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Abstract

Why category theory? - Basic Notions of Category Theory - First steps to a paradigm change? - Natural Transformation and Proemiality - Morphogrammatics as categorifications of semiotics - Polycontextural logics and n-categories - On deconstructing equality

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Categories of the RK-Archive

Categories and Contextures

Rudolf Kaehr

ThinkArt Lab Glasgow 2006

Categories and Contextures

Categories and Contextures

1 Why category theory?

Some citations and hints for further reading are given.

1.1 Motivations

Category theory and mathematical foundational studies:

"I think it is fair to say that most mathematicians no longer believe in the heroic ideal of a single generally accepted foundations for mathematics, and that many no longer believe in the possibility of finding "unshakable certainities" upon which to found all of mathematics." Goguen

http://rbjones.com/rbjpub/philos/bibliog/hatch82.htm
http://rbjones.com/rbjpub/philos/maths/faq004.htm

„Computer scientists have far more flexible view of formalism and sematics than traditional logicians. What is regarded as a semantic domain at one moment may later be regarded as a formalism in need of semantics.“

M.P. Fourman, Theories as Categories, in: Category Theory and Computerpogramming, Springer LNCS 240, p. 435, 1986

A big collection of translations vs. a common language

Uschold, Michael F

"If you want to call these correlations or mappings between languages as expressed in a common meta-language, well that's ok, but it really is in a common language.
If you say "oranges" in L1 are "apples" in L2 and are extensionally (or some other property) equivalent, then you are expressing something in a third language L3.
The power of category theory comes in because you can have the syntax of a logic be related to the semantics of a logic (via morphisms), and then have those related to other objects, such as e.g., another logic (with its syntax and semantics), but the mediating "language" here is category theory (or a corresponding categorical logic).

Leo
ps. Gabbay has also written on labelled deduction systems (probably generalized under fibred logics), which essentially are correlated logics expressed in parallel to work simultaneously but on different aspects of the data (e.g., one portion on the natural language syntax ala categorial grammar, another on the nl semantics ala categorial semantics).
> Heterogeneity vs homogeneity
> ---------------------------
> However how we try to define standards, the world is heterogeneous.
> Sooner or later someone makes a new, incompatible variation. Homogeneity is pretty unstable. So we'll definitely need translations between systems.
> As for how to build them, sure, it's nice if you can find a common language, but that language probably won't capture every feature of every language. It's nice for translating a subset, but sooner or later you think of a nice way to map construct C1 of L1 into construct C2 of L2, and it won't go through the common language. So you have to revert back to individual translations, which are the general case.
> The real point is that we shouldn't get stuck on the idea of a common language-if it emerges naturally, great, but there's nothing wrong with a big collection of translations. The same goes for any collection of format or datatypes or whatever. Look at how we program these days:

More about categories

"Categories originally arose in mathematics out of the need of a formalism to describe the passage from one type of mathematical structure to another. A category in this way represents a kind of mathematics, and may be described as category as mathematical *workspace*.

A category is also a mathematical structure. As such, it is a common generalization of both ordered sets and monoids (the latter are a simple type of algebraic structure that include transition systems as examples), and questions motivated by those topics often have interesting answers for categories. This is category as mathematical *structure*.

Finally, a category can be seen as a structure that formalizes a mathematician's description of a type of structure. This is the role of category as *theory*. Formal descriptions in mathematical logic are traditionally given as formal languages with rules for forming terms, axioms and equations. Algebraists long ago invented a formalism based on tuples, the method of signatures and equations, to describe algebraic structures. Category theory provides another approach: the category is a theory and functors with that category as domain are models of the theory." Barr, Wells, 1999

1.2 Goguen´s Manifesto

Why do we need category theory?

Goguen: Manifesto

www-cse.ucsd.edu/users/goguen/pps/manif.ps

Onto-theology

http://www-cse.ucsd.edu/users/goguen/pps/onto5.pdf

What is a Concept?

http://www-cse.ucsd.edu/users/goguen/pps/iccs05.pdf

What is a logic?

(The identity of a logic is the isomorphism type of its skeleton institution.

http://www-cse.ucsd.edu/users/goguen/pps/nel05.pdf

Environment/interaction/information

http://www.cse.uconn.edu/~dqg/papers/e4mas.pdf

1.3 Category theory and Multi-Agent Systems

J.Pfalzgraf: "On an Idea for Constructing Multiagent Systems (MAS) Scenarios"

This opening talk gives a very brief account on some basic topics of our work on MAS modeling and prospects of intended future work. The concept of logical fiberings provides systems of distributed logics for the logical modeling of a MAS. We are using category theory (CAT) to introduce a unifying mathematical general model of MAS. A new aspect of our work concerns the idea to exploit CAT construction principles with the aim to achieve universal MAS scenario constructions based on CAT notions like limit and colimit. In particular this comprises the special instances of a product and coproduct construction of a possibly large system built up of simple components. Since such universal CAT constructions deal with universal properties, it is currently our hope ("vision") that it is possible to reach the point where we can construct normal forms of scenarios. Similar to the former work on introduction of logical fiberings, we are inspired by the powerful notion and theory of fiber bundles that directed us to the formulation of the "Generic Modeling Principle": Locally described simple components are constructively composed to form a (possibly complex) global system. We try to illustrate the idea by a simple scenario of cooperating robots."

http://www.cosy.sbg.ac.at/~jpfalz/InterSymp-2005.html

2 Basic Notions of Category Theory (H. Peter Gumm)

Categories. A category axiomatizes the abstract structural properties of sets and mappings between sets. Sets are considered as the objects and mappings are called the morphisms or arrows of the abstract category of sets. The language of category theory allows us to talk about arrows, their sources and targets and about their composition (o), of arrows, but not about the internal construction of sets and the nature of their elements. In particular, we cannot talk about the application "f (x)" of a map to an element of a set nor about the way f (x) is evaluated. One might say that sets and arrows are considered atomic particles of category theory and everything that is to be said about sets and mappings must be expressed solely in terms of the notion of composition, source and target.

To every object A, the existence of a particular identity arrow idA (sometimes written as 1_A) is postulated. Categorical language is too weak to axiomatize it using an equation such as e.g. "idA(x) = :x", for this refers to elements x inside the object A and to the application f (x) of f to x. In categorical language rather, idA must be characterized as an arrow satisfying:

- *source(idA) = target(idA) = A*
- *for all morphisms f with source (f) = A we have f o idA = f, and*
- *for all morphisms g with target (g) = A we have idA ° g= g.*

Note that composition is to be read from right to left - in accordance with traditional mathematical habit.

Definition 3.1. A category C consists of a class CO of objects A, B, C, . .. and a class Cm of morphisms or arrows f,g,h,... between these objects together with the following operations:

- *dom: Cm —> Co,*
- *codom: Cm —> Co, and*
- *id:Co - Cm,*

associating with each arrow its source (domain), resp. its target (codomain), and with every object A its identity arrow idA. Moreover there is a partial operation (o) of composition of arrows. Composition of f and g is defined whenever codom(f) = dom(g). The result is a morphism go f with dom(g o f) = dom(f) and codom(g o f) = codom(g). The following lauls have to be satisfied wenever the composition is defined:

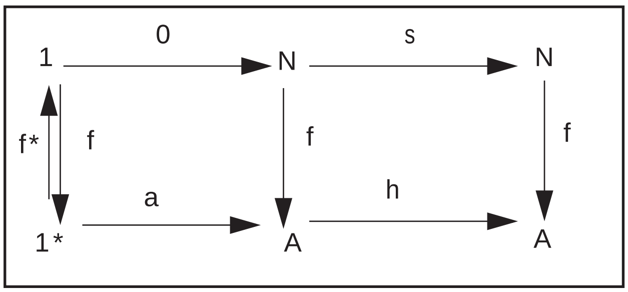
- *(h o g) o f =h o (g o f)*
- *idA o f = f and g = g o idA.*

3.1.1. Commutative Diagrams. Many notions have their origin in the standard example, the category of sets and mappings, so we borrow notions, symbols and graphical visualizations from there. For instance, we write f: A —> B, if f is a morphism with dom(f) = A and codom(f) = B. We use uppercase letters for objects and lower *case letters for arrows.*

It is convenient to draw objects as points and morphisms as arrows between these points. Such a representation is called a diagram. Often, compositions of arrows are not drawn - their presence is implied. A path of arrows represents the composition of the arrows involved. Whenever there are two different paths from an object A to an object B that enclose an area, it is often implied that their compositions are equal. One says that the diagram (or parts of it) commutes.“ Gumm, p.13-14

2.1 Terminal Objects

An important fact is that any two terminal objects (as well as any two initial objects) in a category are uniquely isomorphic. In other words, if T and T‘ are two terminal objects, then there is a unique isomorphism between the two. Because of this, it is customary, to collapse all terminal objects into a representative and talk about the terminal object.



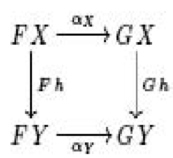
2.2 „up to isomorphism“

„The categorical approach to characterize objects and morphisms in terms of their relation to other objects and morphisms has the particular consequence that universal properties specify objects only „up to isomorphism“.

Definition: Objects A and B are isomorphic if there exists morphisms f: A —> B, f*: B —> A such that f*.f=iA and f.f*=iB

2.3 Natural transformation

Definition 1.24 (Natural Transformation) Given the functors $F, G : \mathcal{A} \to \mathcal{B}$, a *natural transformation* $\alpha : F \Rightarrow G$ consists of a collection of arrows $\langle \alpha_C : FC \to GC \rangle_{C \in \mathcal{A}}$ in $\mathcal{B}$, such that for any arrow $h : X \to Y$ in $\mathrm{arr}(\mathcal{A})$ the diagram



commutes, *i.e.*, $\alpha_Y \circ Fh = Gh \circ \alpha_X$. The diagram above is the *naturality square* associated with h.

