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Discrete Dynamics of Combinatory Logic and morphoCA systems

Presentation of the discret dynamics of morphic and combinatory logic Systems as a preparation for the study of polycontextural calculi

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Abstract

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Categories of the RK-Archive

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| K02 Scientific Essays | K09 Morphogrammatics |
| K03 Polycontexturality – Second-Order-Cybernetics | K10 The Chinese Challenge or A Challenge for China |
| K04 Diamond Theory | K11 Memristics Memristors Computation |
| K05 Interactivity | K12 Cellular Automata |
| K06 Diamond Strategies | K13 RK and friends |
| K07 Contextural Programming Paradigm | |

Discrete Dynamics of Combinatory Logic and morphoCA systems

Presentation of the discret dynamics of morphic and combinatory logic systems
as a preparation for the study of polycontextural calculi

Dr. phil Rudolf Kaehr
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"Rewrite systems prefer to live on computers."
E. Engeler

Contextures prefer to create poly-verses.

Motivations

A Framework of Variant Logic Construction for Cellular Automata

Jeffrey Z.J. Zheng, Christian H.H. Zheng and Tosiyasu L. Kunii

"What are the essential differences between modern binary logic and the I-Ching's dynamic binary structures?"

"Leibniz in as early as 1690 realized that the balanced yin-yang structure proposed by Shao Yong (1050) was equivalent to the binary number system (Hook, 1975; Needham & Wang, 1954-1988). "

"Logic and the development of rules for the expression of logic have provided a language that enabled the construction of today's scientific societies.

In contrast to the binary on - off nature of western logic, oriental culture have been influenced by spiritual traditions of balance and harmony.

The theme of balance can be summarized in the I - Ching or 'The Book of Changes', one of the most influential books of classic oriental literature (Chu & Sherrill, 1977; Cooper, 1981; Govinda, 1981; Hook, 1975; Shchutshii, 1979; Whincup, 1986; Wilhelmi, 1979; Wilhemi, 1979).

"The concept of Yin and Yang forces and the subtle interplay of the two opposing forces yield combinations and permutations of change. Orient philosophy believed that '*the only constant phenomena is change*' and such a world view emphasised the dynamic nature of a system; rather than focusing in the individual states of a system (on, off), prominence was instead placed on operations that yield change (on to off, off to on).

"The structure of thought introduced by the I - Ching allowed change to be systematically documented and analysed.

"Complex interactions, cyclic behaviour and the interplay of nature at all levels of oriental culture–sociology, literature, medicine, astrology and religion–were able to be described using the tools of dynamic logic provided by the I - Ching; the framework remains a complete philosophy as well as a universal language

and has remained unchanged over the past two thousand years (Needham & Wang, 1954 - 1988)."

Nevertheless, Confucianism and its holy trinity is un-stoppable overrunning the western academies. Unfortunately, the Western insights into the history of a fundamental misunderstanding of the I-Ching by the philosopher-mathematician Gottfried Wilhelm Leibniz (1646 -1716) mislead by the French Jesuit Joachim Bouvet is still well put under the carpet in favor of Western digitalism and its adaption by westernized Chinese academics.

The Chinese Challenge :: 中国挑战 ThinkArt Lab

www.thinkartlab.com/CCR/rudys-chinese-challenge.html

<http://the-chinese-challenge.blogspot.co.uk>

Jeffrey Z.J. Zheng, Christian H.H. Zheng and Toshiyasu L. Kunii (2011). A Framework of Variant Logic Construction for Cellular Automata, Cellular Automata - Innovative Modelling for Science and Engineering, Dr. Alejandro Salcido (Ed.), ISBN: 978-953-307-172-5, InTech, Available from: <http://www.intechopen.com/books/cellular-automata-innovative-modelling-for-science-and-engineering/a-framework-of-variant-logic-construction-for-cellular-automata>

"Rewrite systems prefer to live on computers."

E.Engeler, Formal Universes, ETH Zurich, Manuscript draft, November 23, 2014

"Mathematics creates its own universe, not out of chaos or tohu-wabohu as in Genesis, but out of nothingness, the empty set."

"A new and less well-known approach to universalism in mathematics is based on a development that originated in the 1930s in answer to the same "crisis of foundations" sketched above: Combinatory logic, lambda calculus, and type theories."

"Combinatory Logic is open to formal extensions."

"The basic operation is application: Programs may be applied to input data and of course result in data, which may again be programs, programs may be applied to programs, again resulting in data, etc. Indeed, we may admit that all combinations of applications on data result again in data."

https://people.math.ethz.ch/~engeler/Formal_Universes.pdf

Twenty Years of Rewriting Logic

José Meseguer

<http://maude.cs.uiuc.edu/papers/pdf/20-years.pdf>

Theoretical background of the Dynamics of Combinatory Systems in a mono-contextural environment

In general, a generation of combinator X is given by the following actions :

- 1) using B to eliminate the *parentheses* in V;
- 2) using C to re - order the *variables*;
- 3) using W to *eliminate* multiple occurrences of the variables;
- 4) using K to *bring in* the variables which are not present in V.

Consider two basic systems of combinators C, W, B, K and S, K:

Cxyz = xzy, [C = S(BBS)(KK)]

Sxyz = xz(yz),

Wxy = xyy, [W = CSI (= SS(CK) = SS(K(SKK)))]

Kxy = x,

Bxyz = x(yz),

For any syntactic object V , constructed from distinct variables x1, . . . , xn by its applications, one can determine a combinator X, composed from basic combinators, such that:

$$X \ x_1, \dots, x_n = V.$$

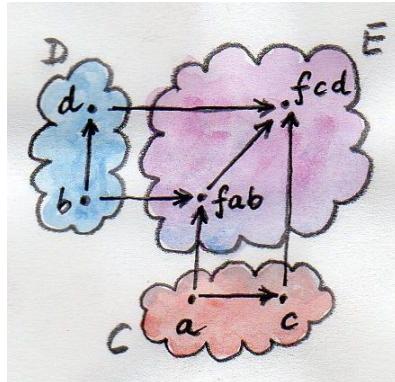
[http://jurinfor.exponenta.ru/papers/Wolfengagen_CLP-2003\(En\).pdf](http://jurinfor.exponenta.ru/papers/Wolfengagen_CLP-2003(En).pdf)

Mono-contextural Bifunctors

"Since functors are morphisms in Cat (the category of categories), a lot of intuitions about morphisms—and functions in particular—apply to functors as well. For instance, just like you can have a function of two arguments, you can have a functor of two arguments, or a *bifunctor*. On objects, a bifunctor maps every pair of objects, one from category C, and one from category D, to an object in category E. Notice that this is just saying that it's a *mapping* from a cartesian product of categories C*D to E."

<http://bartoszmilewski.com/category/functional-programming/>

Mapping



Theoretical background of the Dynamics of Combinatory Systems in a poly-contextural environment

Jumpoid

Notice that this is just saying that it's a *jumpoid* from a cartesian product of categories C^*D to E .

Mapping

mono-c: $C \times D \rightarrow E: (f, g) \circ (f', g') = (f \circ f', g \circ g')$

Jumpoid

poly-c: $C \times D \rightarrow \langle E_C \amalg E_D \rangle: (f \amalg g) \circ (f' \amalg g') = (f \circ f' \amalg g \circ g')$

Dynamics of Combinatory Systems in a mono-contextural environment

Iterative Oscillators

```

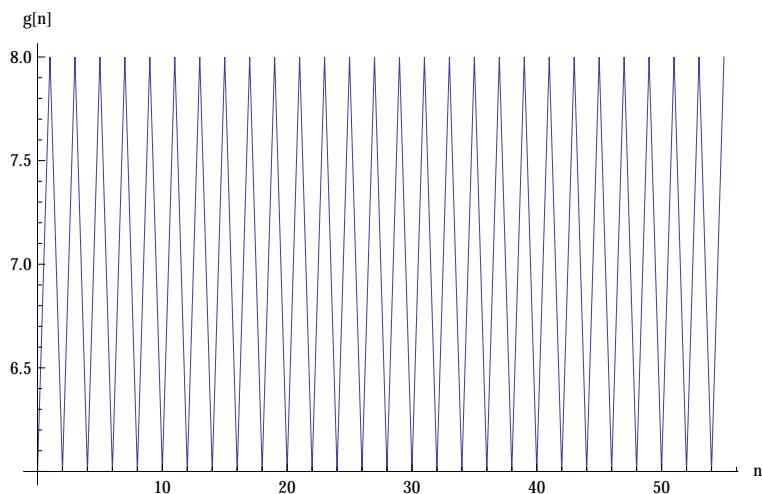
Clear[f]
Clear[g]

SKStep[exp_] := exp /. {s[x_][y_][z_] → x[z][y[z]], k[x_][y_] → x, i[x_] → x,
t[x_][y_] t[u_][v_] → x[u] y[v], w[x_][y_] → x[y][y] }

f[n_] := Nest[SKStep, s[i][i][s[i][i]], n]
g[n_] := LeafCount[f[n]]

```

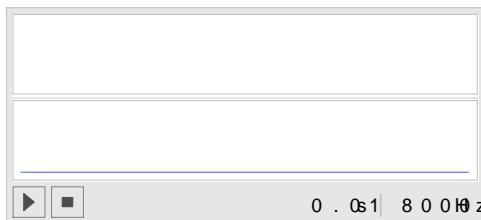
```
ListLinePlot[Table[{n, g[n]}, {n, 0, 55}],
 PlotRange -> All, AxesLabel -> {"n", "g[n]"}]
```



```
Table[f[n], {n, 0, 11}] // MatrixForm
```

```
ListPlay[Table[g[n], {n, 11, 111}]]
```

ListPlay::notound: No sound channel is available.



```
Clear[f]
```

```
Clear[g]
```

```
SKStep[exp_] := exp /. {s[x_][y_][z_] -> x[z][y[z]], k[x_][y_] -> x, i[x_] -> x,
 t[x_][y_] t[u_][v_] -> x[u] y[v], w[x_][y_] -> x[y][y]}
```

```
f[n_] := Nest[SKStep, w[s[i][i]][s[k[s[i][k]]]], n]
```

```
g[n_] := LeafCount[f[n]]
```

```

ListLinePlot[Table[{n, g[n]}, {n, 0, 77}],
 PlotRange -> All, AxesLabel -> {"n", "g[n]"}]

g[n]
50
40
30
20
10
0
n

filter = {s[i][i] -> A, s[k[s[i][i]]] -> B, i[s[k[s[i][i]]]] -> C,
 i[s[k[s[i][k]]]] -> D, s[k[s[i][k]]] -> F, s[i][k] -> H,
 k[s[k[s[i][k]]][s[k[s[i][k]]]]] -> G, s[k[s[i][s[k][s][i]]]] -> K};

Table[f[n], {n, 0, 11}] /. filter // MatrixForm
{W[A][s[k[s[i][k]]]],
 A[s[k[s[i][k]]][s[k[s[i][k]]]]],
 i[s[k[s[i][k]]][i[s[k[s[i][k]]][s[k[s[i][k]]]]]],
 s[k[s[i][k]][s[k[s[i][k]]][s[k[s[i][k]]]]],
 k[s[i][k]][s[k[s[i][k]]][s[k[s[i][k]]][s[k[s[i][k]]]]],
 s[i][k][s[k[s[i][k]]][s[k[s[i][k]]][s[k[s[i][k]]]]],
 i[s[k[s[i][k]]][s[k[s[i][k]]][s[k[s[i][k]]][s[k[s[i][k]]]]],
 s[k[s[i][k]][s[k[s[i][k]]][k[s[k[s[i][k]]][s[k[s[i][k]]]]]],
 k[s[i][k]][k[s[k[s[i][k]]][s[k[s[i][k]]]]][s[k[s[i][k]]][k[s[k[s[i][k]]][s[k[s[i][k]]]]]],
 s[i][k][s[k[s[i][k]]][k[s[k[s[i][k]]][s[k[s[i][k]]]]][k[s[k[s[i][k]]][k[s[k[s[i][k]]][s[k[s[i][k]]]]]]],
 i[s[k[s[i][k]]][k[s[k[s[i][k]]][s[k[s[i][k]]]]][k[s[k[s[i][k]]][k[s[k[s[i][k]]][s[k[s[i][k]]]]]]],
 s[k[s[i][k]][k[s[k[s[i][k]]][s[k[s[i][k]]]]][k[s[k[s[i][k]]][k[s[k[s[i][k]]][s[k[s[i][k]]]]]]]}}

Table[f[n], {n, 0, 22}] /. filter // MatrixForm
{W[A][F],
 A[F][F],
 F,
 F[F][F],
 k[H][F][F[F]],
 H[F[F]],
 i[F[F]][G],
 F[F][G],
 k[H][G][F[G]],
 H[F[G]],
 i[F[G]][k[F[G]]],
 F[G][k[F[G]]],
 k[H][k[F[G]]][G[k[F[G]]]],
 H[F[F]],
 i[F[F]][G],
 F[F][G],
 k[H][G][F[G]],
 H[F[G]],
 i[F[G]][k[F[G]]],
 F[G][k[F[G]]],
 k[H][k[F[G]]][G[k[F[G]]]],
 H[F[F]],
 i[F[F]][G],
 F[F][G],
 k[H][G][F[G]]}

```

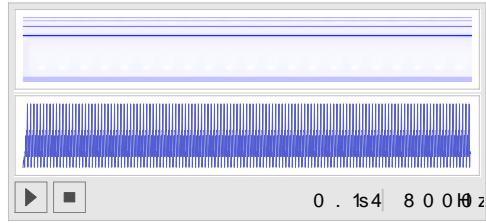
```

ArrayRules[%263]
TableForm[Apply[List, %264, {1}]]

1   W[A][F]
2   A[F][F]
3   F
4   F[F][F]
5   k[H][F][F[F]]
6   H[F[F]]
7   i[F[F]][G]
8   F[F][G]
9   k[H][G][F[G]]
10  H[F[G]]
11  i[F[G]][k[F[G]]]
12  F[G][k[F[G]]]
13  k[H][k[F[G]]][G[k[F[G]]]]
14  H[F[F]]
15  i[F[F]][G]
16  F[F][G]
17  k[H][G][F[G]]
18  H[F[G]]
19  i[F[G]][k[F[G]]]
20  F[G][k[F[G]]]
21  k[H][k[F[G]]][G[k[F[G]]]]
22  H[F[F]]
23  i[F[F]][G]
-
0

```

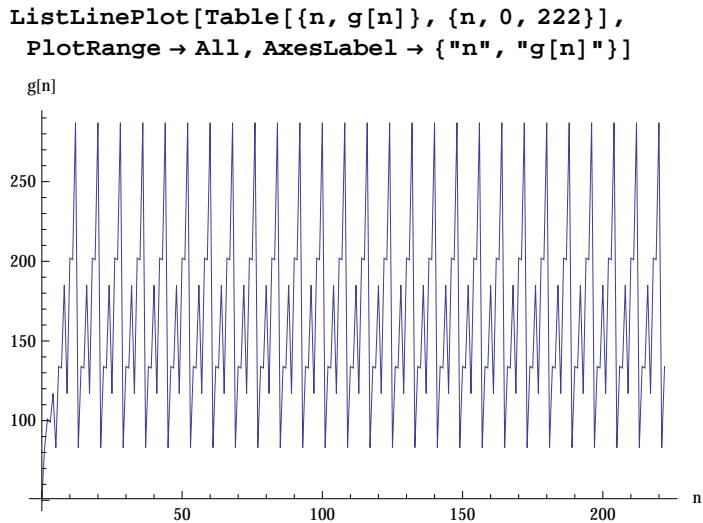
```
ListPlay[Table[g[n], {n, 1111}]]
```



```

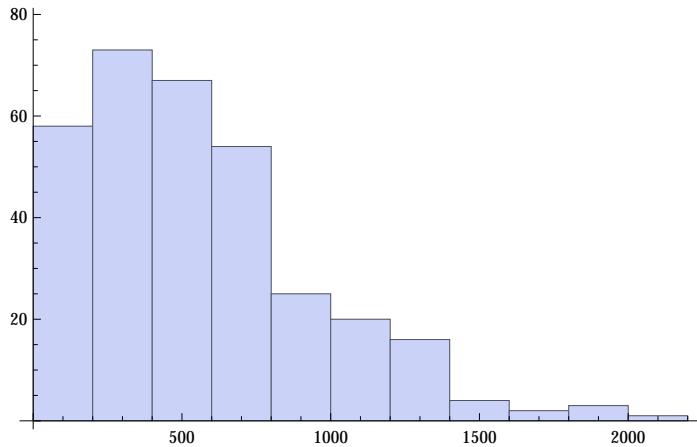
Clear[g]
Clear[h]
SKStep[exp_] := exp /. {s[x_][y_][z_] → x[z][y[z]], k[x_][y_] → x,
          t[x_][y_] t[u_][v_] → x[u] y[v], w[x_][y_] → x[y][y]}
f[n_] := Nest[SKStep, s[s][k][s[s][s][k]]][w], n]
g[n_] := LeafCount[f[n]]

```



```
Table[g[n], {n, 11, 333}]
```

```
Histogram[%117]
```



```
FixedPoint[#, .  
{  
  s[x_][y_][z_] -> x[z][y[z]],  
  k[x_][y_] -> x,  
  i[x_] -> x,  
  t[x_][y_][t[u_][v_]] -> x[u] y[v],  
  w[x_][y_] -> x[y][y]  
 } &,  
 w[s[i][i]][s[s[k[s[i][k]]]]]&, 6] /. filter // MatrixForm
```

```

FixedPointList[# /.
  {
    s[x_][y_][z_] → x[z][y[z]],
    k[x_][y_] → x,
    i[x_] → x,
    t[x_][y_] t[u_][v_] → x[u] y[v],
    w[x_][y_] → x[y][y]
  } &,
  W[s[i][i]][s[s[k[s[i][k]]]]] &, 11] /. filter // MatrixForm


$$\left( \begin{array}{l} W[Q][G] \& \\ Q[G][G] \& \\ i[G][i[G]][G] \& \\ G[G][G] \& \\ s[k[F[k]]][G][G[G]] \& \\ k[F[k]][G[G]][G[G[G]]] \& \\ F[k][G[G[G]]] \& \\ i[G[G[G]]][k[G[G[G]]]] \& \\ G[G[G]][k[G[G[G]]]] \& \\ s[k[F[k]][k[G[G[G]]]][G[G][k[G[G[G]]]]]] \& \\ k[F[k]][G[G][k[G[G[G]]]]][k[G[G[G]]][G[G][k[G[G[G]]]]]] \& \\ F[k][G[G[G]]] \end{array} \right)$$


Clear[f]
Clear[g]

SKStep[exp_] := exp /.
  {
    s[x_][y_][z_] → x[z][y[z]],
    k[x_][y_] → x,
    i[x_] → x,
    t[x_][y_] t[u_][v_] → x[u] y[v],
    w[x_][y_] → x[y][y]
  }

f[n_] := Nest[SKStep, W[s[i][i]][s[s[k[s[i][k]]]]], n]
g[n_] := LeafCount[f[n]]

ListLinePlot[Table[{n, g[n]}, {n, 0, 77}],
  PlotRange → All, AxesLabel → {"n", "g[n]"}]

g[n]


```

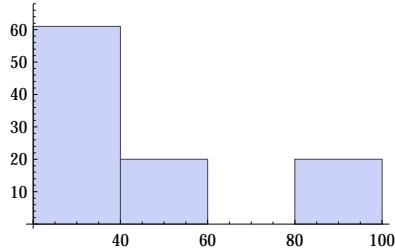
```
Table[f[n], {n, 0, 22}] // MatrixForm
```

```
Table[f[n], {n, 0, 22}] /. filter // MatrixForm
```

