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Materials for a better exploration of the different worlds of mathematics

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Cellular automata in different worlds : CAs in Brownian worlds, CAs in Mersenne worlds, CAs in Stirling worlds

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Abstract

Handouts are for free. These handouts are intended to help teachers and parents to avoid abusive mental suppression of their children by indoctrinating them against their intuition and will to become parrots, fit for the use of their early bank accounts. And it's free! No charge, like for the Soft Start program. But there is also no guarantee included for the mental health of the applicants. They might easily become alienated.

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Categories of the RK-Archive

Kindergarten and Differences/Handouts

Materials for a better exploration of the different worlds of mathematics

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Abstract

Handouts are for free. These handouts are intended to help teachers and parents to avoid abusive mental suppression of their children by indoctrinating them against their intuition and will to become parrots, fit for the use of their early bank accounts. And it's free! No charge, like for the S*oft Start* program. But there is also no guarantee included for the mental health of the applicants. They might easily become alienated.
(work in progress, vers. 0.3, Nov. 2013)

1. Counting in 5 different worlds

1.1. What's about?

1.1.1. How did it start?

All started with the insight that the innocent question of a teacher: *"How much is 2 + 2?"* isn't as trivial as he thought. Before the child answered this simple question it returned it on another level with its own question: "Am I selling or am I buying?"

Everybody knows the games of partitions, permutations and prolongations of sequences for forms, played with shapes of different colors.

Here, 5 different ways of playing such games are introduced.

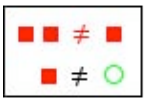
I call them the *Leibniz*, the *Pascal*, the *Brown*, the *Mersenne* and the *Stirling* games.

The differentiation of the games are defined by the different rule sets of the games.

1.1.2. The Leibniz game

The Leibniz game is defined by some strict axioms.

Classical rules



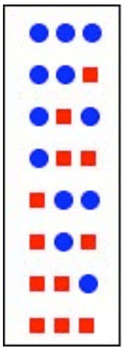
Wording

Two elements are not equal one element.
Different elements are different and not equal.

Little task

Given 2 elements and 3 places, how many different constellations of the two different elements on the 3 places are possible in the Leibniz world?

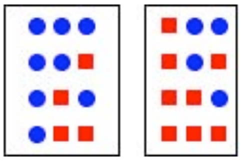
Leibniz(3,2) = 8:



Answer: The Leibnizian order for 2 elements and 3 places has 8 constellations.

Symmetry

There is also a nice symmetry between the first and the second half of the Leibniz constellations.



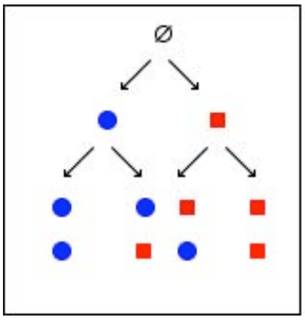
Successors

Alphabet Σ = {•}
succ(•) = • •

Alphabet Σ = {•, •}
succ(•) = {• •, • •}
succ(•) = {• •, • •}

These successors are defining a *binary* tree. With 3 elements the successors are defining a *ternary* tree.

Binary tree for Leibniz



Reversion
As easy as successions are reversions of patterns with 4 elements.
(O • • ▲ ▲) = pattern
rev(O • • ▲ ▲) = (▲ ▲ • • O) : reversion of the pattern.
Hence, rev(O • • ▲ ▲) != (O • • ▲ ▲).

1.1.3. The Pascal game

Between the Leibniz and the Brownian game with its fundamental commutativity of terms, the realm of Pascal partitions has to be placed. The Pascal game is also defined in the general system of graphematics as a *deutero*-structure.

The Pascal game is defined by some strict axioms:

Pascal rules

■ ≠ ○
■○ ≠ ○■

Wording

Different elements are not equal.

Partitions for 3 elements and 3 positions

When we look at parenthesis with order 3 there are 5 possibilities.

()()(); (())(); (()()); ()(()); ((())).