Notation 1.25 A natural transformation $\alpha : F \Rightarrow G$ for functors $F, G : \mathcal{A} \to \mathcal{B}$ is a family of arrows in $\mathcal{B}$ indexed by objects in $\mathcal{A}$. Each arrow of such a family is a *component* of the natural transformation, and we use subscripts to denote them, *e.g.*, $\alpha_C : FC \to GC$. We use angle brackets to denote the family of arrows as in the definition $\langle \alpha_C : FC \to GC \rangle_{C \in \mathcal{A}}$. Naturality squares are often depicted with the arrow which they are associated with on one side, recall that the arrow and the naturality square may be from different categories.

BRICS Lecture Notes, LS-02-catnote, 2002

2.4 Diamond Strategies on objects and morphisms

Given the basic concepts of category theory we are free to apply the Diamond Strategies to re-design the field.

With the basics of objects and morphism naturally 4 positions can be focused.

First, the classic focus, is on objects. The categorial results are statements about objects i categories.

Second, the more modern focus is on morphisms. Here even objects are conceived as special morphisms.

Both thematizations are of equal value especially because the terms "object" and "morphism" are dual.

More interesting are the two further steps of diamondization of the categorial basics "object" and "morphism".

Third, we ask *"What is both at once, object and morphism?"* An answer is given by the distribution and mediation (dissemination) of categories in a poly-categorial framework.

Forth, the question arises: *"What is neither object nor morphism?"*

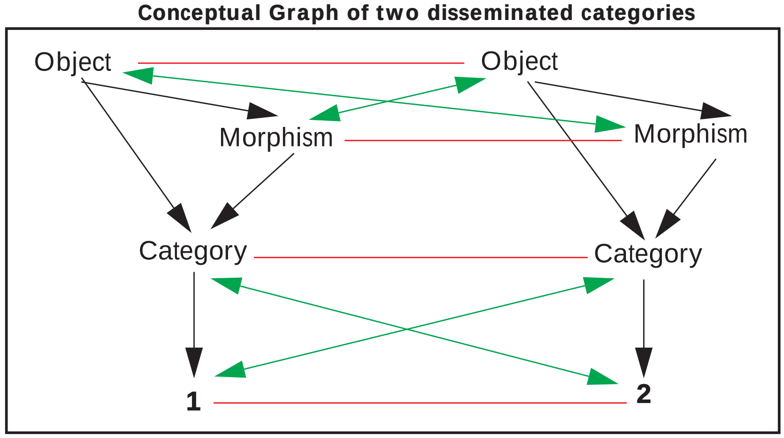
Also the following citation of Gunther does not intent to gives a definitional clear explanation of a *neither-nor* situation it is useful as a hint in the right direction.

> *„Thus the proemial relation represents a peculiar interlocking of exchange and order. If we write it down as a formal expression it should have the following form:*
>
> □ R^{pr} □
>
> *where the two empty squares represent kenograms which can either be filled in such a way that the value occupancy represents a symmetrical exchange relation or in a way that the relation assumes the character of an order.“* Gunther, p. 227

Obviously, the scheme or formula, represents neither an order nor an exchange relation. With this in mind, we can try to think the *neither-nor* of objects and morphisms of category theory as the inscription of the processuality of „categorization“ in itself into a scriptural domain beyond classical formal systems, that is into *kenogrammatics*.

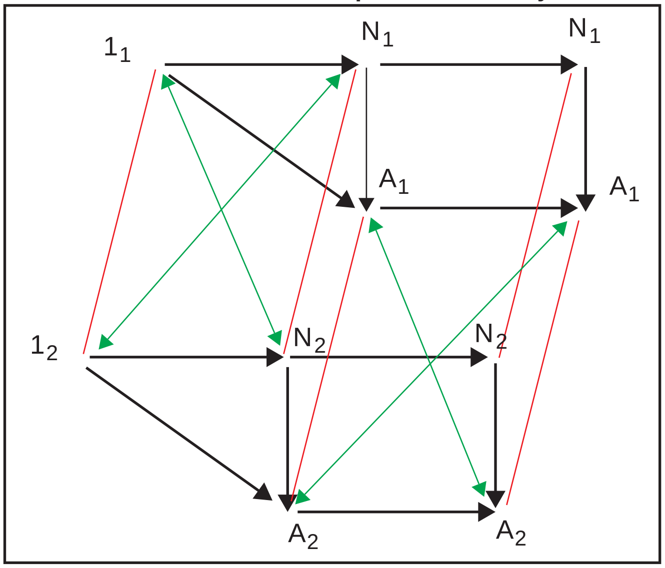
We need this quite wild „anti-concept“ of kenogram and kenogrammatics to deal scientifically and technically with the structure of any change, the proemiality, which is not to catch by any construction based on semiotical identity.

Conceptual Graph of two disseminated categories



2.5 Isomorphismus vs. Heteromorphismen (Dissemination)

Dissemination of isomorphic arithmetic systemes



Characteristics of coloured category systems

Between the arithmetic systems N and A, for i=1,2 a classic isomorphism is established. Between the arithmetic systems S_1 and S_2 with their internal isomorphism between N and A, there is a *hetero-morphism* ruled by the chiastic relations of order, exchange and coincidence which is separating in a mediated way the two arithmetic systems. The separation established by the hetero-morphism is different to the characterization of the arithmetic systems "up to isomorphism". This separation is positioning the arithmetic systems enabling a new kind of concretization of formal systems.

As a metaphor we can introduce *Colored Category Systems*. Colored category systems are mediating the basic concepts of *iterability* as iteration and accretion and the concept of *interaction*.

Beyond use and mention: ab/use

I am using scientific terms and methods to develop and inscribe, formalize, my ideas and at the same time, with the same gesture, I am *abusing* these methods to overpass the limitations of these scientific concepts. Any criticism of my work should keep this double strategy in mind. After that, there is a lot of work to do for all sorts of criticism.

It is not excluded, that all the abuse may be, step by step, by filling some gaps, correcting unnecessary misuse, transformed into a more scientific use of concepts without abandoning the fundamental subversion of the rationality involved.

For example, in the diagram of the conceptual graph of the notion of category as composed by objects and morphisms, I am using arrows which are inscribing the notional dependency structure of the notion of category itself. But in the same sense it could be mentioned that the notional dependencies are a sort of morphisms so I am using arrows as morphisms to explain the dependency structure of objects and morphisms in the notion of category. Usually conceptual graphs are applied to other domains than to themselves as a categorial notion. The conceptual graphs are therefore used in a sort of self-application and the question is open to what system the arrows of the reflexive use of the conceptual graphs belong.

The situation can easily be radicalized. We say a graph consists of the notions *nodes* and *edges*, thus the conceptual graph of the notion *graph* is a graph between nodes and edges. A graph is explained by the use of a graph.

But we all learned that we should distinguish between *use* and *mention*, object- and meta-language, notion and notation, and so on. Is this really always helpful? We can repeat this game of self-application on these terms too. And: Mention mention, use use, mention use and use mention, and neither-nor. Why not?

After having introduced all this ideas and hints to methods, we could start the real work of formalizing the whole stuff in the framework of category theory with the help of some strategies of rule-guided abuse, that is deconstruction.

Hetero-morphism and categorification

The speculations about category theory in the mode of hetero-morphism goes back to my text *"Strukturationen der Interaktivität"* (2003). At this time I didn't know anything about *categorification*. Hence the diamond strategies applied to categories have to be contrasted with the strategy of categorification and de-categorification.

The following experimental thoughts may give some hints for further work to do.

3 First steps to a paradigm change?

O-LUDICS.pdf, Jean-Yves Girard, 2000

One last word : this book has been written during the year 2000, the year of commemorative frenzy. So let me review last century, from the viewpoint of logical foundations.

1900-1930, the time of illusions : Naive foundational programs, like Hilbert's, refuted by Gödel's theorem.

1930-1970, the time of codings : Consistency proofs, monstrous ordinal notations, *ad hoc* codings, a sort of voluntary bureaucratic self-punishment.

1970-2000, the time of categories : From the mid sixties the renewal of natural deduction, the Curry-Howard isomorphism, denotational semantics, system $\mathbb{F}$ ... promoted (with the decisive input of computer science) an approach in which the objects looked natural and reasonably free from foundational anguish.

Proof-theory started as a justification of the rules of logic, as they were given to us, classical logic. The rules became in turn an object of study, inducing their own logic, which is not the original (classical) one. Intuitionistic logic, and later linear logic, not to speak of ludics are part of this logic of rules... whence the subtitle :

From the rules of logic to the logic of rules.

Time is changing quickly, now, we are in 2005, and it seems that category theory has lost its leading function to *polymathematics* with its n-categories.

> "I shall take heart from this dream and extend here a scheme I outlined in Chapter 10 of my book, an amalgamation of a scheme of Sir Michael Atiyah with one of Baez and Dolan, which derives in part from another giant of the twentieth century, Alexandre Grothendieck:
>
> **19th century**
> The study of functions of one (complex) variable
> The codification of 0-category theory (set theory).
> **20th century**
> The study of functions of many variables
> The codification of 1-category theory
> **21st century**
> Infinite-dimensional mathematics
> The codification of n-category theory,
> and infinite dimensional-category theory." (David Corfield)
> http://www.dcorfield.pwp.blueyonder.co.uk/phorem.htm

To connect motivations and metaphors for an introduction of PolyLogics to the 21st century additional to grammatological speculations about Chinese writing, the event of a revolution in category theory could play a significant role. It will still be an analogy and its interpretation still full of risks but easier to handle. The nice symmetry between logic, computation and 1-category theory is in a process of displacement by the new movement of n-category theory, challenges of interactivity in computing (Peter Wegner) and approaches in polycontexturality to transform logic; and more.

The Tale of n-Categories: http://math.ucr.edu/home/baez/week78.html#tale
n-Categories: Foundations and Applications: http://www.ima.umn.edu/categories/

Even if not well studied, we know well that 1-categories are based on triadic concepts. We know well dyadic concepts and their logics. But we still try to understand genuinely triadic concepts (semiotics, categories) in the framework of dyadics. Maybe of combined dyadics. But do we have an idea about n-categories? Are they iterations, even indefinite iterations ("infinite dimensionality") of triadic concepts of 1-category, still based on dyadic logics? Is the term "infinite" not understood as a dyadic and not as a genuine n-category theoretical term? What is the difference in the meaning of the notion "indefinite" in the 3 different conceptualisations; the dyadic, the triadic 1-categorial and the magic n-categorial?