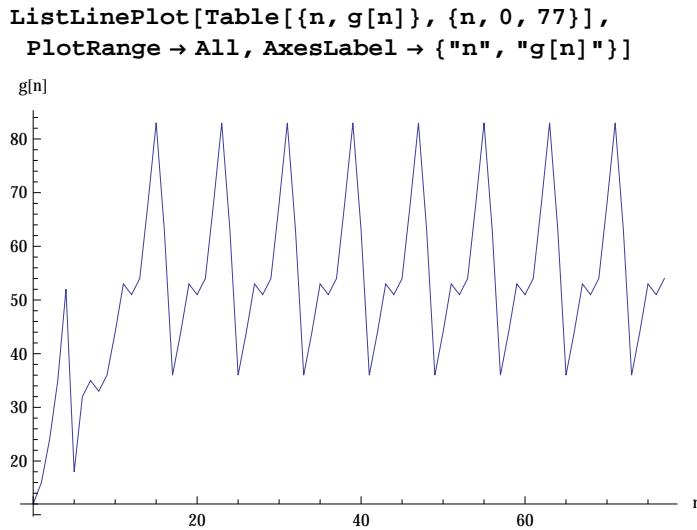
$$\left(\begin{array}{c} W[Q][G] \\ Q[G][G] \\ i[G][i[G]][G] \\ G[G][G] \\ s[k[F[k]]][G][G[G]] \\ k[F[k]][G[G]][G[G[G]]] \\ F[k][G[G[G]]] \\ i[G[G[G]]][k[G[G[G]]]] \\ G[G[G]][k[G[G[G]]]] \\ s[k[F[k]]][k[G[G[G]]]][G[G][k[G[G[G]]]]] \\ k[F[k]][G[G][k[G[G[G]]]]][k[G[G[G]]][G[G][k[G[G[G]]]]]] \\ F[k][G[G[G]]] \\ i[G[G[G]]][k[G[G[G]]]] \\ G[G[G]][k[G[G[G]]]] \\ s[k[F[k]]][k[G[G[G]]]][G[G][k[G[G[G]]]]] \\ k[F[k]][G[G][k[G[G[G]]]]][k[G[G[G]]][G[G][k[G[G[G]]]]]] \\ F[k][G[G[G]]] \\ i[G[G[G]]][k[G[G[G]]]] \end{array} \right)$$

```
Table[g[n], {n, 11, 111}]
```

```
Histogram[%44]
```



```
Clear[f]
Clear[g]
SKStep[exp_] := exp /. {s[x_][y_][z_] → x[z][y[z]], k[x_][y_] → x, i[x_] → x,
 t[x_][y_] t[u_][v_] → x[u] y[v], W[x_][y_] → x[y][y]}
f[n_] := Nest[SKStep, s[s[s][k]][s[i][i]][s[k[s[i][k]]]], n]
g[n_] := LeafCount[f[n]]
```

**cForms**

```
SKStep[exp_] := exp /. {cRules}

f[n_] := Nest[SKStep, cFormula, n]
g[n_] := LeafCount[f[n]]

ListLinePlot[Table[{n, g[n]}, {n, 0, m}],
  PlotRange -> All, AxesLabel -> {"n", "g[n]"}]

Table[f[n], {n, 0, m}] /. filter // MatrixForm
```

Clear[f]**Clear[g]**

```
SKStep[exp_] := exp /. {s[x_][y_][z_] -> x[z][y[z]], k[x_][y_] -> x, i[x_] -> x,
  t[x_][y_] t[u_][v_] -> x[u] y[v], w[x_][y_] -> x[y][y] }

f[n_] := Nest[SKStep, s[s[s][k][s[i][i]][s[k[s[i][k]]]]], n]
g[n_] := LeafCount[f[n]]
```

```
ListLinePlot[Table[{n, g[n]}, {n, 0, 77}],
 PlotRange -> All, AxesLabel -> {"n", "g[n]"}]

g[n]


```

```
Clear[f]
Clear[g]

SKStep[exp_] := exp /. {s[x_][y_][z_] -> x[z][y[z]], k[x_][y_] -> x, i[x_] -> x,
 t[x_][y_] t[u_][v_] -> x[u] y[v], W[x_][y_] -> x[y][y] }

f[n_] := Nest[SKStep, s[s[s][k][s[i][s]][s[k[s[i][k]]]]], n]

g[n_] := LeafCount[f[n]]

ListLinePlot[Table[{n, g[n]}, {n, 0, 77}],
 PlotRange -> All, AxesLabel -> {"n", "g[n]"}]

g[n]


```

```
ListLinePlot[Table[{n, g[n]}, {n, 0, 111}],
  PlotRange → All, AxesLabel → {"n", "g[n]"}]

g[n]
```

n	g[n]
0	15
1	17
2	16
3	18
4	15
5	16
6	17
7	18
8	19
9	20
10	21
11	22
12	21
13	22
14	23
15	42
16	21
17	22
18	21
19	22
20	21
21	22
22	21
23	22
24	21
25	22
26	21
27	22
28	21
29	22
30	21
31	22
32	21
33	22
34	21
35	22
36	21
37	22
38	21
39	22
40	21
41	22
42	21
43	22
44	21
45	22
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69	22
70	21
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74	21
75	22
76	21
77	22
78	21
79	22
80	21
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82	21
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85	22
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87	22
88	21
89	22
90	21
91	22
92	21
93	22
94	21
95	22
96	21
97	22
98	21
99	22
100	21
101	22
102	21
103	22
104	21
105	22
106	21
107	22
108	21
109	22
110	21
111	22

```
Clear[f]
Clear[g]

SKStep[exp_] := exp /. {s[x_][y_][z_] → x[z][y[z]], k[x_][y_] → x, i[x_] → x,
  t[x_][y_] t[u_][v_] → x[u] y[v], W[x_][y_] → x[y][y] }

f[n_] := Nest[SKStep, W[i[s[i][k]]][W[i[W[i]]]], n]
g[n_] := LeafCount[f[n]]

ListLinePlot[Table[{n, g[n]}, {n, 0, 77}],
  PlotRange → All, AxesLabel → {"n", "g[n]"}]

g[n]
```

n	g[n]
0	12
1	11
2	10
3	11
4	10
5	11
6	10
7	11
8	10
9	11
10	16
11	10
12	11
13	10
14	11
15	16
16	10
17	11
18	10
19	11
20	10
21	11
22	10
23	11
24	10
25	11
26	10
27	11
28	10
29	11
30	10
31	11
32	10
33	11
34	10
35	11
36	10
37	11
38	10
39	11
40	10
41	11
42	10
43	11
44	10
45	11
46	10
47	11
48	10
49	11
50	10
51	11
52	10
53	11
54	10
55	11
56	10
57	11
58	10
59	11
60	10
61	11
62	10
63	11
64	10
65	11
66	10
67	11
68	10
69	11
70	10
71	11
72	10
73	11
74	10
75	11
76	10
77	11

```
Table[f[n], {n, 0, 22}] /. filter // MatrixForm
```

```

W[i[H]] [W[i[W[i]]]]
i[H] [W[i[W[i]]]] [W[i[W[i]]]]
H[W[i]] [W[W[i]]]
i[W[W[i]]] [W[W[i]]] [W[W[i]]]
W[W[i]] [k[W[W[i]]]] [W[W[i]]]
W[i] [k[W[W[i]]]] [k[W[W[i]]]] [W[W[i]]]
i[k[W[W[i]]]] [k[W[W[i]]]] [k[W[W[i]]]] [W[W[i]]]
k[W[W[i]]] [k[W[W[i]]]] [k[W[W[i]]]] [W[W[i]]]
W[W[i]] [k[W[W[i]]]] [W[W[i]]]
W[i] [k[W[W[i]]]] [k[W[W[i]]]] [W[W[i]]]
i[k[W[W[i]]]] [k[W[W[i]]]] [k[W[W[i]]]] [W[W[i]]]
k[W[W[i]]] [k[W[W[i]]]] [k[W[W[i]]]] [W[W[i]]]
W[W[i]] [k[W[W[i]]]] [W[W[i]]]
W[i] [k[W[W[i]]]] [k[W[W[i]]]] [W[W[i]]]
i[k[W[W[i]]]] [k[W[W[i]]]] [k[W[W[i]]]] [W[W[i]]]
k[W[W[i]]] [k[W[W[i]]]] [k[W[W[i]]]] [W[W[i]]]
W[W[i]] [k[W[W[i]]]] [W[W[i]]]
W[i] [k[W[W[i]]]] [k[W[W[i]]]] [W[W[i]]]
i[k[W[W[i]]]] [k[W[W[i]]]] [k[W[W[i]]]] [W[W[i]]]
```

cFixedPoints

```
FixedPointList[#, .
{
  crules
} &,
cFormula, steps] /.
filter // MatrixForm
```

```
FixedPointList[#, .
```

```
{
  s[x_][y_][z_] → x[z][y[z]],
  k[x_][y_] → x,
  i[x_] → x,
  t[x_][y_] t[u_][v_] → x[u] y[v],
  w[x_][y_] → x[y][y]
} &,
w[i[s[i][k]]][w[i[W[i]]]], 11] /. filter // MatrixForm
```

```

W[i[F[k]]][W[i[W[i]]]]
i[F[k]][W[i[W[i]]]][W[i[W[i]]]]
F[k][W[W[i]]][W[W[i]]]
i[W[W[i]]][k[W[W[i]]]][W[W[i]]]
W[W[i]] [k[W[W[i]]]] [W[W[i]]]
W[i][k[W[W[i]]]] [k[W[W[i]]]] [W[W[i]]]
i[k[W[W[i]]]] [k[W[W[i]]]] [k[W[W[i]]]] [W[W[i]]]
k[W[W[i]]] [k[W[W[i]]]] [k[W[W[i]]]] [W[W[i]]]
W[W[i]] [k[W[W[i]]]] [W[W[i]]]
W[i][k[W[W[i]]]] [k[W[W[i]]]] [W[W[i]]]
i[k[W[W[i]]]] [k[W[W[i]]]] [k[W[W[i]]]] [W[W[i]]]
k[W[W[i]]] [k[W[W[i]]]] [k[W[W[i]]]] [W[W[i]]]
```

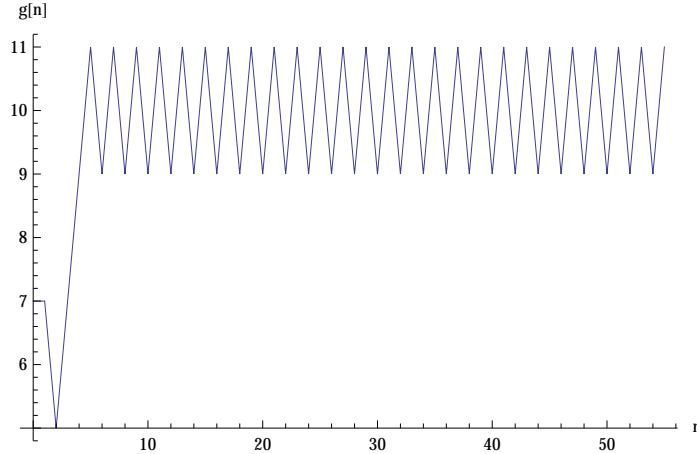
Clear[f]

Clear[g]

```
SKStep[exp_] := exp /. {s[x_][y_][z_] → x[z][y[z]], k[x_][y_] → x, i[x_] → x,
t[x_][y_] t[u_][v_] → x[u] y[v], w[x_][y_] → x[y][y]}
```

```
f[n_] := Nest[SKStep, s[i][i][W][s[i][i]], n]
g[n_] := LeafCount[f[n]]

ListLinePlot[Table[{n, g[n]}, {n, 0, 55}],
 PlotRange → All, AxesLabel → {"n", "g[n]"}]
```

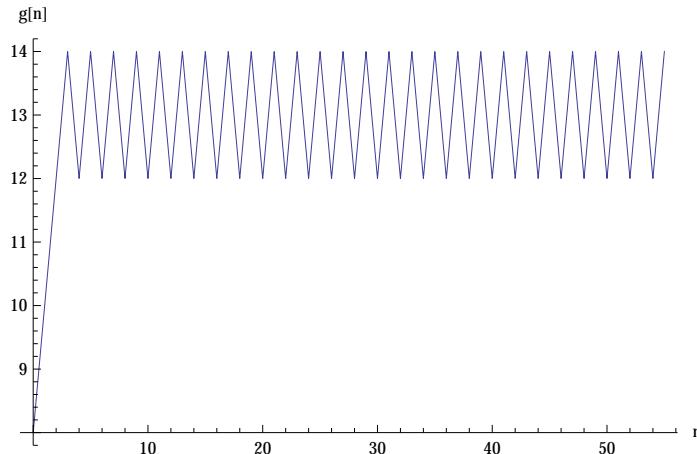


```
Clear[g]
Clear[f]

SKStep[exp_] := exp /. {s[x_][y_][z_] → x[z][y[z]], k[x_][y_] → x,
t[x_][y_] t[u_][v_] → x[u] y[v], w[x_][y_] → x[y][y] }

f[n_] := Nest[SKStep, w[s[s][k]][s[s][k]][w], n]
g[n_] := LeafCount[f[n]]
```

```
ListLinePlot[Table[{n, g[n]}, {n, 0, 55}],
 PlotRange → All, AxesLabel → {"n", "g[n]"}]
```



```

filter = {
  s[s][k][k[s[s][s][k]]][k] → a,
  s[k[s[s][k]]][k][k[k[s[s][k]]][k]] → b,
  k[s[s][k]][k] → c,
  s[s][k] → d,
  k[s][k] → o,
  s[s][k][s[s][k]] → p,
  s[s][k] → q,
  s[s][k][s[s][k]][k[s[s][k]][s[s][k]]] → A ,
  s[s[s][k]][k[s[s][k]]][s[s][k]] → B ,
  i[s[i]] → C,
  s[i] → F,
  s[k][i] → D,
  s[i][i] → Q,
  i[s[i][i]] → V,
  s[i][s[i][s[k][i]]] → W,
  s[s[i][s[k][i]]] → X,
  s[s[k[s[i][k]]]] → G

};

Table[f[n], {n, 0, 11}] /. filter // ColumnForm

Clear[f]
Clear[g]

SKStep[exp_] := exp /. {s[x_][y_][z_] → x[z][y[z]], k[x_][y_] → x, i[x_] → x,
t[x_][y_] t[u_][v_] → x[u] y[v], W[x_][y_] → x[y][y] }

f[n_] := Nest[SKStep, W[s[i][i]][s[i][i]], n]
(*s[s][k][s[s][s][k]]][k]*)

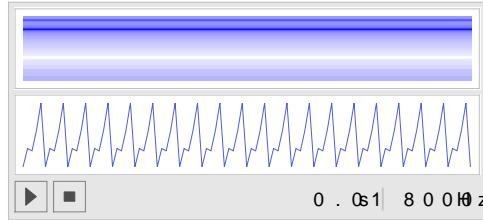
g[n_] := LeafCount[f[n]]

ListLinePlot[Table[{n, g[n]}, {n, 0, 55}],
 PlotRange → All, AxesLabel → {"n", "g[n"]"}]

g[n]


```

```
Table[f[n], {n, 0, 11}] /. filter // ColumnForm
Table[g[n], {n, 11, 111}]
ListPlay[Table[g[n], {n, 11, 111}]]
```



Accelerating oscillators

```
Clear[f]
Clear[g]

SKStep[exp_] := exp /. {s[x_][y_][z_] → x[z][y[z]], k[x_][y_] → x, i[x_] → x,
t[x_][y_] t[u_][v_] → x[u] y[v], w[x_][y_] → x[y][y] }

f[n_] := Nest[SKStep, w[s[i][i]][s[i][s[i][s[k][i]]]], n]
g[n_] := LeafCount[f[n]]

ListLinePlot[Table[{n, g[n]}, {n, 0, 77}],
PlotRange → All, AxesLabel → {"n", "g[n]"}]

g[n]
```

```
Table[f[n], {n, 0, 11}] // ColumnForm
Table[f[n], {n, 0, 22}] /. filter // ColumnForm
Table[g[n], {n, 11, 333}]
Histogram[%79]

Histogram[%79]
```

```
Clear[f]
```

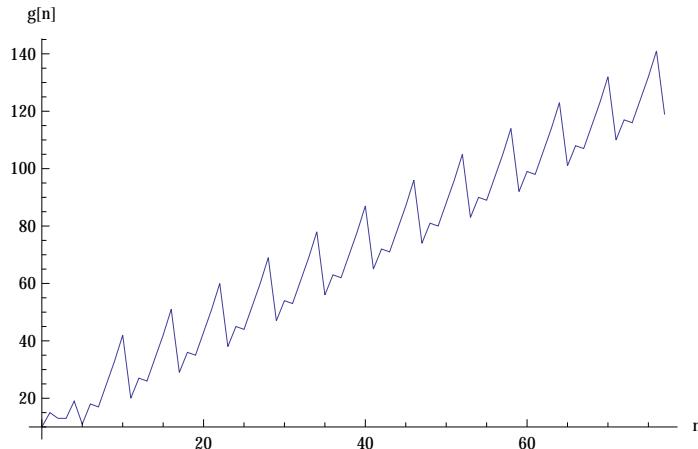
```
Clear[g]

SKStep[exp_] := exp /. {s[x_][y_][z_] → x[z][y[z]], k[x_][y_] → x, i[x_] → x,
t[x_][y_] t[u_][v_] → x[u] y[v], w[x_][y_] → x[y][y] }

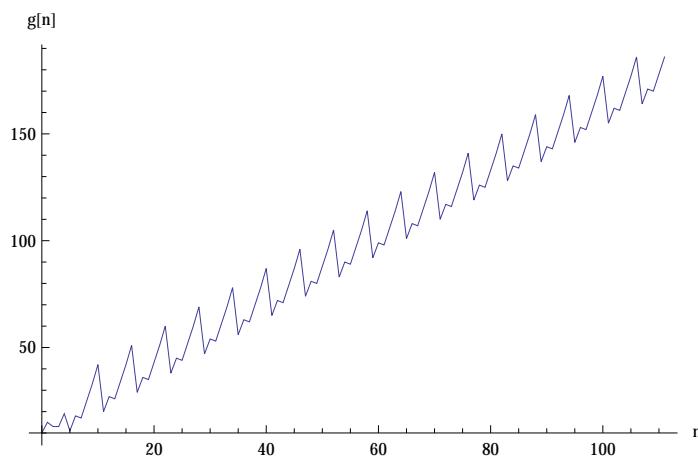
f[n_] := Nest[SKStep, s[i][i][s[s[k[s[i][k]]]]][w], n]

g[n_] := LeafCount[f[n]]

ListLinePlot[Table[{n, g[n]}, {n, 0, 77}],
PlotRange → All, AxesLabel → {"n", "g[n]"}]
```



```
ListLinePlot[Table[{n, g[n]}, {n, 0, 111}],
PlotRange → All, AxesLabel → {"n", "g[n]"}]
```



```
ListLinePlot[Table[{n, g[n]}, {n, 0, 22}],
  PlotRange → All, AxesLabel → {"n", "g[n]"}]

g[n]
60
50
40
30
20
10
0
n
```

```
Table[f[n], {n, 0, 11}] // ColumnForm
Table[f[n], {n, 0, 33}] /. filter // ColumnForm
Table[g[n], {n, 11, 333}]
Histogram[%53]

Histogram[%53]
```

