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Number and Logos

Unforgettable Hours with Warren St. McCulloch

The author of these remembrances (from now on only the 'author') feels painfully that he is in an awkward position. He intends to show a side of Warren McCulloch which is not very well – if it all – known and which hardly becomes visible in the publications of this very great man and first rate scientist: we refer to his importance and profundity as a philosopher. He was aware and very intensely so – of Cybernetics as a discipline *sui generis* that needed a novel philosophic foundation to distinguish it from the conventional disciplines. This conviction of his finally led to the meeting with the author – a contact which lasted almost a decennium. The quandary the author finds himself in stems from the fact that he entertained and still entertains almost identical views about the relation between cybernetics and philosophy as McCulloch and finds it therefore almost impossible to perform a clean separation of his own ideas from those of McCulloch. He is only sure that the thoughts he expressed on cybernetic topics are fully his own up to the publication of his "Cybernetic Ontology and Transjunctional Operations" which came out in 1962. Although McCulloch is already quoted in this essay it was done solely with the intent to appeal to his authority for ideas which the author had entertained for quite a while.

The contact between the author and Warren McCulloch was established after Dr. John Ford, then at the George Washington University, had given McCulloch in 1959 a German paper of the author "Die aristotelische Logik des Seins und die nicht-aristotelische Logik der Reflexion" which had come out in Germany in 1958. He is still intensely grateful to Dr. Ford for having made this connection which was bound to change his total outlook on philosophy. However, it took some time before he really understood what had attracted Warren McCulloch to his paper. It was not so much its potential applicability to cybernetics but a hidden relation that it revealed between number and logical context. When the author wrote it he opined that a non-Aristotelian Logic is nothing but a place value system of innumerable logical sub-systems of Aristotelian (two-valued) character. His interest was at that time wholly conceptual and he did not even dream that a hidden arithmetical issue might lead into deeper foundational layers of Cybernetics. Here McCulloch was far ahead of him.

Their intellectual collaboration started in earnest when some evening the author had made a stop-over on his yearly trip to New Hampshire – McCulloch led the talk to the Pythagoreans and their theorem that numbers describe the ultimate core of Reality. Although the author pressed for a detailed explanation all he was told at that time was that to find out more was exactly his own business. It was the first time that the author encountered a peculiar reticence of McCulloch's regarding ontological or – more precisely – 'metaphysical' questions. It led him to grossly underestimate McCulloch's gifts and intuitions in this direction. He was confirmed in his faulty judgement when he noticed that McCulloch never bothered to make corrective remarks when a paper which was read at a congress or symposium where he was present obviously implied

metaphysical assumptions which had to be partly or totally wrong. First he assumed that McCulloch was not aware of it; later however the author knew better. Nevertheless he must confess that during the whole duration of his acquaintance and – as the author hopes friendship McCulloch never gave up his reluctance to criticize the course cybernetics was taking with relation to Philosophy. Only after McCulloch's death he learned that his mentor in Cybernetics had been as dissatisfied as he himself with the lack of fundamental ontological orientation that characterized – and still characterizes – the pursuit of cybernetic theories. But he came to understand very soon how much McCulloch saw his own endeavors within a novel metaphysical frame. The revelation came one evening when McCulloch started to talk about Martin Heidegger and produced a copy, very shabby and dilapidated from intensive use, of "Sein und Zeit".

The book had originally belonged to his friend and co-worker Eilhard von Domarus, so he explained; he in his turn had studied it carefully and he now wanted to give it to the author for renewed study because the latter had confessed that he did not care very much for Heidegger's philosophy. The expression of thanks for the unexpected present must have sounded rather reluctant because McCulloch grew very eloquent and insisted that the "Nichts" (Nought) in Heidegger's philosophy was precisely the ontological locus where the central problem of cybernetics was located, namely the mapping of the process of Life onto matter per se inanimate. BEING is both: subject and object as well; but western philosophy has fallen into "Seinsvergessenheit" (oblivion of ultimate Reality) since the time of the Greek. Which in McCulloch's view meant: it did not focus on the problem of cybernetics. In classic philosophy mere objectivity without self-reference is mistaken for "Sein". When McCulloch commented on Heidegger with these remarks the author knew he had underestimated his philosophical gifts. His detailed knowledge of "Sein und Zeit" and especially his discussion of this "Nichts" gave the author's metaphysical thinking a new direction and made him look for the roots of Cybernetics in the ultimate and primordial recesses of the Universe.

Since the spiritual contact point between McCulloch and the author happened to be their common interest in the transcendental relevance of logic in other words: how much and what information logic conveys about the world that surrounds us – it was only natural that the author wanted to know from his partner what he meant by the term 'metaphysical'. For a start he was referred to the "Mysterium Iniquitatis ..." and the notions that "prescribe ways of thinking physically about affairs called mental ..." It stands to reason that this answer left the philosopher dissatisfied and it surely did not cover McCulloch's own – very ambivalent appreciation – of Heidegger. This was admitted; and then McCulloch started to express thoughts which went far beyond the metaphysical references imbedded in papers like the "Mysterium Iniquitatis" "Through the Den of the Metaphysician", "What is a Number..." and others. He drew the author's attention to the fact that any logic or calculus Man may ever conceive is nothing but a more or less competent formalization of ontological concepts. This ideas was, of course, not new and may be easily extracted from his writings as ever present implication. But it showed that he had wandered much deeper into the grottoes of metaphysics than he was inclined to express explicitly in his papers. At this juncture the author thinks it fitting to remind the reader of the quotation of Clerk Maxwell appearing in "Through the Den of the Metaphysician" about the relation between thoughts and the molecular motions of the brain: "does not the way to it lie through the very den of the metaphysician, strewn with the bones of former explorers and abhorred by every man of

science?" McCulloch comments this quotation with a "Let us peacefully answer the first half of this question 'Yes', the second half No', and then proceed serenely."

While there can be no doubt that he never abhorred the den of metaphysics his texts show a pronounced reluctance to analyze in detail the accoutrements of Transcendence. On the other hand, this reluctance disappeared almost completely when speculating on the pertinent issues in the presence of a person who was much more at home in the realms of the Transcendental than in the empirical ways of Cybernetics as happened to be the case with the author.

From Heidegger's "Nichts" the discourse went to Kant and Hegel. The author must confess that he was somewhat surprised when he discovered that McCulloch understood that Kant's philosophy closes an epoch of philosophical thought and that Hegel opens a new one. He knew this, of course, himself, – that was after all his business – but he had interpreted it in terms of the distinction between 'Natur- and Geisteswissenschaft' and the pseudo-systematic development of the latter in the Hegel-Renaissance since 1900. Of the Hegel-Renaissance and its concomitant intellectual events McCulloch was hardly aware. Even if he had been familiar with it: the metaphysical gap between matter and mind or subject and object which was emphasized by the Geisteswissenschaft could not be accepted by any cyberneticist, least of all McCulloch. Consequently, he explained the distinction between Kant and Hegel by pointing out the different view of Dialectics entertained in the Critique of Pure Reason and in Hegel's Logic. Kant deals with Dialectics in the sense of the Platonic tradition and in the Critique of Pure Reason the dialectic argument ends in the transcendental illusion as the unavoidable admixture of error that infiltrates all metaphysical assertions. Thus Kant's evaluation of Dialectics is basically negative and the less we imbibe of this poisonous drink the better off we are. For Hegel, on the other hand, he explained, the dialectic structure is a legitimate element of thought as well as of objective existence and it furnished the transcendental link that connects both. Seymour Papert has referred to this situation when he reports in his Introduction to the Embodiments of Mind that McCulloch insisted "that to understand such complex things as numbers we must know how to embody them in nets of simple neurons. But he would add that we cannot pretend to understand these nets of simple neurons until we know – which we do not except for an existence proof – how they embody such complex things as numbers. We must, so to speak, maintain a dialectical balance between evading the problem of knowledge by declaring that it is 'nothing but' an affair of simple neurons, without postulating 'anything but' neurons in the brain. The point is, if I understand him well, that the 'something but' we need is not of the brain but of our minds.. namely, a mathematical theory of complex relations powerful enough to bridge the gap between the level of neurons and the level of knowledge in a far more detailed way than can any we now possess." (p. XIX)

After the author had read this introduction he asked McCulloch whether he really intended to introduce dialectics only in a loose and logically non-coercive manner or whether he realized that Hegel employed the term as a linguistic cover for a hidden exact mechanism which the Universe as a whole employed but which we were still incapable of unravelling. McCulloch remained silent for a few moments and then asked the author to rephrase the question, which the latter did by simply inquiring whether he thought that the term 'dialectics' merely referred to a quirk or weakness of the human mind or whether it indicated an intrinsic property of Reality. This time McCulloch answered that the term should designate an objective quality of the universe and he

added: I think this is what separates Kant from Hegel. The author and McCulloch agreed that the "so to speak" in the lengthy quotation above was not a proper expression because it suggested only a vague analogy. It did not indicate that in the term "dialectical" a very precise systematic foundation problem of mathematical theory was at hand.

