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Günther's Negation Cycles and Morphic Palidromes

Applications of morphic palindromes to the study of Günther’s negative language

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Abstract

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It is proposed that a morphogrammatic understanding of technology is uncovering a level deeper than polycontexturality and is connecting numbers and concepts not just with the will and its praxeological actions but with ‘labour’ (Arbeit) in the sense of Marx’s Grundrisse ("Arbeit als absolute Armut”.) Palindromic cycles are offering a deeper access for the definition of negative languages than the meontic cycles. Morphogrammatics is opening up languages, i.e. writting systems of creativity and labour.

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Gunther’s Negation Cycles and Morphic Palindromes

Applications of morphic palindromes to the study of Gunther’s negative language

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Gunther’s concept of technology is based on a ‘melting’ of number and notion, Begriff und Zahl, in a polycontextural setting. A key to its study is established by the negation-cycles of polycontextural (meontic) logics that are establishing a negative language.

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http://www.wolfram.com/cdf-player/

(work in progress, vers. 0.4, Oct. 2013)

1. Gunther’s Negative Language

1.1. Cycles as palindromes

From a polycontextural point of view, complexity, i.e. polycontexturality is first. Simplicity is a late product of simplification. Domains, contextures or fields are at first considered as discontextural. There is at first no common ground or umbrella to collect the differences into one family and home of sameness or identity.

List of oppositions

hierarchy / heterarchy,
positive / negative,

polycontexturality / morphogrammatics,
negation cycles / palindromes,
Hamilton / journey,
negations / U, O, K, I
combinatorics / number of palindromes
symmetry / asymmetry
permutations / braids

Palindromicity of the names of the negations / permutation of the values of negations.

Complementarity of negation cycles and morphic palindromes.

> *"We begin this time with another Hamilton cycle which exhibits an easily understandable rhythm in the distribution of the negations. "*
> *p = N1-2-3-1-3-2-1-3-1-2-3-1-3-2-1-3-1-2- 3-1-3-2 p (p. 44)*
>
> *"Each individual circle represents a 'word' in a technical dictionary of negative language that does not describe existing – already created – Being in a positive language; rather, each of the 3744 cycles represents a specific instruction, how something can be performed, how something can be constructed."*
>
> *Kreisumfang / circumference : 8 10 12 14 16 18 20 22 24*
> *Anzahl / number : 24 72 264 456 708 920 912 336 44*

http://www.vordenker.de/ggphilosophy/gg_identity-neg-language_billing.pdf

Palindromes with rhythmic repetitions.

1.1.1. Negative languages in action

Negative language in action:
Gotthard Gunther: Philosophical speculations

Berhard Mitterauer, Gerhard Thomas: Computer model, patents, Volitronics

Gerhard Thomas, On permutographs
http://dml.cz/bitstream/handle/10338.dmlcz/701282/WSAA_10-1982-1_28.pdf

Mitterauer: Therapy of decision conflicts,
Der Formalismus der Negativsprache
Therapie von Entscheidungskonflikten
2007, pp 76-80

http://link.springer.com/content/pdf/10.1007%2F978-3-211-71067-8_13.pdf

Patents
https://www.google.com/patents/US4783741

Dirk Baecker: Sociology and LoF, Negativsprachen aus soziologischer Sicht

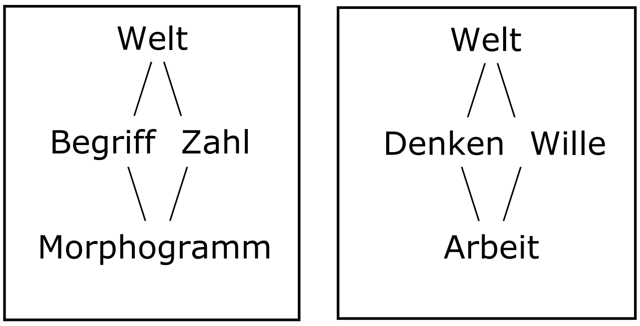
Alfred Toth,
http://www.mathematical-semiotics.com/pdf/Zyklpraesem.pdf,
http://ubdocs.uni-klu.ac.at/open/voll/tewi/AC06761907.pdf
15. Die semiotische Negativsprache

Eberhard von Goldammer:
Vom Subjekt zum Projekt oder vom projekt zur subjektivität !
ftp://www.inf.fh-dortmund.de/pub/professors/vongoldammer/publications/-subjekt-projekt-subjektivitaet.pdf

A funny application by Kaehr:
http://www.thinkartlab.com/pkl/media/Doppelte Buchfuehrung/Doppelte Buchfuehrung.pdf

1.1.2. Polycontexturality of negative languages

Polycontexturality



The presupposition of those approaches was and still is the acceptance of “frozen” values that are easily to be involved in a treatement by mathematical permutations.

Neither the proemiality of the values nor their “braided” permutations as they are common in bi-category theory had been recognized.

According to the newly proposed theory of morphospheres, the permutational negative language is operating on a surface structure while the morphic palindromes are involved with the deep-structure of morphogrammatics.

The surface-structure of negative languages might be highly complex and surpassing everything a positive language is capable to thematize or invoke but it remains nevertheless on the value-level of meontics, covered by polycontexturality, and is not prepared to address the processuality of morphogrammatic structurations on the deep-level of inscription.

Obviously, terms like *deep* and *surface* structures are lend from other disciplines and are applied here as nothing more than "operative" metaphors.

The negative language proposed by Gunther and studied by his followers is covering the surface-structure of semiotic polysemy as it occurs in polycontextural systems.

1.1.3. Auto-cyclicity of negative languages

Auto-cyclicity

There is a surprisingly observation too.
Because negation cycles are based on negations and morphograms are negation-invariant kenogrammatic patterns it turns out that the whole cycle is based on one morphogram, and is therefore morphogrammatically auto-cyclic.

Negation cycles are zooming into the structure of the permutation of the value-theoretical interpretation of a single morphogram.

How can the attempt be saved on a morphogrammatic level if a negation cycle is morphogrammatically auto-referential and is therfore not telling much about its own cyclicity?

Gunther has prepared an answer himself. Albeit never being used, the idea of emanation of morphogrammatic systems gives the clue.

The cycle that is uncovered is the cycle between the levels of emanative disremption: from zero differentiation to full differentiation, and back.

Other cycles are possible too. A mix of *evolutive* and *emanative* morphic cycles is covering an interesting expansion of the idea of cyclic structures on the morphogrammatic level.

1.1.4. Philosophical interpretation: Wille vs. Arbeitskraft

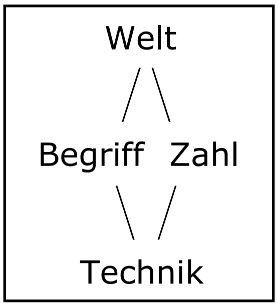
Arbeit, Handlung, Wille, Denken

Labour is not just action.

Gunther writes:

> *"Wir haben hier einen enorm wichtigen Punkt einer Handlungstheorie berührt, in der Begriff und Zahl verschmolzen werden und in dieser Verschmelzung zur Inkarnation des Willens in der*

> *Technik führen.*
>
> ***"****Die Technik ist die einzige historische Gestalt, in der das Wollen sich eine allgemein verbindliche Gestalt geben kann."*



The unknown form A a a A

The German politician Joachim Paul aka Nick Haflinger from the Pirate Party writes:

> *"Ja, ich liebe Marx, und zwar Groucho Marx und seine Brüder."*
> http://www.vordenker.de/blog/?p=753

I'm not a Groucho fan. I love Karl Marx.

Groucho Marx is, as people believe, well known.

Unfortunately, Karl Marx is more or less not known to the students and politicians. Especially in the USA dominated world. What is known are the bourgeois cliches about Marx and, obviously, about Marxism.

Joachim Paul also knows Gotthard Gunther quite well.

This paper will open up some insights and perspectives of thinking beyond classical cliches with the help of a nearly unknow sentence, if it is still a sentence at all, I discovered in the early 1970s in the Grundrisse of Karl Marx on page 203.

The non-apophantic text of Marx reads:

> *"Arbeit als absolute Armut, Armut nicht als Mangel sondern Ermöglichung jeglichen Reichtums."* Karl Marx

This inscription alone offers a kind of a clue to the understanding of Marx as a non-Marxist thinker. It pushes him beyond any Hegelian attempts of understanding society. Like all the bourgeois socologists and economists up to such guys like Peter Krugman.

Gotthard Gunther tried it with his negative language.

Marx was obviously shocked and mesmerized by his inscription too. Therefore, he has given the potential reader some paraphrasing explanations.

It might all be different. But also the paraphrases are un-neccessary at all, the didactic help wasn't recognized and accepted by his followers.

Today, Google gives only a very few hits to the 'magic' sentence.

The paraphrase goes on to explain:

> *"Die Arbeit nicht als Gegenstand, sondern als Tätigkeit, nicht als selbst Wert, sondern als lebendige Quelle des Wertes. Der allgemeine Reichtum, gegenüber dem Kapital, worin er gegenständlich, als Wirklichkeit existiert, als allgemeine Möglichkeit desselben, die sich in der Aktion als solche bewährt."*
>
> K. Marx: Grundrisse der Kritik der politischen Õkonomie. Frankfurt/Wien 1939, S.203

But Marx has also given a much more interesting hint for the very understanding of his 'non-sentence':

> *"Es widerspricht sich also in keiner Weise, oder vielmehr der in jeder Weise sich widersprechende Satz, daß die Arbeit einerseits die absolute Armut als Gegenstand, andererseits die allgemeine Möglichkeit des Reichtums als Subjekt und als Tätigkeit ist, bedingen sich wechselseitig ..."* Marx 1953, S. 203

Again, in a more Hegelian turn, Marx puts it into other words.

> *"Die Arbeit als die absolute Armut: die Armut, nicht als Mangel, sondern als völliges Ausschließen des gegenständlichen Reichtums. Oder auch als der existierende Nicht-Wert und daher rein gegenständliche Gebrauchswert, ohne Vermittlung existierend, kann diese Gegenständlichkeit nur eine nicht von der Person getrennte: nur eine mit ihrer unmittelbaren Leiblichkeit zusammenfallende sein."* (Marx 1953, S. 203.)

As we see, one page from Marx' s Grundrisse says it all.

Unfortunately, with all those explanations, reflections and didactical paraphrases by Marx, the magic is veiled and obscured in favor of a possible academic reader.

The magic of the inscription is forgotten in the delirium of explanations.

The magic is extremely simple: *"Arbeit als absolute Armut"*, a non-sentence, is of the form “AaaA".

The quadrantation of the phonological apophansis.

For non-linguists I remark, a real sentence is of the basic form: “A is B".

The AaaA script has no onto-logical copula “is”. There is no “being” involved, and no ‘meaning’ to capitalize Marx’s formula.

The sentence is in a strict sense not translatable, thus it isn’t a sentence in linguistic terms. In any translation of the content (surface structure), the form it shows evades.

The form could be called a *little chiasm*. In contrast to the big chiasms of the whole Marxian economy: ABBA, ABAB, etcetera.

I have to admit that nearly nobody followed my enthusiasm when I presented my insights in a seminar I organized just for that discovery at the Free University WestBerlin in the early 1970s.

My thesis was and still is, that such a non-sentence is understandable only with the help of Gunther's kenogrammatics and Jacques Derrida’s grammatology.

Gunther:

> *"Since the classic theory of rationality is indissolubly linked with the concept of value, first of all one has to show that the whole "value issue" covers the body of logic like a thin coat of paint.*
>
> *Scrape the paint off and you will discover an unsuspected system of structural forms and relations suggesting methods of thinking which surpass immeasurably all classic theories. This was the purpose of my paper "Time, Timeless Logic and Self-Referential Systems." The trans-classic order which we discover beyond the classic theory of logic was called "kenogrammatic structure."*
>
> http://www.vordenker.de/ggphilosophy/gg_logic_structure.pdf

Derrida:

> *"Je parlerai, donc, d’une lettre.*
> *De la première, s’il faut en croire l’alphabet et la plupart des spéculations qui s’y sont aventurées.*
> *Je parlerai donc de la lettre* ***a****, de cette lettre première qu’il a pu paraître nécessaire d’introduire, ici ou là, dans l’écriture du mot*

différence; et cela dans le cours d'une écriture sur l'écriture, d'une écriture dans l'écriture aussi dont les différents trajets se trouvent donc tous passer, en certains points très déterminés, par une sorte de grosse faute d'orthographe, par ce manquement à l'orthodoxie réglant une écriture, à la loi réglant l'écrit et le contenant en sa bienséance.

http://www.jacquesderrida.com.ar/frances/differance.htm

Politics: Beyond left and right:

"Déconstruire, c'est dépasser toutes les oppositions conceptuelles rigides (masculin/féminin, nature/culture, sujet/objet, sensible/intelligible, passé/présent, etc.) et ne pas traiter les concepts comme s'ils étaient différents les uns des autres."

http://www.signosemio.com/derrida/deconstruction-et-differance.asp

Gunther didn't follow his own advice much further, after he successfully "*scraped the paint of",* and went on, mainly, with his meontic theory of negative languages. He left, officially, the whole body and burden of kenogrammatics to my enjoyment and responsability.

This paper is dedicated to the enterprise to connect the *kenogrammatic* approach with the *meontic* approach of negative languages, concerning negation cycles, and their interpretation for a theory of action (Wille, Handlungstheorie).

http://www.isf-muenchen.de/pdf/isf-archiv/1980-bechtle-betrieb-strategie.pdf

http://publikationen.ub.uni-frankfurt.de/opus4/files/1139/Theorie.pdf by K Lichtblau

1.2. Palindromes as Cycles

Classic palindromes

Palindromes, certainly, are cyclic. They read forward and backward the same.
This so obvious that the conditions are not mentioned necessarily.
Obviosly, the reading means that the conditions of semiotic reading are fully accepted, i.e. two sign sequences are equal if the are equal for all atomic elements at the corresponding positions in the linear

sequence.

Therefore classical palindromes defined by a single comparison of their elements.

A natural generalization of this approach is introduced with a normalization function that translates palindromes into a standard normal form. Hence, a palindrome, like [331122] gets normalized into its normal standard form [112233].

Classical palindrome are symmetric even on their basic level of definition.

Morphic palindromes

Morphograms are not defined by any identity rule of logic or semiotics, neither directly nor indirectly.

Nevertheless there are interesting structures of asymmetric palindromes to discover and to study.

Also morphic palindromes, i.e. morphograms of symmetric and asymmetric patterns, are not symmetric in respect of their concrete patterns, there is a fundamental symmetry of morphic palindromes on a meta-level to dedect: morphic palindromes are not semiotially symmetric but *bi-symmetric* in respect of their ϵ/ν-structure.

Cycles as palindromes.
Are negation cycles even balanced classic palindromes?
Not all negation cycles are palindromic.

Self-cycles in palindromes	palindromic negation cycle
[1,1,1,2,1,2,1,2,2,2]	[1,2,1,2,1,2]

p ≡ N1.2.3.2.3.2.1.2.1. 2.3.2.3.2.1.2.1.2.3.2.3.2.1.2 p.

```
- ispalindrome[1,2,3,2,3,2,1,2,1,2,3,2,3,2,1,2,1,2,3,2,3,2,1,2];
val it = false : bool
```

1.3. Morphic even palindromes

The table collects the results of the prolongation of palindromes of length 4 to palindromes of length 6, e.g rule1([1,2,2,3]) $\Longrightarrow$ [1,1,2,2,3,3].