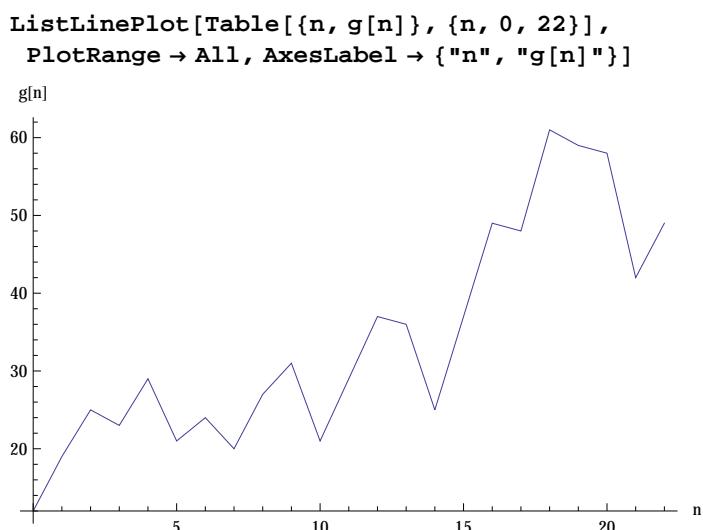
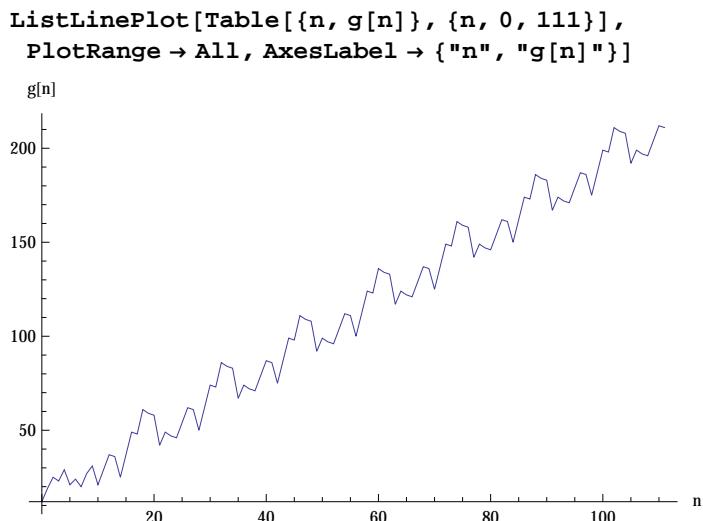
```
Clear[f]
Clear[g]

SKStep[exp_] := exp /. {s[x_][y_][z_] → x[z][y[z]], k[x_][y_] → x, i[x_] → x,
  t[x_][y_] t[u_][v_] → x[u] y[v], w[x_][y_] → x[y][y] }

f[n_] := Nest[SKStep, w[s[i][i]][s[i][s[i][s[k][i][k]]]], n]
g[n_] := LeafCount[f[n]]

ListLinePlot[Table[{n, g[n]}, {n, 0, 77}],
  PlotRange → All, AxesLabel → {"n", "g[n]"}]

g[n]
160
140
120
100
80
60
40
20
0
n
```

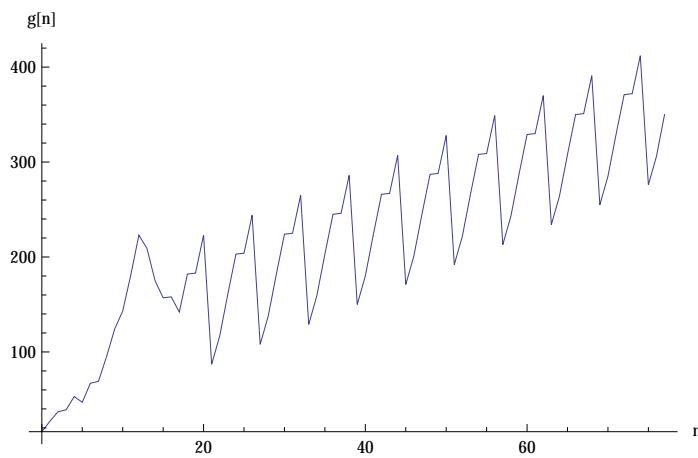


```
Clear[f]
Clear[g]

SKStep[exp_] := exp /. {s[x_][y_][z_] -> x[z][y[z]], k[x_][y_] -> x,
t[x_][y_] t[u_][v_] -> x[u] y[v], w[x_][y_] -> x[y][y] }

f[n_] := Nest[SKStep, w[s[i][i]][s[i][s[i][s[i][i][s[i][s[i]]][k]]]]], n]
g[n_] := LeafCount[f[n]]
```

```
ListLinePlot[Table[{n, g[n]}, {n, 0, 77}],
 PlotRange -> All, AxesLabel -> {"n", "g[n]"}]
```



```
Clear[f]
```

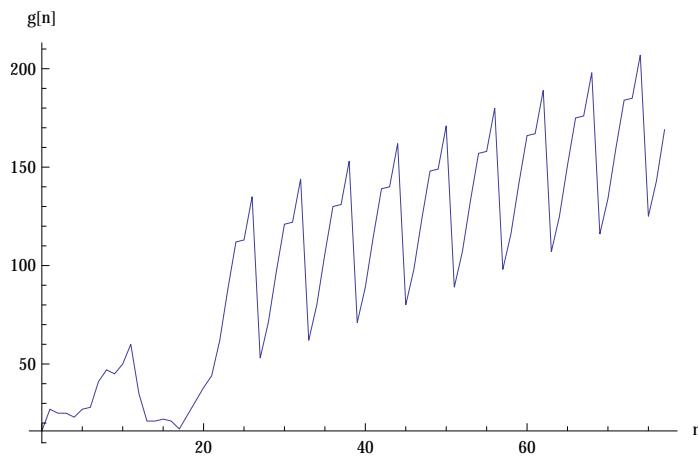
```
Clear[g]
```

```
SKStep[exp_] := exp /. {s[x_][y_][z_] -> x[z][y[z]], k[x_][y_] -> x,
 t[x_][y_] t[u_][v_] -> x[u] y[v], w[x_][y_] -> x[y][y]}
```

```
f[n_] := Nest[SKStep, s[i][i][s[i][s[i][s[i][i][s[i][s[i]]][k]]]][w], n]
```

```
g[n_] := LeafCount[f[n]]
```

```
ListLinePlot[Table[{n, g[n]}, {n, 0, 77}],
 PlotRange -> All, AxesLabel -> {"n", "g[n]"}]
```



```

Table[f[n], {n, 0, 22}] // MatrixForm
{
  {B(B(T(BBT))B)Txyz[B(B(T(BBT))B)Txyz[i][k]][B(B(T(BBT))B)Txyz[i][k]]},

  {B(B(T(BBT))B)Txyz[i][B(B(T(BBT))B)Txyz[B(B(T(BBT))B)Txyz[i][k]][B(B(T(BBT))B)Txyz[i][k]]]},

  {B(B(T(BBT))B)Txyz[i][B(B(T(BBT))B)Txyz[B(B(T(BBT))B)Txyz[i][k]][B(B(T(BBT))B)Txyz[i][k]]]}
}

Table[f[n], {n, 0, 22}] /. filter // MatrixForm
Table[g[n], {n, 11, 333}]

Clear[g]
Clear[f]

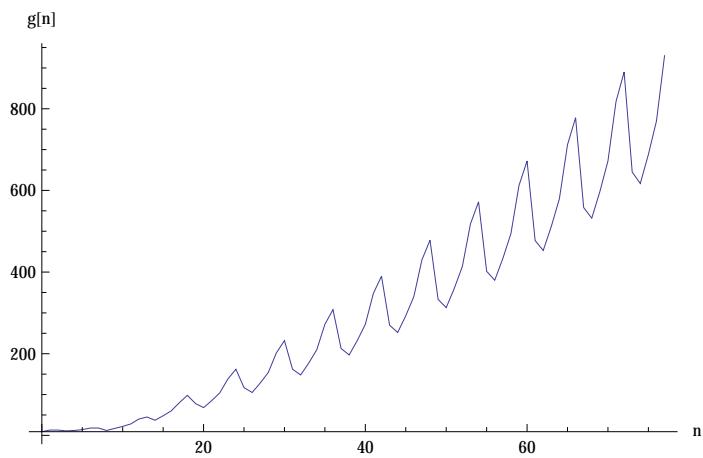
SKStep[exp_] := exp /. {s[x_][y_][z_] → x[z][y[z]], k[x_][y_] → x, i[x_] → x,
t[x_][y_] t[u_][v_] → x[u] y[v], w[x_][y_] → x[y][y]}

(* W = "S(S(KS)(S(S(KS)K)(KI)))(KI)" *)
f[n_] := Nest[SKStep, s[s][k][s[s][s]][k]][w], n]
(*s[s][k][s[s][k]]][k]*)

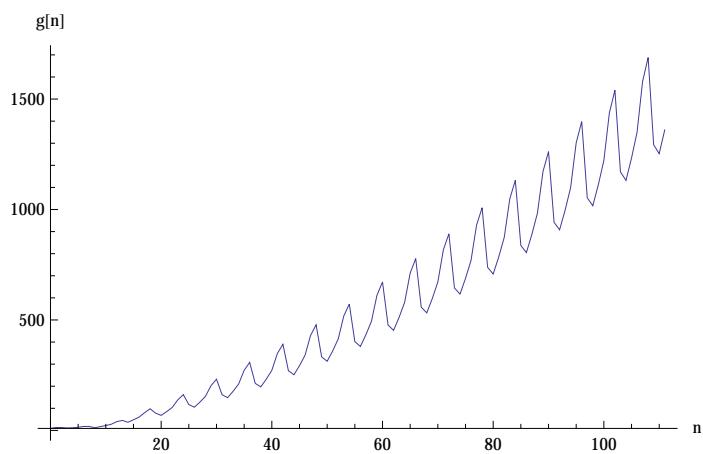
g[n_] := LeafCount[f[n]]

```

```
ListLinePlot[Table[{n, g[n]}, {n, 0, 77}],  
 PlotRange -> All, AxesLabel -> {"n", "g[n]"}]
```



```
ListLinePlot[Table[{n, g[n]}, {n, 0, 111}],  
 PlotRange -> All, AxesLabel -> {"n", "g[n]"}]
```



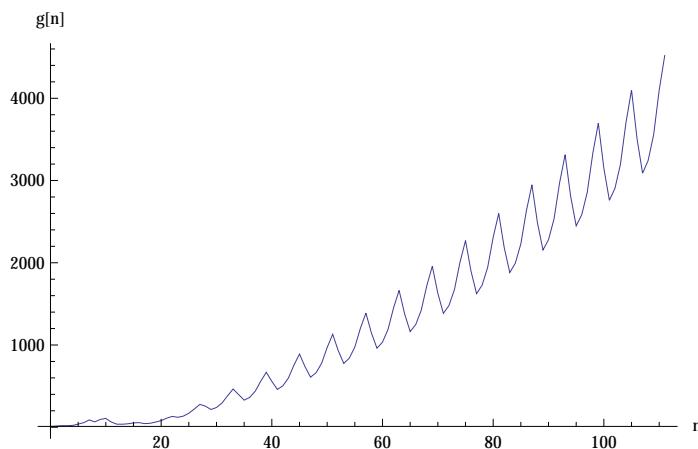
```

Table[f[n], {n, 0, 22}] // MatrixForm
{
  k[W[B(B(T(BBT))B)Txyz[
    B(B(T(BBT))B)Txyz[B(B(T(BBT))B)Txyz][k[k[W[B(B(T(BBT))B)Txyz[B(B(T(BBT))B)Txyz[
      Table[f[n], {n, 0, 22}] /. filter // MatrixForm
      Table[g[n], {n, 11, 333}]

Clear[g]
Clear[f]
SKStep[exp_] := exp /. {s[x_][y_][z_] → x[z][y[z]], k[x_][y_] → x, i[x_] → x,
  t[x_][y_] t[u_][v_] → x[u] y[v], W[x_][y_] → x[y][y] }
(* W = "S(S(KS)(S(S(KS)K)(KI)))(KI)" *)
f[n_] := Nest[SKStep, s[s[s]][k][s[s[s]]][k]][W], n]
(*s[s][k][s[s][k]]][k]*)
g[n_] := LeafCount[f[n]]

```

```
ListLinePlot[Table[{n, g[n]}, {n, 0, 111}],
 PlotRange -> All, AxesLabel -> {"n", "g[n]"}]
```



```
Clear[f]
```

```
Clear[g]
```

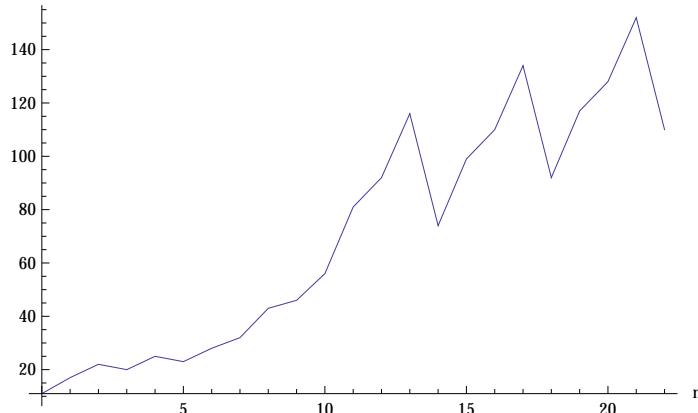
```
SKStep[exp_] := exp /. {s[x_][y_][z_] -> x[z][y[z]], k[x_][y_] -> x, i[x_] -> x,
 t[x_][y_] t[u_][v_] -> x[u] y[v], w[x_][y_] -> x[y][y]}
```

```
f[n_] := Nest[SKStep, w[s[i][i]][s[i][s[i][s[k][i]]]], n]
```

```
g[n_] := LeafCount[f[n]]
```

```
ListLinePlot[Table[{n, g[n]}, {n, 0, 22}],
 PlotRange -> All, AxesLabel -> {"n", "g[n]"}]
```

```
g[n]
```



```
Table[f[n], {n, 0, 22}] /. filter // MatrixForm
```

```
Clear[f]
```

```
Clear[g]
```

```
SKStep[exp_] := exp /. {s[x_][y_][z_] -> x[z][y[z]], k[x_][y_] -> x, i[x_] -> x,
 t[x_][y_] t[u_][v_] -> x[u] y[v], w[x_][y_] -> x[y][y]}
```

```

f[n_] := Nest[SKStep, W[s[i][i]][s[i][s[i][W[k][i]]]], n]
g[n_] := LeafCount[f[n]]

ListLinePlot[Table[{n, g[n]}, {n, 0, 77}],
 PlotRange → All, AxesLabel → {"n", "g[n]"}]

g[n]

```

```

SKStep[exp_] := exp /. {s[x_][y_][z_] → x[z][y[z]], k[x_][y_] → x, i[x_] → x,
 t[x_][y_] t[u_][v_] → x[u] y[v], w[x_][y_] → x[y][y] }

f[n_] := Nest[SKStep, W[s[i][i]][W[s[i][i]]], n]
(*s[s][k][s[s][k]][k]*)

g[n_] := LeafCount[f[n]]

ListLinePlot[Table[{n, g[n]}, {n, 0, 77}],
 PlotRange → All, AxesLabel → {"n", "g[n]"}]

g[n]

```

```

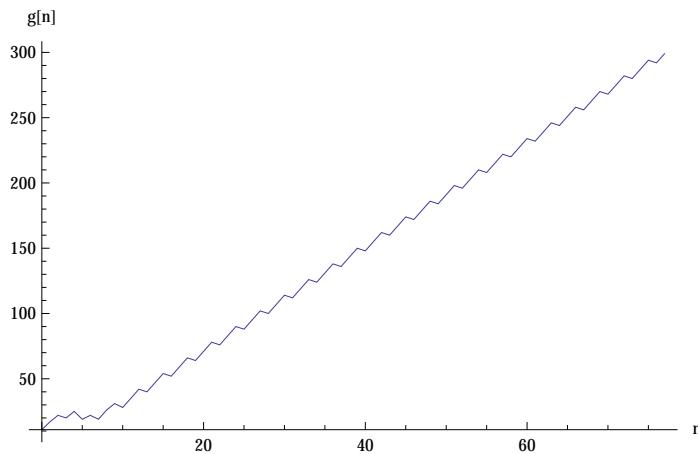
Clear[f]
Clear[g]

SKStep[exp_] := exp /. {s[x_][y_][z_] → x[z][y[z]], k[x_][y_] → x, i[x_] → x,
 t[x_][y_] t[u_][v_] → x[u] y[v], w[x_][y_] → x[y][y] }

f[n_] := Nest[SKStep, W[s[i][i]][s[i][s[i][W[k][i]]]], n]
g[n_] := LeafCount[f[n]]

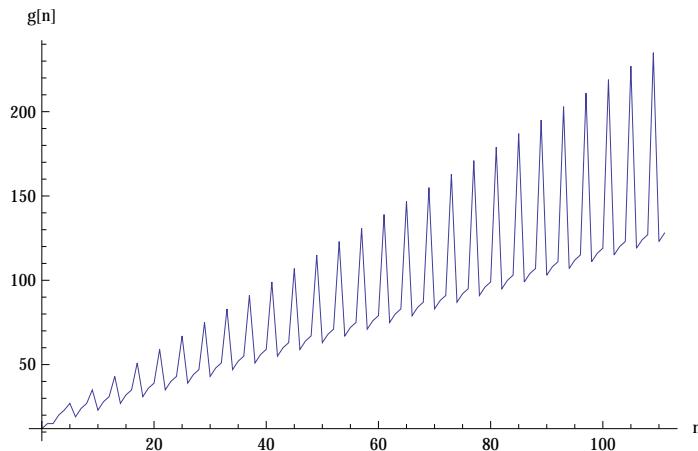
```

```
ListLinePlot[Table[{n, g[n]}, {n, 0, 77}],
 PlotRange -> All, AxesLabel -> {"n", "g[n]"}]
```



```
f[n_] := Nest[SKStep, s[s][k][s[s[s][k]]][t][t[s[t[k]]]], n]
```

```
ListLinePlot[Table[{n, g[n]}, {n, 0, 111}],
 PlotRange -> All, AxesLabel -> {"n", "g[n]"}]
```

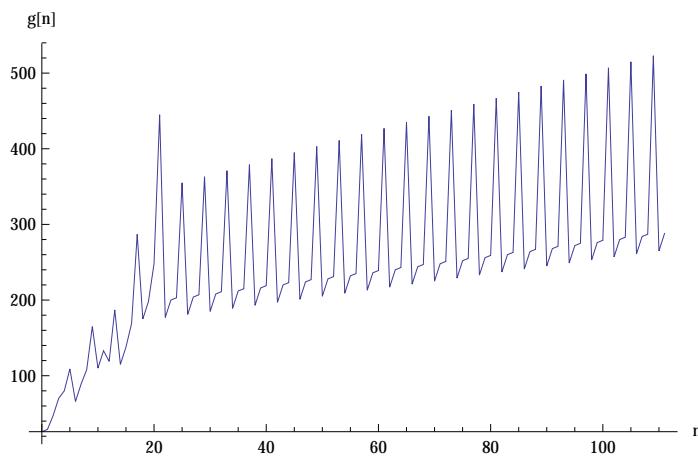


```
f[n_] := Nest[SKStep, s[s][k][s[s[s][k]]][t][t[s[t[k]]]], n]
```