This bracket notation corresponds to the tuple notation:
[1,1,1,1], [2,1,1], [2,2], [1,3], [4].

Partitions for 4 elements and 4 positions

1+1+1+1: ()()()(): • • • • : aaaa [1,1,1,1]
1+1+2: ()() (1): O • • • : aabc [2,1,1]
(2)(2): (1)(1): O O • • : aabb [2,2]
1+3: ()(2): O • • • : abbb [3,1]
4: (3): O • • ▲ : abcd [4]

Combinatorics

In contrast, the Brownian distribution counts 4, and not 5 possibilities for n = 3 according to the immanent commutativity rule: (())() = ()(()).

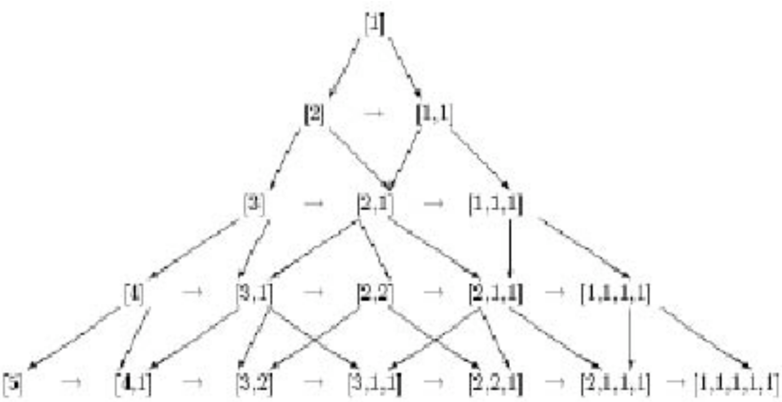
()()(); ()(()); (()()); ((())).

The general number of possibilities is calculated with the partitions of numbers P(n).

$$P(n, k) = \frac{n!}{(n-k)!}$$

n	1	2	3	4	5	6	7	8	9
$P(n)$	1	2	3	5	6	11	15	30	42

(sequence A000041 in OEIS).



Numeric Deutero – number rules

$$R0: \quad \Longrightarrow [1]$$

$$R1.1: [n] \Longrightarrow [n+1] \,\|\, [n, 1]$$

$$R1.2: [1, 1] \Longrightarrow [n+1, 1] \,\|\, [1, 1, 1]$$

$$R1.3: [n, 1] \Longrightarrow [n+1, 1] \,\|\, [n, 2] \,\|\, [n, 1, 1]$$

Examples for deutero – additions

$D = [1], E = [1]$:

$[1] +_d [1] = \{[2], [1, 1]\}$.

$D = [2], E = [1]$:

$[2] +_d [1] = \{[3], [2, 1]\}$.

$D = [2], E = [1, 1]$:

$[2] +_d [1, 1] = \{[3, 1], [2, 1, 1]\}$.

$D = [1, 1], E = [1, 1]$:

$[1, 1] +_d [1, 1] = \{[1, 1, 1, 1], [2, 1, 1]\}$.

$D = [2, 1], E = [1, 1]$:

$[2, 1] +_d [1, 1] = \{[3, 1], [2, 2], [2, 1, 1]\}$.

$D = [3, 1], E = [1]$:

$[3, 1] +_d [1] = \{[4, 1], [3, 2], [3, 1, 1]\}$.

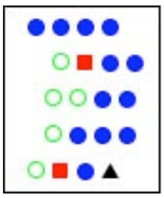
$D = [3, 2, 1], E = [1]$:

$[3, 2, 1] +_d [1] = \{[4, 2, 1], [3, 3, 1], [3, 2, 2], [3, 2, 1, 1]\}$.

Little task

Given 4 elements and 4 places, how many different constellations of the four different elements on the 4 places are possible in the Pascal world?

Pascal(4,4) = 5:



Answer: The Pascal partition order for 4 elements and 4 places has 5 constellations.

Symmetry

There is no obvious symmetry for this partition system. Symmetry is given internally for (●●●●), (○○●●) and (○■●▲).