[1,2,2,3]

[1, 1, 2, 2, 3, 3]	rule1	[1, 1, 2, 2, 3, 3]
[3, 1, 2, 2, 3, 1]]	rule2	[1, 2, 3, 3, 1, 2]
[4, 1, 2, 2, 3, 5]	rule4	[1, 2, 3, 3, 4, 5]
[2, 1, 2, 2, 3, 2]	rule1	[1, 2, 1, 1, 3, 1]

Representation of morphograms:
The single morphogram [1,2,3,4] has 4! = 24 representation on a symbolic or meontic level of permutations.

There are just 7 palindromes of length 4 on the level of morphograms.

genPalindrome(4):
[1,1,1,1]
[1,1,2,2]
[2,1,1,2]
[2,1,1,3]
[2,1,2,1]
[3,1,2,3]
[3,1,2,4]

1.4. Iterability of morphic palindromes

1.4.1. Modi of iterability: inversion, repetition an accretion

Palindromes are traditionally defined by *reversion* and sometimes, for canonical palindromes, by *repetition*.

The concept of morphic palindromes is introducing a new category of iterability for palindromes. Additionally to the category of *reversion* and *repetition* the category of *accretion* is introtruced.

It turns out that the new category of accretion as a mode of iterbility for palindromes is of dominant relevance. Most morphic palidnromes are build by accretion and not by reversion.

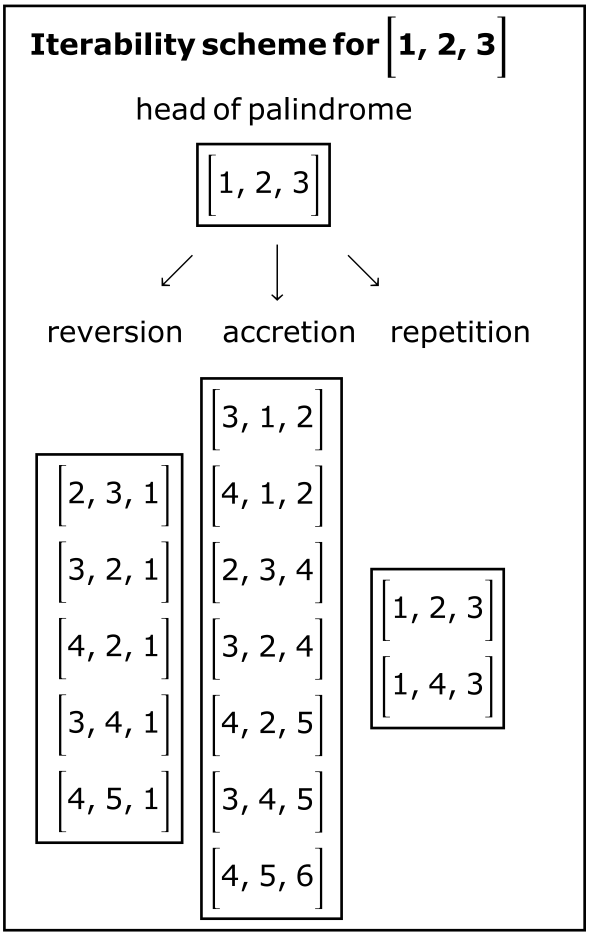


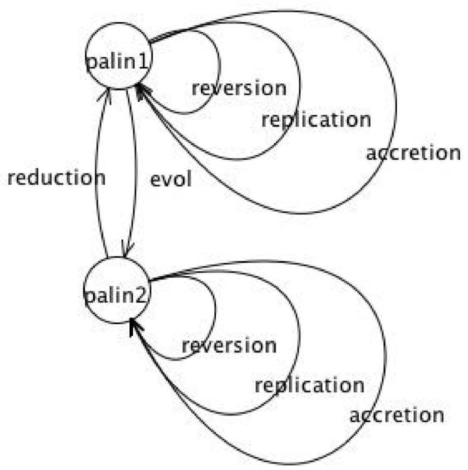
Table view of the palindrome scheme

head of palindrome[1, 2, 3] | 2 | 3 | 4 | All

```
reversion:

  [2,3,1]
  [3,2,1]
  [4,2,1]
  [3,4,1]
  [4,5,1]
```

1.4.2. Iterability as dissemination



Example : **Tcontexture 6**
Tcontexture 6;
–length it;
val it = 203 : int

List.filter **ispalindrome** (Tcontexture 6);
val it =

1. [[1,1,1,1,1,1],[1,1,1,2,2,2],
:2

2. [1,1,2,1,2,2],[1,1,2,2,1,1],[1,1,2,3,4,4],[1,1,2,2,3,3],[1,1,2,3,1,1],
:5
3. [1,2,1,2,1,2],[1,2,1,1,2,1],[1,2,1,3,2,3],[1,2,1,1,3,1],[1,2,1,3,4,3],[1,2,1,3,4,3],
:6
4. [1,2,2,1,1,2],[1,2,2,2,2,1],[1,2,2,3,3,1],[1,2,2,2,2,3],[1,2,2,3,3,4],
:5
5.
[1,2,3,1,2,3],[1,2,3,2,3,1],[1,2,3,3,1,2],[1,2,3,3,2,1],[1,2,3,4,1,2],[1,2,3,4,2,1],[1,2,3,1,4,3] :14

[1,2,3,3,4,1],[1,2,3,2,3,4],[1,2,3,3,2,4],[1,2,3,4,5,1],[1,2,3,4,2,5],[1,2,3,3,4,5],[1,2,3,4,5,6]]

Tcontexture (6) opens up 5 fields of palindromes of length 6.

Number of fields for palindromes of length n : $\sum_{k=1}^{m} Sn(k, m), \; m = n/2.$

There are 5 palindromic fields for 31
palindromes of 203 morphograms from Tcontexture 6 :

$$\sum_{k=1}^{3} Sn(k, 3) = 1 + 3 + 1 = 5.$$

Those fields are prepaired for further classifications and partitions by first and second order refinements.

1.5. Palindromes as e/v-Structures

To avoid interpretational misunderstandings with morphic palindromes written as morphic lists, the e/v-abstraction is opening up a new field of palindromic research.

fun **ENpalindrome** n = map ENstructureEN(List.filter ispalindrome(T-contexture n));

- **ENpalindrome** 6;

2. Combinatorial aspects of Gunther's negative languages

2.1. Gunther's approaches to permutations

Gunther has spend a lot of time to develop the negation cycle of his negative language, discovering Hamilton paths, and many other combinatorial features.

Gunther stubbornly insisted to do this work manually. Only after he succeeded some reasonable steps in the formulation of the field, he accepted computer-assisted support, mainly by Alex Andrew and Gerhard Thomas.

These results and the corresponding speculations about '*negative words'* of a new dictionary, not of meaningful words but of action advices, he called it a dictionary for the *"Code für Handlungsvollzüge"*, was the essence that got some reception and recognition in the literature.

What is missing is the fact that this hard work of manually achieved combinatorics, a kind of paper-pencil permutative meditations, was just the condition for his radical departure/rupture from both: the arithmetical and the notional interpretation of the endeavor.

The motivation for this work was the attempt to mediated number and logos (notion), *Zahl und Begriff.*

Without surprise what was achieved by such a refutation of number and notion was not acceptable for both sides, the philosophical and the mathematical. Simply, because a mechanism of the mediation of number and logos, can not be written in a strict sense, neither in the language of mathematics (and logic) nor in the language of philosophy.

But just this rejection of both, offered a chance to develop his idea of a scripture beyond logos and number.

Hence, the words and letters he used to characterize this intricate mechanism between numbers and notions are to read as neither words nor numbers.

The main results of Gunther's permutational "Negativsprache" and its post-philosophical interpretation is presented in the late oeuvre: "*Identität, Gegenidentität und Negativsprache".*

Accessible via:
http://www.vordenker.de/ggphilosophy/gg_identity-neg-lan-

guage_biling.pdf

This late oeuvre is still mesmerized by the measure of its new dictionary, and the complexity of its 'negative words'.

Only very late he succeeded to return to his main interests: the mediation of number and logos.

The new 'instruments' are in some sense also the oldest ones: Umtausch, Ordnung und Identität. Hence: the new and final 'vocabulary' is: O, U, I plus R (right) and L (left): U, O, I; L, R.

The distinctions of a direction in the permutation by left (L) and right (R) hints to an understanding of permutations that connects with knot theory and braids, and is in conflict with a purely permutative understanding of negations.

Integration von Zahl und Begriff

This vocabulary appeared only rudimentary in his last just mentioned published work.

Later texts, partly produced with some support by Bernhard Mitterauer, had been much more directly focused on the new 'mechanism'.

There are therefore many reasons to risk the statement: *Gunther's meontic negation systems are not based on permutations of values but on interchanges of moves in knot systems.*

This statement gets some support by the interpretative texts of Gunther himself.

The term *knot* of knot systems is applied as a mathematical metaphor and precision of terms like 'chiastic mediation' of bi-objects.

As a strategy I opt for the distinction of the philosophical *intention* and the formal mathematical *realizations* of Gunther's project.

The high ambitions for a new philosophical paradigm of thinking Gunther endorsed with his theory of *Negativesprache* (1974) are expressed and documented at:
http://www.vordenker.de/ggphilosophy/gg_janusgesicht.pdf

Language and complexity

One of the main motives to introduce the idea of a “Negativsprache” is Gunther’s critizism of non-supported philosophical langauge that fails miserably to design complex notional texts.

As examples, Hegel and Heidegger are mentioned.

On the other hand, there are interesting linguistic analysis of natural language constructs that don’t fit into classical logic and formal linguistics: The work of Alfred Toth, and newly, the studies of Barbara Sonnenhauser about: *“Subjectivity, logic and language: the case of double negation.”*

Both are applying Gunther’s negation systems, but are also concretizing the idea of a Ngativsprache.

https://www.academia.edu/4346870/Subjectivity_logic_and_language_the_case_of_double_negation

2.2. Hamilton paths vs. Heideggerian journeys

2.2.1. Preliminaries

This topic of a difference of path and journeys has been studied several times on different levels of formalizations and philosophical interpretations.

Here, I try to connect this difference of *path* and *journey*, or Hamilton and Heidegger experiences, with the difference of meontic or *polycontextural* repetitions (cycles) and *morphogrammatic* returns.

Paths are realized on a pre-defined map. Journeys are realizing their movements by moving (Machado, Heidegger) their ways into maps.

Hence, an application of negative languages in the sense of meontic constructions to societal systems means that the societal systems are perceived as given and the approach is lacking any means for an understanding of transformations and self-modifying events.

Gunther's philosophical aim sounded decisively Heideggerian. But his mathematical approach with permutation systems doesn't pay the bill.

"Each individual circle represents a 'word' in a technical dictionary of negative language that does not describe existing – already created – Being in a positive language; rather, each of the 3744 cycles represents a specific instruction, how something can be performed, how something can be constructed." (Gunther)

Terms of a negative language are more choreographic than representative. They tell the performer how to perform in a performance and evoke its stage that didn't exist and wasn't realized before.

A Hamilton cycle in a landscape of 4 stations (calls) presumes, obviously, a permutation of 4 values of a pre-given set with the end constellation coinciding in identity with the start constellation. This is realized by the equality of the start and the end values. It might have been a nice trip but at the end all remained the same.

The idea, that coming back after the adventure has changed the situation at the departing point is an interpretation of the cyclic permutation that has now equivalent in the formalism.

This condition of equality of the start and the end of a cycle doesn't hold for morphogrammatic situations. A morphic palindrome finds home in a different constellation where the start and the end of the journey aren't equal anymore. Hence, it wasn't a trip from a pre-given place to another pre-given place but a journey. And certainly, there are special classes of classical palindromes too where the start and the end coincide. But they come as a small minority.

Hence, negation cycles are a very special case of palindromic writings. They read forwards and backwards the same where the same is reduced to identical values.

Furthermore, Gunther's negation systems are restricted to a *linear* order of their elements. Hence, cases like *star* patterns are excluded by definition.

The idea of a *negative* theory is not necessarily new. It comes in many attempts and flavors. Negative theology (Via Negativa) and the Negative Dialectics of Adorno are among the most prominent.

It isn't properly seen by the critics of the work of Jacques Derrida that deconstructive writing is not defined by 'negative' work.

It is understood that the main strategy of Gunther's thinking is to deconstruct radically the principle of identity in all its forms and appearances.

This strategy is followed in this paper with the aim to deconstruct the presumptions of the theory of negative language as far as complicities with the identity principle are detected.

Gunther is criticizing the classical semantic concept of double negation,
$\neg(\neg(p)) = p$, on the base of a multitude of negations in a meontic system, e.g. $\neg_1(\neg_2(p)) \neq \neg_2(\neg_1(p))$.

But nevertheless, the presupposition of a double negation for $i \in N$ holds:
$\neg_i(\neg_i(p)) = p$.

A deconstruction of this presupposition might be achieved with an interpretation of negations as *braids* in a dynamic setting.

Therefore, this paper takes the chance to involve some distinctions to deconstruct the identity heritage of the Negativsprache:

firstly, the distinction of *meontic* and *morphogrammatic* accounts,
secondly, the distinction of *permutative* and *dynamic* (braided) negation systems.

Questions

What does it mean for the philosophical concept of a negative language, based on permutations, if its basic structure is determined by *'truncated octahedrons' (*permutohedron) and their generalizations?

In comparison to dyads (of ontology, logic, semiotics, cosmology, etc.), the structure of a permutohedron as a basic concept for reasoning might have some definitive advantages to moel conceptual and procedural complexity.

Nevertheless, octahedrons are not only stable but also extremely symmetric configurations.

> *"The truncated octahedron is one of the 13* ***Archimedean solids****."*
> *"It has 14 faces, 36 edges, and 24 vertices. Since each of its faces has point symmetry the truncated octahedron is a zonohedron."*

http://demonstrations.wolfram.com/ArchimedeanSolids/

Furthermore:

> *"In geometry an* ***Archimedean solid*** *is a highly symmetric, semi-regular convex polyhedron composed of two or more types of regular polygons meeting in identical vertices. They are distinct from the Platonic solids, which are composed of only one type of polygon meeting in identical vertices, and from the Johnson solids, whose regular polygonal faces do not meet in identical vertices."* Wikipedia

Problems with symmetry:

Gunther is criticizing social sciences to not to use in their conceptualizations more complex logical tools than the usual tools based on binary logic, like systems and information theory.

> *"Angesichts dieses Überreichtums an Reflexionsbeziehungen, der sich aus der Tatsache ergibt, dass in der klassischen Logik schon die doppelte Verneinung wieder das Positive ergibt, wird es deutlich, dass hier die Intuition völlig versagt und dass das Denken sich auf die Unterstützung durch den Komputer verlassen muss. Es wird aber auch deutlich, mit welcher begrifflichen Vagheit heute in den sogenannten Kultur- und Sozialwissenschaften gearbeitet wird, wo man auch nicht die entfernteste Vorstellung davon hat, was getan werden müsste, um diese Disziplinen in den*

Rang wirklicher Wissenschaften zu heben; ein Titel, den sie sich heute nur anmaßen." Gunther

Despite the high numbers of permutations there are some limits with this approach. As the results on *permutographs* show, there is a fundamental *symmetry* involved in negation systems formalized as permutographs on all level of complexity. They are all composed just by symmetric subsystems of length 4 and length 6 for the linear and of length 6 for the tree structure.

Also Gunther's approach surpasses significantly the limits of classical conceptualizations, the serious question concerning multi-negational systems for social studies has to deal with the facts and possible limits of this *meta-theoretical symmetry* for negation systems.

In contrast, complexity isn't restricted on the level of morphogrammatic structurations to linear systems.