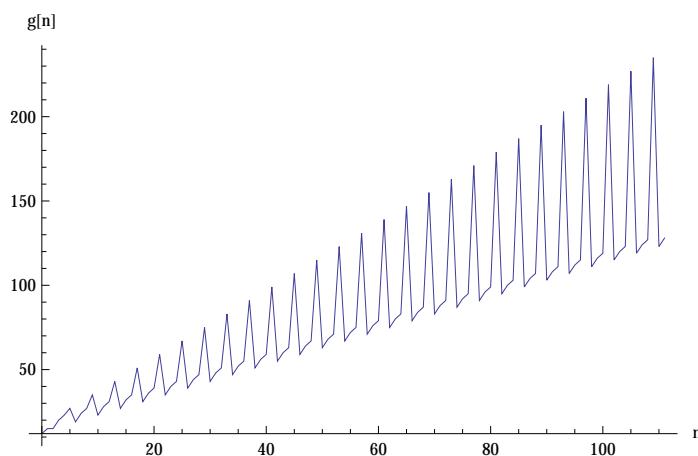
```
Table[f[n], {n, 0, 11}] // ColumnForm
```

```
f[n_] := Nest[SKStep, s[s][k][s[s[s][k]]][s[s[s[s][k]]][s[s[s[s[s][k]]]]][t][[[t[s[t[k]]]]]]], n]
```

```
ListLinePlot[Table[{n, g[n]}, {n, 0, 111}],  
 PlotRange -> All, AxesLabel -> {"n", "g[n]"}]
```

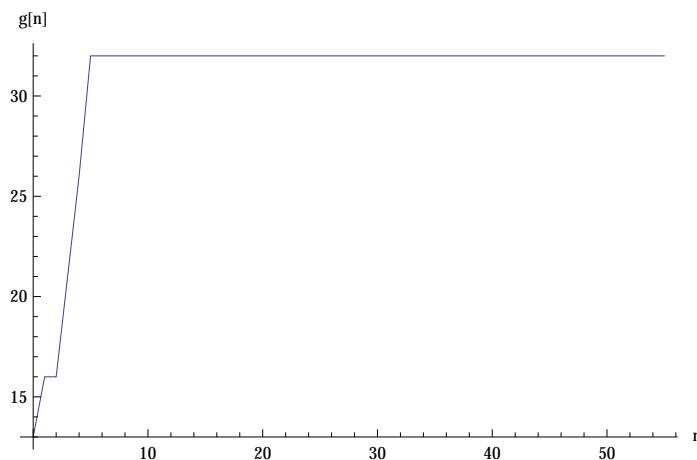


```
f[n_] := Nest[SKStep, s[s][k][s[s[s][k]]][t][t[s[t[k]]]], n]  
ListLinePlot[Table[{n, g[n]}, {n, 0, 111}],  
 PlotRange -> All, AxesLabel -> {"n", "g[n]"}]
```



```
f[n_] := Nest[SKStep, s[s][t][s[s[s][t]]][k][s][t[s[t[s]]]], n]
```

```
ListLinePlot[Table[{n, g[n]}, {n, 0, 55}],
  PlotRange -> All, AxesLabel -> {"n", "g[n]"}]
```



```
Clear[f]
```

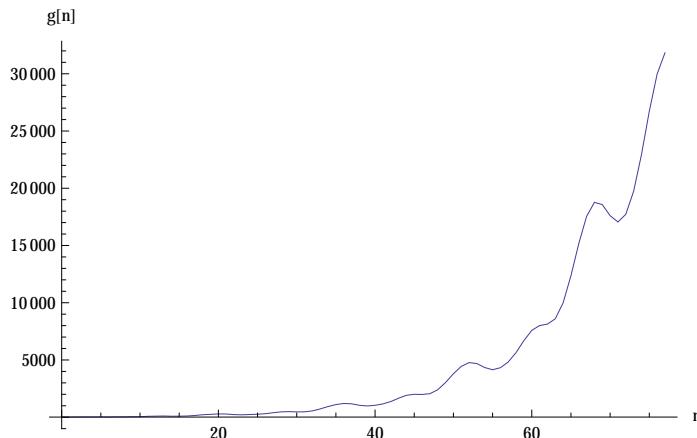
```
Clear[g]
```

```
SKStep[exp_] := exp /. {s[x_][y_][z_] -> x[z][y[z]], k[x_][y_] -> x, i[x_] -> x,
  t[x_][y_] t[u_][v_] -> x[u] y[v], W[x_][y_] -> x[y][y] }
```

```
f[n_] := Nest[SKStep, s[s[s][s][s[i][i]]][s[k[s[i][k]]]], n]
```

```
g[n_] := LeafCount[f[n]]
```

```
ListLinePlot[Table[{n, g[n]}, {n, 0, 77}],
  PlotRange -> All, AxesLabel -> {"n", "g[n]"}]
```



Wolfram example

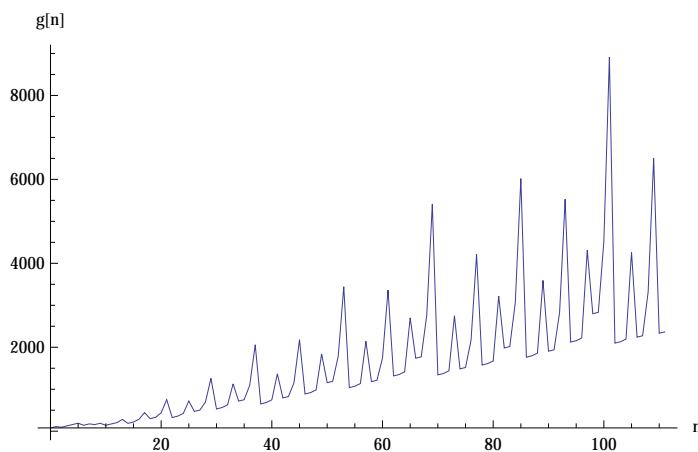
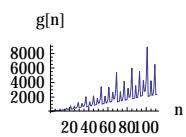
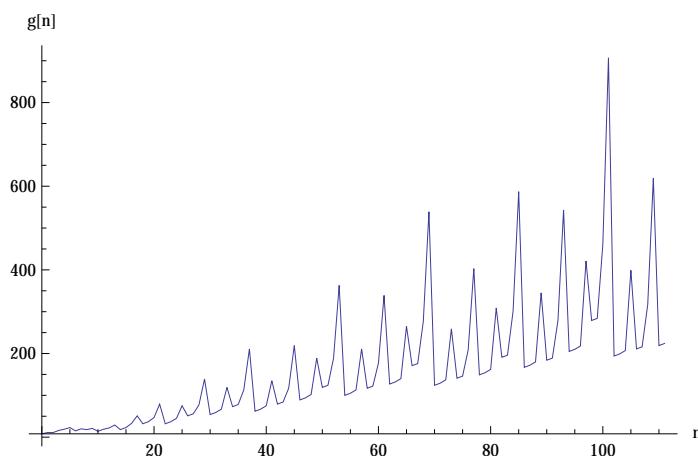
```
Clear[f]
```

```
Clear[g]
```

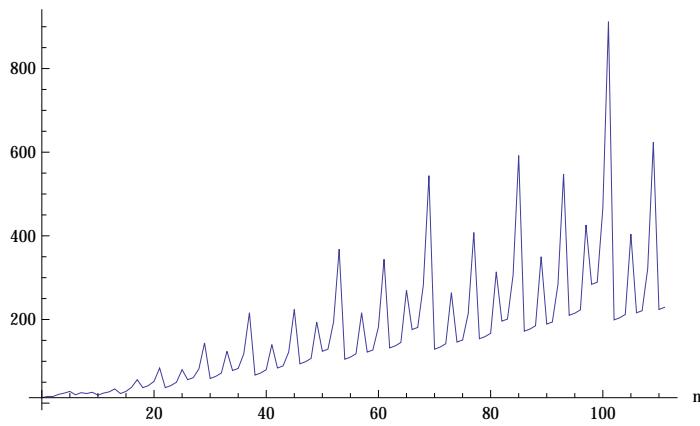
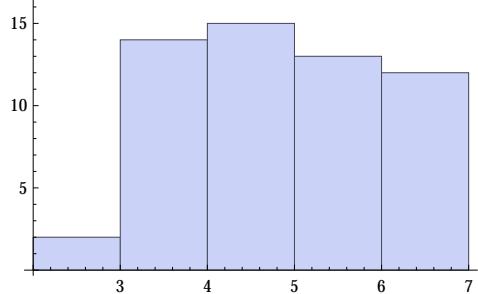
```
SKStep[exp_] := exp /. {s[x_][y_][z_] -> x[z][y[z]], k[x_][y_] -> x}
```

```
f[n_] := Nest[SKStep, s[s][k][s[s[s][k]]][k], n]
g[n_] := LeafCount[f[n]]

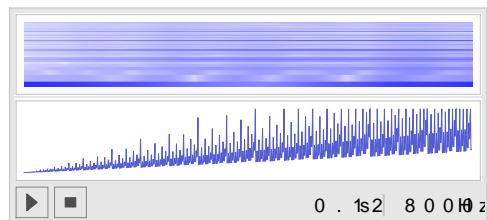
ListLinePlot[Table[{n, g[n]}, {n, 0, 111}],
 PlotRange → All, AxesLabel → {"n", "g[n]"}]
```



```
f[n_] := Nest[SKStep, s[s][k][s[s[s][k]]][k][t][t[s[t[k]]]], n]
```

```
ListLinePlot[Table[{n, g[n]}, {n, 0, 111}],  
 PlotRange -> All, AxesLabel -> {"n", "g[n]"}]  
  
g[n]  
  
  
Table[g[n], {n, 11, 333}]  
  
Histogram[  
 Depth /@ FixedPointList[#, # /. {s[x_][y_][z_] -> x[z][y[z]], k[x_][y_] -> x,  
 t[x_][y_] t[u_][v_] -> x[u] y[v], w[x_][y_] -> x[y][y]} &,  
 s[s][k][s[s][s][k]]][k], 55]]  

```

```
Pane[  
 Quiet@ListPlay[  
 Table[g[n], {n, 1, 999}]]]
```



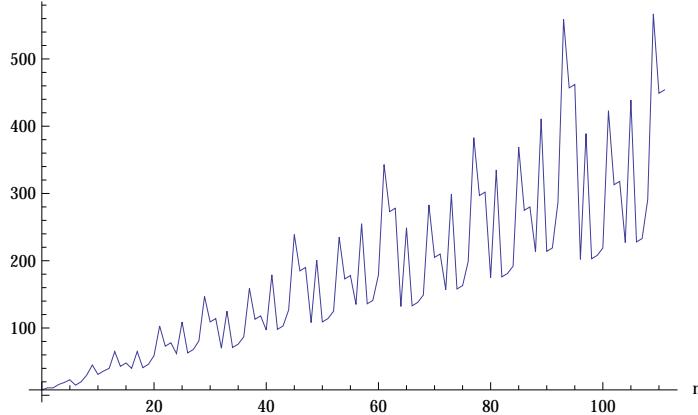
modification with W instead of k

```
Clear[g]  
  
Clear[h]  
  
SKStep[exp_] := exp /. {s[x_][y_][z_] -> x[z][y[z]], k[x_][y_] -> x,  
 t[x_][y_] t[u_][v_] -> x[u] y[v], w[x_][y_] -> x[y][y]}  
  
(* W = "S(S(KS)(S(S(KS)K)(KI)))(KI)" *)
```

```
f[n_] := Nest[SKStep, s[s][k][s[s[s][k]]][W], n]
(*s[s][k][s[s[s][k]]][k]*)
g[n_] := LeafCount[f[n]]
```

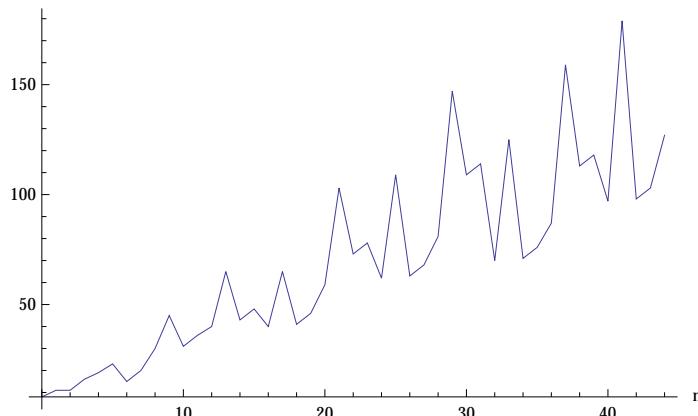
```
ListLinePlot[Table[{n, g[n]}, {n, 0, 111}],
 PlotRange -> All, AxesLabel -> {"n", "g[n]"}]
```

g[n]



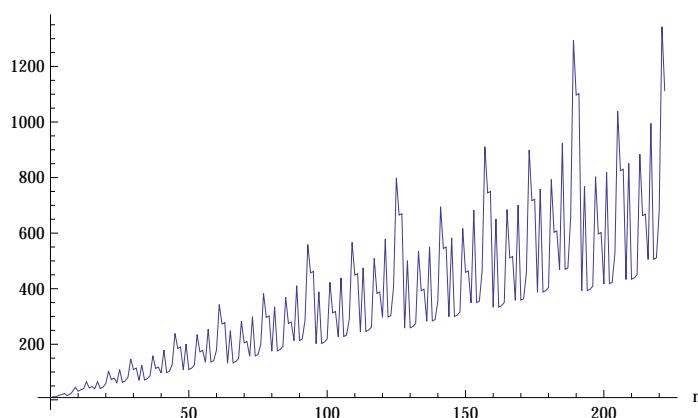
```
ListLinePlot[Table[{n, g[n]}, {n, 0, 44}],
 PlotRange -> All, AxesLabel -> {"n", "g[n]"}]
```

g[n]



```
ListLinePlot[Table[{n, g[n]}, {n, 0, 222}],
 PlotRange -> All, AxesLabel -> {"n", "g[n]"}]
```

g[n]



```

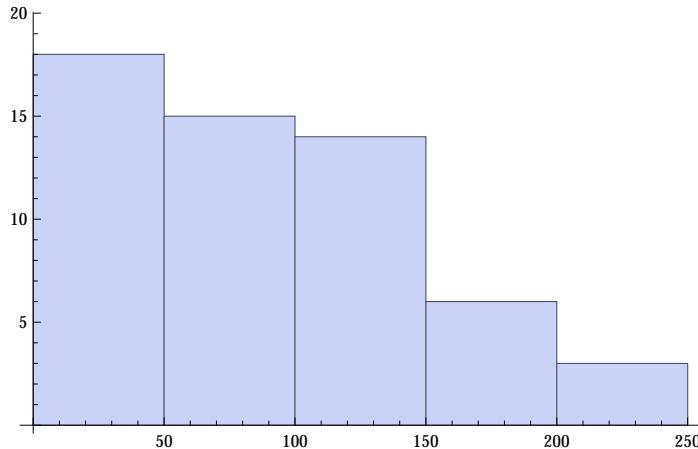
LeafCount /@ NestList[#, {s[x_][y_][z_] → x[z][y[z]], k[x_][y_] → x,
t[x_][y_] t[u_][v_] → x[u] y[v], w[x_][y_] → x[y][y]} &,
s[s][k][s[s][s][k]]][W], 55]
{8, 11, 11, 16, 19, 23, 15, 20, 30, 45, 31, 36, 40, 65, 43, 48, 40, 65, 41, 46, 59, 103,
73, 78, 62, 109, 63, 68, 81, 147, 109, 114, 70, 125, 71, 76, 87, 159, 113, 118,
97, 179, 98, 103, 127, 239, 185, 190, 108, 201, 109, 114, 125, 235, 173, 178}

```

```

Histogram[LeafCount /@ NestList[#, {s[x_][y_][z_] → x[z][y[z]], k[x_][y_] → x,
t[x_][y_] t[u_][v_] → x[u] y[v], w[x_][y_] → x[y][y]} &,
s[s][k][s[s][s][k]]][W], 55]]

```

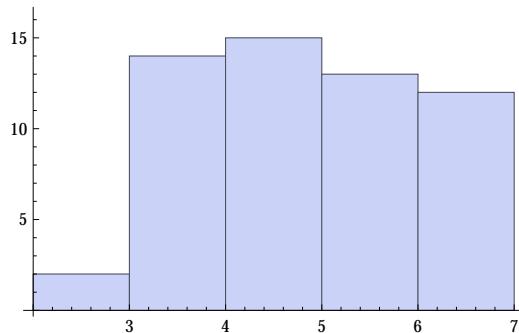


```
Table[g[n], {n, 11, 333}]
```

```

Histogram[
Depth /@ FixedPointList[#, {s[x_][y_][z_] → x[z][y[z]], k[x_][y_] → x,
t[x_][y_] t[u_][v_] → x[u] y[v], w[x_][y_] → x[y][y]} &,
s[s][k][s[s][s][k]]][W], 55]]

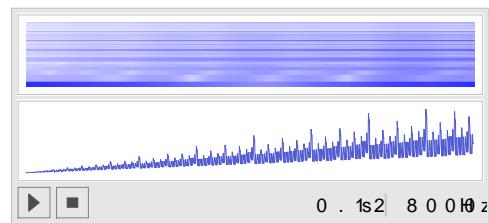
```



```

Pane[
Quiet@ListPlay[
Table[g[n], {n, 1, 999}]]]

```



```

Clear[g]
Clear[h]

SKStep[exp_] := exp /. {s[x_][y_][z_] → x[z][y[z]], k[x_][y_] → x,
t[x_][y_] t[u_][v_] → x[u] y[v], w[x_][y_] → x[y][y] }

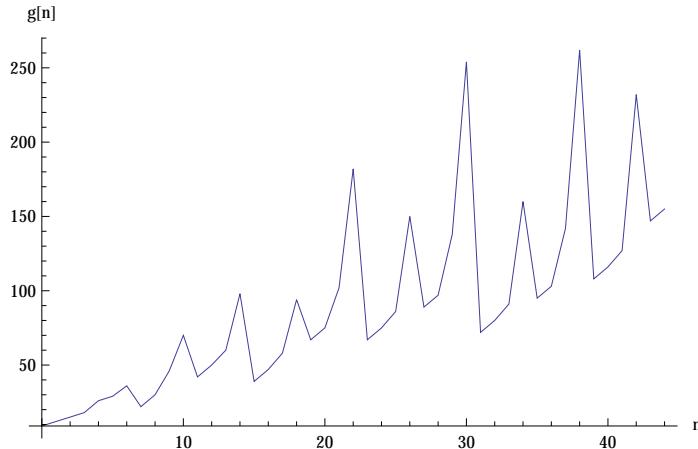
f[n_] := Nest[SKStep, w[s[s][k]][s[s[s][k]]][w], n]

(*s[s][k][s[s[s][k]]][k]*)

g[n_] := LeafCount[f[n]]

ListLinePlot[Table[{n, g[n]}, {n, 0, 44}],
PlotRange → All, AxesLabel → {"n", "g[n"]}]