The author cannot now remember how the talk got to a paper of Barkley Rosser "On Many-Valued Logic", which was published in the American Journal of Physics (Vol.9,4; pp. 207-212, 1941), and from there to the question whether a dialectical analysis of natural numbers might help to bridge the gap between the level of neurons and the level of knowledge which is conveyed by present mathematical theory. Everything was still very vague, and it took an almost nightlong discussion to clear the realm of discourse somewhat. It helped greatly that McCulloch was familiar with the distinction of number by Plato and Aristotle and how much nearer to the Pythagoreans Plato's ideas were than those of Aristotle. And then he surprised the author by saying that, what Hegel meant by number was a not very successful attempt to rebuild again the general concept of numerality which had been divided by the antagonism of Platonic and Aristotelian philosophy. He finally added that Hegel failed to develop a novel theory of mathematical foundation because he thought more about number in the Aristotelian than in the Platonic sense. This was a most astounding conclusion and seemed questionable to the author. He believed that he knew more about Hegel and felt unable to accept McCulloch's thesis. Since the whole history of mathematics from the Greeks to the present time owes all its success to the instinctive acceptance of the Aristotelian way of thinking about numbers McCulloch had to be wrong. The author left Shady Hill Square somewhat dissatisfied and went skiing.

Six weeks later he was back, very contrite and humble. He was not a mathematician, only a logician, moreover reared in the atmosphere of the Geisteswissenschaften. But it had, in the meantime, dawned upon him how much better a philosopher McCulloch was when the mind turned to the problem of the transcendental relation between mathematics and the Universe. Conceding McCulloch his Hegel interpretation the discussion doubled back to the essay of Barkley Rosser. Rosser's attempt seemed now extremely interesting; Rosser had demonstrated in his paper, that one can get numbers from four ideas in two-valued logic which have been formalized in terms of a likewise two-valued calculus. The first idea is 'conjunction' (... and ...); the second idea is 'negation' (not ...); the third idea is 'all'; and the final idea is 'is a member of'. Rosser then suggests a projection of these ideas onto the structure of a many-valued calculus. For the purpose of demonstration and to retain a comparative simplicity he exemplifies his case with a three-valued logic. As values he chooses 'true' (T), 'probable' (?), and 'false' (F). McCulloch and the author agreed that this interpretation of three-valuedness has proved its usefulness in cybernetics and elsewhere but that it could not lead to a trans-classic theory of natural numbers because it has been established since at least 1950 (Oskar Becker) that the introduction of probability or modal values destroys the formal character of a logical system. For if strict formality is insisted on any such spurious many-valued system reduces itself automatically to a two-valued calculus. In order to convince McCulloch that Rosser's approach to the problem needed a weighty correction the author pointed to something which he considered Rosser's second mistake. The latter determines conjunction in classic logic by the following matrix:

	T	F
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T	T	F
F	F	F

and the stipulation that T is not permitted to re-occur in any of the empty places which originate if we extend the places for the functional result from 4 to 9. Thus he defines, in strict analogy, three-valued conjunction by the matrix:

	T	?	F
T	T	.	.
?	.	.	.
F	.	.	.

We repeat: in order to retain the meaning of conjunction T is not to go in any of the empty places which are left open in the above matrix. However (?) and (F) may go indiscriminately in any of the other squares. Since 8 squares are left to be filled and since two choices are available in the case of each square there are 28, i. e. 256 possible choices for filling the squares. in Rosser's opinion all of them represent the general meaning of conjunction in a three-valued logic. This claim was easily refutable if one recognized – as McCulloch did – the interpretation of trans-classic logic as given by the author in his "Cybernetic Ontology and Transjunctional Operations". In order to demonstrate Rosser's too generous interpretation of conjunction the author filled out the matrix in the following way:

	1	2	3
1	1	3	3
2	3	2	3
3	3	3	2

In order to avoid the ontological consequences which are implied in Rosser's use of the symbols T for truth, ? for probability or modality, F for false we have denoted the values in the same order with the first three integers. This choice of values is quite in accordance with Rosser's stipulation for the meaning of conjunction. However, there it not the remotest chance to interpret this arrangement as a matrix of a conjunctive functor. To render a minimum sense of conjunction a three-valued logic would have to retain the structural feature of conjunctivity in at least one of the two-valued alternatives 1 or 2, 2 or 3, or 1 or 3. This is not be case, because or the two-valued system encompassing the first and the second value we obtain the morphogrammatic structure which can only be filled by trans-junctional value-occupancy. For the two-valued system constituted by 2 and 3 we obtain a morphogrammatic structure for value-occupancy which is demanded in the case of equivalence, and for the final two-valued system the morphogrammatic structure of transjunction re-occurs.

But let us, for argument's sake, assume that Rosser is right and we have to deal with 256 possible kinds of conjunction in a three-valued system. What shall we do with this embarrassing wealth? Rosser himself gives the answer: "Apparently the only thing that can be done about the matter is to pick out the 'and' that one likes best, *and try to ignore the rest.*" Emphasis by G. G.). McCulloch pointed out that the arbitrariness which Rosser suggested could not be tolerated in the development of a more basic theory of natural numbers. But he added meditatively: It hints at something in the

relation between matter and form. The author is not quite clear whether this was McCulloch's exact wording; at any rate, he asked his mentor what he meant and McCulloch spun a long tale which seemed to the hearer to go far beyond what he had learned from the essay 'What is a Number that Man may know it ...?'. Finally a spark of tentative understanding jumped from the speaker to the listener. McCulloch was talking about Hermeneutics and about the possibility that, if numbers were subject to hermeneutic procedures in the sense of Dilthey's 'Verstehen' in the Geisteswissenschaften, this would definitely close for the scientist the gap between Nature and Geist. The idea of a basic 'arithmetization' of the Geisteswissenschaften seemed to the author at that time not only bizarre but outrageous and he voiced his violent objections. McCulloch did not answer any of them; he only asked curtly: and what do you make of Rosser's "sidewise motion"? (The reader who is not familiar with this paper should be informed that Rosser said in his somewhat loose manner that the mapping of natural numbers on a many-valued logic produces something like a "sidewise motion" of these numbers.)

It is the purpose of this essay to present the author's theories but to show the philosophic profundity of McCulloch and the author's spiritual indebtedness to him. So we shall return to the remarks McCulloch made about subterranean relations between arithmetic and the hermeneutics of the humanities. From Dilthey McCulloch went back to Hegel as idealist and materialist were equally untenable because Idealism and Materialism both implied that they were sets of statements about *what there is* instead of what the universe *means* to the brain. In any case Hegel's philosophy recognizes an existence as a context of stateable facts. In this respect Hegel was still dependent on Immanuel Kant who "spawned two fertile succubi" as we read in "The Past of a Delusion", One was "the Dynamic Ego as Unconscious Mind. Upon (it) Freud begat his bastard, Psychoanalysis. The other, causality, the Category of Reason, flitted transcendently through Hegel's Dialectical Idealism." Upon Causality herself Karl Marx begat his bastard, Dialectical Materialism. "The author being a stout defender of the Theory of Dialectics then asked McCulloch whose opinion of dialectics in the "Embodiments of Mind" seemed to be extremely low whether dialectics would play a role in a not ontological, but hermeneutical alternative of idealism and materialism. McCulloch conceded that there might be something to it provided a satisfactory interpretation could be found for the "indeterminate duality" ἀόριστος δυνάξ of Greek philosophy. According to Aristotle's metaphysics Plato called the forms numbers and stated that each number has two constituents: the One or unit which Aristotle defines as the formal constituent; and something which he calls a material constituent. This is supposed to be the mysterious ἀόριστος δυνάξ. It stands to reason, of course, that dialectics has its root in a duality. So a renewed and critical analysis of dialectics should start from here. McCulloch seemed to be very well versed in these antecedents of number theory but he voiced some doubt whether the problem of the indeterminate duality was as yet properly understood. He was ready to admit that the testimony of Aristotle seemed to be unimpeachable with regard to what Plato *said* but it seemed to be a different question as to what Plato really *meant*. The author who had studied the relevant passages in Aristotle's metaphysics could not help imparting to McCulloch his impression that Aristotle totally misunderstood Plato's reflections concerning the theory of numbers. Aristotle himself refers to the lectures Plato delivered in the Academy as

the "unwritten doctrine" (ἄγραφα δόγματα) which means that Plato did not produce a written text of his academic teaching. Therefore his listeners handed on several different versions of his famous lecture on "the Good" which has intrigued students of Plato up to the present time.