For a *star* approach to graphs, the theory of Trivalent or *Cubic Graphs* might be of interest.

"A trivalent graph is a graph which has three edges meeting at each vertex."

Zsuzsanna Dancso, On a universal finite type invariant of knotted trivalent
graphs (2011)

http://www.math.toronto.edu/zsuzsi/research/thesis/thesis.pdf

Gunther's concept of negative languages has some restrictions:
• linear order of elements vs. star order,
pre-given value-sets vs. evoking situations,
• permutations as unary operations, and not as binary braids,
• value-systems vs. morphogrammatic systems,
• path in a regular graph vs. journeys in palindromic configurations.

2.2.2. Escapes from symmetry?

One way to escape the immanent symmetry of negation systems was motivated by W. R. Beyer's question to Gunther: *How to mediate circles with spirals?*

http://www.vordenker.de/ggphilosophy/gg_janusgesicht.pdf http://www.vordenker.de/vgo/annotations-negative-language.pdf

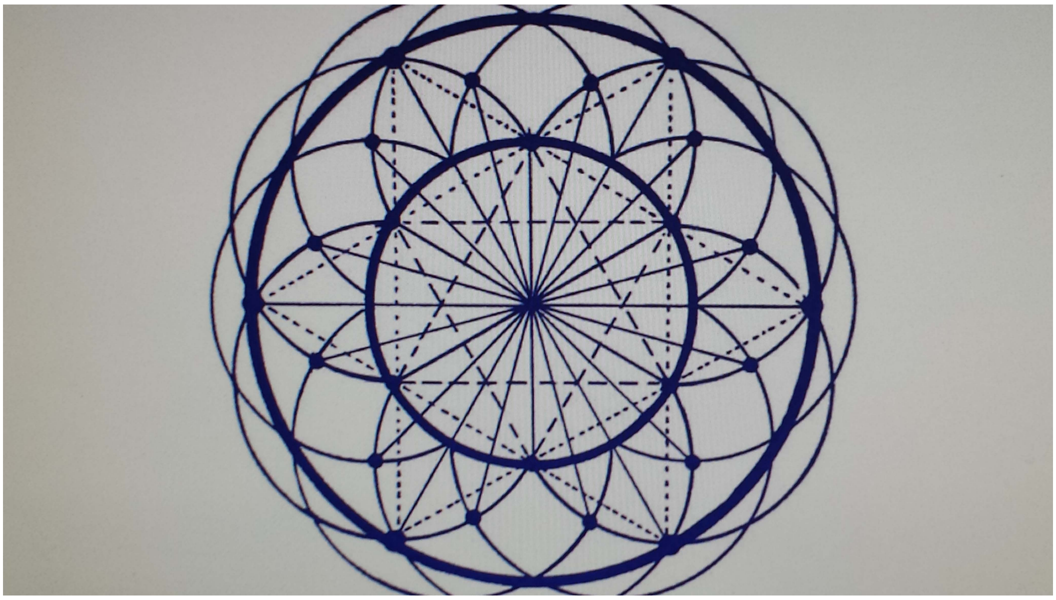
Accretive negation, i.e. Gunther's interpretation and formalization of Hegel's *'zweite Negation'* leads to a successive augmentation of the complexity of the negational system.

Hence, on each level of the negational complexity, symmetry holds. The symmetry is quite simple an regular, and could be called 'octahedral'.

The move between the symmetric negational systems is not symmetric but asymmetric.

Both together are defining an interplay between hierarchical and heterarchical structurations.

In other words, there is an operator necessary to move from a m-valued to a m+1-valued negation system. This operator was seen by Gunther as Hegel's *second* negation*.* The second negation is not a single additional negation but a 'pool' of possible new accretive negations to evoke the negativity of meontics.



http://www.vordenker.de/ggphilosophy/gg_janusgesicht.pdf

The idea of accretive negations in Gunther's **"Identität, Gegenidentität und Negativsprache"**

> *"accretive power of negating systems"*,
> *"because the accretive power of negating systems inexhaustible, counter-*

> *identity renews itself again and again in structural forms of higher and higher accretivity."* (Gunther, p. 9)
>
> *"If one adheres to the diairetic principle, the further symmetrical steps of the pyramid structure follow automatically. However if one distinguishes between iterative and accretive – or in Hegelian terms, between first and second – negation, the diairetic systematic is now interrupted by the cyclical or order principle coming to the fore."* (ibd., p. 10)

http://www.vordenker.de/ggphilosophy/gg_identity-neg-language_biling.pdf

Barbara Sonnenhauser, *Subjectivity, logic and language: the case of double negation*

> "In transclassical logic, however, this active role of the subject is incorporated by the second type of negation, that within subjectivity, which has the *"capacity of accretion"* (Günther2005: 14). Accretive negation may also be iterated, but it is more *"than just the mere repetition of an identical which always stays the same. Something new is produced as well"* (Günther 2005: 9)."

https://www.academia.edu/4346870/Subjectivity_logic_and_language_the_case_of_double_negation

More semiotics at:

https://www.academia.edu/3882654/Subjectivity_in_philosophy_and_linguistics

The whole subject/object drama modeled within Peircian semiotics is well elaborated by Alfred Toth, and published at: http://www.mathematical-semiotics.com

2.2.3. *Mathematica* assisted reconstructions

Mathematica assisted reconstructions

My own contribution to combinatorics in this paper is at first very much restricted to the aim to learn how to use the program *Mathematica* for a reconstruction of some of Gunther's combinatorial concepts and results.

Therefore, the presentation of Gunther's achievements are not complete at all. The reader will find a lot of additional information in the literature (G. Thomas, B. Mitterauer, R. Kaehr).

A well elaborated presentation of some of Gunther's results in "*Identität, Gegenidentität und Negativsprache*" is given with the on-line publication of Gunther's original paper by von Goldammer, with an English translation and properly set tables for permutations, Hamiltonian paths, etc.

Unfortunately, the *Vordenker* edition is (more or less) protected and I don't have the nerves to change this annoying situation. Hence, the reader might have an own look to the tables at:

http://www.vordenker.de/ggphilosophy/gg_identity-neg-language_biling.pdf

An early summary of the combinatorics of Gunther's permutation systems that is not restricted to a linear order is available at:

> Gerhard Thomas, On permutographs
> http://dml.cz/bitstream/handle/10338.dmlcz/701282/WSAA_10-1982-1_28.pdf

There are more results from the 1970s and 1980s around but it isn't my intention to archive it.

Keywords

Permutations, partial permutations (i.e. permutations inside permutations), Hamilton circles, flags, length of permutations, graphs, braids

Hence, the next paragraphs are focused on a reconstruction of Gunther's results with the help of *Mathematica* with the package *Combinatorica* and to the publishing of the results in CDF format (Computable Document Format) and not on a repetition of the PDF-based Vordenker edition.

Because this text is just an exercise in *Mathematica* it should be clear that it has not the intention to fulfill the standards that are stated for the *Wolframs Demonstration Project.*

Therefore, the used programs are generally taken more or less directly from the package *Wolfram Combinatorica,* and are not my own constructs.

The main topics shall therefore be: *Permutations, Braids* and *Hamiltonian cycles* in a dynamic presentation.

2.2.4. Linear and star graphs

The study of negation cycles in Gunther's theory of *Negativsprache* is mainly restricted to a linear order of its elements. In contrast, different orders like stars and combinations of star and linear order are not considered.

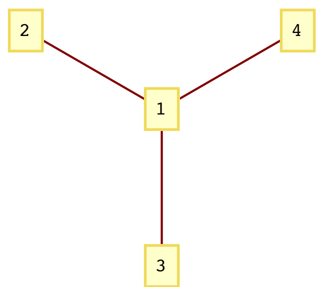
Philosophically, this is simply the traditional situation for fundamental theorems in ontology, epistemology, semiotics and logic.

The trap is easily to detect: for 3 elements, the linear and the star order are collapsing. They appear, at least formally, to be the same.

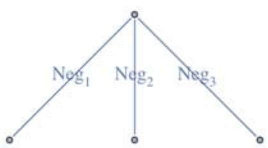
The difference between 3-valued and general systems is decisive. Even for m=4, the difference between linear and star systems is intriguing. That is:

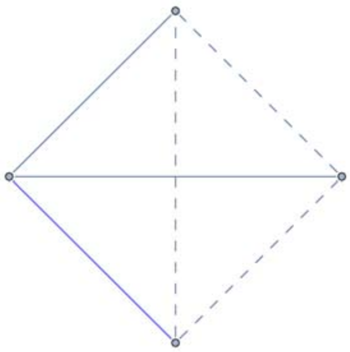
$\binom{3}{2} = 3$, and $\binom{4}{2} = 6$.

```
GraphPlot[StarGraph[4], VertexLabeling → True]
```




```
Graph[star, EdgeLabels → Table[star⟦i⟧ → Neg_i, {i, Length[star]}]]
```





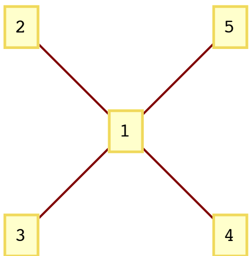
```
GraphPlot[g, VertexLabeling → True, DirectedEdges → True]
```



```
GraphPlot[g, VertexLabeling → True, DirectedEdges → True]
```



```
GraphPlot[StarGraph[5], VertexLabeling → True]
```




```
g = {1 → 2, 2 → 3, 3 → 4}
```

```
GraphPlot[g, VertexLabeling → True, DirectedEdges → True]
```


IsomorphicGraphQ[,   ]

```
False
```

```
PathGraph[{a, b, c}]
```



```
PathGraph[{b, a, c}]
```



```
IsomorphicGraphQ[PathGraph[{a, b, c}], PathGraph[{b, a, c}]]
```

```
True
```

The following application contains Wolfram *Combinatorica*'s complete library of graphs.

g {6, 30}

p Notation

$Aborted

2.2.5. Linear and star negation systems for m=4 (G. Thomas)

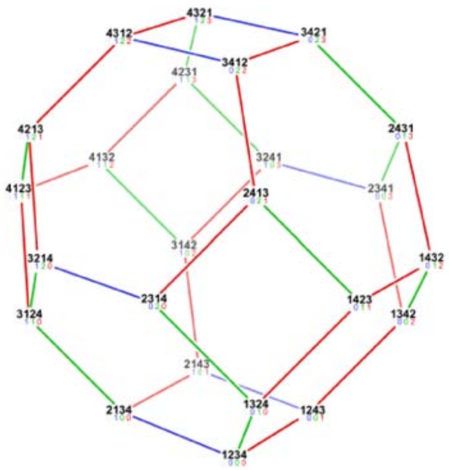
The permutograph for linear structures and m=4 studied by G. Thomas is called in the literature a *permutohedron*, i.e. a *"Cayley graph of S_4, generated by the 3 adjacent transpositions of 4 elements. Only self-inverse permutations are at the same positions as in the permutohedron; the others are replaced by their inverses."*
It is a Symmetric group 4; Cayley graph 1,2,6

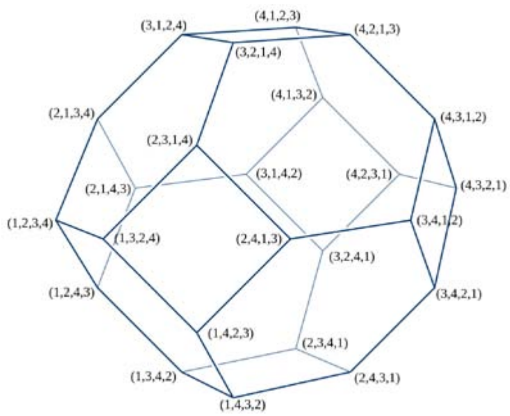
http://en.wikipedia.org/wiki/Permutohedron
http://en.wikipedia.org/wiki/Truncated_octahedron

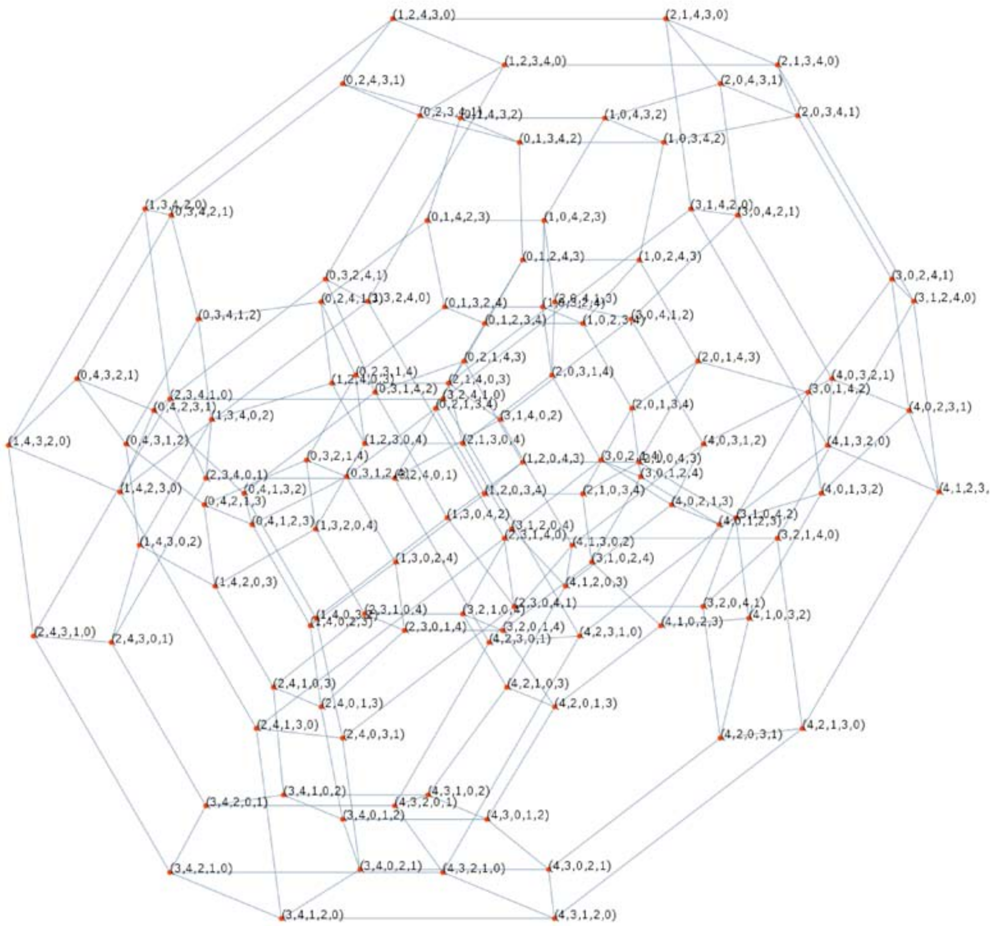
Permutohedron

Definition

The permutohedron is the n-dimensional generalization of the hexagon. The n-permutohedron is the convex hull of all permutations of the vector $(x_1, x_2, \ldots, x_{n+1})$ in $\mathbb{R}^{n+1}$. The number of vertices is $(n+1)!$.







