

The Tradition of Logic and the Concept of a Trans-Classical Rationality

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Nobody can seriously doubt that the development of logic has made enormous strides forward during the last century. This is mostly due to the close alliance which has taken place during that period between this once purely philosophical discipline and modern mathematical methods. Both sides have gained from this union. Especially logic! Its progress has since been phenomenal. However, it will pay to have a critical look at these modern advances and find out what has been gained and in which direction no progress has been made at all. Because, as we will show, there is a field where logic still stagnates as much as it did at the time when Kant made his famous complaint about it in the preface to the second edition of the Critique of Pure Reason.

Let us first have a look at the gains and determine what is their common feature. When, about the middle of the 19th century, mathematicians started to have a closer look at logic they were confronted with a well established formal "system" (with a minimum of symbolism) which dated back to the Organon of Aristotle and to some fundamental concepts first expounded in the dialogues of Plato. A more intimate investigation during the following decades showed that this logical tradition represented anything but a completed and satisfactory system. It could at best be called the fragment of a formal logic. It was incomplete even within its own narrow limits. It lacked sufficient formalization and its operational principles were not well enough defined. Moreover, many procedures which play a dominant part in modern logic were not even discovered. Under the circumstances one is rather tempted to say – with not too much exaggeration –: what the tradition had handed down was just the program or the idea of a formal logic but not the thing itself.

It is obvious that, since the preliminary investigations of De Morgan, Boole and others, the time up to our present day was well filled with implementing this program which the classical Greek thinkers and the medieval tradition had set up for us. The advances made since logic was really developed within the new medium of abstract calculi are so overwhelming that it would take a heavy volume to recount them in detail. It is not necessary anyhow. The professional logician knows them well enough. There is, however, a common feature which all these, new discoveries share – a feature which is rarely in the conscious mind of the practicing logician –: whatever has been accomplished in the modern "revolution" of logic is nothing but the conscientious fulfillment of a plan or program that was conceived and formulated in the hey days of

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classical Greek philosophy. At that ancient time it was mostly expressed by means of mythological or metaphysical concepts. Today we are trying to execute the very same program in purely formal and mathematical terms. In other words: no matter how far advanced and how variegated our modern techniques of logic may be they still hail back to the same ancient metaphysical background.

This implies that the very last philosophical foundations of our logical tradition are neither seriously discussed nor radically questioned. One takes them for granted.

The author of these reflections confesses that he had to muster up some courage to make this statement. All appearances seem to refute him. Do not the schools of Logicism, Formalism and Intuitionism prove by their very existence how intensely the philosophic foundations of modern logic are debated! Is not the deep rift between logical Platonism and Constructionism proof enough that the most basic issues are at stake! Of course, nobody can deny that philosophical controversies are involved in the development of modern calculi. But – and this is our point – they take place only within the confines of classical Rationality. And if the arguments sometimes become so aggressive that recently an outstanding logician had to point out the need for tolerance¹¹ one has to stress the undeniable fact that all these intensive debates and sharp controversies only prove how fanatically all the participants believe in the problem they are trying to solve.

There is a deep and lasting agreement between all modern schools of logic and an almost fanatical consensus that what the tradition has handed down to us as "the" problem of logic is the only problem indeed.

It will be useful to examine this sacrosanct tradition. One can summarize it in the following three "dogmas":

1. the dichotomy of form and matter is relevant for any system of pure logic.
2. the concept <object> is non-ambiguous.
3. the semantic relation between Truth... Falsity (including a scale of intermediate terms) and logical values is unique.

The interdependence of these three tenets is obvious and equally clear is that only a two-valued type of logic can satisfy them all. This accounts for the vacillating attitudes logicians have taken toward the problem of a many-valued logic. The initial enthusiasm with which many-valued systems were tackled when they first came up in 1920 has gradually waned¹². In philosophical logic the animosity against these systems has been almost unanimous, using the unrefutable argument that the principle of many-valuedness violates the classical concept of truth¹³. And it is interesting to note

¹ Cf. Heinrich Scholz und Gisbert Hasenjaeger, *Grundzüge der mathematischen Logik*, 1961, p. 11f.

² I.M. Bochenski, "Die Fachlogistiker, die einst diese Systeme mit Enthusiasmus begrüsst haben, stehen ihnen heute zum grössten Teil sehr skeptisch gegenüber". *Der sowjet-russische dialektische Materialismus*, 1956, p. 132.