McCulloch was intimately familiar with Alfred North Whitehead's essay "Mathematics and the Good". Whitehead keeps quite close to the tradition which connects the Platonic "duality" with the "indefinite" or the "unlimited" (ἄπειρον) of the Pythagoreans. Whitehead interprets this in the following way:

"The notion of complete self-sufficiency of any item of finite knowledge is the fundamental error of dogmatism. Every such item derives its truth, and its very meaning, from its unanalyzed relevance to the background which is the unbounded Universe. Not even the simplest notion of arithmetic escapes this inescapable condition for existence." ("Essays in Science and Philosophy" 1947, p. 101.)

McCulloch could not agree entirely with this viewpoint. Seymour Papert correctly pointed out that the famous 1943 paper by McCulloch and Pitts demonstrated that a logical calculus that would permit the embodiment of any theory of mind had to satisfy "some very general principle of finitude". McCulloch was thinking of some such limitation in the indeterminateness of "indeterminate duality" when he questioned the traditional and conventional interpretations of Plato's ideas on numbers. It was clear to him that in this respect the difference between Plato and Aristotle is basically this that Aristotle permitted only one single concept of number, producing a gradual accumulation of uniform units (μοναδικόζ αριθμόζ), but that Plato's philosophy involved a *second concept* of number resulting from the break between the real of ideas and our empirical existence. He became very insistent that the author should delve deeper into the philosophical aspects of number theory when the latter told him about Hegel's speculation on a "second" system of mathematics "welche dasjenige aus Begriffen (erkennt), was die gewöhnliche mathematische Wissenschaft aus vorausgesetzten Bestimmungen nach der Methode des Verstandes ableitet". (Hegel, ed. Glockner IX, p. 84.) With this idea of a "second" system of mathematics in the background McCulloch began to urge the author to develop his ideas on the connection between number and logical concept further. Very soon an agreement was reached that the starting point should be the fact that the notation of the binary system of numbers coincided in an interesting way with the method by which two-valued truth tables demonstrated in the propositional calculus the meaning of logical concepts like conjunction, disjunction, implication and so on. It was only necessary to reduce the value sequences to their underlying morphogrammatic structures of which eight could be obtained in order to see that there was a peculiar correspondence between the method by which the binary numbers from 000 to 111 were produced and eight 4-place morphograms which used only the idea of sameness between places or difference.

We do not have to repeat all of the next steps here because they have, almost without philosophic background, been reported by the author in Vol. I. in the Journal of Cybernetics. Almost – which means that the formal philosophical concept of *universal contexture* at least was introduced. But neither Plato's ἀόριστος δνάζ nor Hegel's idea of a "philosophische Mathematik", as logically distinct from traditional mathematics, was alluded to. There was also no reference to a general principle of finitude which had been most essential for the production of the afore-mentioned essay in the Journal of

Cybernetics. In fact, the essay could never have been written without the information the author was given by McCulloch about some of his ideas on finitude. The author shall try to repeat what his memory retained because what McCulloch developed in the case of the dialogue seems to deviate from the trend of thought emerging in the "Embodiments of Mind".

After a tentative discussion of Hegel's trans-classic concept of mathematics McCulloch turned back to the problem of finitude referring to a then recent paper by C. C. Chang "Infinite-valued Logic as a Basis of Set Theory". (Logic, Methodology and Philosophy of Science, North Holland Publishing Company, Amsterdam, pp. 93-100, 1965.) He agreed with the author that Chang's paper had to be criticized from the viewpoint of finitude, and that Chang assumed willy-nilly the philosophical theorem of Łukasiewicz that only three systems of logic have ontological relevance: the two-valued system, the three-valued order and a system with an infinite number of values. He admitted that Łukasiewicz's conclusion was quite consistent and reasonable provided one places all values added to True and False "between" these two classical boundary cases of value. That a two-valued logic and a system with an infinite number of values have ontological relevance is beyond question. But why in addition to them only a three-valued system? This assertion of Łukasiewicz may be interpreted as follows: Since the number of values between True and False represents the continuum, any individual value in the middle that is selected out of the totality of values can only be obtained by a Dedekind cut. This cut, and *not* the number obtained by it, is the proposed third value! Thus, if we add a fourth and a fifth and a sixth and so on intermediate value we would only iterate in logical respect the information of the cut. And since – to say it again – the cut itself is the third value and not the results of the cut. The iteration of the cut would, despite a different numerical result, produce logically (and not arithmetically) speaking the same value. Seen from here it makes sense, if Łukasiewicz maintains that only to three systems of logic philosophical meaning can be attached. The talk then turned to the fact that the author had shown in several papers that many-valuedness might be interpreted differently. Denoting all values by integers and starting with 1 one might place all transclassical values not "between" 1 and 2 but 2 "beyond" 2. This "beyond" leads inevitably to a different interpretation of many-valued systems.

At this point the author wants to note that during the initial stage of investigating many-valuedness he had believed that the idea of placing additional values totally beyond the alternative of True and False was the *only* legitimate ontological interpretation of many-valuedness. It was McCulloch who disabused him of this erroneous belief. He drew his attention to the fact that in a many-valued system designed according to the author's concept of many-valuedness being an order of ontological places of two-valuedness any two-valued system could *additionally* contain Łukasiewicz' values between True and False. Later on the author has found this suggestion extremely useful and only recently it has helped him to understand a specific phenomenon of trans-classic logic which, otherwise, might have been uninterpretable.

At this time, however, the new insight in many-valuedness did not lead very far. For the time being there existed only a general agreement between McCulloch and him that the term 'many-valuedness' was ambiguous. The theory had to consider the fact that two

different kinds of many-valuedness had to be distinguished^[1]. Beyond this result there was still much haziness. It was about the time when McCulloch was playing with the idea of the "Triads"^[2], and the author distinctly remembers the day when McCulloch told him: "Gotthard, you can do everything with triads!" The author did not agree; there was too much of the small of Post and Łukasiewicz around this statement. However, he remained silent; McCulloch sounded too emphatic. It must have been the right diplomacy, because later – the author cannot remember the length of the interval – McCulloch declared with equal emphasis when the author based an argument on three-valued relations: "Triads are not enough". The author can guess what caused this change of attitude. First, the return of the discussion to the paper of Chang, and second, a renewed analysis of the meaning of number in the Platonic system. We shall start with Chang. He introduces in his paper a set X which is referred to as the set of truth values of the infinite-valued logic. For the purpose of discussing finite-valued logics he considers a sequence of finite subsets of X, such that for each X_n

$$X_n = \left\{ 0, \frac{1}{n-1}, \frac{2}{n-2}, \dots, 1 \right\}$$

Each set X_n , is regarded as the set of truth values of an n-valued logic. If $n=2$, all functions will, of course, acquire their traditional two-valued character and meaning. The viewpoint underlying this procedure is exactly the same as taken by Łukasiewicz. All values of this pseudo-transclassical logic have their ontological location between the boundary values 0 and 1. In other words: they refer to finite subsets of the continuum. This makes it impossible to eliminate infinity from the basic philosophic theory of logical values.

On the other hand, human awareness as the source of logical-value-and-natural-number theory is a *finite* system of the brain ("Why the mind is in the Head"). Although the system is finite it may produce as its mental content such second order concepts as denumerable and non-denumerable Infinity. If the author understood McCulloch properly then the latter took an extremely revolutionary position. Hitherto philosophers had always – without further questioning – assumed that the Finite is embedded in what we call the Infinite. McCulloch seemed to imply that this order should be reversed and that infinity should be robbed of its primordial rank and only be admitted as a second order product of a finite system of awareness which is a product of the equally finite system of the physical brain. It became clearer and clearer to him that McCulloch's ultimate concept of the entities which made up Reality was not so much the Realm of Ideas – be that in the Platonic or in the Aristotelian-Hegelian sense – but the "Pythagorean" conception of Number although his notion of numerosity had, in the course of the years, drifted away from the position which was taken in "What is a Number, that Man may know it". So at least it seemed to the author. When he first meditated about number it happened against the as yet unquestioned metaphysical background that in order to define Reality one must understand that all Finitude is

¹ Cf. G. Günther, Die Theorie der „mehrwertigen“ Logik: in Philosophische Perspektiven, Ed. R. Berlinger & F. Fink, Frankfurt/ M. 1971; III, p. 131.

² See Christopher Longyear: Towards a Triadic Calculus, I - III, Journal of Cybernetics, 1972, pp.50-65, 7-25 and 51-78.

embedded in the Infinite. When the author saw him last McCulloch seemed to have completely reversed his position. He seemed to believe that ultimate Reality could only be understood in terms of Finitude, and that Reality conceived as infinity was nothing but mythology. The author was led to this conclusion by the discussion of Whitehead's "Mathematics and the Good". Which, of course, led directly to Plato's lecture $\pi\epsilon\rho\iota$ $\tau\acute{\alpha}\gamma\alpha\theta\omicron\upsilon$ and the modern attempts to reconstruct the text.