Is the notion of the infinite chain of 1-categorial concepts constituting n-categories or Z-categories itself a 1-categorial concept?

PolyLogics are both: combinations of dyadic logics and genuinely n-categorial. Because mediation (combination) in PolyLogics is super-additive, decomposition into single dyadic systems is not working without reduction, that is, denying the interactional and reflectional parts of the whole. PolyLogics are based on morphograms. Morphogrammatics: The calculus of kenomic loci.

"Thus, category theory is philosophically relevant in many ways and which will undoubtedly have to be taken into account in the years to come."
Introduction to CT: http://plato.stanford.edu/entries/category-theory/
Manifesto for CT: http://www-cse.ucsd.edu/users/goguen/pps/manif.ps
CT and Computer Science: http://www.cwru.edu/artsci/math/wells/pub/ctcs.html

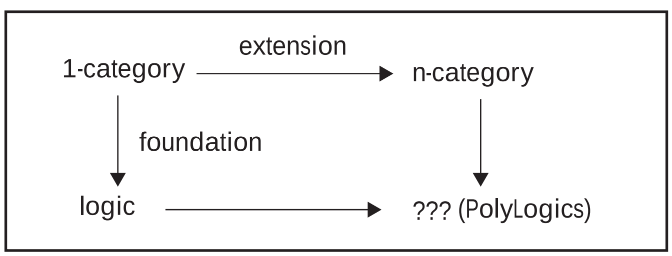
Locus Solum

Only locations matters. Jean-Yves Girard

What's about the location of different n-categorial systems, where are they placed, do they occupy a locus? And what's the calculus of these loci? Locations in the sense of Girard are intra-contextural loci of a system, they are not thematizing the genuine locus of the system itself. Does n-category theory reflect any loci?

Logic and category theory

Classic category theory is well founded in classic logic. On the other hand, logical systems can well be modeled in category theory.
William S. Hatcher: http://www.rbjones.com/rbjpub/philos/bibliog/hatch82.htm



Now, n-category theory claims to be a kind of a revolution transforming the old concepts of 1-category to new concepts of n-categories. My question remains, what are the logics of n-category theory? The plural of logics means the different roles logic can play in the construction of n-categories. What is the *use* of logic in developing n-categories, what is the *deduction* system for n-categories and what is the *foundational* (not fundamentalistic!) role of logic for the new category theory?

Ultimative presentation: Tom Leinster: http://www.maths.gla.ac.uk/~tl/#book
http://www.thinkartlab.com/pkl/lola/PolyLogics.pdf

3.1 Travel Guide to Categorification

Eugenia Cheng and Aaron Lauda are starting the journey with some precautions:

> "Higher-dimensional categories are like a vast mountain that many people are trying to conquer. Some intrepid explorers have made the ascent, each taking a different route and each encountering different hazards. Each has made a map of his route, but do we know how all these maps fit together? Do we know that they fit together at all? In fact, are we even climbing the same mountain?"
> http://www.dpmms.cam.ac.uk/%7Eelgc2/guidebook/guidebook-new.pdf

Viewpoints to Categorification

$Set \longrightarrow Function$: *Set Theory*

| |

$Object \longrightarrow Morphism$: *Basic Category Theory*

| |

$Morphism \longrightarrow Functor$: *Category Theory*

| |

$Functor \longrightarrow Nat\ Transf$: *Functor Theory*

| |

$Nat\ Transf \longrightarrow n-Category$: *Categorification*

| |

$n-category \rightleftarrows (n+1)-categorie$: *(De–)Categorification*

These steps can be read as a journey from the concrete dullness of sets to the noble hospice of categorificational abstraction on top of the mountain.

Historically, many voices for the propaganda of such a readings can be found.

On each level, a further privileging can be propagated: For classic category theorists the focus is on objects. (Post)Modern categorists, say Joseph Goguen, prefer morphisms to objects.

And n-categorists feel more advanced with Natural Transformations and categorification from step to step. The post-modern slogan may be *"Everything is a morphism"* and the n-categorist's avant-garde slogan *"Everything is a categorification"*. The latter sounds less dogmatic than the post-modern version if we involve de-categorification into the game. Both together categorificationn and de-categorification are enabling a specific dynamic between the levels of abstraction.

Nevertheless, there is no scientific need to award a special interpretation. What's wrong with sets? What's so great with functors? Or what's so bad with categories? Etc. Even if it is against the declared pluralism, the decoration of one level seems to be in accordance with the necessity to promote one and only one type of rationality.

3.2 Introducing categorification (John Baez)

"One philosophical reason for categorification *is that it refines our concept of 'sameness' by allowing us to distinguish between isomorphism and equality. In a set, two elements are either the same or different. In a category, two objects can be 'the same in a way' while still being different. In other words, they can be isomorphic but not equal. Even more importantly, two objects can be the same in more than one way, since there can be different isomorphisms between them. This gives rise to the notion of the 'symmetry group' of an object: its group of automorphisms.*

In a marvelous self-referential twist, the definition of '2-category' is simply the categorification of the definition of 'category' ! Like a category, a 2- category has a class of objects, but now for any pair x, y of objects there is no longer a set hom(x, y); instead, there is a category hom(x, y). Objects of hom(x, y) are called morphisms of C, and morphisms between them are called 2-morphisms of C. Composition is no longer a function, but rather a functor:

o: hom(x, y) x hom(y, z) —> hom(x, z).

For any object x there is an identity 1_x *. hom(x, x). And now we have a choice. On the one hand, we can impose associativity and the left and right unit laws strictly, as equational laws. If we do this, we obtain the definition of 'strict 2-category' [42]. On the other hand, we can impose them only up to natural isomorphism, with these natural isomorphisms satisfying the coherence laws discussed in the previous section. This is clearly more compatible with the spirit of categorification. If we do this, we obtain the definition of 'weak 2-category' [12].*

The classic example of a 2-category is Cat, which has categories as objects, functors as morphisms, and natural transformations as 2-morphisms. The presence of 2-morphisms gives Cat much of its distinctive flavor, which we would miss if we treated it as a mere category. Indeed, Mac Lane has said that categories were originally invented, not to study functors, but to study natural transformations!

From 2-categories it is a short step to dreaming of n-categories and even omega-categories — but it is not so easy to make these dreams into smoothly functioning mathematical tools. Roughly speaking, an n-category should be some sort of algebraic structure having objects, 1-morphisms between objects, 2-morphisms between 1-morphisms, and so on up to n-morphisms.
There should be various ways of composing j-morphisms for $1 \leq j \geq n$*, and these should satisfy various laws.*

The first challenge to any theory of n-categories is to give an adequate treatment of coherence laws. Composition in an n-category should satisfy equational laws only at the top level, between n-morphisms. Any law concerning j-morphisms for j < n should hold only 'up to equivalence'. Here a n-morphism is defined to be an 'equivalence' if it is invertible, while for j < n a j-morphism is recursively defined to be an equivalence if it is invertible up to equivalence. Equivalence is generally the correct substitute for the notion of equality in n-categorical mathematics."

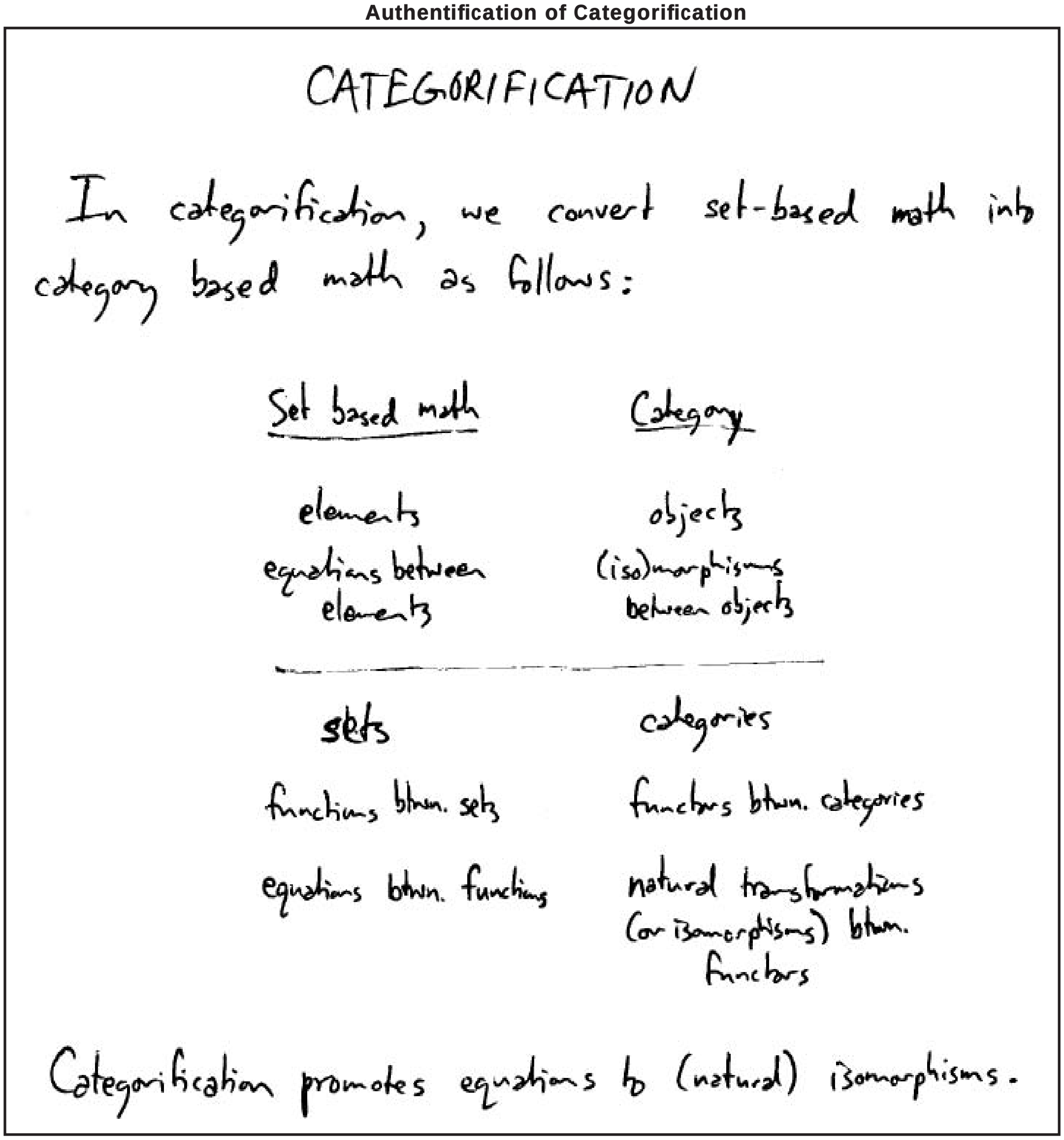
"The second challenge to any theory of n-categories is to handle certain key exam-

ples. First, for any n, there should be an (n+1)-category nCat, whose objects are (small) n-categories, whose morphisms are suitably weakened functors between these, whose 2-morphisms are suitably weakened natural transformations, and so on.