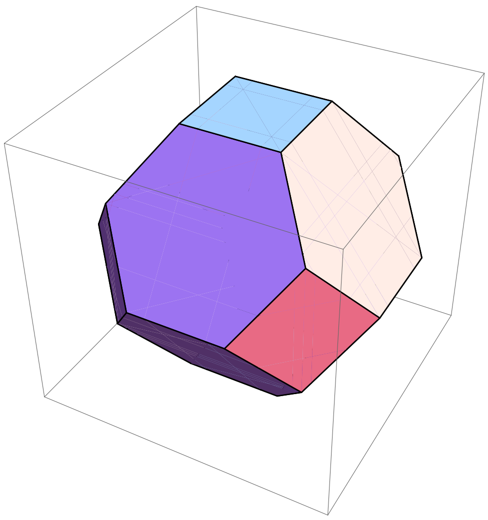
Simple Polyhedron Property Explorer

This explorer delivers all the data necessary to feed further constructions for polyhedrons.

```
Manipulate[Column[{PolyhedronData[g], PolyhedronData[g, p]}],
 {g, PolyhedronData[All]},
 {p, Complement @@ PolyhedronData /@ {"Properties", "Classes"}}]
```

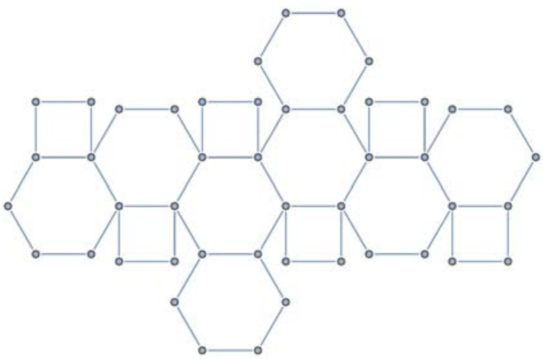
g TruncatedOctahedron

p SkeletonRules

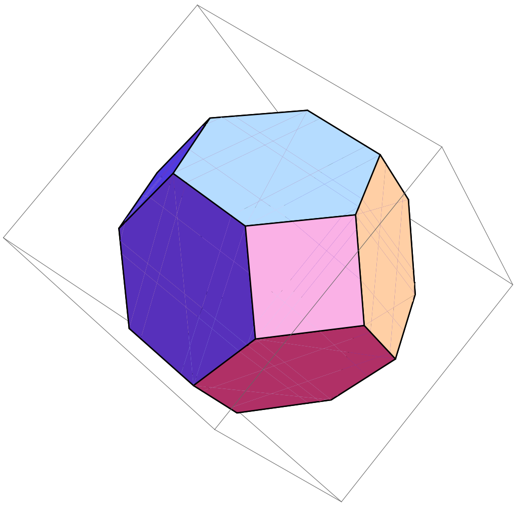


{1 → 2, 1 → 3, 1 → 4, 2 → 5, 2 → 6, 3 → 7, 3 → 8, 4 → 7, 4 → 9, 5 → 10, 5 → 12,
6 → 11, 6 → 12, 7 → 13, 8 → 10, 8 → 14, 9 → 11, 9 → 15, 10 → 16, 11 → 17,
12 → 18, 13 → 19, 13 → 20, 14 → 16, 14 → 19, 15 → 17, 15 → 20, 16 → 21,
17 → 22, 18 → 21, 18 → 22, 19 → 23, 20 → 23, 21 → 24, 22 → 24, 23 → 24}

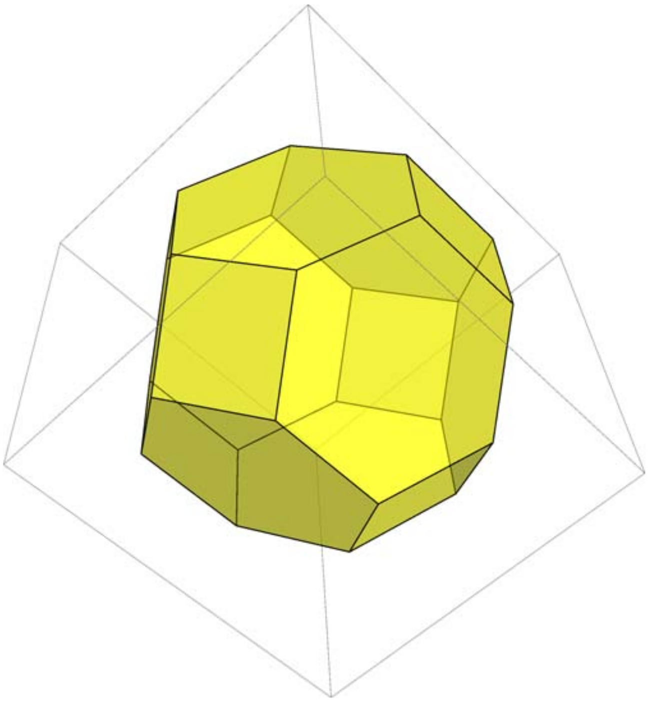
```
PolyhedronData["TruncatedOctahedron", "NetGraph"]
```



```
PolyhedronData["TruncatedOctahedron", "Image"]
```

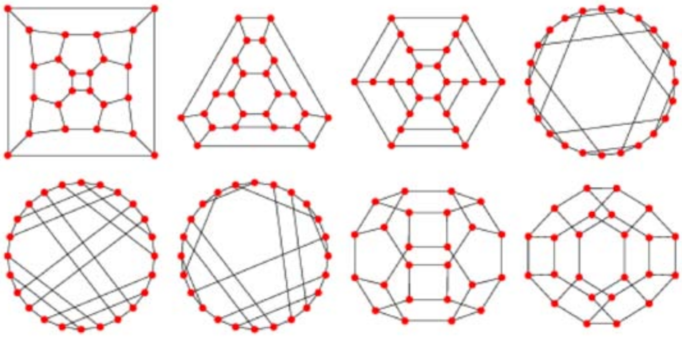


```
Graphics3D[{Opacity[.5], FaceForm[Yellow],
  PolyhedronData["TruncatedOctahedron", "Faces"]}, Lighting → "Neutral"]
```



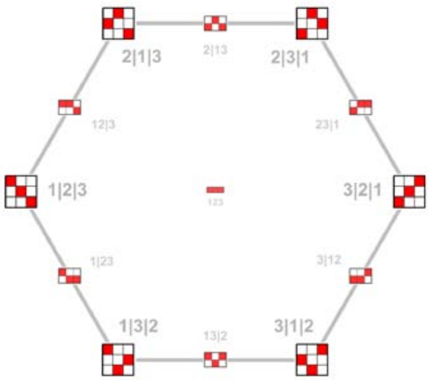
```
PolyhedronData["TruncatedOctahedron", "EdgeCount"]
```

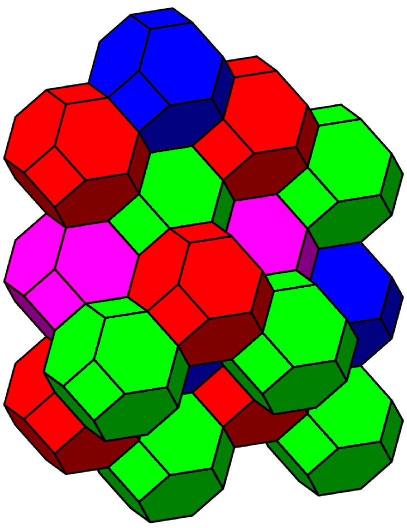
36

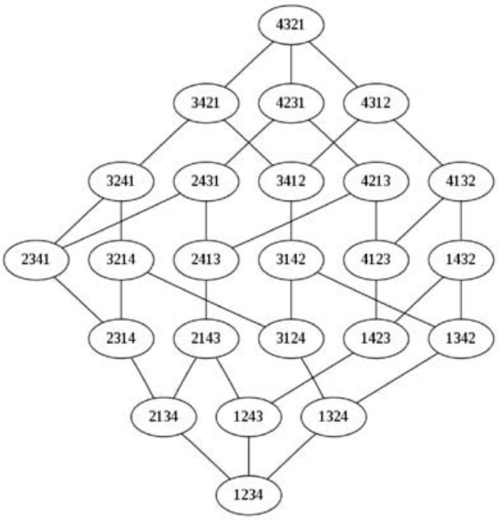


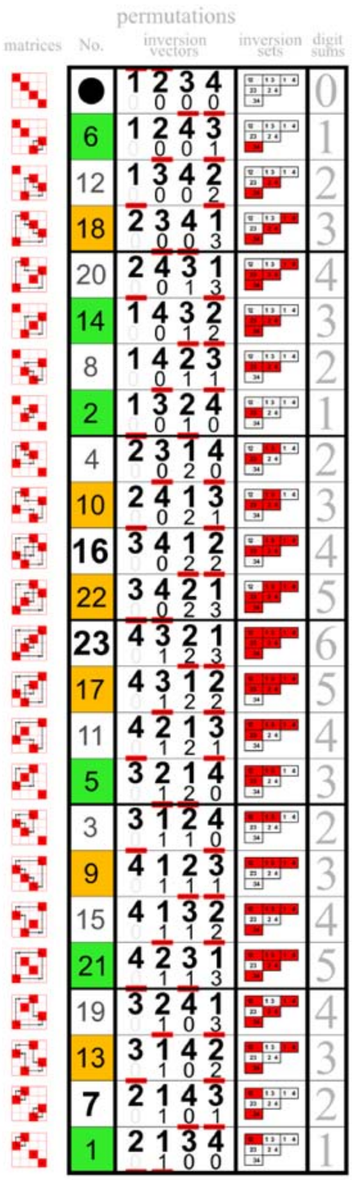
Weisstein, Eric W. “Truncated Octahedral Graph.”
From MathWorld--A Wolfram Web Resource. http://mathworld.wolfram.-com/TruncatedOctahedralGraph.html

Additional information






4321
3421
4231
4312
3241
2431
3412
4213
4132
2341
3214
2413
3142
4123
1432
2314
2143
3124
1423
1342
2134
1243
1324
1234



Reductions

Instead of collapsing all the 24 permutations with the single morphogram MG(4,4), some differentiations and refinements of partitions are at place.

One neat example is a classification of the 24 patterns along their *'digital sums'* as shown in the last table of permutations.

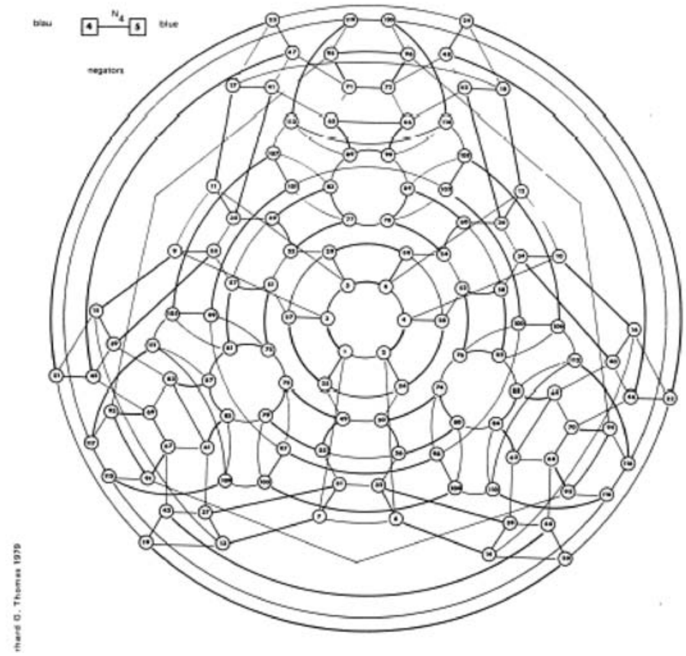
Classifiation by digital sum

dsum(0): (1234)
dsum(1): (1243), (1324), (2134)
dsum(2): (1342), (1423), (2314), (3124), (2143)
dsum(3): (2341), (1432), (2413), (3214), (4123), (3142)

dsum(4): (2431), (3412), (4213),(4132), (3241)
dsum(5): (3421), (4312), (4231),
dsum(6): (4321)

2.2.6. Summary of results from G. Thomas' "*On Permutographs*"

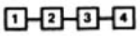
The Negations-Mandala PG1(5)



Negation tables for linear and star graphs

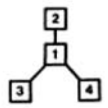
permutation		N1	N2	N3
1234	1	7	3	2
1243	2	8	4	1
1324	3	9	1	5
1342	4	10	2	6
1423	5	11	6	3
1432	6	12	5	4
2134	7	1	13	8
2143	8	2	14	7
2314	9	3	15	11
2341	10	4	16	12
2413	11	5	17	9
2431	12	6	18	10
3124	13	15	7	19
3142	14	16	8	20
3214	15	13	9	21
3241	16	14	10	22
3412	17	18	11	23
3421	18	17	12	24
4123	19	21	20	13
4132	20	22	19	14
4213	21	19	23	15
4231	22	20	24	16
4312	23	24	21	17
4321	24	23	22	18

Table of negations
Permutograph with line contexture

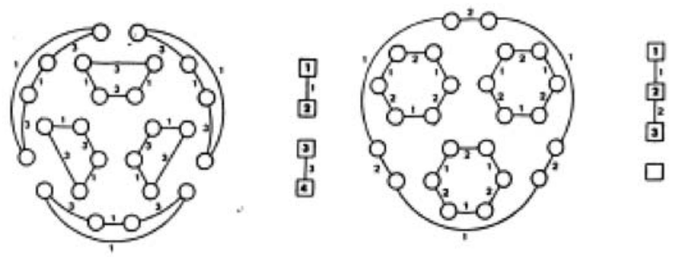


permutation		N1	N2	N3
1234	1	7	15	22
1243	2	8	16	21
1324	3	9	13	24
1342	4	10	14	23
1423	5	11	18	19
1432	6	12	17	20
2134	7	1	9	12
2143	8	2	10	11
2314	9	3	7	10
2341	10	4	8	9
2413	11	5	12	8
2431	12	6	11	7
3124	13	15	3	18
3142	14	16	4	17
3214	15	13	1	16
3241	16	14	2	15
3412	17	18	6	14
3421	18	17	5	13
4123	19	21	24	2
4132	20	22	23	6
4213	21	19	22	2
4231	22	20	21	1
4312	23	24	20	4
4321	24	23	19	3

Table of negations
Permutograph with star contexture



Decompositions into subsystem cycles



Number of different unlabelled trees

In *Harary* [5] you find diagrams of unlabelled trees for n = 1,2,...,10. The following table is an extraction of *Sloane* [7].

n	3	4	5	6	7	8	9	10	11	12	13	14
T(n)	1	2	3	6	11	23	47	106	235	551	1301	3159

number of different unlabelled trees

Tables of PG generating tree-contextures T(n)

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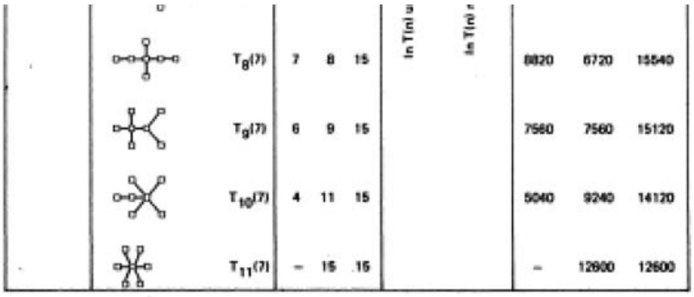
n	PG generating tree-contexture T(n)	Number of different basic cycles ■	●	Σ	Basic cycles (indices of negators) ■	●	Total number of basic cycles in PG ■	●	Σ
2	$T_1(2)$								
3	$T_1(3)$		1	1		121212		1	1
4	$T_1(4)$	1	2	3	1313	121212 · 232323	6	8	14