Plato starts with the question: what are the ultimate building stones of the Universe? The conventional interpretation of Plato is satisfied with the somewhat crude answer that these building stones are the Ideas. But if the ideas represent no ordered system in the shape of a pyramid, with the single idea of the Good on top, and a plurality of other ideas below, the problem of the metaphysical Number emerges and we are carried beyond the domain of Ideas to the ultra-ultimate question: what is the relation between unity and the manifold? In other words: our thinking cannot stop till it reaches the concept of what is conventionally and vaguely known as the natural number. It was immediately clear to McCulloch that our conventional interpretation of the order of natural numbers as a Peano sequence could not satisfy the philosophical reflexion because it was absurd to interpret the order of the Ideas also as a Peano sequence. From the idea of the Good they spread out in an arrangement that was more or less inadequately described as a pyramid. The reports on Plato's lecture unfortunately do not make it clear how Plato himself interpreted the relation between Number and Idea. McCulloch as the cyberneticist interpreted it for purely systematic reasons as a reduction. The analysis of the Ideas leads to a pre-ideative system of only numerically definable relations. An alternative interpretation – traceable back to antiquity – that Ideas are just numbers he did not like. The ideas could not be the ultimate building stones of the universe – they were much too complex. It was unfortunate that neither McCulloch nor the author were aware of the fact that shortly before they entered into their discussion about natural numbers the German philosopher Klaus Oehler had published (in 1965) a paper under the title "Der entmythologisierte Platon" Zeitschr. f. Philos. Forschung XIX, pp. 393-420). This profound essay seems to have anticipated McCulloch's position. What Oehler says is so important that it may be repeated at this point. "Die Entfaltung der Einheit zur Vielheit und die Teilhabe des Vielen an dem übergeordneten Einen bestimmen den gegliederten Aufbau des Ideenkosmos. Nun geht aber weder der Aufstieg zu den umfassenden Begriffen ins Unendliche fort, noch geschieht das bei dem Abstieg zu dem Einzelnen. Der Aufstieg ist begrenzt durch den allgemeinsten und umfassendsten Begriff, das $\acute{\epsilon}\nu$ der Abstieg ist begrenzt durch das jeweils letzte $\epsilon\acute{\iota}\delta\omicron\zeta$. Das bedeutet aber, daß die Ordnung der Ideen zahlenmäßig bestimmt ist. Folglich ist jede Idee durch die Zahl von Inhalten, die sie umschließt und an denen sie teil hat, eindeutig festgelegt. Jede Idee ist also durch eine Zahl bestimmt und ist als solche zahlenmäßig bestimmbar, angebbar. Diese numerische Fixiertheit verleiht der Ordnung der Ideen ihre rationale Klarheit, ihre Durchsichtigkeit und Übersichtlichkeit. Ist das Mannigfache der sinnlichen Wahrnehmung nur durch die Teilhabe an der Idee das, was es ist, so ist die Idee nur durch die Teilhabe an der Zahl das, was sie ist. Mithin muß die Zahl vor der Idee sein. Die Ordnung der Zahlen ist der Ordnung der Ideen übergeordnet, weil überlegen. Das bedeutet aber: die Ideen sind nicht das Letzte und mithin nicht die Prinzipien des Seienden."

(The unfolding of the one into the manifold and the participation of the manifold in the super-ordinated One determine the structure of the cosmos of Ideas. But neither does

the ascent to the comprehensive concepts continue into infinity, nor does this happen in descending to the Particular. The ascent is limited by the most general and the most comprehensive concept, the *év*, the descent is limited by the last particular *είδοζ*. That means that the order of ideas is numerically determined. It follows that each idea is univocally defined by the number elements it contains and in which it participates. Consequently each idea is characterized by a number and is as such numerically describable (and quotable). This numerical fixation endows the order of ideas with its rational clarity, transparency and orientability. If the manifold of sensual perception is what it is only by participation in the idea, then the idea is what it is only by participation in Number. Thus Number must be prior to Idea. The order of Numbers is super-ordinated to the order of Ideas, because it is more potent. This means: the ideas are not ultimate and therefore not the principles of Being.)

It is not difficult to see that Oehler leans toward the notion of finitude, which was so dear to McCulloch, when he points out that the ascent to the One as well as the descent to the Particular are always finite. That does not exclude, of course, that each such finitude may be superseded by numerical increase of the finitude. Infinity, however, is nothing but the everlasting subjective expectation that every given finitude is not the last one. *It is a mistake to ascribe ultimate ontological relevance to the concept the Infinite.* It seems to the author in retrospect that McCulloch in expressing such thoughts moved into the neighborhood of mathematical intuitionism and its criticism of the transfinite or actual (extensional) non-finitude. Existence is constructibility, logically speaking.

Excursus

Before we discuss the quotation from Oehler it will be not only desirable but necessary to introject into the report on McCulloch an excursus on the meaning of the term 'number'; because a modern mathematician will probably object to the way this concept has been handled so far not only by McCulloch but by the author and Oehler as well. The question one has to begin with is the following: why did the concept of number become so important for Plato after the doctrine of Ideas had reached some maturity? The likely answer is, that during the development of the doctrine of Ideas, the quest for the individual ideas lost more and more of its importance in favor of the inquiry into the inter-connectivity and systematic order of *all* the ideas. This led automatically to the search for the most general and, at the same time, elementary form of order. This would, of course, be the linear order mentally accomplished by the simple process of counting. But already the Pythagoreans had discovered – and Plato was familiar with Pythagorean number theory – that this most primitive order was capable of a highly sophisticated treatment which permitted ultimately to encompass any element of ordering the not-yet-ordered.

Such concept of order transcends the principle of quantity by far and such transcendence may be determined in many ways. McCulloch only insisted that any principle of order should be traceable back to the familiar order of natural numbers. Whether we let the natural numbers begin with 0 or 1 is, of course, a mere convention. However, there should be no confusion between the metaphysical Nought and the conventional 0 or 1 in numbers. These

distinctions remained in the discussions with McCulloch always somewhat vague; but he left no doubt that he never considered the gap between number and concept as ultimate but was convinced that it could be bridged. This was for him the significance of transcendental philosophy which he believed would produce the unification of the humanities and the sciences. Both of them – so he argued – start from a common ground: the elementary unit which in its primordially is indistinguishable from any other unit. Thus primordial units are per se unordered and for this very reason they may be used to produce a system of order for the Realm of Ideas. But even at its very beginning Greek mathematics encountered an almost unsurmountable problem: how to understand the relation between unit in the geometrical and in the arithmetical sense. In the Pythagorean mathematics of the fifth century the geometrical point was made to correspond to the arithmetical meaning of 1. In other words: the number 1 that which designated a real point in the objective world. A point is the minimum quantity which we encounter. The difficulties that arose from this viewpoint are too well known to mention them here; it is sufficient to draw the attention to the fact that Aristotle nailed this epistemological attitude down with the formula $\mu\omicron\nu\acute{\alpha}\zeta \acute{\epsilon}\kappa\omicron\upsilon\sigma\alpha \delta\acute{\epsilon}\sigma\iota\nu$ (the unit with location).

At this point the dialectical mechanism of all reflection makes itself visible, and the argument emerges that a point as identified with the number 1 is not a minimum volume of objectivity, but the absence of objectivity. In other words: *to produce as number as a quantity a duality is required*. As soon as this insight is obtained the thought will tend to let the point correspond rather to 0 and not to 1.

If in modern times we insist that it is irrelevant whether we call the first number 0 or 1, this may be a convention in one way; but it is not a convention in a different way because it points to the peculiar relation between primordial unit and Nought.

It would be tempting to spin a consistent yarn how McCulloch connected his many philosophical ideas on Number with each other. Yet this would falsify the situation and the author refrains from doing so.

The connection with Oehler's Plato interpretation seems rather obvious. The difference between the geometrical and the arithmetical meaning of number presents an unresolvable ambiguity which paradoxically renders numbers a suitable structural basis for philosophic thought and thus a possible link between the sciences and the humanities.

Since primordial units are totally indistinguishable from each other they are totality indifferent as building blocks of thought against the distinction between the sciences and the humanities, as we pointed out above. Conceptual distinctions can only be generated by changing the principles of ordering units, and an order is always a matter of interpretation. If the primordial unit is interpreted as a point in space and ontological interpretation is chosen, and if we consider 0 as the idea with which we start our familiar number sequence we have reversed our interpretation and our

first symbol designates – to speak in Platonic terminology – not an objective unit but the subjective act of starting to count (διαίρεσις).