The prospect of exploring this huge body of new mathematics is both exhilarating and daunting. The basic philosophy is simple: never mistake equivalence for equality. The technical details, however, are not so simple — at least not yet. To proceed effcient-ly it is crucial that we gain a clearer understanding of the foundations before rushing ahead with complicated constructions."

http://arxiv.org/pdf/math.QA/9802029

Authentification of Categorification



CATEGORIFICATION

In categorification, we convert set-based math into category based math as follows:

Set based math	Category
elements	objects
equations between elements	(iso)morphisms between objects
sets	categories
functions btwn. sets	functors btwn. categories
equations btwn. functions	natural transformations (or isomorphisms) btwn. functors

Categorification promotes equations to (natural) isomorphisms.

http://math.ucr.edu/home/baez/qg-winter2007/w07week01b.pdf

3.3 Other intros

Eugenia Cheng and Aaron Lauda

Higher-Dimensional Categories: an illustrated guide book
"This work is an illustrated guide book to the world of higher-dimensional categories. A map would be more detailed and precise. An encyclopedia would be more comprehensive. Our aim is neither rigour nor completeness. Our aim is to provide would-be visitors with a sense of what they might nd on arrival; to give them an idea of what landmarks to look out for; to warn them of the hazards of the territory; to introduce them to the language of the place; to whet their appetite for exploring by themselves."

http://www.dpmms.cam.ac.uk/%7Eelgc2/guidebook/index.html
http://www.dpmms.cam.ac.uk/%7Eelgc2/guidebook/guidebook-new.pdf

Tom Leinster

"The heart of this book is the language of generalized operads. This is as natural and transparent a language for higher category theory as the language of sheaves is for algebraic geometry, or vector spaces for linear algebra. It is introduced carefully, then used to give simple descriptions of a variety of higher categorical structures. In particular, one possible definition of n-category is discussed in detail, and some common aspects of other possible definitions are established."
http://www.maths.gla.ac.uk/~tl/#book

A SURVEY OF DEFINITIONS OF n-CATEGORY
"The last five years have seen a vast increase in the literature on higher-dimensional categories. Yet one question of central concern remains resolutely unanswered: what exactly is a weak n-category? There have, notoriously, been many proposed definitions, but there seems to be a general perception that most of these definitions are obscure, difficult and long. I hope that the present work will persuade the reader that this is not the case, or at least does not need to be: that while no existing approach is without its mysteries, it is quite possible to state the definitions in a concise and straightforward way."
Tom Leinster. A survey of definitions of n-category. Theory and Applications of Categories, Vol.10, No.1, pp.1–70, 2002.
www.emis.ams.org/journals/TAC/volumes/10/1/10-01.pdf

Further information:

IMA 2004 Summer Program: n-Categories: Foundations and Applications
http://www.ima.umn.edu/categories/

Where to meet the poeple:

n-category cafe, blog
http://golem.ph.utexas.edu/category/

It is well known that there are different approaches to category theory.Thus, as n-categories are generalizations of categories, even more different approaches to n-category theory are emerging. Independent of this difference, there are many approaches to n-categories as Tom Leinster's *Survey of n-category* shows. In my study, or hallucination, I argue from text to text, constructing and deconstructing my reading, not considering the subtile conceptual differences which may be involved in the basic definitions behind the scenes.

4 Natural Transformation and Proemiality

4.1 Natural transformations as 2-categories

It may be of interest to bring together some not well known statements about natural transformation, category theory and proemiality. One is, that the main interest of Sanders Mac Lane, a founder of category theory, was the study of *"natural transformation"* and that the invention of categories have to be understood as tools to study properly natural transformations. The other one came up with new developments in category theory which shows that the notion of natural transformations in ordinary category theory is not a 1-categories, but belongs, according to John Baez, to the new construction of 2-categories.

> "Saunders Mac Lane, one of the founders of category theory, is said to have remarked, *"I didn't invent categories to study functors; I invented them to study natural transformations."* Just as the study of groups is not complete without a study of homomorphisms, so the study of categories is not complete without the study of functors. The reason for Mac Lane's comment is that the study of functors is itself not complete without the study of *natural transformations."*
> http://en.wikipedia.org/wiki/Natural_transformation#Historical_notes
>
> In fact, MacLane said: *"I did not invent category theory to talk about functors. I invented it to talk about natural transformations."*
>
> Okay, now back to MacLane's cryptic remark. What's a natural transformation?
> Well, natural transformations are things that go between functors. Suppose we have two functors F and G from the category C to the category D. Then a natural transformation N from F to G assigns to each object X in C a morphism N(X): F(X) Å® G(X) such that this diagram commutes:

```
         F(X) -F(f) --> F(Y)
           |               |
   N(X)  |               |  N(Y)
           v              v
       G(X) --G(f) --> G(Y)
```

> In other words, this equation holds:
>
> G(f) N(X) = N(Y) F(f)
>
> If you want to get into deeper waters, think about this question:
>
> What sort of thing is the *"category of all categories"*?
>
> It turns out to be, not just a category, but a 2-category. That means that in addition to objects and morphisms, it has "2-morphisms", that is, morphisms between morphisms. To see how this goes, let's call the 2-category of all categories "Cat". Then the objects of Cat are categories, the morphisms of Cat are functors, and the 2-morphisms are natural transformations!
> http://math.ucr.edu/home/baez/categories.html

***intention* of category theory**

The first statement tells more about the *intention* of category theory, i.e., to study natural transformation, than its way of realization.

Natural transformations in standard category are constructions, build on the base of categorial definitions, i.e., objects and morphisms between objects. Thus the real thing of CT are not natural transformations as such but morphisms. But this is, as we learnt, in some kind, in conflict with the intentions of Mac Lane to build category theory.

Natural transformation: a 2-category

The second statement says that natural transformations per se are not ordinary categories (1-categories) but 2-categories.

A question

Accepting both, the *narrative* and the *construction*, a quite natural question arises:

Why not to start, at least, with 2-categories to realize of Mac Lane's intention (intuition) of natural constructions to build a (more) general category theory?

Ordinary, i.e., 1-categories could then be understood as a *reduction* of the concept of natural transformation and natural transformation would not be a construct, depending of 1-categories but a genuine notion in itself, based on its own intuition.

A trip over the hill

After Baez, the story of the *category of all categories* goes on straight forward with extensions from 1-categories, 2-categories, n-categories to (n+1)-categories and to infinite categories.

> "Let me just say a bit about where things go from here. First of all, it turns out that we can keep playing this game ad infinitum. We can define a notion of "n-category" having objects, morphisms between objects, 2-morphisms between morphisms, and so on up to n-morphisms... and it turns out that "category of all n-categories" is really an (n+1)-category." (Baez)

Is it in the spirit of category theory to enter into endless iterations of the same principle? If yes, what to learn form set theory?

4.1.1 Categorification as a second-order concept

A new operation, introduced by Louis Crane and widely used by Baez, is producing n-categories: *the categorification of categories*.

> *"In a marvelous self-referential twist, the definition of '2-category' is simply the categorification of the definition of 'category' ! Like a category, a 2- category has a class of objects, but now for any pair x, y of objects there is no longer a set hom(x, y); instead, there is a category hom(x, y)."* (Baez)
> http://arxiv.org/pdf/math.QA/9802029

To define or introduce the concept of category is a first-order conceptualization. To categorize categories is obviously a higher order reflection on the notion of category. Self-referential contemplations are leading easily to infinite regresses or as in n+1-category to infinite progresses or, from a logical point of view, to antinomies.

Thus, the question arise, again. Is the logic of categories the same as the logic of the category of categories. Is the activity of categorification on the same epistemological level, on the same level of reflection as the definition of a category?

Up to now, we can collect four observations:
First, natural transformation as the genuine intention of category theory,
Second, natural transformations are 2-categories,
Third, categorification is a second-order concept,
Fourth, second-order concepts are well studied in polycontextural logics.

4.1.2 Categorification as a reflectional thematization

> *"One philosophical reason for* categorification *is that it refines our concept of 'sameness' by allowing us to distinguish between isomorphism and equality."* (Baez)

One of the main problems of philosophy, still is, the problem of identity. Identity rules the possibility to the difference of identical (equal) and diverse (non-equal). Medieval, theological, thought was concerned about the sameness, familiarity, analogy of entities. The leading term was not identity but analogia entis. Wittgenstein introduced the language game of "family similarity" to surpass the abstactness of identity concepts.

> "Categorification is the process of finding category-theoretic analogs of set-theoretic concepts by replacing sets with categories, functions with functors, and equations between functions by natural isomorphisms between functors, which in turn should satisfy certain equations of their own, called `coherence laws'." (Baez)

Thus, categorification is not simply a conservative abstraction, conserving the structure of the situation but also a creational activity, creating the new laws of coherence.