1.1.4. The Spencer-Brown game

The basic rules for the Brownian distinction calculus

○■ = ■○
○○ ≠ ■■

Rule 1. () () = ()
Rule 2. (()) = ∅
3. Substitution rules

Wording

Rule1: A distinction of 2 distinctions is a distinction.
Rule2: A distinction of a distinction is no distinction.

In colors

Rule1. • • = •
Rule2. [■] = ∅

Especially:

(( )) ( ) = ( ) (( )) : [■] • = • [■] .
Red in red kills red, [■]=∅ and red • saves red •
equal
red • with red in red [■] kills red [■]=∅ and saves red •.
Hence, [■] • = • [■] = •.

Superpositions

(() ((()))) = [■ [[■]]]
(() ()) = [■ ■]
(()) = [■]
(()) = ∅

In words:

[■ [[■]]] : Red with red in red in red kills red and red saves red: [■ ■] .
[■ ■] : Red with red in red saves red : [■] .
[■] : Red in red kills red ∅ .

Little task

Given 2 elements and 3 places, how many different constellations of the two different elements on the 3 places are possible in the Brownian world?

Brown(3,2) = 4

●●●
●●■
●■■
■■■

Answer: The Brownian order for 2 elements and 3 positions has 4 constellations.

The number of forms (not possibilities) of degree 3 is 4 and not 5 as for the Pascal partitions. The forms are:

()()(); ()(()); (()()); ((()))

Normal form

The written table is accepting the normal form for Brownian constellations. Hence, the two constellations ●■●, ■ ●● are represented by the single constellation ●●■ in Brownian normal form.

Symmetry

In contrast to the Pascal world, there is a nice symmetry between the first and the second half of the patterns based.on the commutativity of the forms.

●●●	●■■
●●■	■■■

Successor

Alphabet Σ = {•, •}
succ(•) = {(• •), (• •), (• •) }

Addition Sum

sum (•, ∅) = •
sum(• •, •) = {• • •, • • •}.
sum(• •, •) = {• • •}
sum(• •, •) = {• • •}.

Reversion for Brownian patterns

rev(• •) = (• •) and (• •) = (• •).

1.1.5. The Mersenne game

The basic rules of the calculus of differentiations

○■ ≠ ■○
○○ = ■■

Rule 1. () () = ∅
Rule 2. (()) = ()
3. Substitution rules

Wording

Rule1: A differentation between 2 differentiations is an absence of a differentiation.
Rule2: A differentiation of a differentiation is a differentiation.

In colors

Rule1. • • = ∅
Rule2. ▣ = •

Other wording

Blue with blue kills blue.
Blue in blue saves blue.
The rules are also well understood as oriented actions.
Rule1. • • = ∅ is an equational notation for to corresponding actions:
Rule1a. • • ==> ∅ and
Rule1b. • • <== ∅
Rule2. ▣ = • this also holds for Rule2
Rule2a ▣ ==> •
Rule2b. ▣ <== •

Some examples

1. ()()() = (()(())(()) : the same are the same, thus there is no differentiation.
• • • = ▣ ▣ ▣
• • • = •

▣ = •
Thus, • • • = ▣ ▣ ▣.

2. ( ) ( ) () = () : rule1 : • • • = •
(( )) ( ) = ∅ : rule2, rule1 : ▣ • = ∅
((())) = () : rule2 : [▣] = •

Especially
3. (( )) = ( ) () () : ▣ = • • •
Proof of ▣ = • • •
[• •] • : brackets
[∅] • : rule1
• : rule1.
▣
• : rule2

Wording

In a Mersenne universe, the order of 2 different elements is relevant. In contrast to the Brownian universe, they are therefore different.
But a constellation of two same elements is equal to another constellation of two same elements.

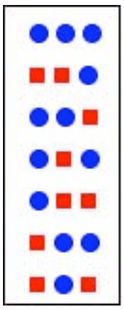
Alternative wording

Red and green together are safe.
Two greens together are killed by two reds.