	$T_2(4)$	–	3	3	 121212 131313 232323	–	12	12
5	$T_1(5)$	3	3	6	1313 121212 1414 232323 2424 343434	90	60	150
	$T_2(5)$	2	4	6	1313 121212 1414 232323 242424 343434	60	80	140
	$T_3(5)$	–	6	6	121212 131313 141414 232323 242424 343434	–	120	120
6	$T_1(6)$	6	4	10	1313 1414 121212 1515 232323 2424 343434 2525 454545 3535	1080	480	1560
	$T_2(6)$	5	5	10	1313 121212 1414 232323 1515 343434 2424 353535 2525 454545	900	600	1500
	$T_3(6)$	3	7	10	121212 232323 1313 242424 1414 252525 1515 343434 353535 454545	540	840	1380
	$T_4(6)$	4	6	10	121212 1414 131313 1515 232323 2424 343434 2525 353535 454545	720	720	1440

Table: **Basic cycles in permutographs (with tree-contextures) on n-permutations**

n	PG generating tree-contexture T(n)	Number of different basic cycles ■	●	Σ	Basic cycles (indices of negators) ■	●	Total number of basic cycles in PG ■	●	Σ
continue 6	$T_5(6)$	5	5	10	1414 1515 2525 3434 4545	121212 131313 232323 242424 353535	900	600	1500
	$T_6(6)$	–	10	10		121212 131313 141414 151515 232323 242424 252525 343434 353535 454545			
7	$T_1(7)$	10	5	15	neighboured edges form 4-cycles	neighboured edges form 6-cycles	12600	4200	16800
	$T_2(7)$	9	6	15			11340	5040	16380
	$T_3(7)$	9	6	15			11340	5040	16380
	$T_4(7)$	9	6	15			11340	5040	16380
	$T_5(7)$	8	7	15			10080	5880	15960
	$T_6(7)$	8	7	15			10080	5880	15960
	$T_7(7)$	7	8	15			8820	6720	15540



$T_8(7)$	7	8	15	ln T(n) u	ln T(n) n	8820	6720	15540
$T_9(7)$	6	9	15			7560	7560	15120
$T_{10}(7)$	4	11	15			5040	9240	14120
$T_{11}(7)$	–	15	15			–	12600	12600

Gerhard Thomas, On permutographs, 1982
http://dml.cz/bitstream/handle/10338.dmlcz/701282/WSAA_10-1982-1_28.pdf

2.2.7. Permutations

A representative citation recapitulating the Guntherian approach on *meontics,* i.e. a theory of negativity based on permutations of values, is given e.g. by Mitterauer:

> *"Nach Günther (1980) kann eine Negativsprache in einem Permutationssystem formalisiert werden. Allgemein ausgedrückt, ist eine Permutation von n-Objekten als eine Anordnung aller Mitglieder der Menge — einmal genommen — entsprechend der Formel n! (!: bedeutet Fakultät) definiert. Tabelle 9 zeigt ein vierwertiges Permutationssystem in einer lexikographischen Ordnung. Es besteht aus den Zahlen 1 bis 4. Die Anzahl der Permutationen ist 24 (4! = 1 × 2 × 3 × 4 = 24). Die Permutationen der Elemente können durch die Anwendung von 3 verschiedenen Negationsoperatoren (N_1, N_2, N_3) erzeugt werden, indem benachbarte Zahlen (Werte) nach dem folgenden Schema umgetauscht werden: [...]."*
> Mitterauer

Three-valued permutations

```
Column[Table[Permutations[Range[3], {3}]], Center]
```

```
{1, 2, 3}
{1, 3, 2}
{2, 1, 3}
{2, 3, 1}
{3, 1, 2}
{3, 2, 1}
```

```
t = Table[Permutations[Range[3], {3}]]
```

```
Grid[t, Frame → All]
```

1	2	3
1	3	2
2	1	3
2	3	1
3	1	2
3	2	1

Four-valued permutations: Permutation group NegSys(4)

```
t4 = Permutations[Range[4], {4}]
```

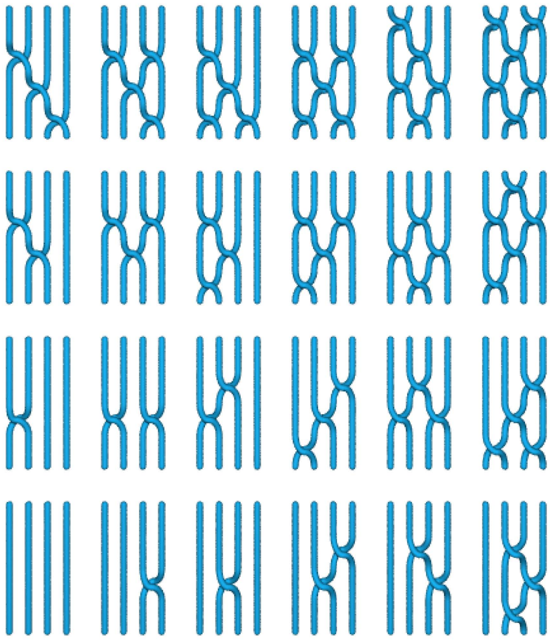
```
Length[Permutations[Range[4], {4}]]
```

```
24
```

```
Grid[t4, Frame → All]
```

1	2	3	4
1	2	4	3
1	3	2	4
1	3	4	2
1	4	2	3
1	4	3	2
2	1	3	4
2	1	4	3
2	3	1	4
2	3	4	1
2	4	1	3
2	4	3	1
3	1	2	4
3	1	4	2
3	2	1	4
3	2	4	1
3	4	1	2
3	4	2	1
4	1	2	3
4	1	3	2
4	2	1	3
4	2	3	1
4	3	1	2
4	3	2	1

Permutation group NegSys(4) as braids



The 24 elements of a permutation group on 4 elements as braids.
Note that all crossings shown are of the left over – right sort
and other choices are possible.
WiKI, http : // en.wikipedia.org / wiki / Braid_theory

Three-valued subsystems of four-valued permutations

```
Permutations[Range[4], {3}]
```

```
Length[Permutations[Range[4], {3}]]
```

```
24
```

```
Permutations[Range[5], {4}]
```

2.2.8. Hamiltonian Cycles

Keywords:

Hamiltonian cycles, table of shortest paths, flags,

Hamiltonian cycles as Negationszyklen

"*Struktureigenschaft der Vierwertigkeit auch das Verhalten der Dreiwertigkeit im Rahmen vierwertiger Relationen beeinflusst. Wir wollen unsere Hinweise über die Rolle der Dreiwertigkeit im vierwertigen System mit meinigen flüchtigen Bemerkungen über die Hegelschen "Knotenpunkte" beschließen.*

"Nun ist in der Tat in dem Übergang von der Triadik zur Vierwertigkeit die Kreiskonstruktion schon vielfältig involviert, in unserer bisherigen Darstellung aber nicht in dem Sinn sich überschneidender Kreise. Tatsächlich jedoch sind solche Überschnei-

dungen im Spiel, wenn man den Übergang vom drei- zum vierwertigen Kreis nicht als Sprung, sondern mit dem Element der Vermittlung charakterisieren will.

"Laut Auskunft der Maschine muss man mit 88 solcher Äquivalenzen zwischen positivem p und einer Negationsfolge, die wieder zum Ausgangspunkt zurückkehrt, rechnen. Vorausgesetzt ist dabei, dass die Folge der Verneinungen sämtliche Permutationen des vierwertigen Negationssystems durchläuft, aber keine Permutation bis zum Schlußschritt wiederholt wird."

Unser Beispiel für die erste Kategorie hat die folgende Gestalt:

p ≡ N1.2.3.1.3.2.3.2.3.1.3.2.3.2.3.1.3.2.1.2.3.2.3.2 p
p ≡ N2.3.2.3.2.1.2.3.1.3.2.3.2.3.1.3.2.3.2.3.1.3.2.1 p
p ≡ N3.2.3.1.3.2.1.2.3.2.3.2.1.2.3.1.3.2.3.2.3.1.3.2 p

Für die dritte Kategorie ergibt sich als entsprechende Reflexionssituation

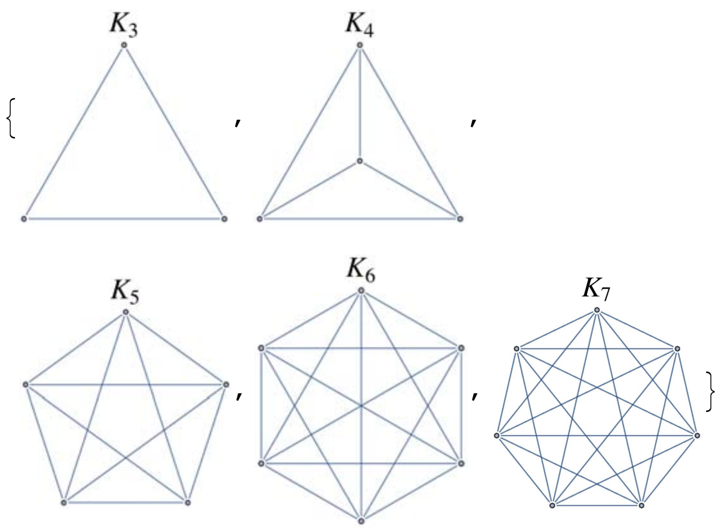
p ≡ N1.3.1.2.3.1.3.2.1.3.1.2.3.1.3.2.1.3.1.2.3.1.3.2 p
p ≡ N2.3.1.3.2.1.3.1.2.3.1.3.2.1.3.1.2.3.1.3.2.1.3.1 p
p ≡ N3.1.3.2.1.3.1.2.3.1.3.2.1.3.1.2.3.1.3.2.1.3.1.2 p

(G. Gunther, Janusgesicht, 1974, p.22/24)
http://www.vordenker.de/ggphilosophy/gg_janusgesicht.pdf

More about graphs

```
<< Combinatorica`
```

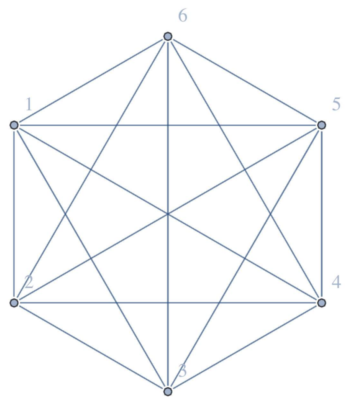
```
Table[CompleteGraph[i, PlotLabel -> Ki], {i, 3, 7}]
```



```
Length[CompleteGraph[4]]
```

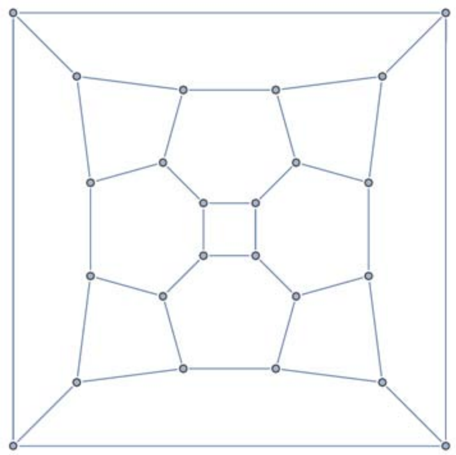
2

```
k = CompleteGraph[6, VertexLabels → "Name", ImagePadding → 10]
```



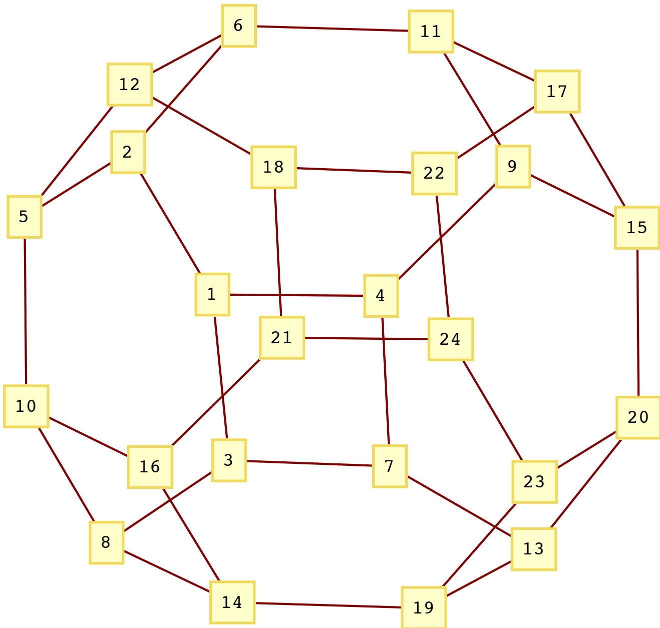
Graphs for m=4

l =

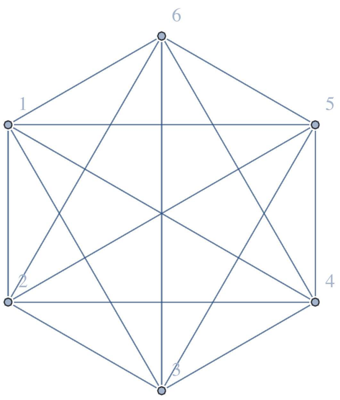


http : // commons.wikimedia.org / wiki / File : Symmetric_group _ 4;
_permutation _list; _Steinhaus–Johnson–Trotter_permutohedron.svg

http : // pageperso.lif.univ -
mrs.fr / ~luigi.santocanale / TRECOLOCOCO / TRECOLOCOCO.pdf



Permutation system for m=3

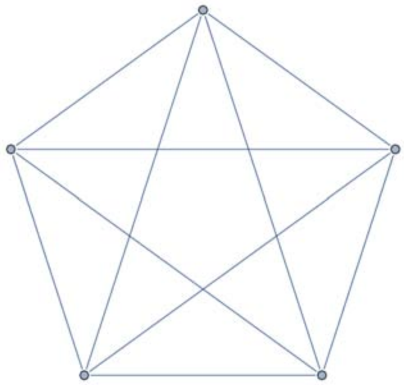
HighlightGraph[

, {1}, GraphHighlight → {2, 2 ⟷ 3}]

TableForm[ToAdjacencyMatrix[CompleteGraph[6]]]

0	1	1	1	1	1
1	0	1	1	1	1
1	1	0	1	1	1
1	1	1	0	1	1
1	1	1	1	0	1
1	1	1	1	1	0

g = AdjacencyGraph[$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$]



```
M[CompleteGraph[6]]
```

```
15
```

```
ToOrderedPairs[CompleteGraph[6]]
```

```
First[FindHamiltonianCycle[6]]
```

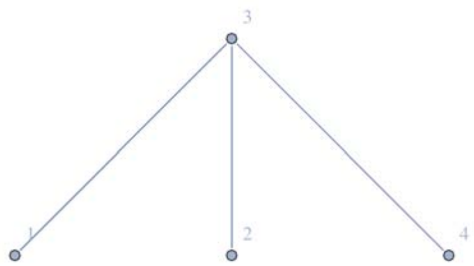
```
FindShortestPath[k, 1, 120]
```

```
ExtractCycles[k]
```

```
{{5, 3, 4, 5}, {5, 1, 4, 2, 5}, {3, 1, 2, 3}}
```

```
FindCycle
```

g = AdjacencyGraph[$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$, **VertexLabels → "Name", ImagePadding → 10**]



g = AdjacencyGraph[$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$, **VertexLabels → "Name", ImagePadding → 10**]

Negation system for 3 element linear negation

```
Graph[{1, 2, Style[3, Red]}, {1 ⟷ 2, 2 ⟷ 3, Style[3 ⟷ 1, Blue]}]
```

No	state	N1	N2
1	1 2 3	3	2
2	1 3 2	6	1
3	2 1 3	1	5
4	2 3 1	2	6
5	3 1 2	6	3
6	3 2 1	5	4

```
TableForm[HamiltonianCycles[k, 11]]
```