```



```

Clear[g]
Clear[h]

SKStep[exp_] := exp /. {s[x_][y_][z_] → x[z][y[z]], k[x_][y_] → x,
t[x_][y_] t[u_][v_] → x[u] y[v], w[x_][y_] → x[y][y] }

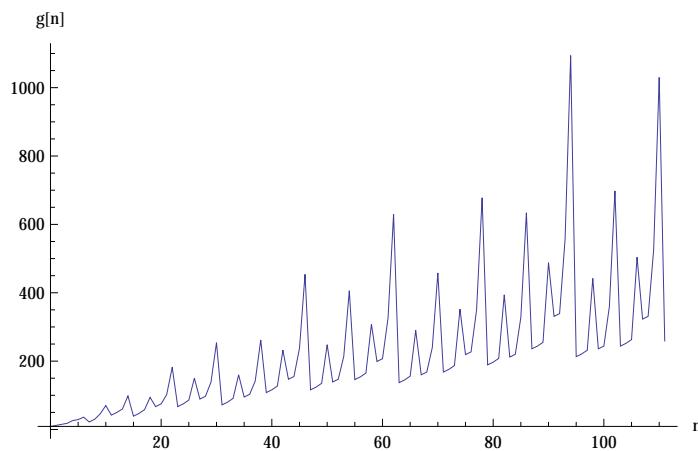
f[n_] := Nest[SKStep, w[s[s][k]][s[s[s][k]]][w], n]

(*s[s][k][s[s[s][k]]][k]*)

g[n_] := LeafCount[f[n]]

```

```
ListLinePlot[Table[{n, g[n]}, {n, 0, 111}],
 PlotRange -> All, AxesLabel -> {"n", "g[n]"}]
```



```
Table[f[n], {n, 0, 11}] // ColumnForm
```

```
Table[f[n], {n, 0, 11}] /. filter // ColumnForm
```

```
Clear[g]
```

```
Clear[h]
```

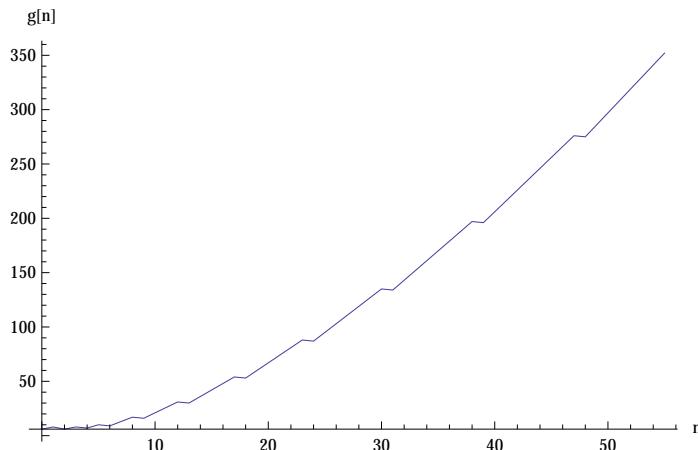
```
SKStep[exp_] := exp /. {s[x_][y_][z_] -> x[z][y[z]], k[x_][y_] -> x, i[x_] -> x,
 t[x_][y_] t[u_][v_] -> x[u] y[v], w[x_][y_] -> x[y][y]}
```

```
f[n_] := Nest[SKStep, s[i][i][s[i][w]], n]
```

```
(*s[s][k][s[s][k]]][k]*)
```

```
g[n_] := LeafCount[f[n]]
```

```
ListLinePlot[Table[{n, g[n]}, {n, 0, 55}],
 PlotRange -> All, AxesLabel -> {"n", "g[n]"}]
```



Curves

Wolfram

```

Clear[f]
Clear[g]

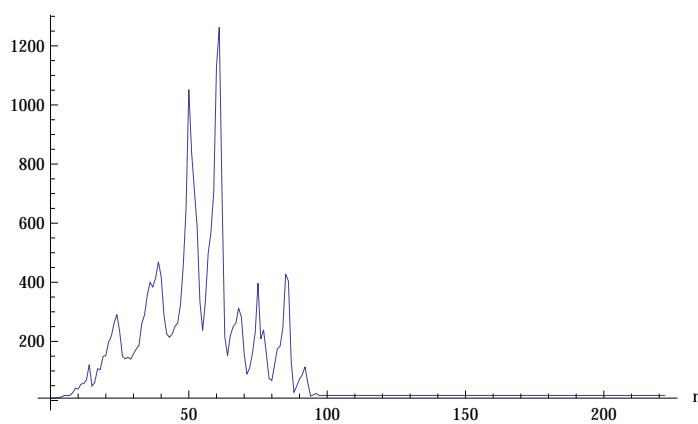
SKStep[exp_] := exp /. {s[x_][y_][z_] → x[z][y[z]], k[x_][y_] → x, i[x_] → x,
t[x_][y_] t[u_][v_] → x[u] y[v], w[x_][y_] → x[y][y]}

f[n_] := Nest[SKStep, s[s[s[s]]][s][s][k], n]

g[n_] := LeafCount[f[n]]

ListLinePlot[Table[{n, g[n]}, {n, 0, 222}],
PlotRange → All, AxesLabel → {"n", "g[n"]}]

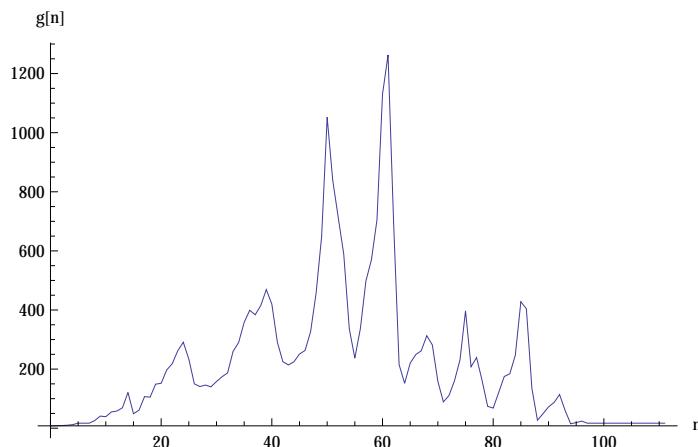
```



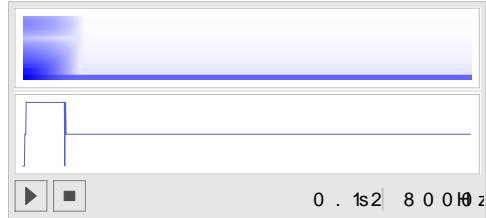
```

ListLinePlot[Table[{n, g[n]}, {n, 0, 111}],
PlotRange → All, AxesLabel → {"n", "g[n"]}]

```



```
Pane[
 Quiet@ListPlay[
 Table[g[n], {n, 1, 999}]]]
```



```
filter = {
 s[s[s[s][s]]][s][s][k] → C,
 k[s[s[s]][k]] → B,
 s[s[s[s]][s[s[s]]]] → A,
 k[s[s[s]]] → F,
 s[s[s]] → D,
 s[k][k[k]] → G,
 s[k][k[k]] → V
};
```

```
FixedPointList[#, /.
 {
   s[x_][y_][z_] -> x[z][y[z]], k[x_][y_] -> x, i[x_] -> x,
   t[x_][y_] t[u_][v_] -> x[u] y[v], w[x_][y_] -> x[y][y]
 } &,
 s[s[s[s][s]]][s][s][k], 11] // MatrixForm
```

```
FixedPointList[#, /.
 {
   s[x_][y_][z_] → x[z][y[z]], k[x_][y_] → x, i[x_] → x,
   t[x_][y_] t[u_][v_] → x[u] y[v], w[x_][y_] → x[y][y]
 } &,
 s[s[s[s][s]]][s][s][k], 11] /. filter // MatrixForm
```

```
SKStep[exp_] := exp /. {s[x_][y_][z_] → x[z][y[z]], k[x_][y_] → x, i[x_] → x,
 t[x_][y_] t[u_][v_] → x[u] y[v], w[x_][y_] → x[y][y] }

(* W = "S(S(KS)(S(S(KS)K)(KI)))(KI)" *)
f[n_] := Nest[SKStep, s[s[s]][k][s[i[s[i]]]][k]][W], n]
(*s[s][k][s[s[s][k]]][k]*)
```

```
g[n_] := LeafCount[f[n]]
```

```

ListLinePlot[Table[{n, g[n]}, {n, 0, 22}],
 PlotRange → All, AxesLabel → {"n", "g[n]"}]
g[n]

n

ListLinePlot[Table[{n, g[n]}, {n, 0, 111}],
 PlotRange → All, AxesLabel → {"n", "g[n]"}]
g[n]

n

Clear[f]
Clear[g]

SKStep[exp_] := exp /.
 {
  s[x_][y_][z_] → x[z][y[z]],
  k[x_][y_] → x,
  i[x_] → x, i[x_] → y, i[x_] → z,
  t[x_][y_] t[u_][v_] → x[u] y[v],
  w[x_][y_] → x[y][y]
 }

f[n_] := Nest[SKStep, s[s[s]][s][s][s][k], n]
g[n_] := LeafCount[f[n]]

```

```

ListLinePlot[Table[{n, g[n]}, {n, 0, 22}],
 PlotRange -> All, AxesLabel -> {"n", "g[n]"}]

g[n]
25
20
15
10
5
0
n
10
15
20

Table[f[n], {n, 0, 22}] /. filter // MatrixForm
Clear[f]
Clear[g]

SKStep[exp_] := exp /. {s[x_][y_][z_] -> x[z][y[z]], k[x_][y_] -> x,
t[x_][y_] t[u_][v_] -> x[u] y[v], w[x_][y_] -> x[y][y]}

f[n_] := Nest[SKStep, s[i][i][s[i][s[i]][s[i][i][s[i][s[i]]][k]]][s[w]], n]
g[n_] := LeafCount[f[n]]

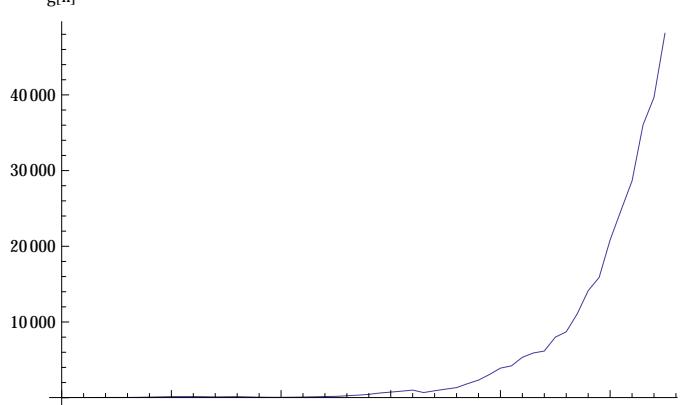
ListLinePlot[Table[{n, g[n]}, {n, 0, 22}],
 PlotRange -> All, AxesLabel -> {"n", "g[n]"}]

g[n]
140
120
100
80
60
40
20
0
n
10
15
20

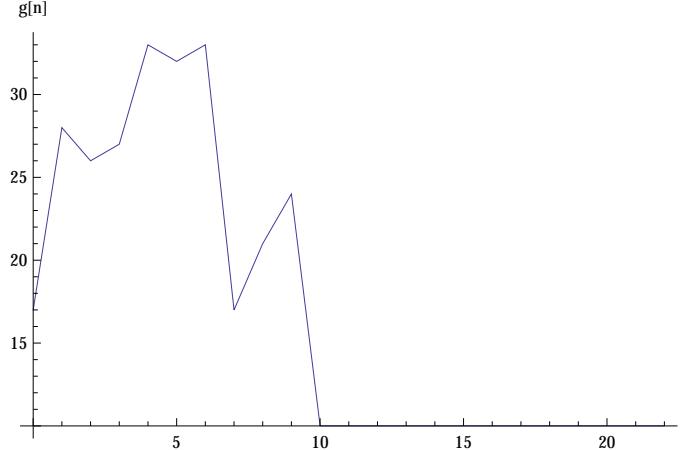
```

```

ListLinePlot[Table[{n, g[n]}, {n, 0, 55}],
 PlotRange -> All, AxesLabel -> {"n", "g[n]"}]
g[n]


f[n_] := Nest[SKStep, s[i][i][s[i[s[i]]][s[i][i][s[i[s[i]]][k]]]][k[w]], n]

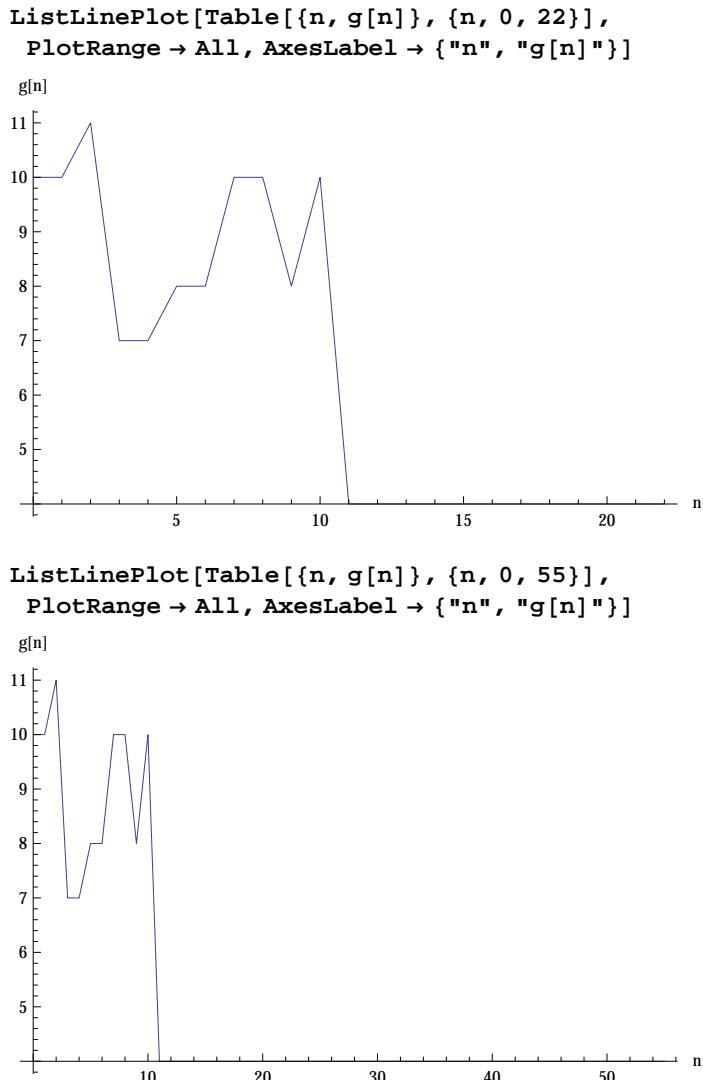
ListLinePlot[Table[{n, g[n]}, {n, 0, 22}],
 PlotRange -> All, AxesLabel -> {"n", "g[n]"}]
g[n]


Clear[f]
Clear[g]

SKStep[exp_] := exp /. {s[x_][y_][z_] -> x[z][y[z]], k[x_][y_] -> x, i[x_] -> x,
 t[x_][y_] t[u_][v_] -> x[u] y[v], w[x_][y_] -> x[y][y] }

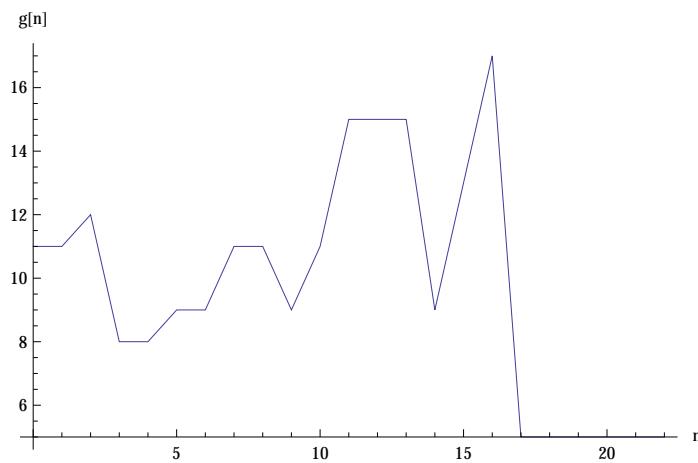
f[n_] := Nest[SKStep, s[s[k]][s][w][s][s][k][k][s], n]
g[n_] := LeafCount[f[n]]

```

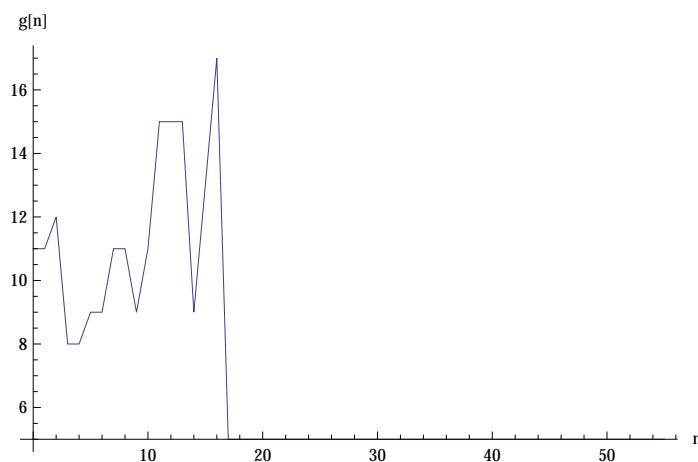


```
Clear[f]
Clear[g]
SKStep[exp_] := exp /. {s[x_][y_][z_] -> x[z][y[z]], k[x_][y_] -> x, i[x_] -> x,
t[x_][y_] t[u_][v_] -> x[u] y[v], w[x_][y_] -> x[y][y]}
f[n_] := Nest[SKStep, s[s[k]][s][w][s][s][k][w][k][s], n]
g[n_] := LeafCount[f[n]]
```

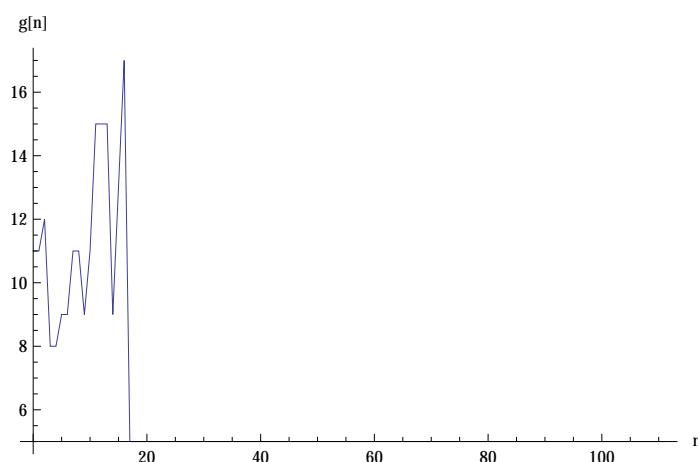
```
ListLinePlot[Table[{n, g[n]}, {n, 0, 22}],  
 PlotRange -> All, AxesLabel -> {"n", "g[n]"}]
```



```
ListLinePlot[Table[{n, g[n]}, {n, 0, 55}],  
 PlotRange -> All, AxesLabel -> {"n", "g[n]"}]
```



```
ListLinePlot[Table[{n, g[n]}, {n, 0, 111}],  
 PlotRange -> All, AxesLabel -> {"n", "g[n]"}]
```



```
Clear[f]
```

```

Clear[g]

SKStep[exp_] := exp /. {s[x_][y_][z_] → x[z][y[z]], k[x_][y_] → x, i[x_] → x,
t[x_][y_] t[u_][v_] → x[u] y[v], w[x_][y_] → x[y][y] }

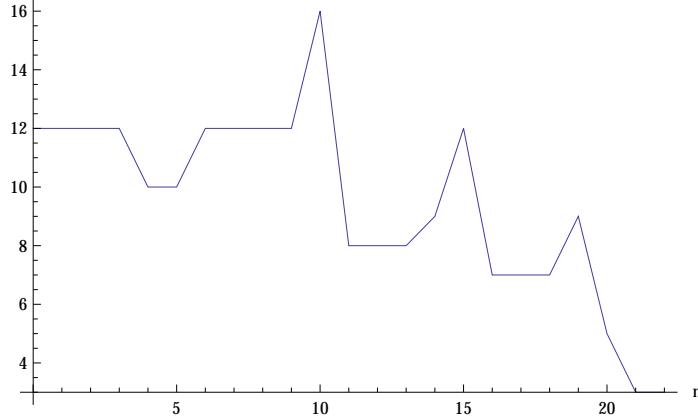
f[n_] := Nest[SKStep, s[s[W[k]][s]][W][s][s][k][W][k][s], n]

g[n_] := LeafCount[f[n]]