Little task

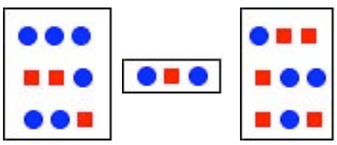
Given 2 elements and 3 places, how many different *situations* of the two different elements on the 3 places are possible in the Mersennian world?

Mersenne(3,2) = 7



Answer: The Mersenne order for 2 elements and 3 positions includes 7 situations.

Symmetry
The nice symmetry of the whole set as we have seen for Leibniz and Brown is broken.



Successor

Alphabet Σ = {•, •}
succ(●) = {(● ●), (● ■), (■ ●)}.

Addition Sum

sum(●, ∅) = ●
sum(● ●, ●) = {● ● ●, ● ● ■, ■ ■ ●}

sum(● ■, ●) = {● ■ ●, ● ■ ■}
sum(■ ●, ●) = {■ ● ●, ■ ● ■}.

Reversion for Mersenne

rev(ab) = (ba) and (ab) != (ba)
rev(● ■) = (■ ●) and (● ■) != (■ ●).

Comparison Brown and Mersenne

Interestingly, there are some coincidences between both calculi. Both are deducing form the 3 brackets one resulting bracket: ( ) ( ) () = ().
But the way they are doing it is differently organized according to the 2 different rule sets.
It is a common failure to not to recognize this crucial difference.

Mersenne : ( ) ( ) () = () :
by rule1 :
• • • = • :
(• •) • = (∅) • = •
(• (• •) = • (∅) = •
Hence, • • • = • .

Brown: ( ) ( ) () = () :
by rule1 :
• (• •) = • (•) = •
(• •) • = (•) • = •
Hence, • • • = •.

In **contrast**:
[■] = • • •
[■] = •
• • • = •

Hence, [■] = • • •.
[■] != • • •
[■] = ∅
• • • = •
Thus, ∅ != •.

1.1.6. The Stirling game

A Stirling blend

For a Stirling approach, the fact that the concept of *patterns,* i.e. ordered strings or configurations of identity-free elements, is crucial, leads to the following rules.

Those rules shall be understood as a *blend* of Brownian, Rule3, and Mersennian, Rule2, rules. A blend always produces also something new: Rule1 and Rule4.

Rule1. () = (())
Rule2. () () = (()) (())
Rule3. () (()) = (()) ()
Rule4. ()()(()) != ()(())() != ()(())(()) != ()(())((())).

In colors
Rule1. ● = ■
Rule2. ● ● = ■ ■
Rule3. ● ■ = ■ ●
Rule4. ● ● ■ != ● ■ ● != ● ■ ■ != ● ■ ○.

Another setting:
Rule1. ● ≡ [●]

Rule2. ● ● = [●] [●]
Rule3. ● [●] = [●] ●
Rule4. ● ● [●] != ● [●] ● != ● [●] [●] != ● [●] [[●]] .

Wordings of constellations

For a Stirlingian game with 3 elements, some typical situations occur.

1. • • • ≡ ■ ■ ■ ≡ ○ ○ ○,
   ● ● ■ ≡ ■ ■ ● ≡ ○ ○ ●
   etcetera
2. ● ● ■ ≡ rev(● ■ ■) : reversion
3. ● ■ ● ≡ rev(● ■ ●) : self-symmetry
   ● ● ● ≡ rev(● ● ●)|
   ● ■ ○ ≡ rev(● ■ ○)

Wordings of rules for Stirling(3,3)

● ≡ ■: Blue kills red.
● ■ ≡ ■ ●: Blue together with red kills red together with blue.
● ● ■: Two blue together with one red, and
● ■ ●: one blue together with one red and one blue, and
● ■ ■: one blue with two reds, are safe in the Stirlingian world.
● ■ ○: As well as blue and red and green together.
This constitutes a kind of safety in groups.

Little task

Given 3 elements and 3 places, how many different *patterns* (morphograms) of the three different elements on the 3 places are possible in the Stirlingian world?