"… one does not merely replace equations by isomorphisms. One also demands that these isomorphisms satisfy some new equation of their own, called 'coherence laws'." And further, emphasizing the creative aspect: *"Finding the right coherence laws for a given situation is perhaps the trickiest aspect of categorification."* (Baez)

Thus, the question of a symmetry or duality between categorification and de-categorification can be placed again. The new equation, *coherence laws*, obviously, has no equivalent in the old categorified situation. Therefore, it seems clear that a de-categorification of the new constellation is not in symmetry to the categorified situation.

Another asymmetry: algebras and co-algebras

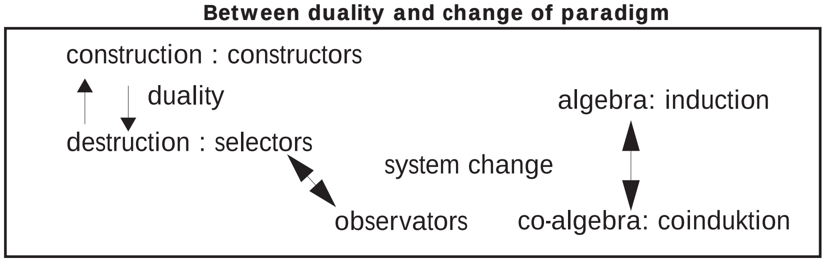
There is a similar situation to observe between algebras and co-algebras. On one side, a strict dualism between the terms of algebra and co-algebra exists. On the other hand, this dualism is not as symmetric as it suggest at a first glance. There are asymmetric situations too. A term like bisimulation in co-algebra has no counter-part in algebra from which it could be a dual.

Duality

Algebra	Co-Algebra
induction	co-induction
initial	final object
constructor	destructor
total	partial functions
algebra	coalgebra
visible	hidden
structure	behavior
well founded	non well founded sets
Turing Machine	Persistent TM
Horn clauses	liveness axioms

Co-algebra is in some sense a dual concept to algebra, but as Peter Gumm clearly pointed out, it comes with some transformations, producing new and not simply dual concepts. Co-algebra is not only dual to algebra but in some sense also subversive to it. You have the choice to focus on its duality or more on its subversiveness; it's up to you.

Between duality and change of paradigm



„But the theory is not just a simple minded dual to universal algebra. Structures such as e.g. bisimulations*, that don't have a classical counterpart in universal algebra, but that are well known from computer science, figure prominently in the new theory."* Peter Gumm (Cf.: Universelle Coalgebra, in: Th. Ihringer: Universelle Algebra, Heldermann Verlag, Berlin 2003.

R. Kaehr, Strukturationen der Interaktion
http://www.thinkartlab.com/pkl/media/SKIZZE-0.9.5-medium.pdf

4.1.3 Categorification and natural transformation

"Categorification is the process of finding category-theoretic analogs of set-theoretic concepts by replacing sets with categories, functions with functors, and equations between functions by natural isomorphisms between functors, which in turn should satisfy certain equations of their own, called `coherence laws'. Iterating this process requires a theory of `n-categories', algebraic structures having objects, morphisms between objects, 2-morphisms between morphisms and so on up to n-morphisms." (Baez, Dolan)
http://arxiv.org/pdf/math.QA/9802029

If we accept to focus on natural transformation as a basic interest of category theory we can use this construct as an irreducible fundamental concept or unit of categorial formalization. Categorification of categories would then turn into a categorification of natural transformation. A further step could be to conceive natural transformation as a chiasm, i.e., as a proemial relationship, and step-wise categorification of categories would then be an application of proemiality as an operator onto itself. That is, chiasms of chiasm, hierarchically and heterarchically organized, could be understood as a polycontextural explication of the idea of categorization.

Categorification, as introduced by Baez, seems still to be connected to a "linear" order of step-wise enlargements of categories from n-categories to n+1-categories. The application of categorization as an operator of reflection seems to be bound by the model of the linear order of natural numbers, may be as a natural model of subjective reflection. This seems to be legitimized, at first, by the restriction to *"iterated categorifications and stabilizations of some of the very simplest algebraic structures: the natural numbers and the integers. However, one can also categorify many other concepts [...]."*

Thus, there is no reason for a limitation in the application of the genuine concept of categorification. Further to the mentioned more complex examples of categorification, the modi of Iterability of categorization itself could be enlarged, without conflicts, at once, to hierarchic as well as to heterarchic modi of iterability.

Such a tabular order of the organization of general categorification could be supported by the similar order of chiastic iterability. That is, the iterability of chiasms is at once iterative and accretive, producing the tabularity of hierarchic and heterarchic dynamics. In other words, accretive categorification, in contrast to iterative categorification, could operate as a creative process of deliberating, say the concept of natural numbers, from its "set-theoretical" linearity in favor to a tabular, iterative and accretive, structure. Thus, the refinement of "our concept of 'sameness' by allowing us to distinguish between isomorphism and equality" itself could be refined into the additional distinction between isomorphisms and 'hetero-morphisms'.

Because, say, natural numbers, are anyway characterized in set-theory only "up to isomorphism" and not to equality, a categorification of such an isomorphic object has not to be restricted to an iterative structure excluding its possible accretive dynamics. The distinction between isomorphism and hetero-morphism and equal, iteration and accretion, can be clarified by the introduction of two distinctive modi of iterability and abstraction: the *is-abstraction* and the *as-abstraction*.

Categorification seems to be neutral to this distinction. In other words it is applying the is-abstraction without giving any reasons for this decision. That is, the action of categorifying a structure is categorifying this structure as this structure and not as something else. Obviously, categorification of the concept of natural numbers is different from the categorification of the concept of Hilbert spaces. But to categorify something as something else is producing reflectional differences into the concept of sameness of this object and is not confusing it with other different objects. Categorification can be explained as a specific kind of thematization.

As an additional metaphor to Baez' tip of an iceberg metaphor *"It is clear, therefore, that the set-based mathematics we know and love is just the tip of an immense iceberg of n-categorical, and ultimately omega-categorical, mathematics."* the uniqueness of the ultimate tip has to be deconstructed in favor of a multitude of "equally" ultimate tips of a monster of even greater immensity.

This new distinction of iterative and accretive categorification, is still in concordance with Baez' declared basic philosophy: *"never mistake equivalance for equality"*.

In simplifying words, the unique linearity of natural numbers is, from the reflectional and polycontextural view-point of n-categories, not guaranteed.

Creative asymmetry?

It could be said, that the thematizations of categorification and de-categorification are not necessarily symmetric operations.

Only in abstract terms we can say that the decategorification of an (n+1)-category is producing an n-category and the categorification of an n-category is producing an (n+1)-category. What is missing in this argument is the asymmetrical and "creative" aspect involved in categorification, i.e., that *"these isomorphisms satisfy some new equations of their own, called 'coherence laws'."* The complementary happens with decategrification as a process of concretization of the situation from (n+1)-categories to n-categories. Here too, some creativity is involved albeit in a reverse direction to (re)store the relative concreteness of the categorified category. From this level of reflection, all categories are of equal relevance. The *relevance* of a category is not necessarily augmented by a higher level of categorification. All levels are having their own characteristics producing different kinds of relevance.

Thus, what exactly is the relationship between categorification and decategorification in categorial terms? Obviously, this question belongs to a kind of a metatheory of category theory. Mostly, mathematicians are very proud to find a duality property or even a duality principle in their theories. There is, surely, one in ordinary category theory. Thus, how is this duality transformed inside the new theory of n-categories? The new complexity of a general duality hence seems to be *first*, duality inside each n-category, *second*, duality between different levels of categories, i.e., in fact, between categorification and decategorification.

We can play this game further and involve some questions from the Diamond Strategies. Between categorification and decategorification exists an interesting relation, maybe a (a)symmetric) duality. Both part of the duality are well established and accessible to further studies. What could the field of research be which is, from the point of view of the activities of (de)categorification, *neither* categorification *nor* decategorification? And in the "same" manner, what, in this sense, could be *both at once* categorification and decategorification for mathematical activities? A new field could emerge, at first called, *dynamic category theory*, playing with the intertwined mechanism of chiasms between categorification and decategorification as simultaneous and autonomous activities. The complementary activity could be called *morphic category theory* studying the kenomic patterns of the dynamics of (de)categorification. From the point of view of the double thematization "(de)categorification" there should be no priority of relevance between categories of different levels. Nor should one of the four Diamond positions between categorification and de-catogorification have a prority to another postion.

Dichotomy, trichotomy and 4-foldness

Ordinary category theory is based on the epistemology and semiotics of triadic-trichotomic activity of formalization. Also it is often understood in a reductive way as dyadic-dichotomic, because it is build on only two entities: objects and morphisms. But this reduction is denying the triadizity of objects, morphisms and compositions.

Categorification of ordinary categories can be modeled a as a semiotic activity of building super-signs, i.e., signs of signs. Repeating the triadic-trichotomic structure of signs.

Natural transformation as a basic principle is best conceived as a tetradic and 4-fold structure of grammatological or graphematic activity. It is surpassing the model of signs, thus it isn’t anymore based on semiotics and logic. Challenges of appropriate notational systems to cope with such a situation are not yet understood.

This way of thinking becomes more accessible if we connect it with the design of proemiality.

4.2 Chiasms as natural transformations

There was never a problem to understand a chiasm in polycontextural logic as a natural transformation. What was in conflict to it was its definition as a construction on the base of morphisms. Chiasms, on the other hand, are introduced conceptually as genuinely 4-fold structures, thus not based on triadic or dyadic categorial terms.

Chiasms, nevertheless, are introduced and constructed as a composition of order-, exchange and coincidence relations.