From this dichotomy the way leads either to the sciences or to the humanities.

With this thesis that not the Finite is embedded in the Infinite but that the Infinite – be it conceived as potential or actual – is, in the metaphysical sense, only a subordinated element of Finitude McCulloch showed himself to be a first rate metaphysician. This view of Metaphysics had never occurred to the author though he had always prided himself of having effected in his: "Cybernetic Ontology..." a metaphysical breakthrough from classic tradition by means of the rejection value. But McCulloch went much farther with his reversal of the mutual role of Finitude and the Infinite. Whenever classic tradition through the history of Philosophy discussed the meaning of the Absolute a philosopher would have deemed to have lost his senses if he had proclaimed that the Absolute is a Finitude and that the main characteristic of the empirical world is its Infinity. Unfortunately, McCulloch did not elaborate this point in detail. And the author did not press him very much because he hoped to have, later on, a better occasion to elicit a detailed explanation of this startling and paradoxical theorem. Alas, this opportunity never came.

There was just a hint of an explanation in his evaluation of the Platonic confrontation of the One and the ἀόριστος δυνάς, the indeterminate duality. He approved of Aristotle's opinion that this duality was nothing but a material constituent. To put it differently: a number is an entity which is produced by the actual determination of determinable potentiality. And the vehicle of the determination is always the One. McCulloch agreed with this Aristotelean interpretation but not wholeheartedly. He told the author again and again that this way of thinking overlooked something and did not account clearly for the difference between the step from 1 to 2 in the familiar sense of Peano sequence and the step from Oneness to Duality in the other sense that Duality already implied an unbounded manifold. It had been noted before that Aristotle seems to be confused about the difference between the "indeterminate duality" and the number 2 (A.E. Taylor; Plato, N.Y. 1927, p.512); knowing this McCulloch's arguments gained a greater weight with the author than they would have done otherwise. He decided, startled by the novel metaphysical viewpoint of McCulloch, to attempt a new interpretation of natural numbers on the basis of a many-valued logic with a kenogrammatic background. He sought and obtained McCulloch's agreement not to follow the way of Barkley Rosser but to choose a different method. There was nothing in Rosser's paper on undetermined duality, whereas McCulloch and the author agreed that the meaning of this term was the key to the whole problem. Aristotle's lack of the understanding of the problem led to a position where he could only recognize what he called 'mathematical number' which is nothing but what we have called Peano numbers. The other numbers, the numbers of Platonic ideality, which define the Platonic order of ideas would not possess any logical legitimacy if we wanted to follow Aristotle. This, according to McCulloch, was unacceptable because the order of the Peano numbers was intrinsically incapable to reproduce the conceptual wealth of the system of Ideas. In McCulloch's opinion Rosser was still and Aristotelian in his number theory. When the author, with some trepidation, decided to leave pure logic for the time being and tackled number theory he was warned

from some other side that his lack of mathematical training could only lead to an abysmal failure. With his first sketch which he called proto-numbers he went to McCulloch and told him of the warning he had received and made no bones about his mathematical incompetence. However, he was at the same time able to point out that the same argument could have been applied to the corresponding efforts of the mathematicians. Since Frege there had been strenuous efforts to give mathematics safe logical foundation but it could hardly be denied that the logic underlying these efforts nowhere went beyond Leibniz at best and that neither the transcendental turn effected by German Idealism nor the problem of dialectics and its distinction between Platonic and Hegelian dialectics was properly understood on the side of the mathematicians. Here stood incompetence against incompetence and it could only be hoped that a better cooperation between mathematics and philosophy would produce something worth while. McCulloch encouraged the author to continue who took it as part of the encouragement that McCulloch invited two or three friends and collaborators of his to whom the author should present his ideas. He has now forgotten who else attended but he remembers that Professor Manuel Blum was present. Taking into consideration everything McCulloch had said about the indetermined duality and also including the result of discussions on Hegel the author took the following step toward a transclassic theory of natural numbers. Guided by Hegel's dialectics he said that the process of adding 1 to a preceding number was ambiguous: it could either be interpreted as "iterative" or as "accretive". Starting from 1 and proceeding to 2 the duality thus obtained was indeed indeterminate but not in the sense which Plato, according to his interpreters, might have intended. Interpreters have usually been of the opinion that for Plato going from 1 to 2 was only the step from Oneness to Manifoldness and that the indeterminacy of the manifold which this step established was not positively fixed. It could be anything: 2, 3, 4 and so on.

The argument against this interpretation is that it does not lead to dialectics and Plato was a dialectician. His doctrine of ideas clearly shows a dialectic structure and if the order of the ideas is determinable by numbers then *the numbers themselves must display a dialectic structure* also. This was a consequence McCulloch had not only admitted in the discussions with the author. More so: he had pointed it out to him before the latter had become aware of it. The dialectical treatment of natural numbers – 'dialectic' in the combined meaning of Plato and Hegel – implied that the process of addition $1 + 1 = 2$ should be interpreted in two ways: one could either look at the two 1's as being identical or as being non-identical. This could be done by either ignoring the fact that the second 1 was a repetitor of the first 1 or by not ignoring the repetitional character of the second unit. The result is different in both cases. No matter which interpretation was chosen the result would, of course, always be a duality. But duality would carry two meanings; it was important to express this in a way that the difference in meaning would become computable.

At this point the author was helped by a stray remark McCulloch had made a year ago the importance of which the author had previously overlooked. McCulloch said that the difference of meaning seemed to him a difference of quality in the sense in which Hegel differentiated at the beginning of his Logic between Being and Nothingness as antithetical qualities. Only in this way could one understand how dialectics might finally turn qualities into quantities. The author found this remark extremely cryptic and asked McCulloch how this dialectic transition might happen. He got the

disappointing answer: This is for you to find out. At a renewed attempt to extract at least some shreds of information pertinent to the problem the author was only reminded of a former discussion about Heidegger and his treatment of the Nichts³. This he considered no help at all. But then he found his attention drawn back from the concept of number and directed towards the idea of the kenogram. Kenograms are empty places which may or may not be occupied by values. Up to this point the author had always believed that only one value at a time could occupy a single kenogram. Not it occurred to him that a kenogram might behave differently in the ease of numbers, and that it might be the ontological locus not just for a single number but for a total Peano sequence of natural numbers. And since a Peano sequence is of infinite extent such numerical order would be a demonstration of McCulloch's startling metaphysical thesis that not the Finite is encompassed in the Infinite but that all Infinity must be understood as a subordinated element of Finitude, i.e. a kenogram. The author was so excited by his brainwave that he did what he had never done before and as far as he can remember never did afterwards, he rang McCulloch up to ask his opinion. Contrary to his expectation McCulloch was not swept off his feet but asked all sorts of question how a single kenogram could be defined as an all-encompassing domain accomodating a never ending process of counting. There was nothing in the original conception of a kenogram, so McCulloch reminded the author, that would suggest such property. The author must confess that he felt deflated when he hung up. But his respect for McCulloch's mental acuity was so great that he settled down immediately to think the problem over. Very soon his initial disappointment turned into deep gratitude, because out of McCulloch's critical remarks the concept of the universal contexture was born. The author is convinced that he would never have found this idea if he had not been privileged to listen to McCulloch's thoughts about the metaphysical rank of Finitude and the information given over the telephone. Re gratefully acknowledges that McCulloch is as much the creator of the concept of universal Contextuality as opposed to mere context as the author of this essay. For this reason it seems to be fitting to describe here the difference between a mere context and a universal Contexture.

If, e.g. in court the question is raised whether the defendant is guilty or not guilty, it would be non-sensical to answer: no, he is broad-shouldered. In other words: the alternative guilty or not guilty is enclosed in the context described by the statutes of criminal law. On the other hand: the question: 'Is the growth in this person malignant or non-malignant?' cannot be answered by: 'No, he is a poet', because the alternative which has been raised belongs to the context of pathology. In both cases the answer must be guided by a tertium-non-datur which refers to a superordinated viewpoint which in our first ease was criminal law and in the second pathology. The alternates of a context may be very narrow and again they may be of ever increasing generality, the alternative still constitutes a mere context as long as it is possible to determine a superordinated viewpoint. A context changes into a universal contexture only on condition that it is impossible on principle to find a superordinated viewpoint which defines the meaning of the tertium-non-datur for the opposites for which the superordinated common viewpoint has been sought. The classical example for this situation is Hegel's "alternative" between Being (Sein) and Nought (Nichts). They are alternatives which exclude each other. Nobody can deny it. Yet nobody can conceive of a metaphysical

³ See also: Martin Heidegger, Was ist Metaphysik? Frankfurt/M. 1951, pp.22 to 38.

concept that would be of greater generality than both of them. In other words: both constitute separate universal contextures. We are not able to understand the distinction between Sein and Nichts as alternatives *within a context*. The question: of what context? must in this case remain unanswered. Similarly we read in Lenin's works that for the opposition of Mind and Matter no common denominator of higher generality can be found. Mind and Matter are not elements of a context. They are universal contextures, capable of encompassing contexts with limited alternations. Lenin concludes from this insight that the thinker who has arrived at this alternative has come to an end of his theoretical way. He is only left with the decision to declare himself either an idealist or a materialist. This is not the place to sit in judgment of the legitimacy or illegitimacy of Lenin's conclusion but his example shows that the situation Hegel discusses at the beginning of his Logic can turn up under radically different aspects^[4].