```

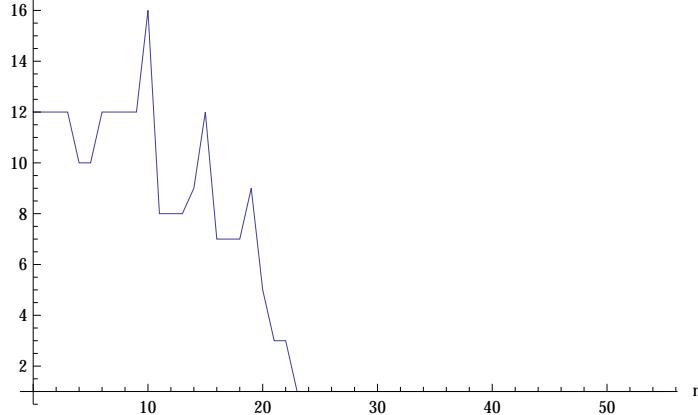
ListLinePlot[Table[{n, g[n]}, {n, 0, 22}],
PlotRange → All, AxesLabel → {"n", "g[n]"}]

g[n]



ListLinePlot[Table[{n, g[n]}, {n, 0, 55}],
PlotRange → All, AxesLabel → {"n", "g[n]"}]

g[n]



Clear[f]

Clear[g]

```

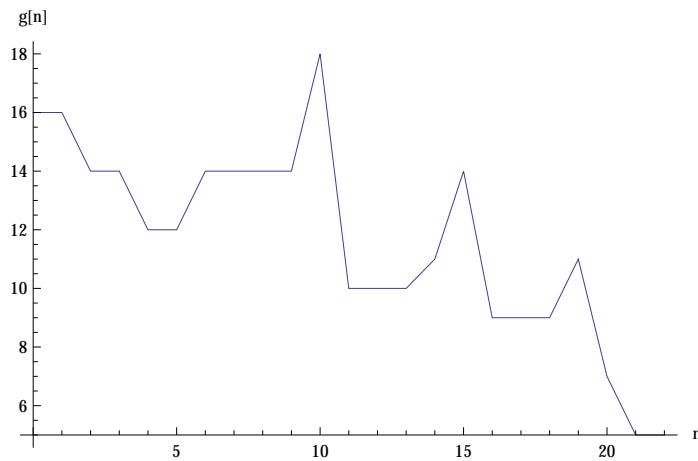
SKStep[exp_] := exp /. {s[x_][y_][z_] → x[z][y[z]], k[x_][y_] → x, i[x_] → x,
t[x_][y_] t[u_][v_] → x[u] y[v], w[x_][y_] → x[y][y] }

```

f[n_] := Nest[SKStep, s[s[W[k]][s]][W][s][s][k][W][k][s][W[k][s]], n]

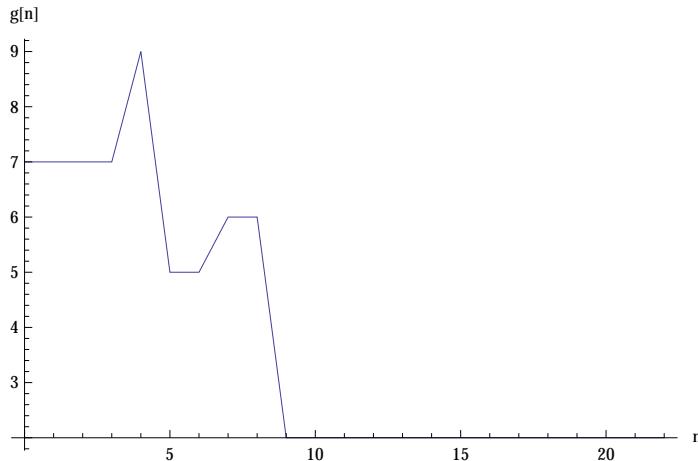
g[n_] := LeafCount[f[n]]

```
ListLinePlot[Table[{n, g[n]}, {n, 0, 22}],  
 PlotRange -> All, AxesLabel -> {"n", "g[n]"}]
```



```
Clear[f]  
  
Clear[g]  
  
SKStep[exp_] := exp /. {s[x_][y_][z_] -> x[z][y[z]], k[x_][y_] -> x, i[x_] -> x,  
 t[x_][y_] t[u_][v_] -> x[u] y[v], w[x_][y_] -> x[y][y] }  
  
f[n_] := Nest[SKStep, w[s][s][w][k][s][s], n]  
  
g[n_] := LeafCount[f[n]]
```

```
ListLinePlot[Table[{n, g[n]}, {n, 0, 22}],  
 PlotRange -> All, AxesLabel -> {"n", "g[n]"}]
```



```
Clear[f]  
  
Clear[g]  
  
SKStep[exp_] := exp /. {s[x_][y_][z_] -> x[z][y[z]], k[x_][y_] -> x, i[x_] -> x,  
 t[x_][y_] t[u_][v_] -> x[u] y[v], w[x_][y_] -> x[y][y] }  
  
f[n_] := Nest[SKStep, w[s][s[w[k]][s]][w][s][s][k][w][k][s], n]
```

```

g[n_] := LeafCount[f[n]]

ListLinePlot[Table[{n, g[n]}, {n, 0, 22}],
 PlotRange → All, AxesLabel → {"n", "g[n]"}]

g[n]


```

```

ListLinePlot[Table[{n, g[n]}, {n, 0, 55}],
 PlotRange → All, AxesLabel → {"n", "g[n]"}]

g[n]


```

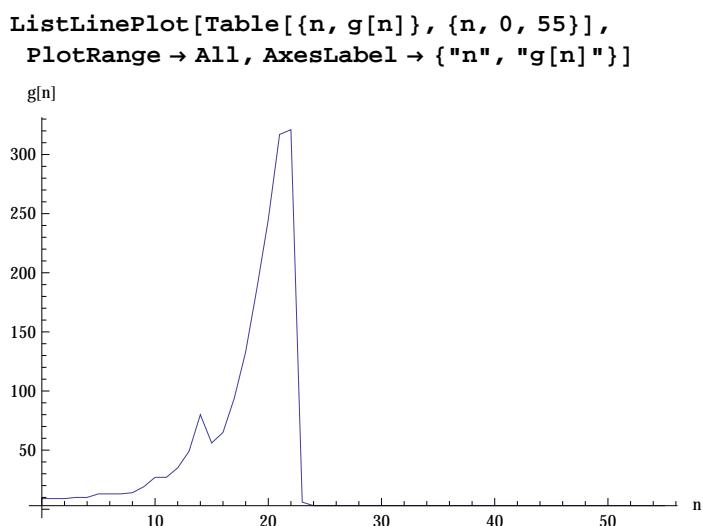
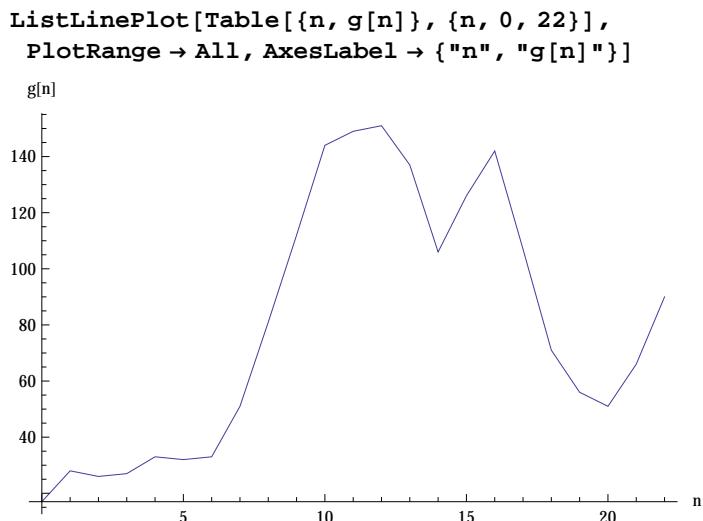
```

Clear[f]
Clear[g]

SKStep[exp_] := exp /. {s[x_][y_][z_] → x[z][y[z]], k[x_][y_] → x, i[x_] → x,
 t[x_][y_] t[u_][v_] → x[u] y[v], w[x_][y_] → x[y][y] }

f[n_] := Nest[SKStep, w[s[s[s]][s]][s][s][k][k], n]
g[n_] := LeafCount[f[n]]

```

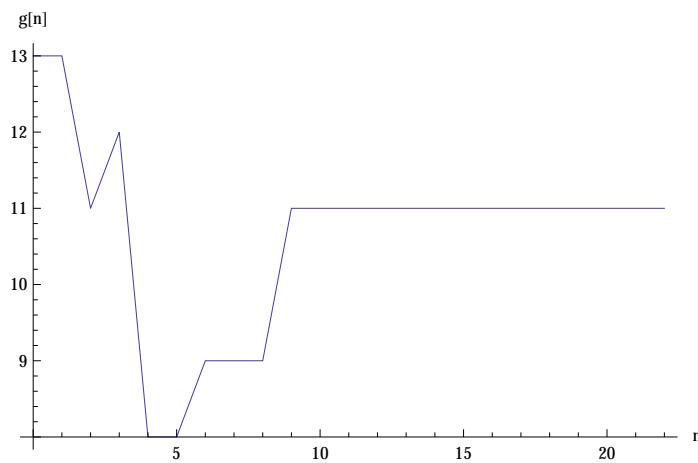


```
Clear[f]
Clear[g]

SKStep[exp_] := exp /. {s[x_][y_][z_] -> x[z][y[z]], k[x_][y_] -> x, i[x_] -> x,
t[x_][y_] t[u_][v_] -> x[u] y[v], w[x_][y_] -> x[y][y] }

f[n_] := Nest[SKStep, w[s[s[k]][s]][w][s][s] w[k][k][s], n]
g[n_] := LeafCount[f[n]]
```

```
ListLinePlot[Table[{n, g[n]}, {n, 0, 22}],
  PlotRange → All, AxesLabel → {"n", "g[n]"}]
```



```
Clear[f]
```

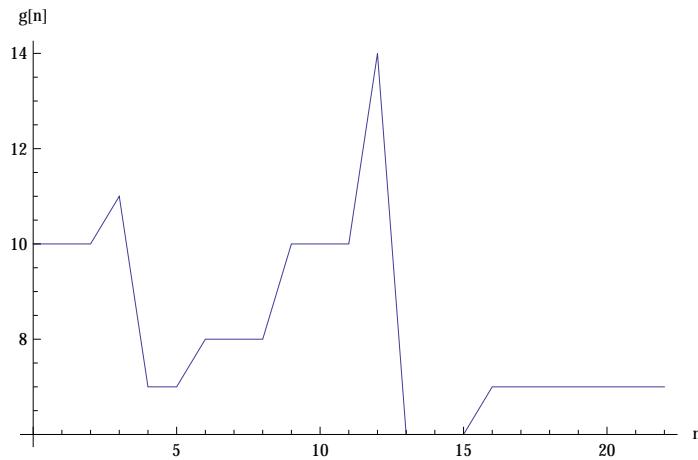
```
Clear[g]
```

```
SKStep[exp_] := exp /. {s[x_][y_][z_] → x[z][y[z]], k[x_][y_] → x, i[x_] → x,
  t[x_][y_] t[u_][v_] → x[u] y[v], w[x_][y_] → x[y][y] }
```

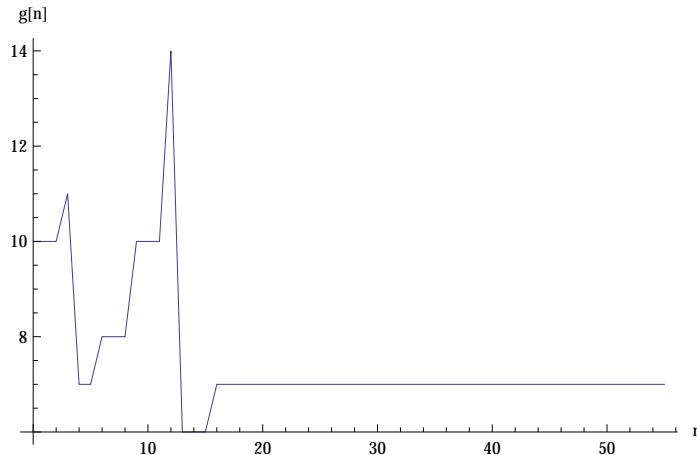
```
f[n_] := Nest[SKStep, w[s[s[k]][s]][w][s][s][k][s], n]
```

```
g[n_] := LeafCount[f[n]]
```

```
ListLinePlot[Table[{n, g[n]}, {n, 0, 22}],
  PlotRange → All, AxesLabel → {"n", "g[n]"}]
```



```
ListLinePlot[Table[{n, g[n]}, {n, 0, 55}],
 PlotRange -> All, AxesLabel -> {"n", "g[n]"}]
```



```
Clear[f]
```

```
Clear[g]
```

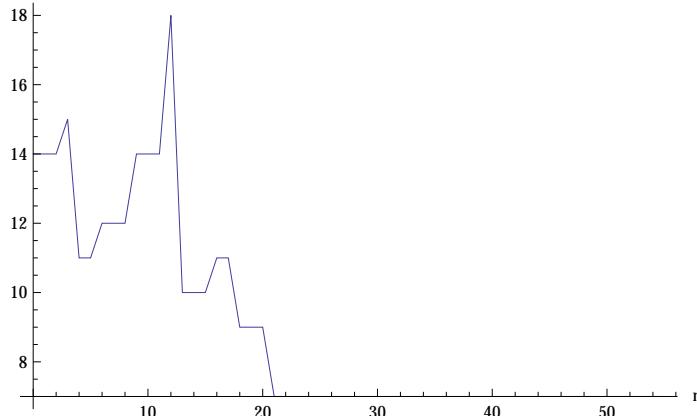
```
SKStep[exp_] := exp /. {s[x_][y_][z_] -> x[z][y[z]], k[x_][y_] -> x, i[x_] -> x,
 t[x_][y_] t[u_][v_] -> x[u] y[v], w[x_][y_] -> x[y][y] }
```

```
f[n_] := Nest[SKStep, w[s[s[k]][s]][w][s][s][k][s][t][s][s][k], n]
```

```
g[n_] := LeafCount[f[n]]
```

```
ListLinePlot[Table[{n, g[n]}, {n, 0, 55}],
 PlotRange -> All, AxesLabel -> {"n", "g[n]"}]
```

g[n]



```
Clear[f]
```

```
Clear[g]
```

```
SKStep[exp_] := exp /. {s[x_][y_][z_] -> x[z][y[z]], k[x_][y_] -> x, i[x_] -> x,
 t[x_][y_] t[u_][v_] -> x[u] y[v], w[x_][y_] -> x[y][y] }
```

```
f[n_] := Nest[SKStep, w[s[s[k]][s]][w][s][s][k][w][w][s][k], n]
```

```

g[n_] := LeafCount[f[n]]

ListLinePlot[Table[{n, g[n]}, {n, 0, 22}],
  PlotRange → All, AxesLabel → {"n", "g[n]"}]

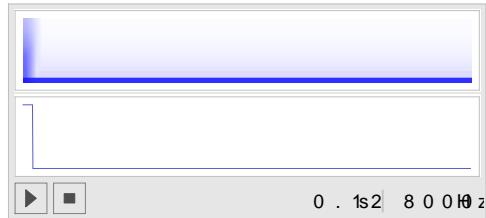
g[n]

```

```

Pane[
 Quiet@ListPlay[
  Table[g[n], {n, 1, 999}]]]

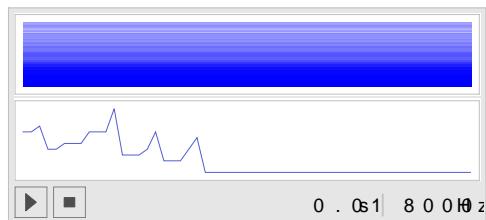
```



```

sa = Pane[
 Quiet@ListPlay[
  Table[g[n], {n, 1, 55}]]]

```



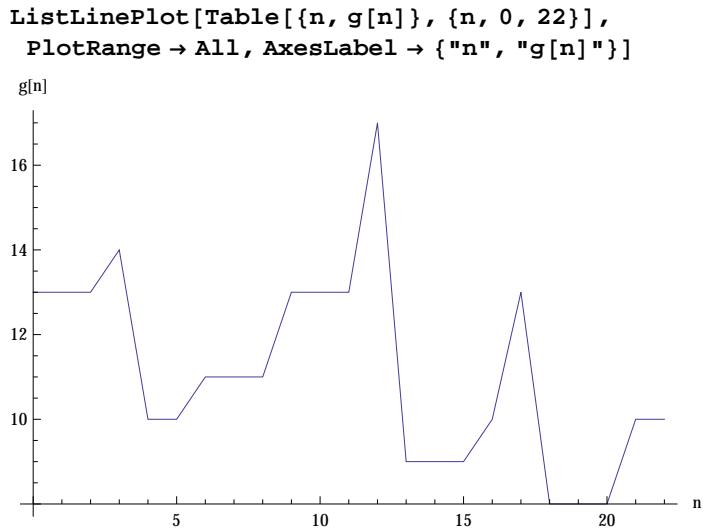
```

Clear[f]
Clear[g]

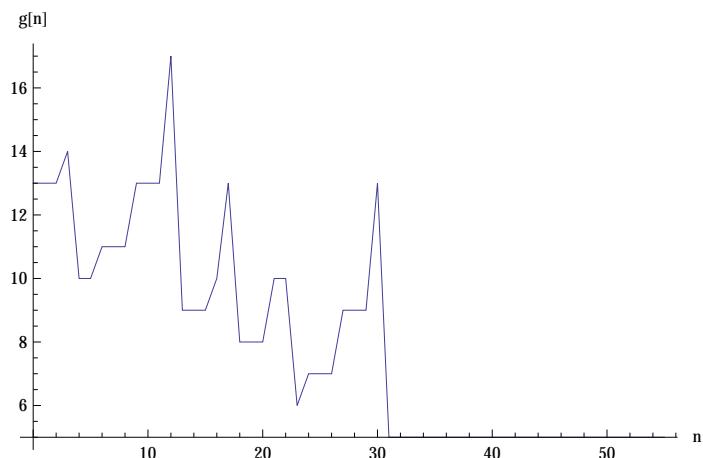
SKStep[exp_] := exp /. {s[x_][y_][z_] → x[z][y[z]], k[x_][y_] → x, i[x_] → x,
  t[x_][y_] t[u_][v_] → x[u] y[v], W[x_][y_] → x[y][y] }

f[n_] := Nest[SKStep, W[s[s[k]][s]][W][s][s][k][W][s][s][k], n]
g[n_] := LeafCount[f[n]]