Obviously, morphisms can be considered as order relations, especially:
`F(X) ---F(f) ---> F(Y)` and `G(X) ---G(f) ---> G(Y)`

"Well, natural transformations are things that go between functors."
`F(X) -- N(X) --> G(X)` and `F(Y) -- N(Y) --> G(Y)`

The natural transformation N can be interpreted as the coincidence relation in the definition of chiasms. That is, the coincidence guarantees the analogy or sameness between the two order relations (analogia entis). Sameness in chiasms are the equivalences in contrast to equality in category theory. Natural transformations are harmonizing, i.e., establishing analogy and coincidence between two morphisms as coincidence relations in chiasms are naturalizing between morphisms.

What is not mentioned in the concept of natural transformations are the exchange relations of chiasms. But this seems to be obvious because between the objects F(X) and G(Y) as well between the objects F(Y) and G(X) a kind of a conceptual inversion holds. These inversions can be modeled as exchange relations between the involved terms.

The construction of natural transformation then is summarized or boiled down to the equation:

`G(f) N(X) = N(Y) F(f)`

Which defines the formal equivalence or isomorphism between the functors F and G. Natural transformations are describing the "behaviour" of functors, as functors are describing the "behavior" of categories, which are a composition of objects and morphisms.

It is not wrong to ask stubbornly to which equational logic the equation term (=) itself belongs.

> "A natural transformation is a relation between two functors. Functors often describe "natural constructions" and natural transformations then describe "natural homomorphisms" between two such constructions. Sometimes two quite different constructions yield *"the same"* result; this is expressed by a natural isomorphism between the two functors."
> http://www.fact-index.com/c/ca/category_theory.html

Natural transformations are focussed on the isomorphism between functors F and G. In contrast, chiasms are focussed on the *hetero-morphism* between F and G. The new idea of hetero-morphism is not denying the isomorphism between functors but is additionally emphazising the *positionality* of the different functors. In other words, the equation `G(f)N(X)=N(Y)F(f)` which holds for functors in a category is replaced by *mediation* in polycontexturality theory. Distributed functors which are isomorph are still different in their sameness as positioned at different (notational) loci.

Because chiasms are not based only on order-, exchange and coincidence relations but also on the *positionality* of the relations, the separation of isomorphism and hetero-morphism for natural transformations is possible. This argumentation seems to be supported by the understanding of natural transformation as 2-catgories.

"One philosophical reason for categorification *is that it refines our concept of 'sameness' by allowing us to distinguish between isomorphism and equality."* (Baez) It might be a further step of refinement, differentiation and deconstuction of identity and equality to introduce additionally to equality and isomorphism (equivalence) the abstraction of hetero-morphism.*"Equivalence is generally the correct substitute for the notion of equality in n-categorical mathematics*." (Baez) A deconstruction of equality and difference was given in earlier papers and German wordings as the chain of *"Selbigkeit, Gleichheit, Verschiedenheit"* which itself is involved in dynamic interpretations.

Positionality in polycontexturality theory is the mechanism to put thoughts from the mind, subjective or objective, to the blackboard (Berthold Brecht). Blackboarding is preventing from subjective idealism.

On the base of this modeling of the concept of chiasm as a natural transformation we can now continue to study complex chiasms and their, say logical interpretations in polycontextural systems. This may be of interest in itself but is not corresponding with the intentions of the introduction of chiasms, say to disseminate, not only morphisms or logical systems, but category theories, too.

What makes this interpretation of chiasms as natural transformations interesting is the new idea to understand them not in the classical sense of 1-category theory but as genuine 2-categories. With that we are not forced to reduce the idea of chiasms as natural transformations to the mono-contexturality of 1-categories and their logic.

On the other hand there is not much information to find about the relationship between the new idea of n-categories and logic. Despite of the quite allergic reactions to logic, understood as foundational studies, FOL, at least in the context of deductions, logic, of what ever kind, has to enter the game.

Hence, it would be interesting to connect the concept of n-categories with polycontextural logic. To each n-category a corresponding m-contextural logic could offer the necessary deductional apparatus for n-categorial developments. Such a link between n-categories and polycontextural logics would be a possibility to prevent n-categories to be reduced to mono-contexturality.

What is not considered by categorial constructions is the super-additivity as it emerges in polycontextural systems. Thus, to each n-category s(n)-contextures are involved. On this stage of a possible modeling questions about the architectonics of disseminated contextures are not yet considered.

4.3 Natural transformations as chiasms

It seems that the real intention of Lane in the development of category theory was not a theory of morphisms but a formalization of natural transformation. Category theoreticians are celebrating today the deliberating abstractness of morphisms, its departure from set theory and its formal operativity as a modeling and translation language.

> "Eilenberg / MacLane have said that their goal was to understand natural transformations; in order to do that, functors had to be defined; and to define functors one needed categories."
> http: / / www.fact-index.com / c / ca / category_theory.html

The intuition of chiasm (proemial relationship) is realized as a composition of relation, i.e., order, exchange and coincidence relations. But it makes not much sense to think that therefore chiasms are special compound relations based on dyadic relations of classic relation theory (Peirce, Schröder). In a similar sense it could be postulated that natural transformation, despite its construction out of objects and morphisms, i.e., defined as a category, is per se not reducible to morphisms.

Hence, it would be conceived as a formal explanation of a new intuition with the help of known or specially invented methods belonging to the traditional way of thinking and formalizing. In other words, as it turned out, the idea of natural transformation is a 2-category formalized with the tools of 1-category.

Thus, the notion of natural transformation can play a double game. It can be part of category theory as a special category, i.e., as a functor category.

> "Our slogan proclaimed: With each type of Mathematical object, consider also the morphisms. So, what is the morphism of functors; that is, a morphism from F to G where both F and G are functors F, G: **C** –> **D** between categories **C** and **D**?" MacLane, p. 390

Or it can play the role of the starting point of a new concept of formality and operativity, first thematized as irreducible 2-category.

Thus the introduction of categories and functors are not more than the tools to define and *"understand natural transformations"*. To concentrate on functors and categories makes the tools the theory they should support. In traditional category theory, the servants are becoming the masters.

Classic category, also it is trying to abandon its classical heritage, like set theory and first-order logic, is still too much relied on its rejected past. There is no big paradigmatic jump from set theory to category theory. But it would be a remarkable paradigm change to start, conceptually and with its corresponding operative apparatus, with “natural transformation”. A first step into this direction may be opened up by the movement of n-categorial studies.

In polycontextural terms, the two categories, compared and brought into relation by the functor of natural transformations are two different contextures. Their difference is basic and best understood as dis-contexturality. Each contexture is giving place for its own formality, i.e., formal rationality: logic, semiotics, category theory, etc.

Chiasm of categorial composition

Another approach to connect category theory with proemial relation is introduced by the observation I published in the 80s as a very short sketch that the rule and mechanism of composition of morphisms can be read as a chiasm between domain and co-domain of morphisms. Compositions are crucial for categories, i.e., they are part of the axiomatics of category theory (Identity, Commutativity, Associativity of composition). Isolated morphisms wouldn't make much sense.

In terms of *domain* and *co-domain* of morphisms, composition simply means a coupling of a codomain of a first morphism with the domain of a second morphism. This is a quite innocent notion. In the sense of deconstructive (ab)use it can be involved into a more intriguing game. With the four terms, domain, co-domain, first, second, a chiasm easily can be constructed. This construction, again, can be generalized in the sense that not only morphisms of a category are involved but categories as such. Thus the simple composition of morphisms inside category theory can be generalized to a mediation of a multitude of category theories. Such a multitude of mediated category theories, a polycontextural category theory or in other words, category theory in polycontextural situations, could be considered as a theory of n-categories.

Again, such a concept of categories in polycontextural situations would naturally involve the necessary and adequate semiotics, arithmetic and logics not only to round up the scenario but to prevent it from unnecessary reductions back to the old paradigm of house holding.

4.4 Proemiality and chiasms of n-category theory

As a complementary concept to isomorphism, *hetero-morphism* may be considered. The positionality of distributed isomorphisms, i.e., hetero-morphism can be supported by plurality of distributed categories of n-categories.

An aspect of chiasms not yet thematized is its proemiality. Not only the fact that categories are distributed has to be considered but also the insight that there is no category, single or plural, i.e., 1-category or n-category, without being placed, taking a position, thus being distributed. Therefore, we can change focus and concentrate on the distributedness and the in-between of categories. The in-between of categories itself is not a category. It pre-faces categories, hence, it is its proemiality.

Isomorphism is producing ideality; hetero-morphisms are generating positionalities. Isomorphisms are generating abstractness, hetero-morphisms concreteness in formal systems.

The notion of natural transformations between functors can be understood as a type of *abstraction* for functors. Complementary, *proemialization* can be considered as a morphic *subversion* opening up the domain of morphogrammatic patterns of functorial behaviours.

Different kinds of morphic abstractions

Morphic abstraction (subversion) had been introduced by Gunther as *"negation-invariant"* patterns. Thus, this kind of morphic abstraction is depending on negations. This is well studied in [Kaehr, Mahler, 1995]. Another, more formal or metatheoretic foundation of morphic abstraction can be introduced over the *duality* principle of a formal theory. Thus, for this kinf of morphic abstraction, say, disjunctional and conjunctional logical operations have the same pattern, i.e., the same morphogram. To connect this step of abstraction with Gunther's terminology, it could be called a *reflector-invariant* pattern.

5 Morphogramatics as categorifications of semiotic systems

Morphogrammatics are conceived as the result of a "morphic abstraction" of semiotic systems which is in fact more a subversion then an abstraction in the strict sense. Because morphic abstraction is not only constructing a more abstract pattern but is also reversing the systematic order of the concepts. It is a kind of "renversement" and "deplacement" of the structure of sign systems. Morphic abstraction can be specified as a kind of "categorification", i.e., of building the category of the category of sign systems.

5.1 What is a sign system?

Sign systems, short, semiotics, are well conceptualized or categorized as categorial systems (Joseph Goguen).

In more detail, a **sign system** consists of:

- a set S of **sorts** for signs;
- a partial ordering on S, called the **subsort** ordering;
- a set V of **data sorts**, for information about signs, such as colours, locations, and truth values;
- a partial ordering of sorts by **level**;
- for each level n, a set C_n of **constructors** used to build level n signs from data and signs at level n or less (**constants** take zero arguments);
- a (partial) **priority ordering** on each C_n;
- some relations and functions on signs; and
- a set of **axioms**, constraining the possible signs.