³ Cf. Paul F. Linke, *Die mehrwertigen Logiken und das Wahrheitsproblem*, *Ztschr. f. Philos.* III (1948) p. 378 ff. and p. 530 ff. Also: B. v. Freytag-Löringhoff, *Logik*, 1955, p. 177 ff.

that already in 1932 a scholar who is equally at home in philosophic as in symbolic logic declared with reference to many valued theories: "The attempt to include all modes of classification, and all resultant principles, would produce, not a canon, but chaos".^[4]

This was written 30 years ago and it cannot be said that the situation has improved since. We shall cite only one more witness for the most recent time with the statement: Bei "Einführung von mehr als zwei Wahrheitswerten ... gelangt man, sofern man sie wirklich als *Wahrheitsmodi* auffassen will, zu offenbaren Aporien der Interpretation, die sich auf keine zwanglose Weise überbrücken lassen."^[5]

This is the point where, despite the rapid advances during the last century, an area of stagnation is still visible within the domain of logic. The stubborn adherence to the three philosophic "dogmas" of classic tradition, and the consequent obsession with the idea that any logical value of any system whatsoever must be interpreted as a truth value, has blinded scholars in the realm of logic to the fact that the acceptance of many-valued procedures constitutes an actual defection from the classic basis of scientific thought. In view of the fact that quantum mechanics has also departed from this basis^[6] it seems natural that logic should not put the clock back nor refrain from sailing further into the treacherous waters of many-valuedness. But if this venture is really undertaken that last pool of stagnation in logic must be finally stirred up by a thorough investigation into the limits of the generality of our three classic "dogmas" of logic.

Since 1953 this author has tried to make a start in this direction with a series of publications^[7] all of which attempt to deal with the proposition that the so far uncontested classic definition of logic should be abandoned in favor of a broader one. As philosophical maxims for this new trans-classic logic we suggest:

- 1a. the dichotomy of form and matter does not hold in n-valued systems where $n > 2$.
- 2a. the concept of 'object' is amphibolic^[8] when $n > 2$.
- 3a. the disjunction truth/falsity applies as value designation if and only if $n = 2$.

⁴ C. I. Lewis, *Alternative Systems of Logic*, The Monist XLII, 4 (1932), p. 507.

⁵ H. Arnold Schmidt, *Mathematische Gesetze der Logik I*, (1960), p. 125, also p. 370 ff.

⁶ The half-hearted attempt of Hans Reichenbach (*Philosophic Foundations of Quantum Mechanics*, 1946) to demonstrate this departure logically could not really succeed because he was not able to rid himself of the classic prejudice that 'logical value' and 'truth value' are synonymous.

⁷ *Die philosophische Idee einer nicht-aristotelischen Logik*, XI. Int. Congr. Philos., Brussel (1953), V; p. 44-50 – *Dreiwertige Logik und die Heisenbergsche Unbestimmtheitsrelation*, Int. Congr. Philos. of Science, Zürich (1954), II; p. 53-59. – *Metaphysik, Logik und die Theorie der Reflexion*, Arch. Philos. (1957), VII., 1/2; p. 1-44 – *Die Aristotelische Logik des Seins und die nicht-Aristotelische Logik der Reflexion*, Ztschr. f. Philos. Forsch. (1958), XII, 3; p. 360-407 – *Ein Vorbericht über die generalisierte Stellenwerttheorie der Logik*, Grundlagenstudien aus Kybernetik und Geisteswissenschaft (1960), I, 4; p. 99-104.

⁸ Cf. I. Kant., *Die Kritik der reinen Vernunft*. B, 316 ff. (Von der Amphibolie der Reflexionsbegriffe).

In the first volume of his "Idee und Grundriss einer nicht-Aristotelischen Logik" (1959) this author has endeavored to outline the historic antecedents and to develop – on a purely philosophic basis – the systematic concept of a field of genuine trans-classic rationality. There are abundant historic antecedents in Kant (his *Transzendente Dialektik*) Fichte, Hegel and Schelling, and since they all converge in that enigmatic product which Hegel calls "Logik" it seemed advisable to concentrate on him. However, that should not be construed as an attempt to vindicate the "spekulative Logik" in the eyes of modern symbolic Logic or even to amalgamate the two. This is clearly impossible. On the other hand: there can be no doubt that the *Deutsche Idealismus* has discovered a new systematic problem for Logic! It is the phenomenon of self-reflection. Kant, Fichte, Hegel and Schelling have stoutly maintained that this phenomenon, although "logical", is not capable of formalization.

