bottomless iterativity.

In contrast to the traditional viewpoint – which has its own legitimacy not contested on these pages – the theory of many-valued logic suggests that in a different respect the iterative capacity of a thinking subject has certain limits. Without such limits we might be able to think but we would not be able to be aware of our process of iterative reflection. Awareness of iteration implies the power to put a stop to it. Such capacity to step out of the iterative process making it an arrested object of our reflection might be dependent on the richness of ontological structure a subject faces. Especially as the structure of consciousness itself must be an exact corollary of the ontology it possesses. The logical systems which follow their ontologies in our Table V may, on the other hand, in a different sense also be considered iterations since a four-valued system e. g. contains and repeats in its own structure one-, two-, and three-valuedness. However, iteration in this case implies both a repetition of the old and a gain of something new . . . a gain in additional structure, albeit such gain is limited by the scarcity or richness of structure which the preceding ontology implies.

So far we have assumed that each logical system following a specific ontology is monothematic. It "focuses" on only one theme, may that be two-, three-, four-, or any other singularity of valuedness. We expressed this assumption by stipulating that, for the time being, the excess of values available for non-designation should not be split up into smaller groups of values. We shall now drop this requirement. The issue of dividing the excess becomes topical in the second interval which contains two systems of logic. The four-valued system has only an excess of one value but in the case of five-valuedness the excess is two. The preceding ontology is dia-thematic. It represents a one- and a two-valued theme. But if we would split up the two-valuedness of non-designation in the five-valued system into two separate non-designative values, we would obtain, in Table V, the following pattern of redundancy with regard to non-designation for the one-valued system. For convenience, the following Table VI illustrates the redundancy.

Table VI

m	designation	non-designation
5	1 2	1 1
	repetition	
	repetition	

The case is different if the splitting up of the value-excess does not result in redundant patterns of non-designation. We shall consider, e.g., a logic with 20 values. Table VII displays its possible modes of non-designation presented separately.

Table VII

m	designation	non-designation
20	1 2 3 4 5	2 3
	repetition	
	repetition	
20	1 2 3 4 5	1 4
	repetition	
	repetition	
20	1 2 3 4 5	5
	repetition	

In the first two cases for m = 20, the value-excess available for non-designation is distributed once over two- and three-valuedness and then over one- and four-valuedness. In the third case the value-excess has suffered no splitting up. Table VII differs from Table VI insofar as, in the latter, no redundancy occurs in the assignment of values (or systems) of non-designation to different distributions. However, as Table VI shows, not all such partitions are admissible. Only those which are partitioned into unequal parts. Table VIII displays unrestricted partitions of integers from 1 to 5. The partitions admissible for non-designational purposes have been underlined.

Table VIII (unrestricted partitions)

_1_	_2_ _1.1_	_3_ _2. 1_ 1.1.1	_4_ _3.1_ 2.2 2.1.1 1.1.1.1	_5_ _4.1_ _3.2_ 3.1.1 2.2.1 2.1.1.1 1.1.1.1.1

The question remains to be answered – what philosophical problem is involved in the partitions of the non-designational value-excess? We linked many-valuedness with self-reference. No self-reference is possible unless a system acquires a certain degree of freedom. But any system is only free insofar as it is capable of interpreting its environment and choose for the regulation of its own behavior between different interpretations. The partitions of the value-excess indicate exactly such choices. The fifteen-valued ontology in our system of twenty values is capable of three different interpretations according to which partition of the value-excess is chosen (we consider excess divided by one also a partition). Since the ontology in question has a range of five object-classes certain combinations are permissible for designation. The richness of choice depends on the magnitude of the value-excess offered by the logic which follows.

We shall conclude our presentation with an example of such value choice and a possible logical interpretation of it. Let us take a universe where we may distinguish between bona fide objects (a stone, for instance) as one ontological class and subjective thoughts as another. Such three-valued ontological structure is followed by two logical systems with either an excess-value of one or two. Since bona fide objects are sufficiently designated by a single value, and thoughts, to be distinguished from them, must at least be designated by two values choosing an interpretation of objectivity entails a switch from one logical system to another. Obviously a three-valued ontology is not rich enough in structure to allow for the distinction between two ontological classes of Being in one and the same system of logic. In order to achieve it, we would have to proceed to interval III. But the details of such procedure go beyond the space allotted to this paper.

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