1	2	3	4	5	6
1	2	3	4	6	5
1	2	3	5	4	6
1	2	3	5	6	4
1	2	3	6	4	5
1	2	3	6	5	4
1	2	4	3	5	6
1	2	4	3	6	5
1	2	4	5	3	6
1	2	4	5	6	3
1	2	4	6	3	5

```
HamiltonianCycles[k, All]

Length[HamiltonianCycles[k, All]]

120
```

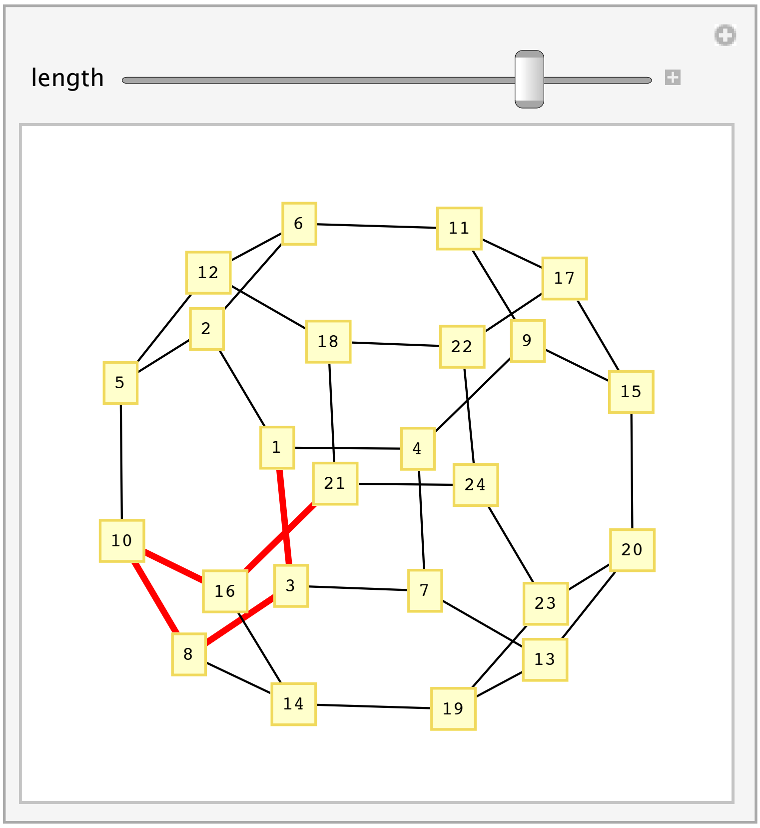
Negation system for 3 element star negation

No	state	N1	N2
1	1 2 3	3	2
2	1 3 2	6	1
3	2 1 3	1	5
4	2 3 1	2	6
5	3 1 2	6	3
6	3 2 1	5	4

Negation systems for m=4

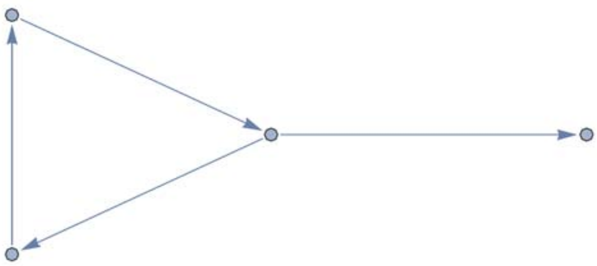
```
h24 = CompleteGraph[24, VertexLabels → "Name", ImagePadding → 10]

- Graph:< 276,24,Undirected >-

Column[HamiltonianCycles[h24, 11], Left]

Column[HamiltonianCycles[h24, AllPairsShortestPath], Left]
```

```
Needs["GraphUtilities`"]

PathGraph[{1, 2, 3},
 {1⟶3, 1⟶2, 2⟶6, 2⟶1, 3⟶1, 3⟶5 , 4⟶1, 4⟶5, 5⟶6, 5⟶3 , 6⟶5, 6⟶4}]

g = {1 → 2, 2 → 3, 3 → 4, 4 → 5, 5 → 6, 6 → 4,
   6 → 3, 6 → 2, 6 → 1, 2 → 6, 3 → 1, 3 → 2, 3 → 4, 3 → 5, 3 → 6}

h = {1 → 2, 1 → 3, 1 → 4, 1 → 5, 1 → 6, 2 → 1, 2 → 3, 2 → 4, 2 → 5,
   2 → 6, 3 → 1, 3 → 2, 3 → 4, 3 → 5, 3 → 6, 4 → 1, 4 → 2, 4 → 3, 4 → 5,
   4 → 6,  5 → 1, 5 → 2, 5 → 3, 5 → 4, 5 → 6,  6 → 1, 6 → 2, 6 → 3, 6 → 4, 6 → 5 }
```



Flags

```
Graph[{1 ↔ 2, 2 ↔ 3, 3 ↔ 1, 3 ↔ 4, 4 ↔ 4 }]
```



```
g = {1 → 2, 2 → 3, 3 → 4, 4 → 5, 5 → 6, 6 → 15,
    15 → 7, 6 → 1, 7 → 8, 8 → 9, 9 → 10, 10 → 11, 11 → 12, 12 → 7};
```

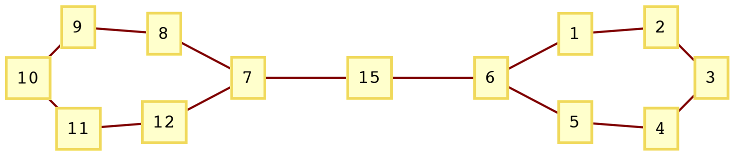
```
path = GraphPath[Flatten[{g, Map[Reverse, g]}], 1, 9]
```

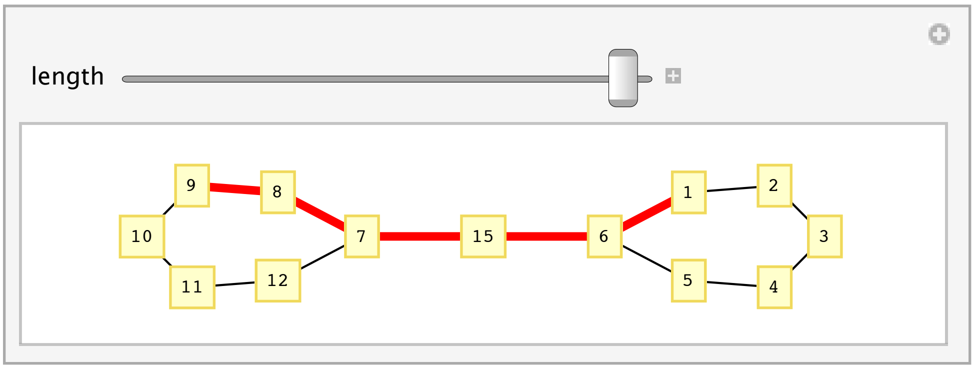
```
{1, 6, 15, 7, 8, 9}
```

```
path = GraphPath[Flatten[{g, Map[Reverse, g]}], 1, 11]
```

```
{1, 6, 15, 7, 12, 11}
```

```
GraphPlot[g, VertexLabeling → True]
```






```
cs = Transpose[{h, RotateRight[h]}]; GraphPlot[k,
 EdgeRenderingFunction → (If[MemberQ[cs, #2] || MemberQ[cs, Reverse[#2]],
      {Red, Thickness[.01], Line[#1]}, {Black, Line[#1]}] &), VertexLabeling → True]
```

```
cs = Transpose[{g, RotateRight[g]}]; GraphPlot[k,
 EdgeRenderingFunction → (If[MemberQ[cs, #2] || MemberQ[cs, Reverse[#2]],
      {Red, Thickness[.01], Line[#1]}, {Black, Line[#1]}] &), VertexLabeling → True]
```

```
g = {1 → 2, 1 → 3, 1 → 4, 1 → 5, 1 → 6, 2 → 1,
   2 → 3, 2 → 4, 2 → 5, 2 → 6, 3 → 1, 3 → 2, 3 → 4, 3 → 5, 3 → 6}
```

```
HamiltonianCycles[g, All]
```

```
Length[HamiltonianCycles[g, All]]
```

```
2
```

```
h = PolyhedronData["TruncatedOctahedron", "SkeletonGraph"];
FindHamiltonianCycle[h]
```

"The truncated octahedron graph is the cubic Archimedean graph on 24 nodes and 36 edges that is the skeleton of the truncated octahedron.
It is implemented in *Mathematica* as GraphData["TruncatedOctahedral-Graph"]."

http://mathworld.wolfram.com/TruncatedOctahedralGraph.html

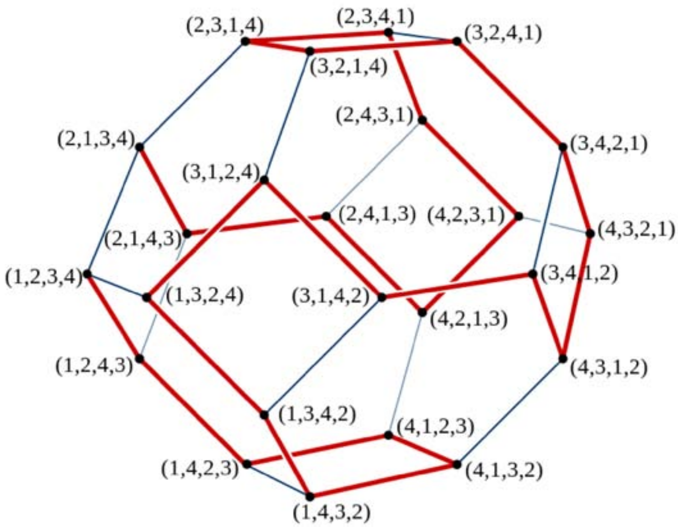
Gunther's 88 Hamiltonian cycles for the linear order m=4 com-

puted by another machine

```
Column[HamiltonianCycles[h, All], Left]
FindShortestPath[h, 2]
{1, 2, 5, 10, 16, 21, 24}
```

HamiltonianPathCount

Permutograph(4) with a Hamiltonian cycle

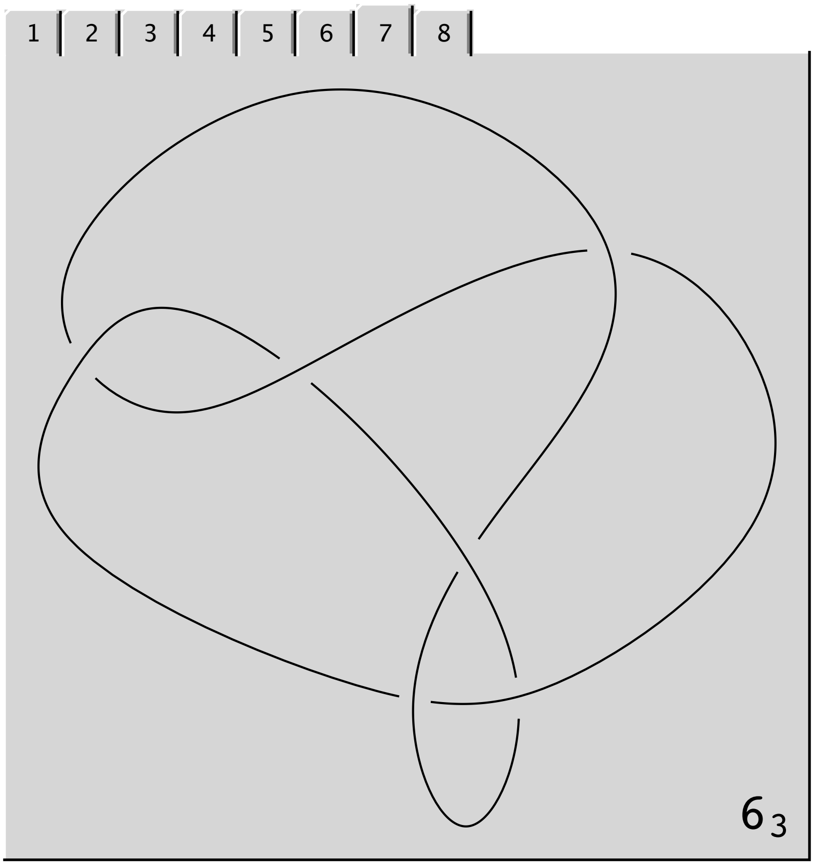


Hence, a negational transformation from (1,2,3,4) to (2,1,3,4) might take a longest path through the graph and realizing a Hamiltonian cycle that delivers the anticipated result (2,1,3,4). A result that could be obtained in a single step by the negation N_1: N_1(1,2,3,4) = (2,1,3,4).

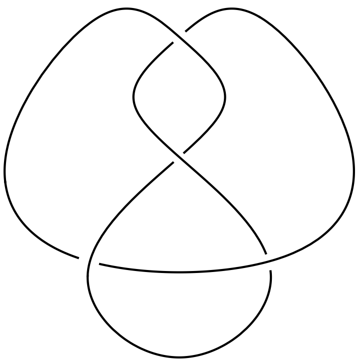
This arises the question: How many cycles of different length are leading back to (1,2,3,4)?

2.3. Combinatorial knots and braids

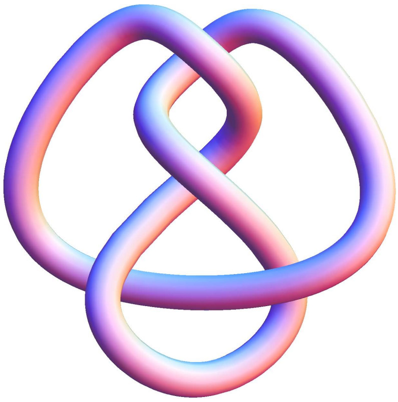
http://homepages.math.uic.edu/~kauffman/KFI.pdf
http://www.maths.manchester.ac.uk/~grant/knotschap4.pdf
http://katlas.math.toronto.edu/wiki/Main_Page




```
Graphics[KnotData["FigureEight", "KnotDiagramData"]]
```



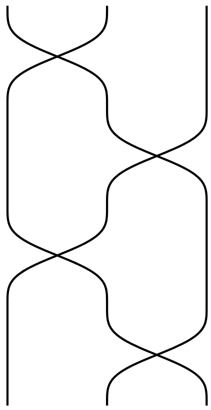
```
KnotData["FigureEight"]
```



```
KnotData["FigureEight", "ColoringNumberSet"]
```