It is the main thesis of "Idee und Grundriss..." that the datum of self-reflection (consciousness) is fully amenable to formalization. The resulting calculus would be the backbone of the New Logic. Its basis would be represented by the trans-classical maxims, 1a, 2a and 3a containing the classic tenets 1, 2, 3 as the ontologic subsystem. This author is convinced that many propositions of Hegel's logic would lend themselves to treatment within a calculus⁹. However, in view of the main goal this is incidental and it would be the business of the mathematician but not that of the philosopher!

The phenomenon of reflection has, of course, always played its part in symbolic calculi. Yet no formal criterion for self-reflection has been discovered. We see the nearest approach to it in the theory of Intuitionism. The emphasis on construction is a sort of self-reflection which the more traditional methods lack. But here too the tenacious adherence to the idea of equating value and truth-value has impeded the final deliverance from the Greek Tradition. Thus – in the principal philosophic sense in which we use the term – Intuitionism still belongs to classic mathematics! It follows that many statements that mathematicians make about intuitionistic procedures cannot be accepted at their face value.

The crux of the matter is, of course, the question: what is self-reflection and why can its laws not be developed in two-valued logic? Let us first tackle the semantic side of the question. With the alternative: is this true or false, we miss the whole point of the problem. Because as soon as we begin to talk about self-reflection we have ceased to refer to the original classic situation where a thinking subject naively (= without reference to itself) faces a universe of (thought) objects. Instead of it we want to know: what laws of reflection govern the opposition between Subject and Object? It is easy to see where the fundamental difference between classic and trans-classic theory of thought lies. The first, not referring to the subject of reflection, uses reflectional structures exclusively for the description of objects in the most general sense of the term. The second refers expressly to the phenomenon of subjectivity and investigates the tripartite division between individual subject, general subjectivity and object. This obviously calls for a three-valued formal system. Its advantages for a theory of reflection are clear. It can, provide us with a formalized language that permits us to

⁹ Cf. A. Speiser, *Elemente der Philosophie und der Mathematik*, 1952, Esp. from p. 83 on.

treat a term as subjective as well as objective reflection in the very same context. In the classic system it was always an exclusive either-or. But this conjunction of Object and Subject is exactly what confronts us in the world that surrounds us. As everybody knows, it is not made up of bona fide objects alone but also of subjects which are, like everyone's own ego, centers of reflection and claim to be treated as such. From a logical point of view, however, they are to me theoretical objects of thought. In other words: subjectivity is a phenomenon that appears in distributed as well as in non-distributed form. In its non-distributed form we call it an individual subject. If it is in distribution we refer to it as the intersubjective medium of general subjectivity. In contrast to it: the objectivity of an object is never distributed. The isolated object is fully identical with itself. For a subject this is an unattainable ideal. In its non-distributed form it is merely our thought-object and not a subject in its own right.

It should now be understood if we say that the classic, two-valued logic describes our system of formal rationality as an undistributed order of concepts. This is done by vigorously excluding any reference to the thinking subject^[10]. Or – to express the same fact in syntactical terms – by permitting just one negational operator as a means to establish a symmetrical exchange relation between two values. This leads to, the famous *coincidentia oppositorum* of Nicolas Cusanus as Reinhold Baer has pointed out^[11]. Apart from earlier philosophical attempts it has so far been mostly L. E. J. Brouwer's criticism of the Tertium non datur which can be taken as a symptom that the need for a distributed system of rationality was more or less clearly felt. However, it seems that Intuitionism does not go far enough. The principle of distribution should not only affect the Tertium non datur but Contradiction and Identity as well. This can only take place in a genuine many-valued system and Intuitionism is not such a system.^[12]

There seems to be only one way to effect a consistent distribution of rationality for Identity as well as for Contradiction and Tertium non datur, namely, the introduction of a general m-valued system where $m > 2$. But our proposal differs fundamentally from all previous attempts. Instead of interpreting a many-valued system as a true-false theory with an intermediate sequence of indeterminacies or modalities we declare any m-valued logic to be a place-value system of all subsystems of the value order m-1, m-2, m-3, m-n, such that $m-n = 2$. It can be easily seen that a three-valued logic provides three "places" for the classical two-valued logic, because the latter is represented in the larger order by the subsystems $1 \leftrightarrow 2$, $2 \leftrightarrow 3$, and $1 \leftrightarrow 3$.^[13] Similarly a four-valued logic offers 6 places for two-valued and 4 places for three-valued

¹⁰ This is what E. Schrödinger calls "the principle of objectivation" which demands that "we exclude the Subject of Cognizance from the domain of nature that we endeavour to understand," *Mind and Matter*, 1958, p. 38.