More formally, a **semiotic morphism** from a sign system S_1 to a sign system S_2 consists of partial functions mapping

- sorts of S_1 to sorts of S_2,
- constructors of S_1 to constructors of S_2,
- predicates and functions of S_1 to predicates and functions of S_2,

such that the mapping of sorts to sorts preserves the subsort ordering and does not change data sorts, and arguments and result sorts of constructors and predicates are preserved (modulo the sort mapping).

http://www-cse.ucsd.edu/users/goguen/new.html

http://www-cse.ucsd.edu/users/goguen/papers/sm/smm.html

"A good (semiotic) morphism should preserve as much of the structure in its source (sign) system as possible. Certainly it should map sorts to sorts, subsorts to subsorts, data sorts to data sorts, constants to constants, constructures to constructors., etc."
Goguen, Algebraic Semiotics, p. 11

Definition 2: Given sign systems S_1, S_2, a **semiotic morphism** $M: S_1 \to S_2$, from S_1 to S_2, consists of the following partial functions (all denoted M):

1. sorts of $S_1 \to$ sorts of S_2,
2. constructors of $S_1 \to$ constructors of S_2, and
3. predicates and functions of $S_1 \to$ predicates and functions of S_2,

such that

1. if $s \leq s'$ then $M(s) \leq M(s')$,
2. if $c: s_1 ... s_k \to s$ is a constructor (or function) of S_1, then (if defined) $M(c): M(s_1)...M(s_k) \to M(s)$ is a constructor (or function) of S_2,
3. if $p: s_1...s_k$ is a predicate of S_1, then (if defined) $M(p): M(s_1)...M(s_k)$ is a predicate of S_2, and
4. M is the identity on all sorts and operations for data in S_1.

More generally, a semiotic morphism can map source system constructors and predicates to compound terms defined in the target system[13]. □

Semiotics and Ontologies

Semiotics, as the general theory of signs, would seem a natural place to seek a general HCI framework. However

(1) semiotics has not developed in a precise mathematical style, and hence does not lend itself well to engineering applications;

(2) it has mostly considered single signs or systems of signs (e.g., a novel, or a film), but not representations of signs from one system by signs from another, as is needed for studying interfaces;

(3) it has not addressed dynamic signs, such as arise in user interaction; and

(4) it has not paid much attention to social issues such as arise in cooperative work.

A new project to address such problems has so far developed precise algebraic definitions for sign systems and their representations, and a calculus of representation providing laws for operations that combine representations as well as precise ways to compare the quality of representations.

Joseph Goguen, Algebraic Semiotics and User Interface Design, 2000

http://www.isr.uci.edu/events/dist-speakers00-01/goguen00.html

Goguen’s general overview:

http://www.cs.ucsd.edu/users/goguen/projs/semio.html

5.2 Sign systems as 1-categories

Thus a sign system is defined as a 1-category. It will turn out that a categorification of sign systems are evolving as 2-categories.

In an analogous sense as signs are abstracted from their graphemic substrate, morphograms can be seen as abstractions from the structure of semiosis, i.e., the process of realizing signs. Instead of stressing on the concepts of abstraction and generalization the techniques of "categorification" (Baez) could be introduced to define sign systems out from graphemic inscriptions and morphograms as the "elements" of morphogrammatics.

Categorification is the reverse process of decategorification. Decategorification is a systematic process by which isomorphic objects in a category are identified as equal. Whereas decategorification is a straightforward process, categorification is usually much less straightforward, and requires insight into individual situations.

Sign systems are categorifications of sign repertoires, i.e., sets of tokens. Such an abstract understanding of sign systems is based on the abstractness of alphabetism. In fact, the sign repertoire of a formal sign system can be reduced to two elementary signs: the stroke-sign and the nil-sign. In other words: to the binarity of 1 and 0.

Sign systems and semiotics in the tradition of Peirce are triadic-trichotomic structures. Thus, they are easily conceptualized as categories in the sense of mathematical category theory. This was stressed by Max Bense in the 70s.

The semiosis of semiosis, i.e., the categorification of semiosis is producing n-categories. n-categories are highly complex objects which can not be described at once from one and only one point of view. The topology of n-categories is highly interwoven and knotted. Thus, morphograms are appearing as categorifications of complex sign systems, i.e., of semiotic morphisms.

Signatures and the invariance of truth under change of notation

To speak about alphabetism in formal systems, with its atomicity, linearity, iterability, and ideality is not forgetting the conceptual move from alphabets as *sign repertoires* to the more abstract concept of *signatures* of institutions introduced by Goguen. This move is connected with the move from set to category theoretic conceptualizations.

> Institutions accomplish this formalization by passing from "vocabularies" to signatures, which are abstract objects, and from "translations among vocabularies" to abstract mappings between objects, called signature morphisms;
> then the parameterization of sentences by signatures is given by as assignment of a set Sen(S) of sentences to each signature S, and a translation Sen(f) from Sen(S) to Sen(S') for each signature morphism f: S -> S', while the parameterization of models by signatures is given by an assignment of a class Mod(S) of models for each signature S, and a translation Mod(S') -> Mod(S) for each f: S -> S' (please note the contravariance here).
>
> More technically, an institution consists of an abstract category Sign, the objects of which are signatures, a functor Sen: Sign -> Set, and a contravariant functor Mod: Sign -> Setop (more technically, we might uses classes instead of sets here).
>
> Satisfaction is then a parameterized relation $|=_S$ between Mod(S) and Sen(S), such that the following satisfaction condition holds, for any signature morphism f: S -> S', any S-model M, and any S'-sentence e: $M \; |=_S f(e)$ iff $f(M) \; |=_{S'} e$
> ***This condition expresses the invariance of truth under change of notation*.**
>
> http://www.cs.ucsd.edu/users/goguen/projs/inst.html

Ideality: Abstractness of sign systems

Signatures are even better realizing alphabetism than sign repertoires because they are empathizing the abstractness of alphabetical signs, that is, the *ideality* of signs, and sign systems, in contrast to concrete occurrence of signs, independent of the content of the sign repertoire, i.e., the concrete notational material. That is, sign systems are not only characterized by atomicity, linearity, iterability, but also by *ideality*. Ideality is the medium of realization of signs. Sign systems are not concrete systems but ideal systems. Notational systems of sign systems are, to some degree, the concrete realizations, that is, the representations of abstract sign systems. And signatures as they are defined in the theory of institutions are the themes of thematizations.

Chinese Gödel nummbers?

Does it make any sense for notational systems, like Chinese hieroglyphs, but also for morphograms, that their truth is *"invariant under the change of notation"*?

Obviously not at all. Simply because morphograms are categorial abstractions from sign systems. And Chinese hieroglyphs are holistic patterns, where the meaning of the involved strokes are at once context-depending and context-enabling.

The kind of abstractness of both, the morphograms and the hieroglyphs, are different from the ideality constituting the abstractness of sign systems. In some sense, the hieroglyphs are ultra-concrete, and the morphograms are ultra-formal. Thus, sign systems are in-between the conceptuality of hieroglyphs and morphograms.

As a first consequence, we have to understand, that Gödelization (Arithmetization) of morphogrammatic and hieroglyphic systems doesn't make sense.

This is not in conflict with attempts to codify hieroglyphs by a four-byte coding for practical reasons.

The body of signs

Befor entering into a full definition of signs as representamen we can study the body of signs,i.e., the carrier of the carried meaning and significance of signs.

Signs as morphisms between graphems.

Between tokens and types a morphism over the set of tokens is defined.

6 Sign Systems as Concretizations of morphogrammatics

In analogy to the cristallization metaphor, sign systems can now be considered as cristallizations, i.e., concretisations of morphogrammatic constellations.

7 Polycontextural logics and n-categories

There are some interesting correspondences between n-categories and different types of polycontecturality. Inn realizing the approach of a polycontextural matix 4 types of polycontexturality can be introduced: 1. elementary, 2. interactional and reflectional, 3. interventional and 4. anticipatoy modi of polycontexturality

7.1 1-categories

7.1.1 Classic logics as 1-categories

As well known, logic corresponds to 1-categories.

Objects as propositions.
Arrows as deductions.

The law of associativity of morphisms is guaranteeing linearity.

http://www.arxiv.org/PS_cache/math/pdf/9802/9802029.pdf

7.1.2 Linear polycontextural logics as 1-categories

Distribution of logics along a linear ordered index category is a reasonable way of modeling polycontextural logics with a linear architectonic.

1. One approach is realized by Jochen Pfalzgraf with his *fibre bundle theory* and a modeling of distributed and mediated logics in a topology of a linear ordered index category.

http://racefyn.insde.es/Publicaciones/racsam/indices/vol98_1.htm
http://racefyn.insde.es/Publicaciones/racsam/art%C3%ADculos/racsam%2098_1/2004-pfalzgraf.pdf

2. Another approach is realized by the construction of *mediated logics*.

Objects as logics.
Morphisms as mediations.

Associativity of morphisms guarantees linearity.

Morphisms as mediations is, as typical for category-theoretical approaches, a very abstract and external thematization of the mechanism of mediation.

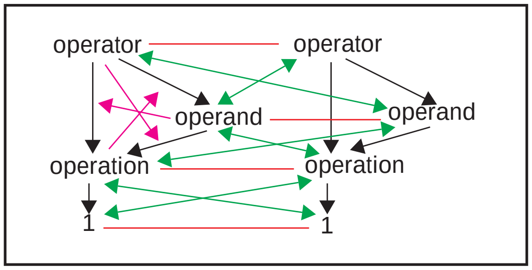
Both cases are not dealing properly with the features of reflectionality and interactivity. Features, which are not conform to linearity. They are also treating mediation from an external and abstract point of view. And are restricted to linear distributions of logical systems.