{5}

```
KnotData["FigureEight", "BraidDiagram"]
```

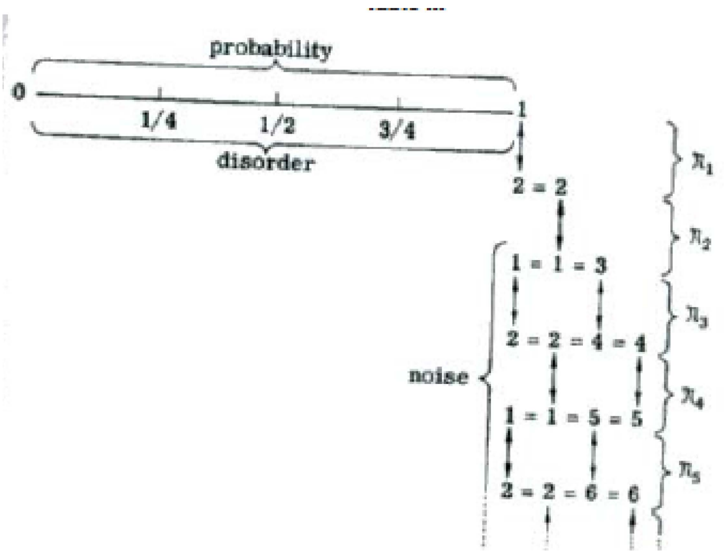


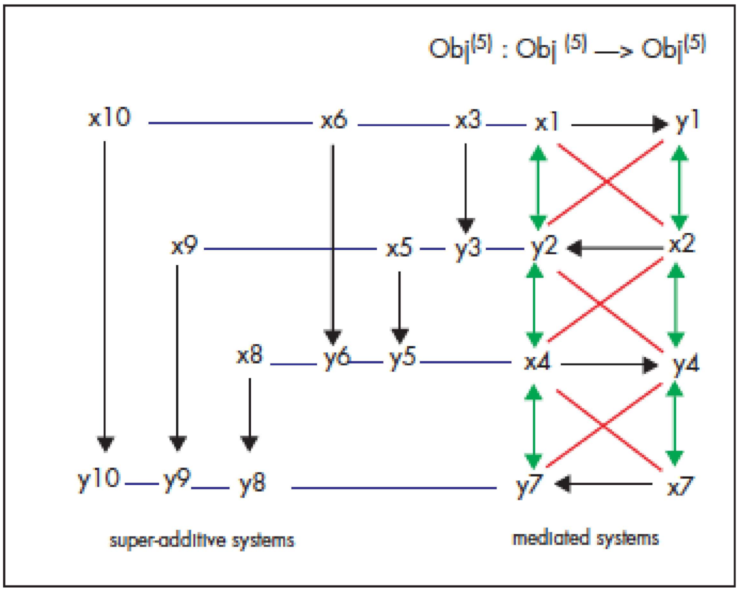
[1,2,3] $\Longrightarrow$ [2,3,1]

```
KnotData["FigureEight", "BraidWordNotation"]
```

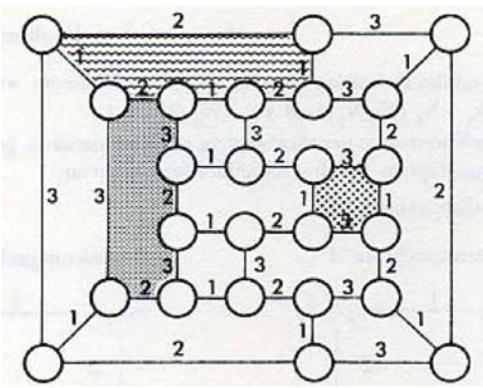
$\sigma_1 \sigma_2^{-1} \sigma_1 \sigma_2^{-1}$

2.3.1. Gunther's hidden knots





2.3.2. Gunther's permutation systems as braids



Algebra of NegSys(4)

$N_i(N_i(X)) = X$, i=1,2,3
$N_1(N_3) = N_3(N_1)$

$N_1(N_2(N_1)) = N_2(N_1(N_2))$
$N_2(N_3(N_2)) = N_3(N_2(N_3))$

Gunther’s new interpretation of permutation groups for dialectical negation systems.

p	U	N_1 p
p	Kl	$N_{1 \cdot 2}$ p
p	O	$N_{1 \cdot 2 \cdot 1}$ p
p	Kr	$N_{1 \cdot 2 \cdot 1 \cdot 2}$ p
p	U Is	$N_{1 \cdot 2 \cdot 1 \cdot 2 \cdot 1}$ p $N_{1 \cdot 2 \cdot 1 \cdot 2 \cdot 1 \cdot 2}$ p

"Dabei bedeuten U Umtausch*, Kl Kreis mit* Linksdrall*, O* Ordnungsverhältnis*, Kr Kreis mit* Rechtsdrall *und Is* Identität *in Spiegelung.*

"Wie wir aus der Aufstellung der Negationsverhältnisse ersehen können, haben wir es mit drei logischen Konzeptionen zu tun, von denen zwei sowohl als kontemplativ *zu verstehende Begriffe als auch als* aktive *Reflexionsereignisse gedacht werden können. Wir meinen damit symmetrischen Umtausch (U) und hierarchische Ordnung.*

"Es fällt schwerer, der Kreisrelation (K) dieselbe Doppeldeutigkeit zuzuschreiben. Sie kann kaum als begrifflich fixierbares Sein mit eigener Stabilität verstanden werden. Bezeichnenderweise tritt sie sofort mit den Superskripten (Kr und Kl) auf, die auf inverse Bewegungsvorgänge hinweisen. Für einen Kreis ist das einzig Ruhende eigentlich nur sein Mittelpunkt, der indifferent gegenüber Rechts- und Linksdrall ist. Diese Indifferenz ist das, was der coincidentia oppositorum in der diairetischen Logik der Positivsprache entspricht."

The primary assumption of Gunther’s negation systems is the decision for a non-related set of elements of permutation.

With that, negation is defined as *unary* mapping of the set.

This holds automatically for the combinatorial approach of knots and braids too.

Indexed objects

In contrast, a polycontextural approach of indexed objects takes the relational character of the elements into account. The properties of the relational terms are defined by their indices.

Hence, just for m= 3 there is a coincidence of the number of elements and the number of relations.

For m>3, there are $\binom{m}{2}$ different values in the game.

For the example of m=4, there are 6 differentiations of the elements to consider.

```
Binomial[4, 2] // TraditionalForm
```

6

The tuple (1,2,3,4), interpreted as the truth-values of a 4-valued system, is involved into 6 interpretations written as tuples: (1;1,3,6), (2;1,2,5), (3;3,2,5), (4;4,5,6).

```
1 – 2 – 3 – 4
 \/   \/   \/
 s1   s2   s4
   \/   \/
   s3   s5
     \/
     s6
```

```
(1; 1, 3, 6) – (2; 1, 2, 5) – (3; 3, 2, 5) – (4; 4, 5, 6)
        \/              \/              \/
        s1              s2              s4
              \/              \/
              s3              s5
                     \/
                     s6
```

N_1(1;1,3,6) = (2;1,2,5)
N_1(2;1,2,5) = (1;1,3,6)
N_1(3;3,2,5) = (3;2,3,5)
N_1(4;4,5,6) = (4;5,4,6)

Subsystem tuples

P = (s1,s2,s3,s4,s5,s6):
N_1(s1,s2,s3,s4,s5,s6) = (s^{-}1,s3,s2,s4,s5,s6)

N_2(s1, s2, s3, s4, s5, s6) = (s3, s^- 2, s1, s5, s4, s6)
N_3(s1,s2,s3,s4,s5,s6) = (s1, s5, s6, s^- 4, s2, s3)

Corresponce table for $B4$

Negation system properties	**Braid words**	**Gunther**
$N_i(N_i(X)) = X$, i=1,2,3	: $\sigma_1 \sigma_1^{-1} = 1$	: Is (mirror identity)
$N_1(N_3) = N_3(N_1)$	: $\sigma_1 \sigma_3 = \sigma_3 \sigma_1$	: K (circle), L, R
$N_1(N_2(N_1)) = N_2(N_1(N_2))$	: $\sigma_1 \sigma_2 \sigma_1 = \sigma_2 \sigma_1 \sigma_2$	: O (order relation)
$N_2(N_3(N_2)) = N_3(N_2(N_3))$	: $\sigma_2 \sigma_3 \sigma_2 = \sigma_3 \sigma_2 \sigma_3$	: O
And: $N_i(X)$ (exchange relation).	: σ_i	: U, L, R

The classical case for m=3

Trinitarian or triadic systems have a fundamental significance in Western culture. Here I mention just the intriguing fact that trinity is the only system where the number of knots and edges are equal. This has produced enormeous confusions in the debates of dichotimic thinking and triadic relegion.

For indexed objects, i.e. elements in a relational and contextural system, the table of possibilities and permutations is well defined.

The tuple (1,2,3) becomes ([T,1,3], [F,1,2], [**F,** 2,3]).

That is: 1 ⟶ 2 ⟶ 3 becomes the 'linear' chiastic structure:

t3 ≡ t1	⟶	f1
↓ ↕		↕
f3 ≡ f2	⟵	t2

With t3 ≡ t1 : [T;1,3]
f1 ↔ t2: [F;1,2]
f3 ≡ f2 : [**F;** 2,3]

Hence, the permutation group for m=3 is:

Permutation Indexed permutation Negation Braid notation

1	2	3
1	3	2
2	1	3
2	3	1
3	1	2
3	2	1

T; 1, 3	F; 1, 2	**F**; 2, 3
T; 3, 1	**F**; 3, 2	F; 2, 1
F; 1, 2	T; 1, 3	**F**; 3, 2
F; 2, 1	**F** ; 2, 3	T; 3, 1
F; 3, 2	T; 3, 1	F; 1, 2
F; 2, 3	F; 2, 1	T; 1, 3

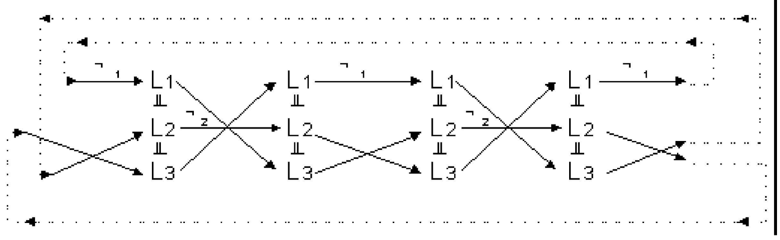
X
N2
N1
N2 .1
N1 .2
N1 .2 .1

–	si s – i	–
–	s2	–
s1	–	–
s2	s1	–
s1	s2	–
s1	s2	s1

Negation circle NZ_1 for m=3

$$NZ_1 : L^{(3)} \longrightarrow L^{(3)}$$

$$NZ_1(X) = N_1(N_2(N_1(N_2(N_1(N_2(X))))))$$



http : // www.vordenker.de / ics / poly.htm (1989)

Triadic braids

Braid generators N_1 and N_2

N1: **N2:**

```
T;1,3  F;1,2   F;2,3      T;1,3  F;1,2   F;2,3
    \ /        |          |          \ /
    / \        |          |          / \
 F;1,2  T;1,3  F;3,2      T;3,1   F;3,2  F;2,1
```

N1(N1): **N2(N2):**

```
T;1,3  F;1,2   F;2,3      T;1,3  F;1,2   F;2,3
    \ /        |          |          \ /
    / \        |          |          / \
 F;1,2  T;1,3  F;3,2      T;3,1   F;3,2  F;2,1
```

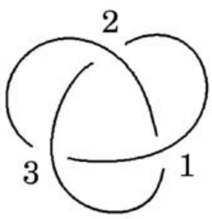
```
    \ /       |         |         \ /
    / \       |         |         / \
T;1,3  F;1,2  F;2,3     T;1,3  F;1,2  F;2,3
```

N2(N1): **N1(N2):**

```
T;1,3  F;1,2  F;2,3     T;1,3  F;1,2  F;2,3
    \ /       |         |          \ /
    / \       |         |          / \
 F;1,2  T;1,3 F;3,2     T;3,1  F;3,2  F;2,1
  |         \ /            \ /        |
  |         / \            / \        |
T;1,3  F;1,2  F;2,3     T;1,3  F;1,2  F;2,3
```

N2(N1(N2)): **N1(N2(N1)):**

```
T;1,3  F;1,2  F;2,3     T;1,3  F;1,2  F;2,3
    \ /       |         |          \ /
    / \       |         |          / \
 F;1,2  T;1,3 F;3,2     T;3,1  F;3,2  F;2,1
  |         \ /            \ /        |
  |         / \            / \        |
T;1,3  F;1,2  F;2,3     T;1,3  F;1,2  F;2,3
   \ /        |            |       \ /
   / \        |            |       / \
T;1,3  F;1,2  F;2,3     T;1,3  F;1,2  F;2,3
```

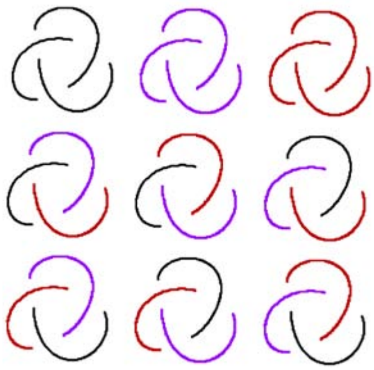


O1U2O3U1O2U3

"Figure 14 shows all colorings of the trefoil knot by the three-element kei $\boldsymbol{R}_3$ *with operation table given by*

▹	**1**	**2**	**3**
1	1	3	2
2	3	2	1
3	2	1	3

Figure 14. Kei colorings of the trefoil by $\mathbf{R}_3$.



http://www.ams.org/staff/jackson/fea-nelson.pdf

The operation table for Figure 14 has additionally many different interpretations. One is the interpretation of the table as the logical table for a full transjunction in a 3-contextural logic.

With a different coloring the table corresponds the transjunction:

⊗	1	2	3
1	1	3	2
2	3	2	1
3	2	1	3

http://homepages.math.uic.edu/~kauffman/KFI.pdf
http://www.maths.manchester.ac.uk/~grant/knotschap4.pdf

Closure of a braid

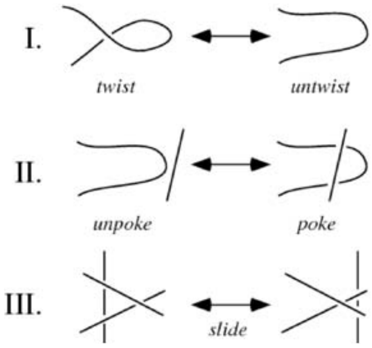
The connections between knots and braids is established by procedure to connect the point of at the top to those at the bottom. The resulting knot is called the closure of the braid. (Alexander)

Composition of braids are not commutative but associative

https://web.math.princeton.edu/~baldwinj/GurnaniNotes.pdf

Reidemeister moves

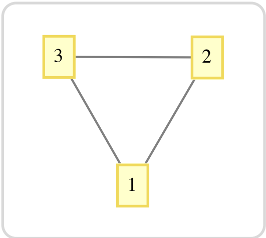
Reidemeister moves applied to the negation braids results in $N_i(N_i X)) = X$, i=1,2.
The iterated negations: $N_1(N_1(N_1(X)))$ corresponds to σ_1^3 and by Reidemeister move to X.



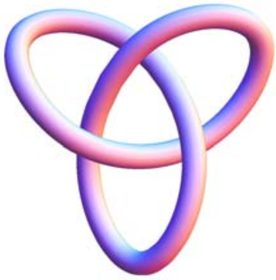
Weisstein, Eric W. “Reidemeister Moves.”
From MathWorld--A Wolfram Web Resource.
http://mathworld.wolfram.com/ReidemeisterMoves.html

Basic exmple: Trefoil

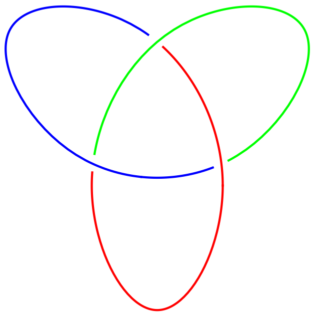
```
KnotData["Trefoil"]
```



becomes



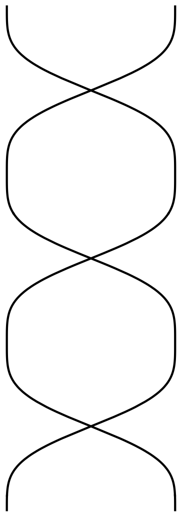
```
Graphics[
 Transpose[{{Red, Blue, Green, Red}, KnotData["Trefoil", "KnotDiagramData"]}]]
```



```
KnotData["Trefoil", "ColoringNumberSet"]
```

```
{3}
```

```
KnotData["Trefoil", "BraidDiagram"]
```



```
KnotData[{3, 1}, "BraidWordNotation"]
```

$"\sigma"_1^3$

Negations: N1(N1(N1)) :: σ_1^3

```
KnotData[{3, 1}, "BraidImage"]
```

The classical case for m=4

> *"Here hopeless confusion rules with Hegel. On one hand he permits only a second negation, which allows for a single circle. On the other hand he speaks of circles of circles. These, however, can only be produced if at least a third negation is introduced – and thus a minimum of four values. Schelling's approach permits this.*
> *Hegel's does not. His Absolute is the Christian Trinity as non-extendable system."* (G.G., p. 29)

The tuple (1,2,3,4) becomes ([T;1,3,6], [F;1,2,5], [**F**; 3,2,5], [𝔽;4,5,6]), or numeric:

(1;1,3,6) - (2;1,2,5) - (3;3,2,5) - (4;4,5,6).