```



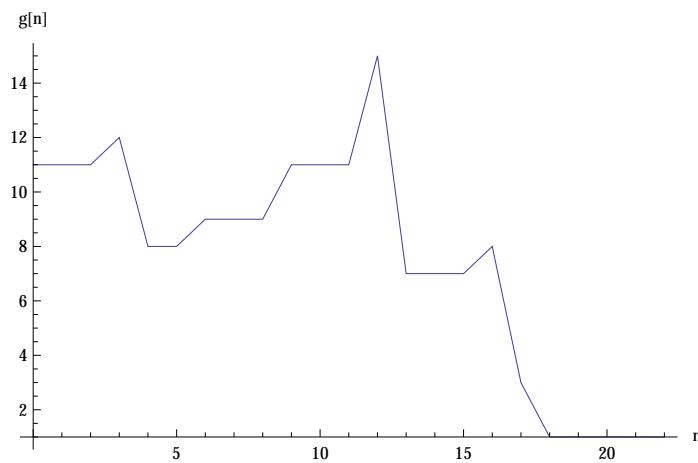
```
ListLinePlot[Table[{n, g[n]}, {n, 0, 55}], PlotRange -> All, AxesLabel -> {"n", "g[n]"}]
```



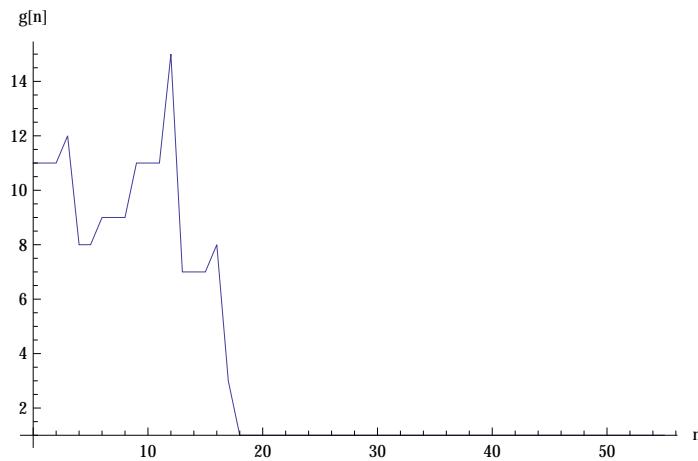
```
Clear[f]
Clear[g]
SKStep[exp_] := exp /. {s[x_][y_][z_] -> x[z][y[z]], k[x_][y_] -> x, i[x_] -> x,
t[x_][y_] t[u_][v_] -> x[u] y[v], w[x_][y_] -> x[y][y] }

f[n_] := Nest[SKStep, w[s[s[k]][s]][w][s][s][k][k][s], n]
g[n_] := LeafCount[f[n]]
```

```
ListLinePlot[Table[{n, g[n]}, {n, 0, 22}],
 PlotRange → All, AxesLabel → {"n", "g[n]"}]
```



```
ListLinePlot[Table[{n, g[n]}, {n, 0, 55}],
 PlotRange → All, AxesLabel → {"n", "g[n]"}]
```

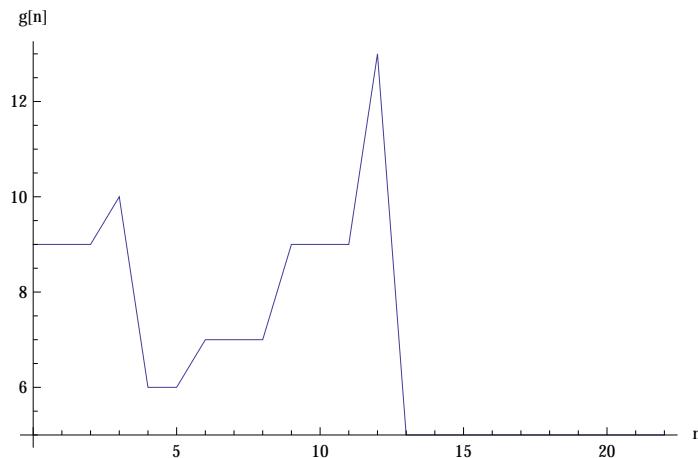


```
Clear[f]
Clear[g]

SKStep[exp_] := exp /. {s[x_][y_][z_] → x[z][y[z]], k[x_][y_] → x, i[x_] → x,
t[x_][y_] t[u_][v_] → x[u] y[v], w[x_][y_] → x[y][y] }

f[n_] := Nest[SKStep, w[s[s[k]][s]][w][s][s][k], n]
g[n_] := LeafCount[f[n]]
```

```
ListLinePlot[Table[{n, g[n]}, {n, 0, 22}],
 PlotRange -> All, AxesLabel -> {"n", "g[n]"}]
```



```
Clear[f]
```

```
Clear[g]
```

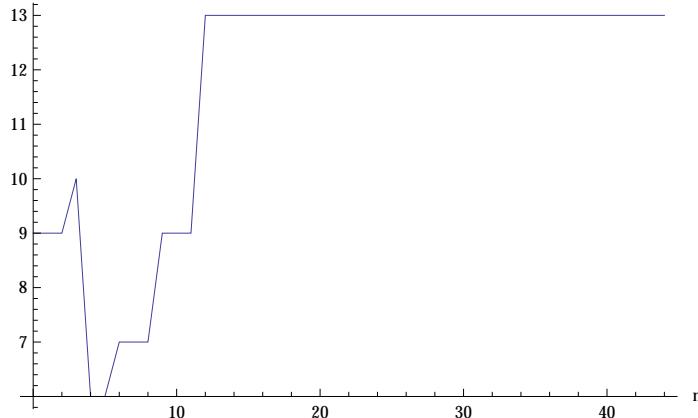
```
SKStep[exp_] := exp /. {s[x_][y_][z_] -> x[z][y[z]], k[x_][y_] -> x, i[x_] -> x,
 t[x_][y_] t[u_][v_] -> x[u] y[v], W[x_][y_] -> x[y][y] }
```

```
f[n_] := Nest[SKStep, W[s[s[k]][s]][W][s][s][s], n]
```

```
g[n_] := LeafCount[f[n]]
```

```
ListLinePlot[Table[{n, g[n]}, {n, 0, 44}],
 PlotRange -> All, AxesLabel -> {"n", "g[n]"}]
```

g[n]



```
Clear[f]
```

```
Clear[g]
```

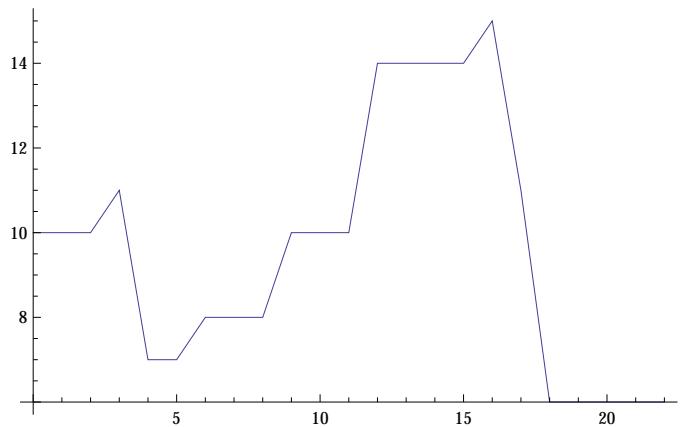
```
SKStep[exp_] := exp /. {s[x_][y_][z_] -> x[z][y[z]], k[x_][y_] -> x, i[x_] -> x,
 t[x_][y_] t[u_][v_] -> x[u] y[v], W[x_][y_] -> x[y][y] }
```

```
f[n_] := Nest[SKStep, W[s[s[k]][s]][W][s][s][s][k], n]
```

```

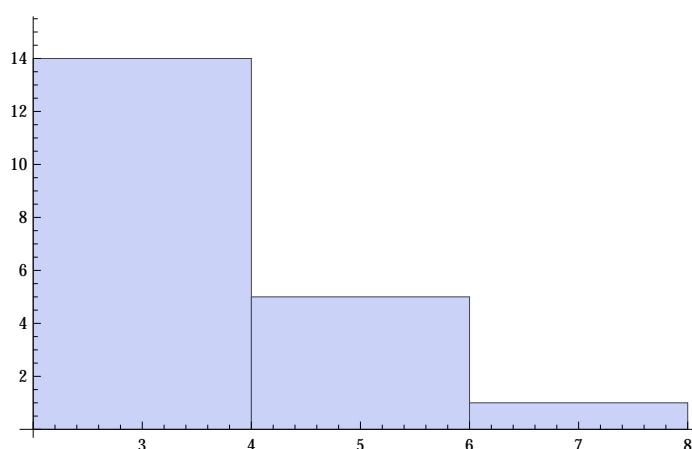
g[n_] := LeafCount[f[n]]

ListLinePlot[Table[{n, g[n]}, {n, 0, 22}],
 PlotRange -> All, AxesLabel -> {"n", "g[n]"}]

g[n]


```

```

Histogram[
 Depth /@ FixedPointList[#, {s[x_][y_][z_] -> x[z][y[z]], k[x_][y_] -> x,
 t[x_][y_][u_][v_] -> x[u] y[v], w[x_][y_] -> x[y][y]} &,
 w[s[s[k]]][s][w][s][s][k], 55]]


```

```

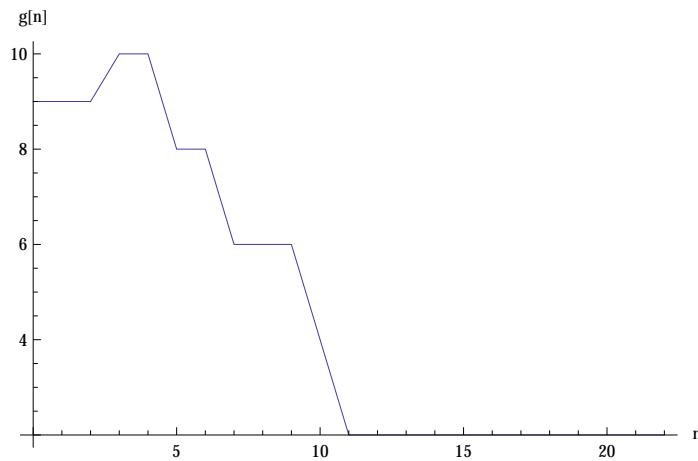
Clear[f]
Clear[g]

SKStep[exp_] := exp /. {s[x_][y_][z_] -> x[z][y[z]], k[x_][y_] -> x,
 t[x_][y_][u_][v_] -> x[u] y[v], w[x_][y_] -> x[y][y]}

f[n_] := Nest[SKStep, w[s[s[s]]][k]][w][k][s][s], n]
g[n_] := LeafCount[f[n]]

```

```
ListLinePlot[Table[{n, g[n]}, {n, 0, 22}],
 PlotRange -> All, AxesLabel -> {"n", "g[n]"}]
```



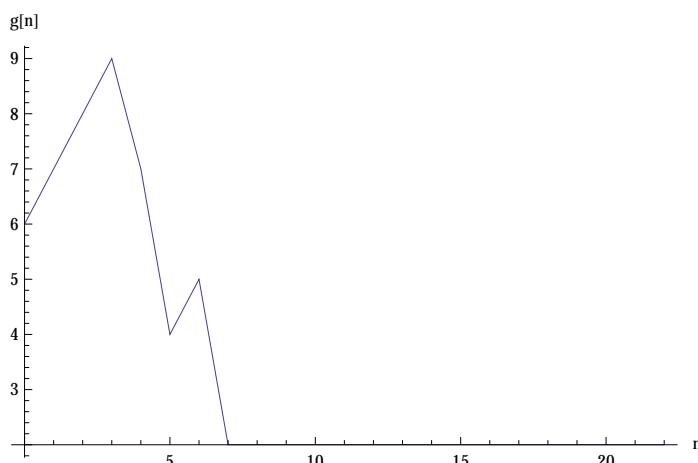
```
Clear[f]
Clear[g]

SKStep[exp_] := exp /. {s[x_][y_][z_] -> x[z][y[z]], k[x_][y_] -> x, i[x_] -> x,
t[x_][y_] t[u_][v_] -> x[u] y[v], w[x_][y_] -> x[y][y] }

f[n_] := Nest[SKStep, w[s[s][k]][w[k]], n]

g[n_] := LeafCount[f[n]]
```

```
ListLinePlot[Table[{n, g[n]}, {n, 0, 22}],
 PlotRange -> All, AxesLabel -> {"n", "g[n]"}]
```



```
Clear[f]
Clear[g]

SKStep[exp_] := exp /. {s[x_][y_][z_] -> x[z][y[z]], k[x_][y_] -> x, i[x_] -> x,
t[x_][y_] t[u_][v_] -> x[u] y[v], w[x_][y_] -> x[y][y] }

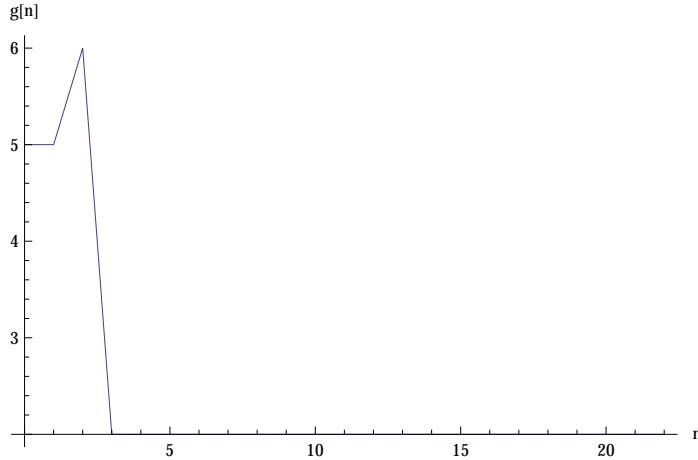
f[n_] := Nest[SKStep, w[s][k][w[s]], n]
```

```

g[n_] := LeafCount[f[n]]

ListLinePlot[Table[{n, g[n]}, {n, 0, 22}],
  PlotRange → All, AxesLabel → {"n", "g[n]"}]

```



```

Clear[f]

Clear[g]

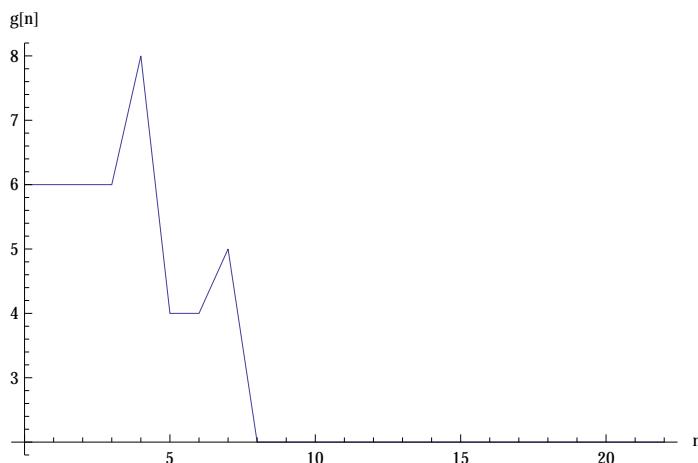
SKStep[exp_] := exp /. {s[x_][y_][z_] → x[z][y[z]], k[x_][y_] → x, i[x_] → x,
  t[x_][y_] t[u_][v_] → x[u] y[v], w[x_][y_] → x[y][y] }

f[n_] := Nest[SKStep, w[s][s][w][k][k], n]

g[n_] := LeafCount[f[n]]

ListLinePlot[Table[{n, g[n]}, {n, 0, 22}],
  PlotRange → All, AxesLabel → {"n", "g[n]"}]

```



```

Clear[f]

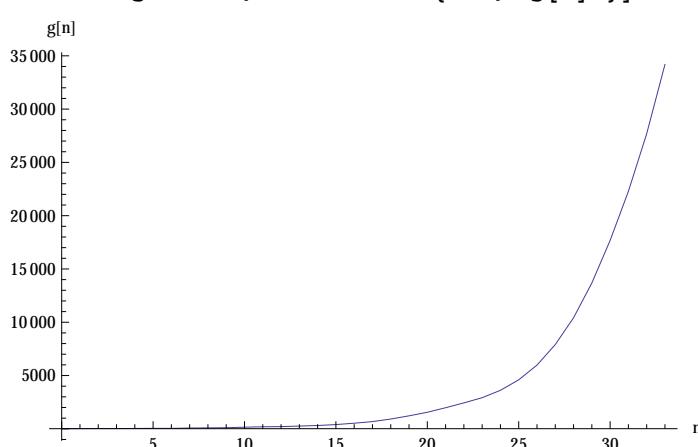
Clear[g]

SKStep[exp_] := exp /. {s[x_][y_][z_] → x[z][y[z]], k[x_][y_] → x, i[x_] → x,
  t[x_][y_] t[u_][v_] → x[u] y[v], w[x_][y_] → x[y][y] }

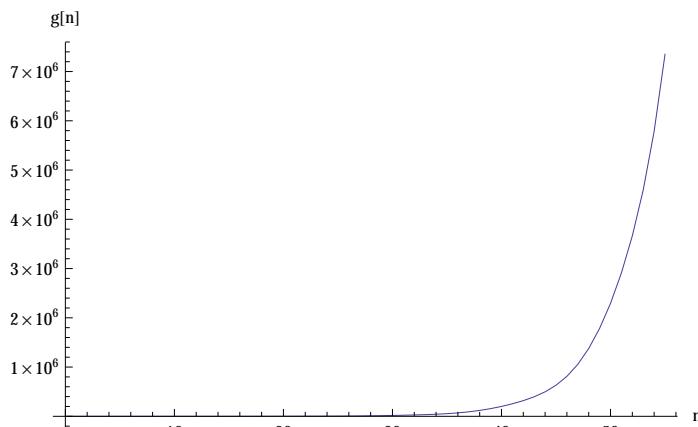
```

```
f[n_] := Nest[SKStep, W[s[i][i]][s[s[k[s[i][k]]][s[i][i]]], n]
g[n_] := LeafCount[f[n]]
```

```
ListLinePlot[Table[{n, g[n]}, {n, 0, 33}],
PlotRange -> All, AxesLabel -> {"n", "g[n]"}]
```

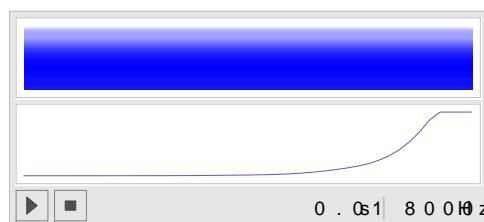


```
ListLinePlot[Table[{n, g[n]}, {n, 0, 55}],
PlotRange -> All, AxesLabel -> {"n", "g[n]"}]
```



```
ListPlay[Table[g[n], {n, 11, 77}]]
```

```
Pane[
Quiet@ListPlay[
Table[g[n], {n, 1, 44}]]]
```



```
Clear[f]
```

```
Clear[g]
```

```

SKStep[exp_] := exp /. {s[x_][y_][z_] → x[z][y[z]], k[x_][y_] → x, i[x_] → x,
t[x_][y_] t[u_][v_] → x[u] y[v], w[x_][y_] → x[y][y] }

f[n_] := Nest[SKStep, w[s[s][k]][w[s]], n]

g[n_] := LeafCount[f[n]]

ListLinePlot[Table[{n, g[n]}, {n, 0, 22}],
PlotRange → All, AxesLabel → {"n", "g[n]"}]

g[n]