Sn (3,3) = 5

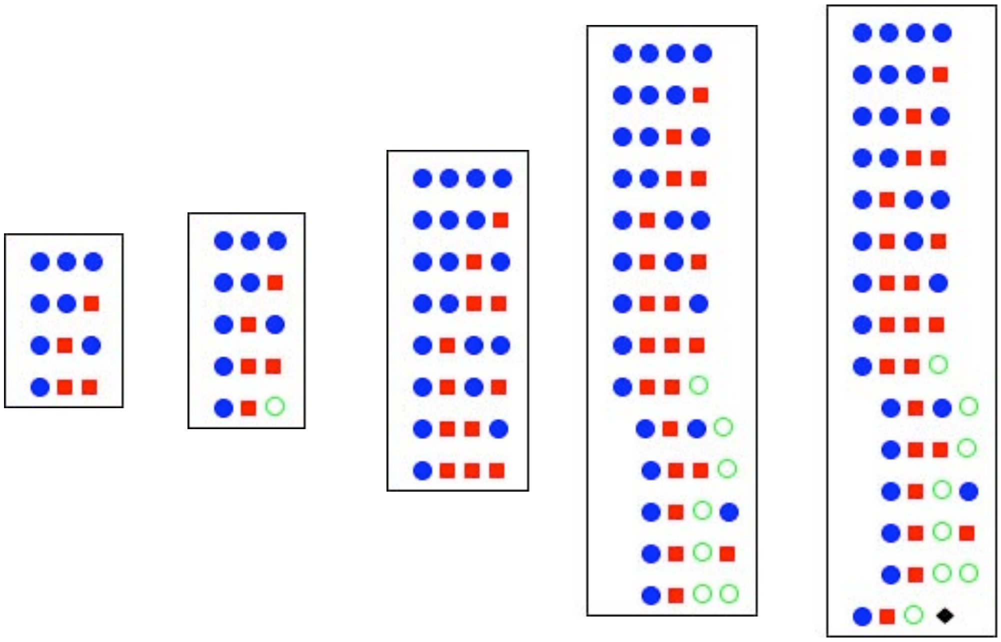
numeric *symbolic alphabetic*
$[1^3]$: • • • : aaa
$[1^2 2^1]$: • • • : aab
$[1^1 2^1 1^1]$: • • • : aba
$[1^1 2^2]$: • • • : abb
$[1^1 2^1 3^1]$: • • O : abc

Answer: The Stirling order for 3 elements and 3 positions for distribution is 5.

Hence, there are 5 different morphograms for 3 elements and 3 positions. The choice of the color of the elements, here as blue, red and green is arbitrary and ruled by its normal form.

Further tasks for more complex situations

Sn (3,2) = 4 **Sn** (3,3) = 5 **Sn** (4,2) = 8 **Sn** (4,3) = 14 **Sn** (4,4) = 15



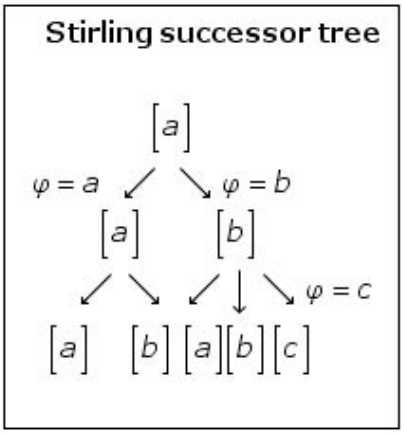
Symmetry

Here, again, the symmetry of the set of the basic patterns is broken.
But there are some nice internal symmetries left.
rev(●●■) = (●■■), that is rev(●●■) = (■ ●●) but (■ ●●) = (●■■).
Self-symmetric patterns: (●●●), (●■●), (●■○).

Successors, addition and multiplication

The function ϕ is iterative if it repeats a given element, and accretive if it adds a new element. There is no recurs to a pre-given alphabet necessary. The successor operation is recurring retrograde to the predecessor elements and iterates the produced elements iteratively and adds accretively a new element to the system.

Resulting in the production of the 5 trito-patterns with 3 elements:
[aaa], [aab], [aba], [abb], [abc].