If the reader thinks that these reflexions are far from what we read in the "Embodiments of Mind" he may be reminded of the insight the essay "A Hierarchy of Values determined by the Topology of Nervous Nets" conveys. There we learn that "an organism possessed (at least of six neurons) is sufficiently endowed to be unpredictable from any theory founded on a scale of values.. It has a heterarchy of values, and is thus interconnectively too rich to submit to a *summum bonum*."

A *summum bonum* requires an ultimate hierarchy of values with an absolute value at the summit. Logically this means that there must be a *tertium-non-datur* crowned by a final common denominator of 'Sein' and 'Nichts'. If somebody insists that such a denominator is inconceivable the hierarchist will willingly agree but explain that this ultimate common denominator is nothing but God himself, as the Lord of a *monocontextural* Universe. McCulloch's heterarchy of values, on the other hand, postulates a reality that is only conceivable in a poly-contextural sense. In other words: the world we live in cannot be understood as an unbroken universal context. In fact, the term 'universal context' is in itself a *contradictio in adjecto*. It may be true that the author finally formulated the difference between context and contexture, but it is also true that he could never have done it without the spade work McCulloch had provided.

In fact, there is another way to show how near McCulloch came to develop the distinction between context and contexture. He had an amazing knowledge of medieval logic and he once referred to the famous ninth chapter of Peri Hermeneias and its influence on medieval logic up to William Occam. Aristotle had stated that in logical terms the difference between Past and Future could be defined by the fact, that the *tertium-non-datur* is valid for and applicable to all the Past. With regard to any Future the *tertium-non-datur* is equally valid, but it is not applicable. McCulloch considered this distinction very important for the understanding of the present, and it shows how near he came to distinguish between context and contexture because, if we refer to the Past, we refer to what has happened in a context. Thinking about the Past we always mean the actual contents of a contexture, thinking about the Future, however, we can only refer to an as yet empty universal frame which has not yet been filled with any contents because, if it were, it would not be the Future. Writing down these lines the

⁴ Cf. G. Günther, Life as Poly-Contextuality in: Wirklichkeit und Reflexion. Festschrift for Walter Schulz (H. Fahrenbach Ed.), Pfullingen (Neske) 1973, pp.1 87-210.

author wonders how far he is perhaps plagiarizing McCulloch. Because he is convinced that his own thoughts might not have gone in this direction if he had never had the good fortune to have those long nocturnal talks with McCulloch.

It was not always easy to listen to him, because his way of thinking was seasoned, as Seymour Papert rightly remarks, "with a very personal flavor" which not unfrequently led to misunderstandings. One example was his pronouncement that Finitude should be given metaphysical priority over the Infinite. The author is by no means sure that he has caught the full meaning of what McCulloch really intended by this statement. It is much too simple an assertion to describe an involved situation correctly. But it was one of the suggestions which helped him to arrive at his own distinction between a contextuality and its potential contents. A universal contexture is a finitude insofar as it is only one piece in a patch-work of an unbounded multitude of contextures. It is limited by its borderline to a neighboring contextural domain, but its capacity for content is unlimited owing to the peculiar character of its *tertium-non-datur*. When talking about the metaphysical priorities of finitude and infinity McCulloch casually mentioned Heidegger's "Seinsvergessenheit". If the author understood him properly – which is by no means certain since the morning was dawning and he was overtired – then Heidegger's "Seinsvergessenheit" must not be understood as a term referring to the contexture 'Sein' but to its contents only. On the other hand, when the talk focussed on Heidegger's 'Nichts' it was a foregone conclusion that the contextural frame was referred to, because it would have been nonsensical to speak of the actual contents which nothingness might encompass.. Further, it must be understood that the expression 'universal contexture' was understood that the expression 'universal contexture' was not used either by McCulloch nor the author at that time because neither was ready for it. Instead of it rather involved circumlocutions were used. However, trying to distill from his memory what seems to him the essence of the discussion the author finds it easier to use this more precise term which assuredly was a result of the mental exchanges between McCulloch and the present reporter.

During the last meeting the author had just returned from his yearly skiing vacation – it was agreed that he should write a paper on natural number theory within the frame of trans-classic logic for the next meeting of the ASC in Gaithersburg. The author remembers he had grave doubts that his paper would be ready for the third Annual Symposium of the American Society for Cybernetics. In consequence of his misgivings he informed McCulloch that he did not yet know whether he would be able to offer something in time to the Society. It turned out later that his pessimism was unjustified and he completed within the deadline the second part of the text which later appeared in the July/September issue 1971 of the Journal of Cybernetics. McCulloch did not know it; he had been in Europe during this period and when he returned he asked Dr. Edmund Dewan whether the promised paper had been handed in. This the author was told by Dr. Dewan on the first day of the Symposium which McCulloch could not attend because he had died on Sept.24, 1969 in Old Lyme, Conn.

When the paper was finally published with a Part I preceding the original text now designated Part II the writer added a footnote that the ideas expressed in the first part were to a great extent the result of a night session he had with McCulloch toward the end of February 1969. Since then 5 years or more have past. and his memories of McCulloch have gained a new dimension. He knows now how much more he owes to McCulloch than this footnote expresses. The maturing of his memories has shown him

among other things that McCulloch's influence did not only extend to one part of the aforementioned essay but to the other part as well. It was one of the remarkable gifts of this great man and scholar that he developed in his associates ideas and mental trends which they themselves might never have brought to fruition unassisted. The author of these remembrances has endeavoured to show how McCulloch, by delving deep in the philosophic aspect of Finitude elicited from the brain of his listener the conception that the Universe we live in is not mono-contextural but a network of Finitudes, partly bordering, partly overlapping, and in the case of compound textures even encircling elementary contexturalities, in short: a polycontextural Universe. He deeply regrets that McCulloch never saw the final text in order to give or deny it his imprimatur. He feels that the philosophical impact of McCulloch's thinking is still vastly underrated even by his admirers and disciples. He was such a many-sided thinker that he appeared enigmatic, never showing all facets of his mind to a single partner in discourse. To a neurologist he was an innovator in neurology: to a psychiatrist he revealed new ideas on psychiatric problems; with a mathematician he would discuss the mathematical aspects of his work, and when he met the author it was in the den of the metaphysician.

The quantity of topics McCulloch liked to talk about was enormous and his roving mind led the listener, sometimes quite unexpectedly to connections which went far beyond conventional associations. But wherever he turned to the problem of ultimate or penultimate foundations he looked for his data in the realm of numbers and number was for him invariably linked with Finitude.

Once the general topic of discussion had been a passage in "Why the Mind is in the Head?" concerning the relation between quantity and number. There we read that 2 in so-called analogical contrivances a quantity of something ... is replaced by a number... or, conversely, the quantity replaces the number." When the author suggested that, following the example of Hegel's Logic, the triadic relation between a quantity, number, and quality would also deserve a closer look, McCulloch switched to the question: why in primitive societies the capacity of counting was often very limited. The most elementary system of counting would, of course, work only with three hazy concepts: oneness, duality, and general manifoldness. McCulloch insisted that something was conceptually wrong when Plato according to tradition included general manifoldness in the concept of duality only because duality was not longer oneness. This improper inclusion was due to the fact that classic logic permitted only two values and nothing beyond. But then McCulloch continued that, if a finite system of numbers increased by the addition of one more numerical concept it would no longer be the same system to which a new numerical unit had been added, but it would be, logically speaking, in its totality a new system of counting! And every time one more unit was added this was not an adding process in the conventional sense in which we increase a given quantity by adding just 1. Instead, by addition we abandoned the numerical representation of a given conceptual order and moved to a different conceptual relationship with a somewhat higher complexity. This means that – let us say – the number 3 in a numerical order that went up to 4 was logically no longer identical with the 3 that occurred in a system which permitted you to count up to 5. To melt all these logically distinct systems of finite counting together into an unending Peano sequence one had to suppress most of the logical distinctions which number as a metaphysical concept implied. For this very reason number as a medium of thought had fallen into disrepute

in ontology and was forced to make room for conventional language to represent metaphysical concepts.