```

n	g[n]
0	1
5	1
10	1
15	1
20	1000
22	30000

Wolfram, NKS p. 712

```

Clear[f]

Clear[g]

SKStep[exp_] := exp /. {s[x_][y_][z_] → x[z][y[z]], k[x_][y_] → x, i[x_] → x,
t[x_][y_] t[u_][v_] → x[u] y[v], w[x_][y_] → x[y][y] }

f[n_] := Nest[SKStep, s[s[s]][s][s][s], n]

g[n_] := LeafCount[f[n]]

ListLinePlot[Table[{n, g[n]}, {n, 0, 33}],
PlotRange → All, AxesLabel → {"n", "g[n]"}]

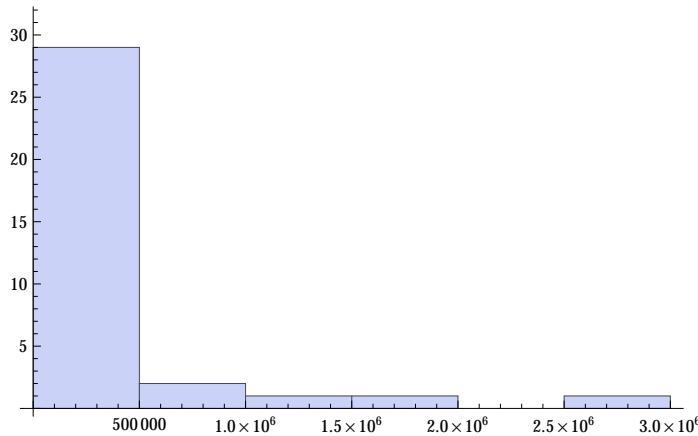
g[n]

```

n	g[n]
0	1
5	1
10	1
15	1
20	1
25	1000
33	40000

```
Table[f[n], {n, 0, 11}] // ColumnForm
```

```
Table[g[n], {n, 11, 44}]
```

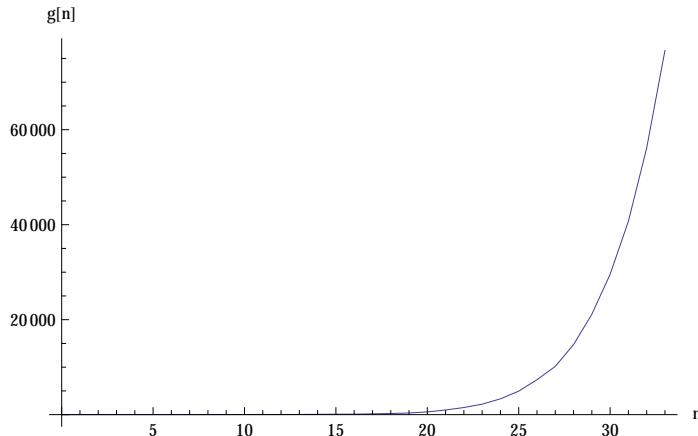


```
Clear[f]
Clear[g]
SKStep[exp_] := exp /. {s[x_][y_][z_] → x[z][y[z]], k[x_][y_] → x, i[x_] → x,
t[x_][y_] t[u_][v_] → x[u] y[v], W[x_][y_] → x[y][y]}
```

```
f[n_] := Nest[SKStep, s[s[s]] W[s][s][s], n]
```

```
g[n_] := LeafCount[f[n]]
```

```
ListLinePlot[Table[{n, g[n]}, {n, 0, 33}], PlotRange → All, AxesLabel → {"n", "g[n]"}]
```



```
Clear[f]
Clear[g]
```

```
SKStep[exp_] := exp /. {s[x_][y_][z_] → x[z][y[z]], k[x_][y_] → x, i[x_] → x,
t[x_][y_] t[u_][v_] → x[u] y[v], W[x_][y_] → x[y][y]}
```

```
f[n_] := Nest[SKStep, s[s[s[i][k[u]]][k[y]][k[v]]], n]
```



```
Table[f[n], {n, 0, 11}] // ColumnForm
```

```

filter = {
    s[k[s]] [k] → A,
    s[s[k[s]] s[k[k]]] → B,
    s[k[s[s[k][k]]]] [k] → C,
    s[s[s[s]]] → D,
    s[k[k]] → H}

```

```
Table[f[n], {n, 0, 11}] /. filter // ColumnForm
```

CL versus morphoCA systems

Initialization Code for ruleSets

StaticMorphoRule Set

DynamicMorphoRule Set

Indicational Rule Set

Filters

```

filter = {
    s[k[s]][k] → A,
    s[s[k[s]] s[k[k]]] → B,
    s[k[s[s[k][k]]][k]] → C,
    s[s[s[s]]] → D,
    s[k[k]] → H}

filter = {
    s[s[s[s][s]]][s][s][k] → C,
    k[s[s[s]]][k] → B,
    s[s[s[s]]][s[s[s]]] → A,
    k[s[s[s]]] → F,
    s[s[s]] → D,
    s[k][k[k]] → G,
    s[k][k[k]] → V
};

filter = {
    s[s][k][k[s[s[s][k]]][k]] → a,
    s[k[s[s[s][k]]][k]][k[k[s[s[s][k]]][k]]] → b,
    k[s[s[s][k]]][k] → c,
    s[s[s][k]] → d,
    k[s[s][k]] → o,
    s[s][k][s[s][k]] → p,
    s[s][k] → q,

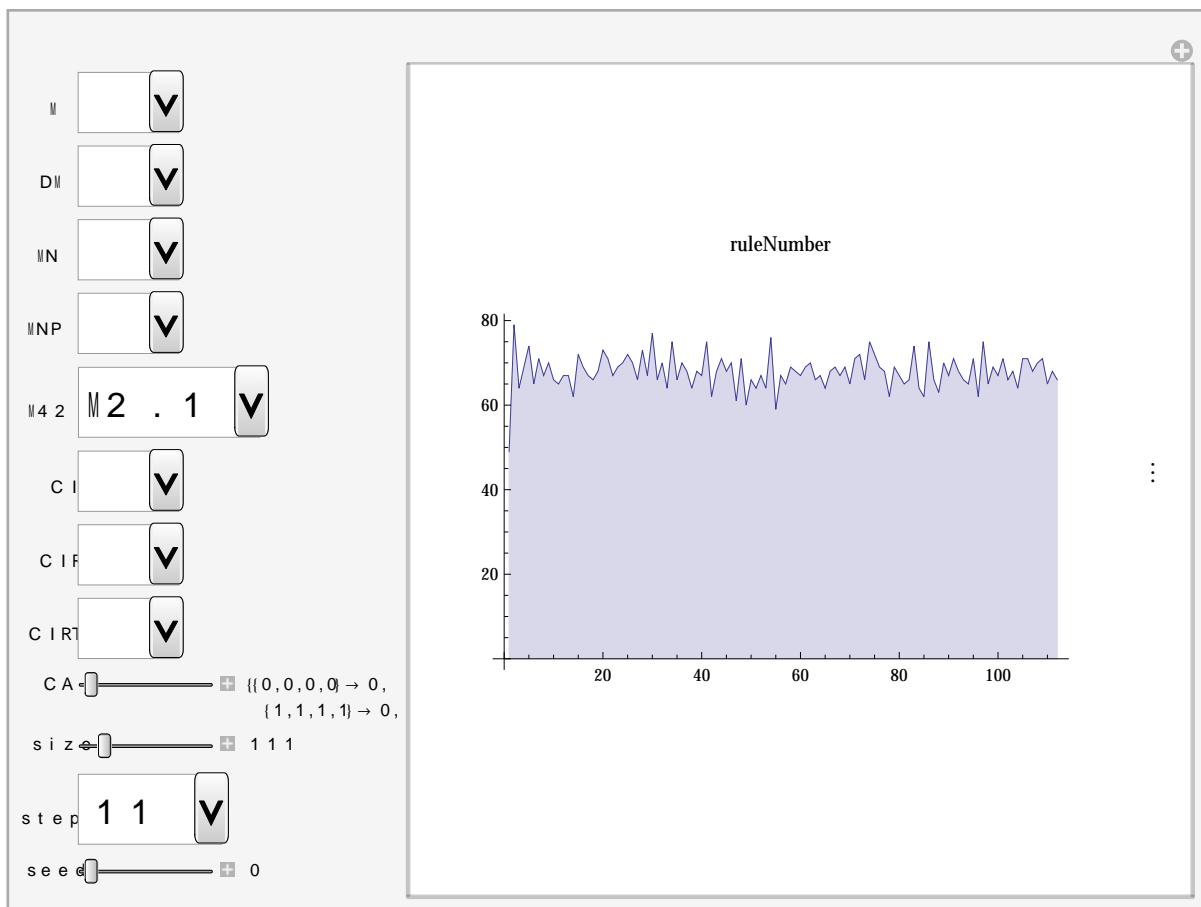
    s[s][k][s[s][k]][k[s[s][k]][s[s][k]]] → A ,
    s[s[s][k]][k[s[s][k]]][s[s][k]] → B ,
    i[s[i]] → C,
    s[i] → F,
    s[k][i] → D,
    s[i][i] → Q,
    i[s[i][i]] → V,
    s[i[s[i][s[k][i]]]] → W,
    s[s[i][s[k][i]]] → X,
    s[s[k[s[i][k]]]] → G

};

```

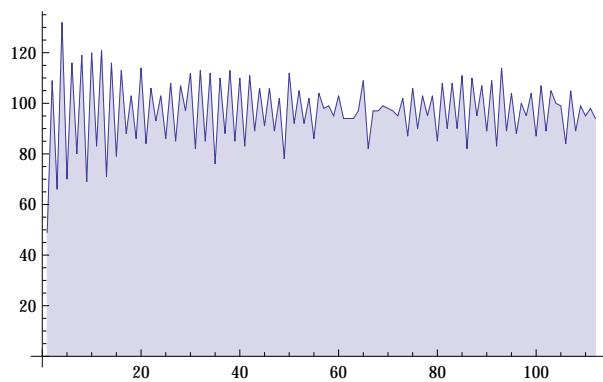
ListLinePlots of morphoCA rules

ListLinePlot Box



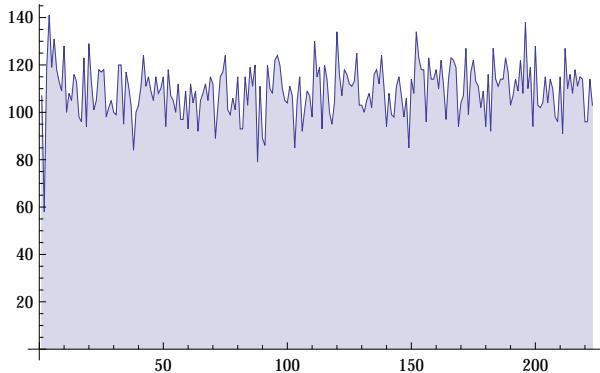
ruleMN15

CA Dynamics



ruleMN[{1, 7, 3, 13, 10, 15}] -> "MN28"

CA Dynamics



[Table of ListLinePlots of ruleDM](#)

[Table of ListLinePlots of ruleCI](#)

Arbitrary Genealogies

by Jaime Rangel-Mondragón

Method to obtain the genealogy of arbitrary birds from birds S, K and I by computing α -free terms.

[Genealogies](#)

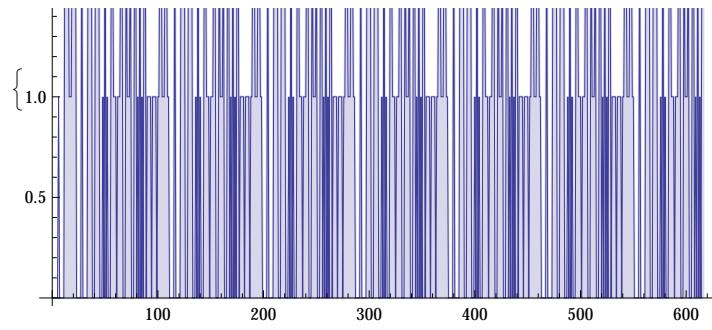
[Derivations](#)

[NKS Programs \(p. 896\)](#)

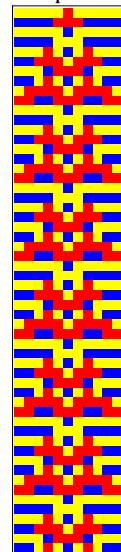
Comparisons: CL vs. CA

Dynamic Kaleidoscope: Iconography of the dynamics of classical, morphic and indicational CAs



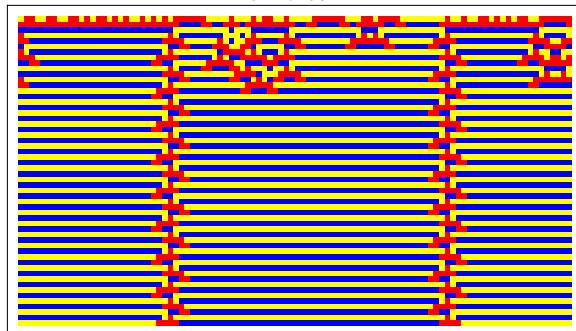


morphoCA

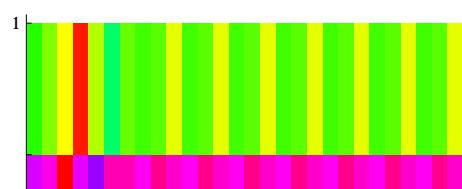


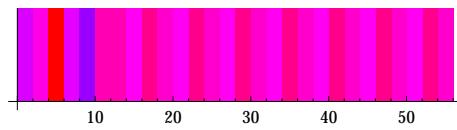
MNP	<input type="checkbox"/>
M 4 2	<input type="checkbox"/>
C I	<input type="checkbox"/>
C I R:	<input checked="" type="checkbox"/>
C I R T	<input type="checkbox"/>
L i s	<input checked="" type="checkbox"/>
m C A	<input checked="" type="checkbox"/>
S t y	<input checked="" type="checkbox"/>
Trans	<input checked="" type="checkbox"/>
s i z	<input checked="" type="checkbox"/>

CA random

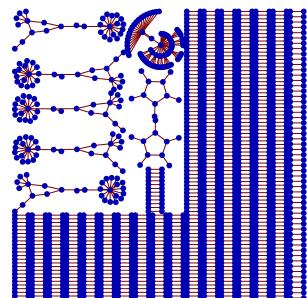


CA dynamics wavelet

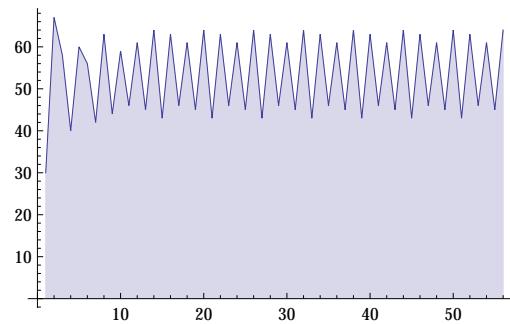




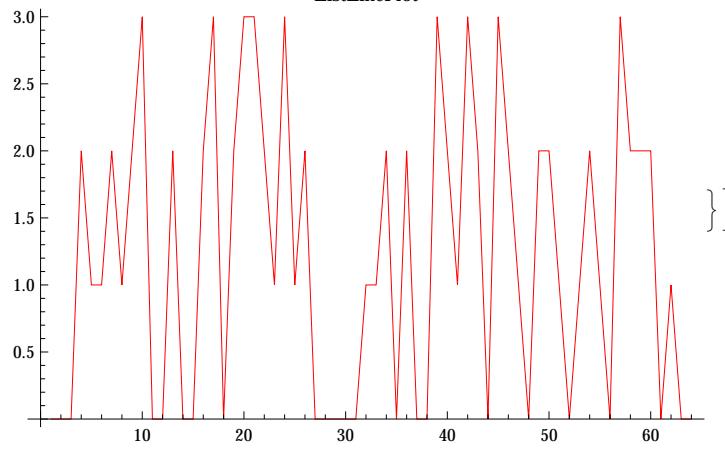
Transitions



caDynamics

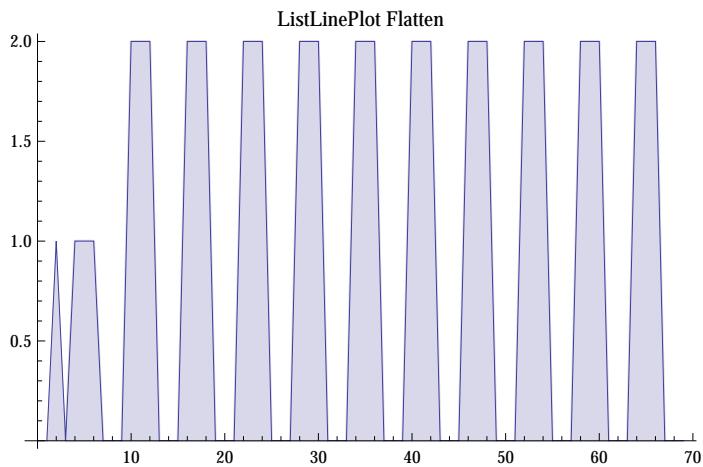


ListLinePlot





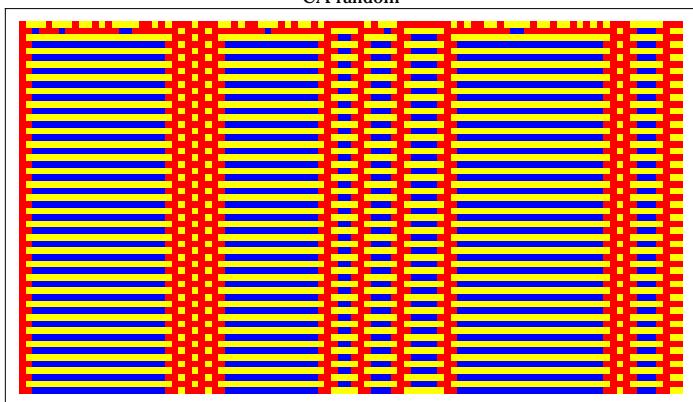
<http://iconicmath.com/logic/lawsofform/>



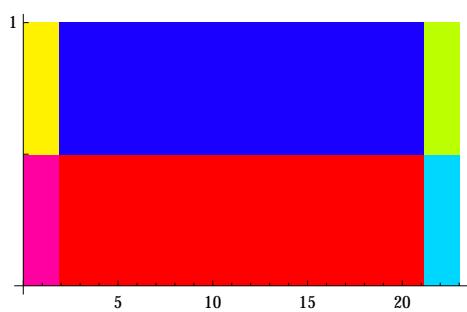
morphoCA

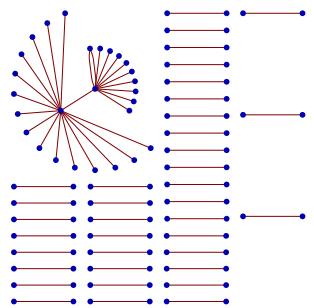
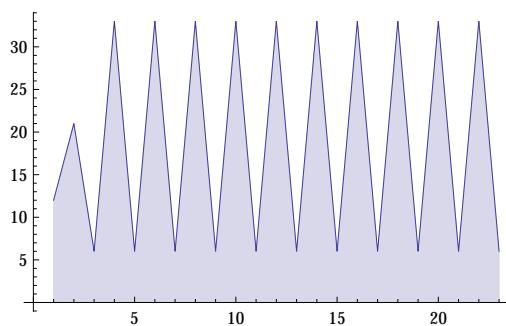


CA random



CA dynamics wavelet



Transitions**caDynamics****ListLinePlot**