¹¹ Reinhold Baer, *Hegel und die Mathematik*, In: Verhandlung. d. 2. Hegelkongresses v. 18-21. X. 1931. Ed. B. Wigersma, Tüb. 1932.

¹² This was stated only recently by H. Arnold Schmidt who remarked "dass die intuitionistische Logik überhaupt keine 'mehrwertige' Logik ist!" *Mathematische Gesetze der Logik* 1,(1960), p. 370.

¹³ More relevant details are given in the following publications of the author: "*Die Aristotelische Logik des Seins und die nicht-Aristotelische Logik der Reflexion*", and also in the "Vorbericht", Cf. footnote 7.

sub-systems. Generally speaking, the numbers of all possible subsystems of any m -valued structure are identical with the corresponding numbers in the Table of Binomial Coefficients.^[14]

It stands to reason that no m -valued structure ($m > 2$) that is interpreted as a logical place-value system can be used in the classical truth functional sense. Since the very same two-valued logic may turn up in any number of places, and since nobody will sincerely subscribe to the "atrocious monstrosity" (Schrödinger) of a roof-mind having an indefinite number of sub-minds which semi-independently judge statements as true or false, the only natural solution is to understand these larger systems as inter-subjective patterns of reflection that distribute our unique, classic two-valued logic over the total range of Object and Subject. It is absurd to assume that any individual consciousness could ever manifest itself in anything but a two-valued logic. In this sense the classic system of thought is archetypal and canonical! But if any thinking subject faces the world it discovers that its environment displays this very same logic in a wide (possibly infinite) pattern of distribution. This pattern possesses a primordial polarity: the opposition of the I and the It. But since the Universe for any given center of thought (scil. subject) contains not only bona fide objects but other centers of thought (scil. objective subjects) too, the reflectional pattern of our archetypal logic is distributed over all these centers.

The classic Tradition has ignored this fact as far as formal logic was concerned. It has done so with very good reason. Because as long as the pure and undiluted objective character of the Universe – which is faced by all thinking subjects alike – seems to be the sole theme of theoretical thought there is no problem about the inter-subjective generality of our conceptual terms. It is evident that, if any two subjects A and B agree about a given object O they also agree with each other. Consequently A, B and any other subject C that is in the same position can be treated as a single logical subject. The result is the absolute dichotomy of subject and object to which the two-valued system precisely corresponds. It all boils down to the time-hallowed concept of a universal, absolute (divine?) Self into which all individual minds merge if they think in strict logical terms. It is obvious that, if subjectivity, qua subjectivity, plays any part at all in this logic it does so only in its non-distributed form... having one, solitary center of reflection. It should be equally obvious, that, if computer-theory aims at defining a mind in rigidly objective terms, our classic Tradition is principally insufficient. Or shall we assume that an automaton that is catching up with us in handling problems of logic also undergoes a mysterious merging into an absolute Subject? This is absurd if not blasphemous.

Ergo, we have no choice but to assume that, if the bona fide object also handles formal logic and establishes theoretical agreement with us (or we with it), inter-subjective communication which "transcends" the shell of the isolated individual self is based on a distributed system of Rationality where the very same logic may be activated (as a complete system) in a minimum of three ontological "places": (1) in the individual isolated subject as apart from the world; (2) in the isolated object. And (3) in a system that describes the difference between (1) and (2). A three-valued logic fulfills these

¹⁴ I am indebted to Professor Heinz von Foerster (University of Illinois) for having drawn my attention to this fact.

minimum requirements. But since there is more than one individual subject in the Universe the subsequent introduction of higher valued systems is also required. They all serve the same purpose: to distribute our unique classic order of two-valued rationality over larger and larger place-value systems. Their infinite order implements the concept of a trans-classical (non-Aristotelian) rationality. The rationality of Reflection which embraces that of Being.

To sum it up: A non-Aristotelian or trans-classical logic is a system of distributed rationality. Our traditional (two-valued) logic presents human rationality in a non-distributed form. This means: the tradition recognizes only one single universal subject as the carrier of logical operations. A non-Aristotelian logic, however, takes into account the fact that subjectivity is ontologically distributed over a plurality of subject-centers. And since each of them is entitled to be the subject of logic human rationality must also be represented in a distributed form. The means to do this is to interpret many-valued structures as place-value systems of our two-valued logic. In any m-valued logic our classical system is distributed over $\frac{m^2 - m}{2}$ places.

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