The author must confess that for one reason or another he had forgotten these remarks when he wrote "Natural Numbers in Trans-Classic Systems". But the memory came painfully back to him when later on he tried to apply his number concept to Hegel's system of Dialectics. Only then did he realize that McCulloch's startling statement that a 3 in a system which permits counting only up to 4 is logically not identical with the 3 in a system where the count up to 5 is permitted was linked to the fact that even in its own order of numerality a given number loses something of its rigid identity when the numbers are mapped onto a many-valued logic. It was obvious that, even by mapping numbers onto a trans-classic system of logic, they could not change their positions, "lengthwise". A 3 remained always a 3 and could not move to the place of 4. Thus $1+1$ remained always 2, but if the position of 2 was not a fixed point on a, so to speak, horizontal line, one could always ask: at which locus of the line the 2 was located. Thus, according to the location, the number could have different meanings. In other words: any number system of finite length represented itself to a philosopher as a hermeneutical order. Thus even the number 2 was already open to conceptual interpretation. Seen from here it was obvious that a system of higher numerality offered more chances of interpretation in a metaphysical sense and that therefore every time a successor number was added the previous system was semantically discarded, which meant that each specific world concept had its own numerical system fitting its own philosophical requirements. If at this stage we use the term 'number' it should be understood that we do not mean what Aristotle calls "mathematical" number or "number made of 1's" (μοναδικὸς ἀριθμὸς) but what we shall call here the esoteric number following terminological usage in which the lectures of Plato which he did not write down himself have been frequently called his esoteric doctrine. The indeterminate duality, e.g. is such an esoteric number. And so is any number which measures the distance between the universal One (έν) and the last particular εἶδος pertinent to the occasion. It is obvious that the Aristotelian numbers count empirical things or data of the world we live in and that the esoteric (Platonic) numbers are only concern with the realm of Ideas.

Many comments made on the difference between counting in the Aristotelic and the Platonic sense remained very hazy to the author at the time he heard them and he is not certain how much of what he has still to report on the philosophy of numbers is McCulloch's or his own understanding of the problem. It should also be added – and this troubles him very much in retrospect – that in his talks with McCulloch neither ever referred to the concept of a kenogram⁵. This has been very annoying to him in two respects: first, in order to get on paper what he had learned from McCulloch on numbers he found it unavoidable to use kenogrammatic structures and second, since not even the term was ever used, there was not opportunity to ask McCulloch what he made of the difference between numbers *within* the space of a kenogram and numbers *counting* the kenograms. Since then, the issue has become extremely important, much more than the author had anticipated in former years, and this again impedes his

⁵ Except in a phone-call.

memories of McCulloch's fundamental philosophic concepts. He is only certain that McCulloch during his last period would have agreed with Klaus Oehler's statement: "If the manifold of sensual perception is what it is only by participation in the Idea, then the Idea is what it is only by participation in Number. Thus Number must be prior to Idea. The order of Numbers is superordinated to the order of Ideas because it is more potent".

This association of esoteric Number with Idea seems to require another agreement with Oehlers Plato interpretation that esoteric number sequences are completely dominated by the principle of finitude. When we refer in every day life to natural numbers we assume automatically that they form an unending sequence. But if we trust Oehler's interpretation no Platonic system of esoteric numbers ascends an endless way toward the One, nor can it happen that it descends into the bottomless.

Thus peculiar dialectic situation is produced for the earthly thinker. He has the choice of interpreting the Peano sequence of numbers as an ultimate dilution of the orders of esoteric numbers to a degree where they become unfit for the representation of philosophic problems and where they are only good for showing money amounts in cash registers or temperature grades on the scales of thermometers and for similar trivial tasks. But we can also look at them as the material from which we build up orders of esoteric numbers starting from systems with minimal complexity to ever increasing structures of higher order. This produces a scale that proceeds from finitude to finitude! An infinite system of esoteric numbers is inconceivable. If trying to think it we cannot help but apply the numbers of the Peano sequence – which means: we drop out of the realm of metaphysics.

What has just been said is important to elucidate the philosophical radicality of McCulloch's principle of finitude which finally led him to the observation that the finite, metaphysically speaking, is not embedded into an infinite Absolute but that wherever we meet concepts of transcendence the latter will be finite and the Infinite will be is subordinated content.

McCulloch not infrequently remarked that it was necessary 'to lay the ghost of the Absolute', since in the philosophical tradition the Absolute and Infinity are invariably equated. Heidegger's treatment of the Nichts seemed to him a confirmation of his views. This was very difficult to understand, especially for somebody who was constantly aware of Heidegger's contempt for a thinking that arithmetizes (*rechnendes Denken*) and who could not forget the severe criticism McCulloch as a psychiatrist had at a different occasion launched against Heidegger and his work. The author was bewildered; but he regained some understanding when McCulloch casually remarked that Peano's definition of a progression, applied to the system of natural numbers, tacitly assumed that we know what Zero is. It was this remark which helped the author very much when, following McCulloch's trend of thought, he developed a system of trans-classic numbers.

In order to make clear how the author tried to implement McCulloch's comment on Zero and Nothingness it will be useful to start with Leibniz' dyadic method of counting:

Table I

1				(1)					
1	1			(2)	(3)				
0	1								
1	1	1	1	(4)	(5)	(6)	(7)		
0	0	1	1						
0	1	0	1						
1	1			(8)	(9)	.	.	.	
0	0								
0	0								
0	1	.	.						
.	

The left side of Table I displays the sequence of natural numbers expressed in the binary fashion; on the right side we note (always in parentheses) the same sequence in the conventional decimal fashion of writing. If we extend the method of Leibniz to write numbers to a ternary sequence of notation we obtain

Table II

1										
1	1	1	2	2	2					
0	1	2	0	1	2					
1	1	1		1	1	1	1	1	2	
0	0	0	1	1	1	2	2	2	0	
0	1	2	0	1	2	0	1	2	0	.
1	1									.
0	0									
0	0									
0	1	.	.							

Both Tables have two characteristics in common:

- a) 0 never turns up in the first place of a vertical sequence; and
- b) any numeral, belonging to the system, (except 0) may turn up at any place of the vertical sequence.

Yet there is a significant distinction between both Tables: since no sequence is permitted to begin with 0 it is impossible that there will ever be structural redundancy in Table I; in other words: as long as we stick to two symbols our representation of a Peano sequence cannot be negated, without violating our first rule. Table II shows a different picture. We notice at once that in the group of the two-place sequences (this time written horizontally for convenience' sake) 1 0, 1 2, 2 0 and 2 1 are structurally (morphogrammatically) identical; so are 1 1 and 2 2. In other words: what Table II displays is not a sequence composed of kenograms. This redundancy of structural characteristics would also occur in quaternary, quinary and any subsequent Leibnizian notation of counting.

It stands to reason that in both cases (represented by Table I and II) 0 is given a very specific interpretation: it is assumed a limine that an unlimited supply of zeros is

available forming an indifferent background against which numbers can be written. But zero may be interpreted differently.

However, if one attempts to write down with more or less chance of success an adequate representation of the esoteric numbers of Plato one has to abide (using as a mere convention the same kind of symbols) by two principles: first, every number must begin with 0 – as an initial symbolic expression, designated as such and no other symbol may be placed in the notation unless the symbol of counting in our conventional order of signs for counting 0, 1, 2, 3... has turned up at least once. This means that, e.g., a fourplace sequence, 0 1 2 1, is a legitimate expression. 0 2 1 1 is not, because it only repeats the morphogrammatic structure of the first four-place sequence. It follows that a system of esoteric numbers would have an approximately pyramidic shape and that every horizontal layer would represent a relatively independent numerical system beginning with 0 and ending with the highest number which is structurally permissible in the system.

Peano had used three primitive notions:

nought
number
successor.

Since nought represented no quantity, it was self-understood that his expressions had always to begin with a number denoting a measurable quantity. Nought represented only a boundless background against which numbers could be placed. This meaning of nought, of course, changes, when the distinction between foreground and background becomes irrelevant in an attempt to use a quantitative order of symbols to represent *structure*. It stands to reason that such a combination of quantity and structure must always have a highest number. And since McCulloch had at least approved of the distinction between iteration and accretion it was always a question how many structural differences can be accommodated between the 0 of accretion and its maximum.

Table III represents an attempt to display a Platonic system of esoteric numbers for a maximum of four places. It is the equivalent of one section of Table VII in Part II of "Natural numbers in Trans-classic Systems". Whether it would have found the approval of McCulloch as a representation of some of his ideas we will, alas, never know.

Table III of this report gives at least an inkling of what McCulloch might have meant with his ruminations that every way to understand the Absolute must be finite; but, on the other hand, Table III also suggests that some caution is needed if we want to reverse the classical thesis that all earthly existence is finitude and as such encompassed in the infinite Absolute. It is true that whenever and wherever we try to confront the Absolute the face it shows is that of finitude. But Table III also demonstrates that it belongs to the attributes of the Absolute that every finite aspect of it which we discover is followed by an unending sequence of aspects of higher complexity.