7.2 What kind of logics could correspond 2-categories?

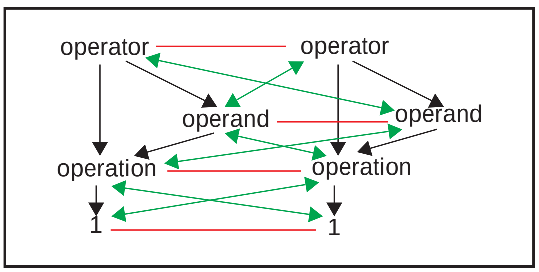
A modeling of polycontexturality into a matrix of *reflectional* and *interactional* dimensions is a candidate for a 2-category realization of disseminated logics.

Again, one possibility could be a double indexed fibring of logics.

Another, more polycontextural modeling, is realized by a 2-dimensional mediation of distributed logics.



Reflectionality comes into the general game if we thematize the relationality or operativity of the proemial construction from the point of view of an *internal* description/construction. An internal description has to consider all given concepts of a construction and to re-construct the build construction out from the inside. An *external* description is realized by an external observer of the construction knowing the rules of construction. A full polycontextural description has furthermore to take into account the complementarity of internal and external descriptions of its constructions.



Interactivity, which is not changing the structure of architectonics, can be seen as a kind of reflectionality, reflection-onto-others. In other words, with a stable architectonics which is excluding metamorphosis and evolution/emanation, both concepts are complementary. That is, reflectionality can be seen as an interactivity in the modus of replication into itself. Both activities are complementary to each other and have to be distinguished properly. In polycontextural logic interactivity is mainly realized by different kinds of *transjunctions*. But interactivity is a general concept and is not reduced to logical operations only.

7.3 3-category

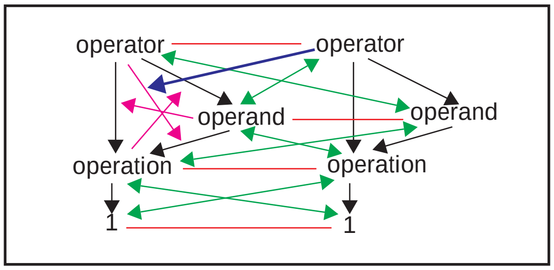
To continue this idea of modeling, obviously a next step could be a dissemination of logics, not only along reflectional and interactional dimensions, but additionally over the dimension of *intervention*.

Thus, realizing a 3-category.

Interaction as reflection: reflectional interactivity (intervention)

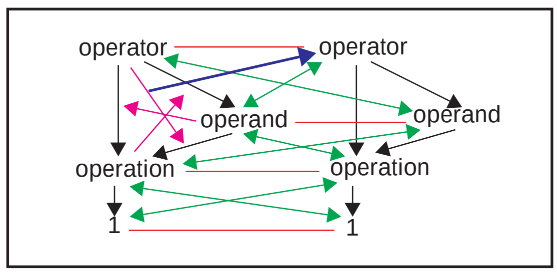
Reflectional interactivity can be understood as an interaction unto the reflectional patterns of a neighbor agent or into the acting agent itself, therefore it can be called intervention and self-intervention.

Interventions are anticipating the behavior of an agent and try to influence it and to change its plans and motivations maybe to avoid conflicting situations.



Intervention is re-programming the reflectional system of the neighbor system and not the system itself. The self-image of the neighbor system is re-programmed and not the system itself as it appears in an interactional context to the interacting agent and also not as the reflectional image of the neighbor in the internal environment of the agent. This is a further specification of subjectivity in the I/Thou-relation of togetherness or the proemiality of (cognition, volition, I, Thou) in the sense of Gunther.

Interventions may be realized in two directions of reflection and interaction, reflectional interactions.

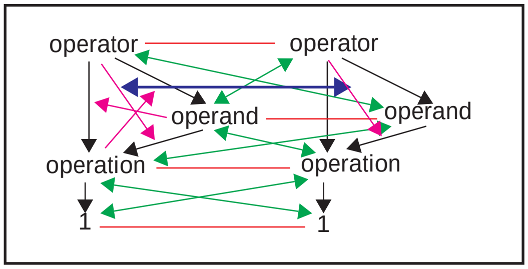


7.4 4-category

A 4-category could therefore model a dissemination of logics along the dimension of reflectionality, interactivity, intervention and *anticipation*.

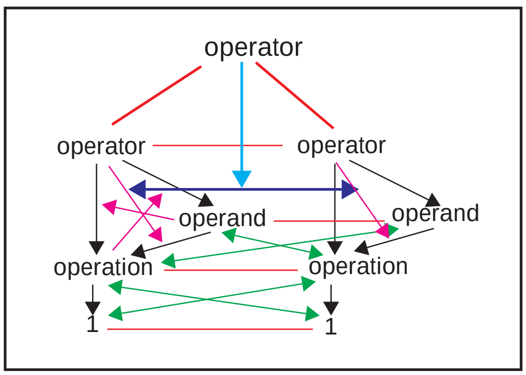
More dimensions would offer possibilities to the application of general n-categories.

Reflection as interaction: interactional reflectionality (interlocution, anticipation)



Interactional reflectional can been seen as a one-directional or a mutual interaction between two reflectional activities. Plans, motivations and strategies are directly involved with the aim to interact or change each others intentions and self-interpretations.

Inner and outer description of the arena of reflections and interactions.



Francis Jacques, l'espace logique de l'interlocution, puf, 1985, Paris

8 The Circus all together

Tetraktys, Proemiality, Natural Transformation, and n-Categories

All together are attempting to escape two fundamental structures of thinking, the *circle* and the *line*.

Linearity is the success of modern Occidental thinking culminating in digitalism *"up to self-destruction"* with great scientific and technological operativity.

Circularity is encircling Ancient, New Age and Second-Order Cybernetic thinking in its material and qualitative way of thinking the self, the world and the cosmos without a working operativity.

Both, linearity and circularity, are not opening up futures, necessary today, here and now.

9 On deconstructing equality

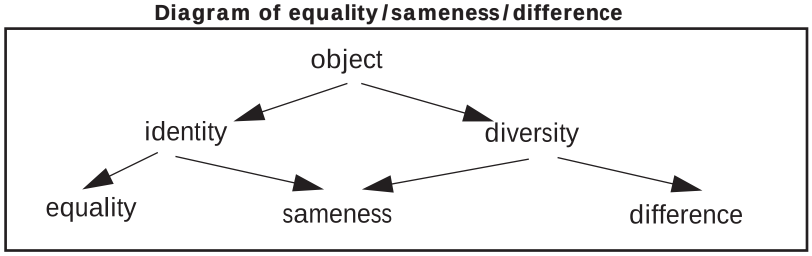
The endless attacks against a rigid philosophy of identity and its consequences for logic, math and semiotics has a new companion in the n-category approach.

equality, equivalence, morphism, bisimulation

"The basic philosophy is simple: never mistake equivalence for equality" (Baez).

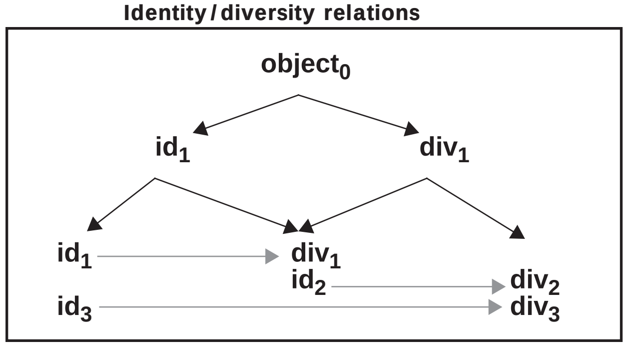
9.1 Diagrams of equality, sameness and difference

Diagram of equality / sameness / difference



The diagram shows a general scheme of an extension of the difference of identity / diversity to the differences of equality / sameness / difference.

Identity / diversity relations



This extension can be modeled as a distribution of the original identity / diversity relation over 3 loci. Thus, iterating the difference: equality = {id_1, id_3}, sameness = {div_1, id_2}, difference = {div_2, div_3}. Obviously, all these id / div-relations are semantically founding a base of a logical system, delivering different negations.

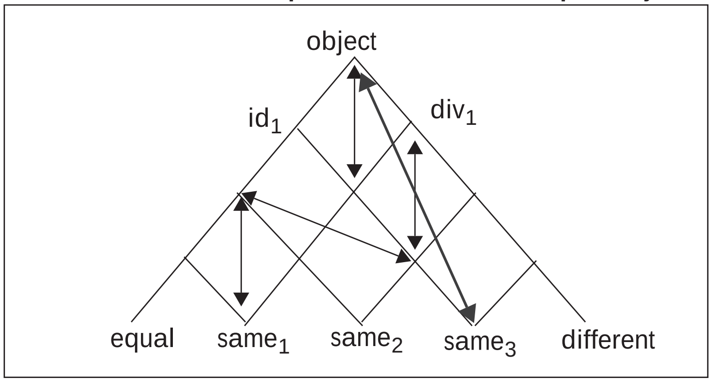
Negations in polycontextural logics have two function:

1) *inversion* of the values (id / div) and

2) *permutations* of the subsystems involved.

Thus, $N_1(id_{1.3}, div_1 / id_2, div_{2.3}) = (div_1 / id_2, id_{3.1}, div_{3.2})$. That is, the values of $subsystem_1$ are inverted to (id_1, div_1) => (div_1, id_1) and the $subsystems_{2.3}$ are permuted to subsystems $_{3.2}$.

Differences in the concept of sameness of a complex object



Different paths through the graph of the determination of an objects' complex identifying structure can be studied and linked to multi-negational operations.

Without doubt, the ambiguity can also be distributed over the terms "equal" or "different". Thus, different interpretations of the id/div-relation are possible.

E.g, (equal$_1$, equal$_2$, same, different$_1$, different$_2$) or

(equal$_1$, same$_1$, same$_2$, different$_1$, different$_2$).

With additional terms for id/div-clusters new wordings are avaiable.

http://www.thinkartlab.com/pkl/media/SKIZZE-0.9.5-medium.pdf

Each scriptural level of kenogrammatics has its own "identity principle".