That is: 1 ⟶ 2 ⟶ 3 ⟶ 4 becomes the 'linear' chiastic structure Chi(4):

Chi(4)

t6 ≡ t3 ≡ t1 ⟶ f1
↓ ↓ ↕ ↕
↓ f3 ≡ f2 ⟵ t2 ≡ t5
↓ ↕ ↕ ↓
f6 ≡ ≡ t4 ⟶ f4 ≡ f5

Operator system of Chi(4) : {N1, N2, N3}

N1 **N2** **N3**
T;1,3,6 F;1,2,5 **F**;2,3,4 𝔽;4,5,6 T;1,3,6 F;1,2,5 **F**;2,3,4 𝔽;4,5,6 T;1,3,6
F;1,2,5 **F**;2,3,4 𝔽;4,5,6
\ / | | | \ / | |
| \ /
/ \ | | | / \ | |
| /\
F;1,2,5 T;1,3,6 **F**;3,2,4 𝔽;4,5,6 T;3,1,6 **F**;3,2,4 F;2,1,5 𝔽;4,5,6 T;1,6,3
F;1 5,2 𝔽;3,2,4 **F;**4,3,2

Introduction to Braids

> *"No longer merely symbols standing in for topological or geometric objects, the knot diagrams themselves have become mathematical objects of interest."*

http://www.ams.org/staff/jackson/fea-nelson.pdf

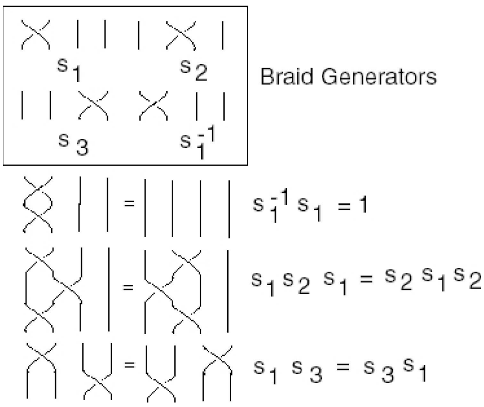
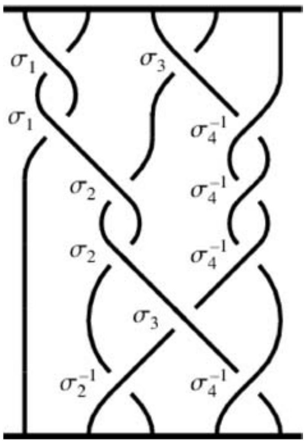


Figure 2. Braid generators and relations.

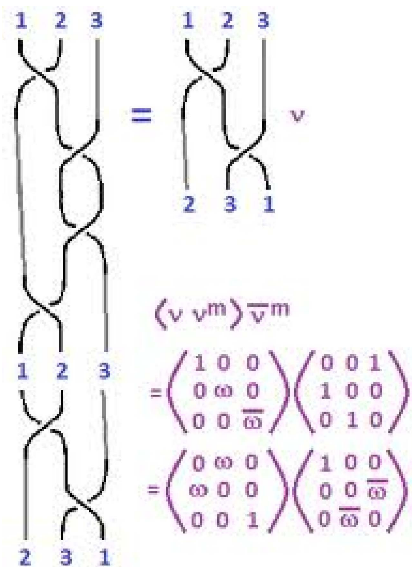
"In figure 2 we have restricted the illustration to the four-stranded braid group B4. In that figure the three braid generators of B4 are shown, and then the inverse of the first genera-

tor is drawn. Following this, one sees the identities s1s1 - 1 = 1 (where the identity element in B4 consists in four vertical strands), s1s2s1 = s2s1s2, and finally s1s3 = s3s1.
With this interpretation, it is apparent from figures 1 and 2 that the second braiding relation (above) is formally the same as the Yang-Baxter equation."

http://iopscience.iop.org/1367-2630/6/1/134/fulltext/



" An ordered combination of the and symbols constitutes a braid word.
For example,
σ1σ3σ1σ - 4 σ2σ - 4 σ3σ - 2 σ - 4 is a braid word for the braid illustrated above, where the symbols can be read off the
diagram left to right and then top to bottom."
Weisstein, Eric W.Braid. From *MathWorld*-- A Wolfram Web Resource.
http : // mathworld.wolfram.com / Braid.html



Braids and logic

Braids from geometry, topology, algebra, category theory to *polycon-*

textural logics.

A possible transition from the topological and algebraic understanding of knots and braids to a *logical* interpretation might open up a new field of studies in polycontextural logic that are focusing more than before on the process-oriented and interactive character of polycontextural negation and dualization/conjugation systems.

There is an interesting development from classical, permutation-oriented logical negation to a dialog-based logic as introduced by Paul Lorenzen. Taken the dialogical construct seriously, the interaction between opponent and proponent in respect of winning strategies gets a reasonable interpretation as chiasms, (Opp, Propp, win, lose), formalized for negation dialogs as braids (start, end; left, right).

http://mathoverflow.net/questions/87002/what-is-the-metamathematical-interpretation-of-knot-diagrams

Correspondence between the braid algebra and Gunther's negation systems.

Corresponce table for $B4$

Negation system properties	**Braid words**	**Gunther**
$N_i(N_i(X)) = X$, i=1,2,3	: $\sigma_1 \sigma_1^{-1} = 1$	: Is (mirror identity)
$N_1(N_3) = N_3(N_1)$	: $\sigma_1 \sigma_3 = \sigma_3 \sigma_1$	: K (circle), L, R
$N_1(N_2(N_1)) = N_2(N_1(N_2))$	: $\sigma_1 \sigma_2 \sigma_1 = \sigma_2 \sigma_1 \sigma_2$	: O (order relation)
$N_2(N_3(N_2)) = N_3(N_2(N_3))$	: $\sigma_2 \sigma_3 \sigma_2 = \sigma_3 \sigma_2 \sigma_3$	: O
And: $N_i(X)$ (exchange relation).	: σ_i	: U, L, R

http://math.uchicago.edu/~mann/permutations.pdf http://www.ms.unimelb.edu.au/publications/Chiodo.pdf

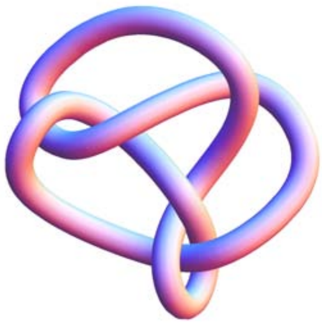
***Mathematica* Braid properties**

KnotData
Image
BraidWord
BraidWordNotation
BraidDiagram

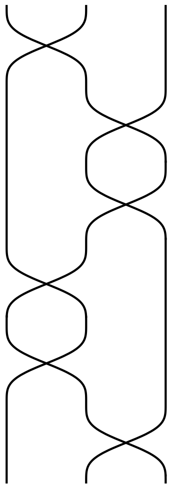
BraidImage

Example "KnotData[{6,3}, 'property']

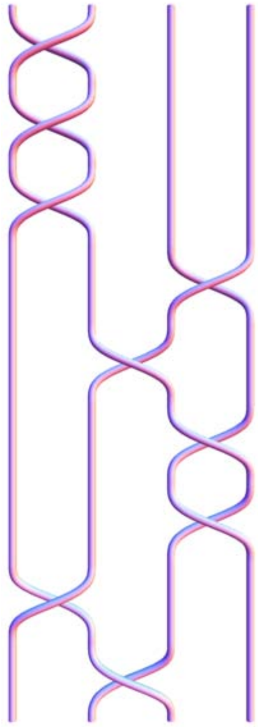
```
KnotData[{6, 3}, "Image"]
```



```
KnotData[{6, 3}, "BraidDiagram"]
```



```
KnotData[{6, 3}, "BraidImage"]
```



[1, 2, 3, 4] ⟹ [2, 3, 4, 1]

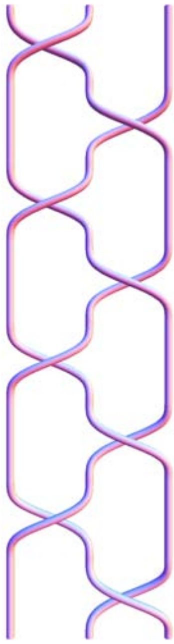
```
KnotData[{6, 3}, "BraidWordNotation"]
```

$\sigma_1^{-1}\sigma_2^2\sigma_1^{-2}\sigma_2$

```
KnotData["Amphichiral"]
```

```
{{0, 1}, {4, 1}, {6, 3}, {8, 3}, {8, 9}, {8, 12}, {8, 17}, {8, 18},
 {10, 17}, {10, 33}, {10, 37}, {10, 43}, {10, 45}, {10, 79}, {10, 81},
 {10, 88}, {10, 99}, {10, 109}, {10, 115}, {10, 118}, {10, 123}}
```

```
KnotData[{8, 18}, "BraidImage"]
```



```
N2 .1[1, 2, 3] ⟹ [3, 1, 2]

Negations : N2 (N1 (p))

KnotData[{8, 18}, "BraidWordNotation"]
```

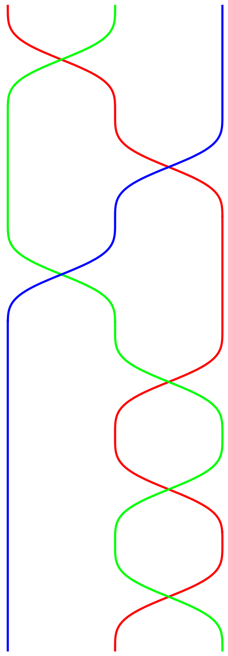
$\sigma_1\sigma_2^{-1}\sigma_1\sigma_2^{-1}\sigma_1\sigma_2^{-1}\sigma_1\sigma_2^{-1}$

```
KnotData[{6, 2}, "BraidWordNotation"]
```

$\sigma_1^{-1}\sigma_2\sigma_1^{-1}\sigma_2^3$

```
data2 = KnotData[{6, 2}, "BraidDiagramData"];
```

```
Graphics[Transpose[{{Red, Green, Blue}, data2}]]
```



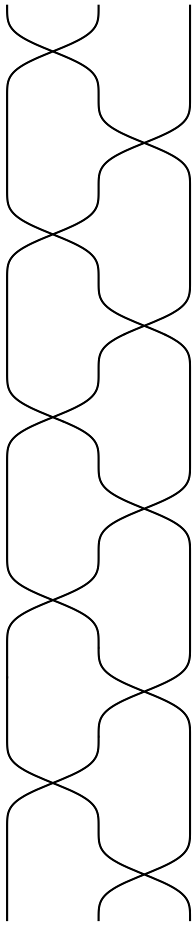
N1.2.1.2 [1,2,3] $\Longrightarrow$ [2,3,1]

Negation notation :

$\sigma_1^{-1}\sigma_2\sigma_1^{-1}\sigma_2^3$ **:**

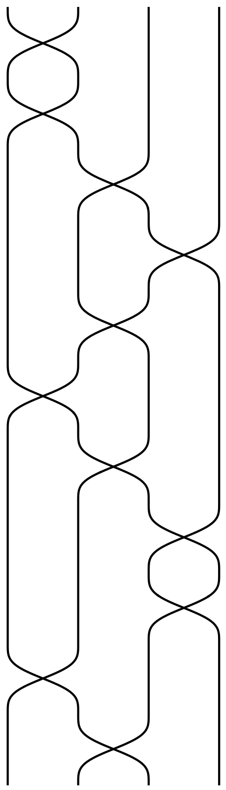
```
Nl .1 (N2 (Nl .1 (N2 (N2 (N2 (X)))))) = N1 (N2 (N1 (N2 (p)))) = N2 .1 (p)
```

```
KnotData[{10, 123}, "BraidDiagram"]
```



```
KnotData[All]
```

KnotData[{10, 165}, "BraidDiagram"]



Braid word notation

KnotData[{10, 123}, "BraidWordNotation"]

$\sigma_1\sigma_2^{-1}\sigma_1\sigma_2^{-1}\sigma_1\sigma_2^{-1}\sigma_1\sigma_2^{-1}\sigma_1\sigma_2^{-1}$

KnotData[{9, 12}, "BraidWordNotation"]

$\sigma_1\sigma_2^{-1}\sigma_1^{-2}\sigma_3\sigma_2^3\sigma_4^2\sigma_3\sigma_4^{-1}$

KnotData[{0, 1}, "BraidWordNotation"]

σ_1

KnotData[{10, 165}, "BraidWordNotation"]

$\sigma_1^2\sigma_2\sigma_3^{-1}\sigma_2\sigma_1^{-1}\sigma_2\sigma_3^2\sigma_1^{-1}\sigma_2$

KnotData[{4, 1}, "Properties"] // Short

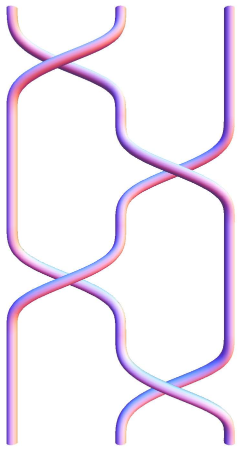
{AlexanderBriggsList, ≪62≫, UnknottingNumber}

```
KnotData[{0, 1}, "BraidImage"]
```

```
KnotData[{0, 1}, "BraidWordNotation"]
```

σ_1

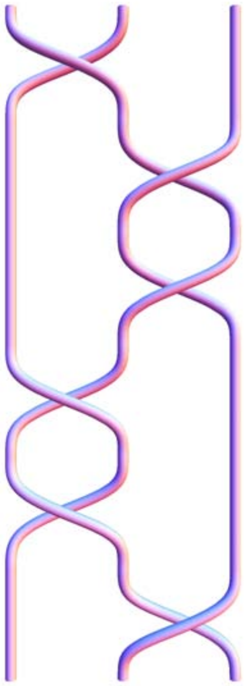
```
KnotData[{4, 1}, "BraidImage"]
```



```
KnotData[{4, 1}, "BraidWordNotation"]
```

$\sigma_1\sigma_2^{-1}\sigma_1\sigma_2^{-1}$

```
KnotData[{6, 3}, "BraidImage"]
```



[1, 2, 3] [3, 1, 2]

KnotData[{**6, 3**}, "BraidWordNotation"]

$\sigma_1^{-1}\sigma_2^2\sigma_1^{-2}\sigma_2$

Braid notation : $\sigma_1^{-1}\sigma_2^2\sigma_1^{-2}\sigma_2$
Negation notation :
$N_2\ (N_{2.1}\ (N_{2.1}\ (N_2\ (N_2\ ((N_{1.1}\ (X))))))).$