At this point an intricate problem of number theory evolves as the numbers which make up the increase of accretion are the esoteric numbers. For the numbers available to us when counting the sequence of the esoteric number systems are the numbers of the non-esoteric Peano order.

Table III

A short report of certain consequences of McCulloch's thinking on a domain remote from cybernetics may illustrate its philosophical relevance.

It is the area of philosophical hermeneutics as applied in history and other branches of the humanities. For the time being it seems absurd to approach hermeneutics as Dilthey and his successors understood it with arithmetical procedures. A number is always what it is, and the result of an arithmetical operation is either true or false – or undecidable. There is not the slightest room for 'interpretation'. But if we look at the numerical system evolved in the manner in which Table III demonstrates it is no longer enough to say: This is 2, this is 3, this is 4 etc. Because even if we add 1 to 1 equals 2, the question already will haunt us: which 2 do you mean? 2 in the iterative, or 2 in the accretive sense? If we read Table III from top to bottom there is no case in which a number has just one successor; it has at least two mostly, however, more. In Table III the fully accretive version of 4 would e. g. have five successors. In order to obtain this situation nothing has been done but apply the elementary dichotomy of sameness or otherness. This has the effect that, beginning with 0, an ever increasing amount of Peano sequences of non-esoteric numbers are spreading out in different sequences of esoteric numbers. However, as far as a given system of esoteric numbers is concerned the principle of successorship is not the one which we have just describes. In these finite number sequences which we have to read horizontally every "esoteric" number has just one and only one successor – except the last which is fully accretive; it has therefore no successor at all. Correspondingly, the first, which is fully iterative, possesses no predecessor. It follows *that the principle of hermeneutics originates only the transition from one finite system to the subsequent one with increasing structural properties*. But as long as we remain on a given esoteric level the principle of single successorship holds unconditionally.

If we want to express ourselves in Platonic terms we may say that the esoteric numbers partake ($\mu\acute{\epsilon}\delta\epsilon\xi\iota\zeta$) of the "mathematical" numbers of Aristotle ($\mu\omicron\nu\alpha\delta\iota\kappa\omicron\iota$). On the other hand, if we look at Table III and follow a sequence not horizontally but vertically we observe that the increasing multiplicity of Peano sequences is determined by the fact that every one of them crosses the horizontal order of esoteric numbers at different points. It is this concatenation of two different numerical orders that endows Number with properties which make it a useful tool for philosophy in general and especially for hermeneutics. Unless very specific and limiting conditions occur it is no longer sufficient to ask what is number, but in how many ways can it be interpreted, hermeneutically. A first step in this direction is an observation made almost simultaneously by Heinz von Foerster and the logician von Freytag-Löringhoff (Tübingen). They informed the author that the distinction between a fully iterative and a completely accretive number could be interpreted as the difference between cardinality and ordinality. In conventional mathematics it would, of course, be hard to see a hermeneutic issue in this contrast. What makes it hermeneutic is the fact that the cardinal and the ordinal number are connected by "mediative" numbers that have a cardinal and ordinal component. This requires a different way of thinking about numbers, a circumstance of which McCulloch was probably more ware than any other scientist of his time.

It had to be so. When Rufus Jones, the Quaker, asked him in his youth what he wanted to do in his life, he told him that the guiding star of his thinking would be the question

of numerosity. When the author met him in the evening hours of his life McCulloch had remained true to the self-dedication of his youth.

The reference to the Platonic numbers might suggest that McCulloch was basically a Platonist. However, such judgment would be far from the mark. He was well aware that Platonism in its narrow sense belongs to an epoch of philosophic thought which had seen its heyday. For him philosophy still oscillated between two fundamental inquiries: 'is reality rooted in a last irresolvable discord or in a final coincidence and reconciliation of all contradictions? The "Embodiments of Mind" give the impression that he leaned more toward the concept of a final resolution. In the "Mysterium Iniquitatis" we read that "cybernetics has helped to pull down the wall between the great world of physics and the ghetto of the mind" and "so we seem to be groping our way toward an indifferent monism". But the author, during the very late sixties, heard sometimes statements which were not exactly in accordance with the last quotation. The author remembers one occasion when McCulloch attacked psychoanalysis with a degree of animosity and the author drew his attention to a short sentence in the "Past of a Delusion" where he had read: "Upon Causality herself Karl Marx begat his bastard, Dialectical Materialism." The author who never considered himself a Marxist but an Hegelian stoutly defended Dialectics (and never mind the distinction between dialectic idealism and dialectic materialism). For him any transcendental theory of the universe had to have dialectic structure McCulloch denied the validity of this position but he was interested enough in the issue that some sort of discussion ensued. In its course he developed some ideas which fitted in ill with his leanings toward monism. The author is not sure whether they expressed some real convictions and new philosophical insights or whether they were merely argumentative stratagems to win over his opponent and disabuse him of dialectics. The author is inclined to believe the first: but he is by no means sure about it.

McCulloch casually referred to the Buddhistic Nirvana and insisted that European concepts of Reality were too deeply associated with the idea of „Substance" at the expense of "Relation". As always when he talked with the author he drew his exemplifications rather from formal logic and abstract number theory and not from cybernetics proper. Commenting on his suspicion that the concept of substantiality played too large a role in Western philosophy at the expense of the problem of relationship he speculated what philosophy would look like if we stopped talking so much about ultimate building blocks of the Universe and postulated that there were no such things and that every assumed last unit was nothing but a relation of even more fundamental units and that this splitting of the building blocks was a process that could never end. As a firm believer in dialectics the author could only agree. It fits in quite well with McCulloch speculations about numbers and Finitude. On the other hand, his musings on Substance and relation do not harmonize with the concept of an "indifferent monism" because there is no transcendental 'space' in which the difference between relator and his relata may ever disappear⁶].

⁶ Cf. C. Günther, Cognition and Volition in: Cybernetics Technique in Brain Research and The Educational Process. 1971 Fall Conference of American Society for Cybernetics. pp. 119-135.

Unfortunately, there remains a rest of doubt. McCulloch showed as usual an extraordinary reluctance to criticize the arguments of his opponent and to reveal much of his own philosophic forays into the Ultimate.

One thing seems certain, however – the philosophic position displayed in the "Embodiment of Mind" does not fully reflect what McCulloch thought during the last years of his life. He was no longer certain – as we still read in "Through the Den of the Metaphysician" – that "the seeming contradictions vanish in the grace of greater knowledge". His concept of metaphysics had deepened and he frequently made statements that were difficult to reconcile with the remark in the "Mysterium Iniquitatis of Sinful Man" that notions are metaphysical if "they prescribe ways of thinking physically about affairs called mental". Many of his remarks during the very last years would have suggested that by metaphysical terms he understood concepts which refer to a situation in which it was on principle impossible separate object and subject, including the thinker.

The author is led to this conclusion by McCulloch's reflections on the mutual logical position of Substance and Relation. There is no way in which Relation can ever be dissolved in a term of substantiality and vice versa. On the other hand, a relator and its relata depend functionally on each other, neither makes sense without reference to the other. They are – as Hegel would say – dialectically connected, and the problem of this connection defines the realm metaphysical. The author believes that McCulloch might lastly have agreed. If one shifts from the distinction between 'physical' and 'mental' in his former definition of what he would be willing to call "metaphysical" to the radically logical contrast between relation and relator it is obvious that the meaning of the term 'metaphysical' must also change. In the sense of Hegel's logic the distinction between relator and relatum can never "vanish in the grace of greater knowledge". While only relata may designate substance metaphysically the relator refers for ever to an act of subjectivity. This requires a deeper insight into the philosophical problem than cybernetics possesses at the present moment.

When the author was told that McCulloch was seriously dissatisfied with the development of cybernetics he could well understand it. But while writing this essay and trying to trace McCulloch's philosophic reflections into greater depths he has also learned to understand his reluctance to criticize the turn cybernetics has been taking. In his last years he was experimenting with new thoughts but had not reached the degree of certainty where his scientific conscience would have permitted him so speak aloud of his doubts and misgivings.

It might be possible to draw a clearer picture of McCulloch's last philosophical reflections; but this would require a greater amount of interpretation by the author – in other words: it would have been progressively more difficult to distinguish between what McCulloch had been thinking and what the author thought he did think. For this reason greater clarity and coherence has been sacrificed to the aim of at least approximate historical accuracy. The author is sure that he has not succeeded in the desired degree. He only knows that apart from Plato, Aristotle, Leibniz, Kant and Hegel – no modern philosophical thinker has exerted a greater influence on him than Warren McCulloch whose memory he shall always cherish and revere.