Successors in colors

$$\mathrm{succ}(\bullet) = \begin{pmatrix} \bullet\,\bullet \\ \bullet\,\blacksquare \end{pmatrix}$$

$$\mathrm{succ}(\bullet\,\bullet\,\bullet) = \begin{pmatrix} \bullet\,\bullet\,\bullet\,\bullet \\ \bullet\,\bullet\,\bullet\,\blacksquare \end{pmatrix}$$

$$\mathrm{succ}(\bullet\,\bullet\,\blacksquare) = \begin{pmatrix} \bullet\,\bullet\,\blacksquare\,\bullet \\ \bullet\,\bullet\,\blacksquare\,\blacksquare \\ \bullet\,\bullet\,\blacksquare\,\circ \end{pmatrix}$$

Null

$$\mathrm{succ}(\bullet\,\blacksquare\,\bullet) = \begin{pmatrix} \bullet\,\blacksquare\,\bullet\,\bullet \\ \bullet\,\blacksquare\,\bullet\,\blacksquare \\ \bullet\,\blacksquare\,\bullet\,\circ \end{pmatrix}$$

$$\mathrm{succ}(\bullet\,\blacksquare\,\blacksquare) = \begin{pmatrix} \bullet\,\blacksquare\,\blacksquare\,\bullet \\ \bullet\,\blacksquare\,\blacksquare\,\blacksquare \\ \bullet\,\blacksquare\,\blacksquare\,\circ \end{pmatrix}$$

$$\mathrm{succ}(\bullet\,\blacksquare\,\circ) = \begin{pmatrix} \bullet\,\blacksquare\,\circ\,\bullet \\ \bullet\,\blacksquare\,\circ\,\blacksquare \\ \bullet\,\blacksquare\,\circ\,\circ \\ \bullet\,\blacksquare\,\circ\,\blacklozenge \end{pmatrix}$$

Left successor

Example

$$\mathrm{l-succ}(\bullet\,\blacksquare\,\blacksquare) = \begin{pmatrix} \bullet\,\bullet\,\blacksquare\,\blacksquare \\ \blacksquare\,\bullet\,\blacksquare\,\blacksquare \\ \circ\,\bullet\,\blacksquare\,\blacksquare \end{pmatrix} = \begin{pmatrix} \bullet\,\bullet\,\blacksquare\,\blacksquare \\ \bullet\,\blacksquare\,\bullet\,\bullet \\ \bullet\,\blacksquare\,\circ\,\circ \end{pmatrix}$$

$$\mathrm{succ}(\bullet\,\blacksquare\,\blacksquare) \neq \mathrm{l-succ}(\bullet\,\blacksquare\,\blacksquare)$$

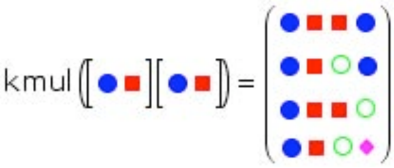
Null

left – succ	pattern	right – succ
$\bullet\,\bullet\,\blacksquare\,\bullet$	$\boxed{\bullet\,\blacksquare\,\bullet}$	$\bullet\,\blacksquare\,\bullet\,\bullet$
$\bullet\,\blacksquare\,\bullet\,\blacksquare$	–	$\bullet\,\blacksquare\,\bullet\,\blacksquare$
$\bullet\,\blacksquare\,\circ\,\blacksquare$	–	$\bullet\,\blacksquare\,\bullet\,\circ$

Addition

$$\mathrm{add}(\bullet\,\bullet,\ \bullet\,\blacksquare) = \begin{pmatrix} \bullet\,\bullet\,\bullet\,\blacksquare \\ \bullet\,\bullet\,\blacksquare\,\bullet \\ \bullet\,\bullet\,\blacksquare\,\circ \end{pmatrix}$$

Multiplication

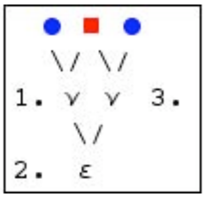


Difference notation for morphograms

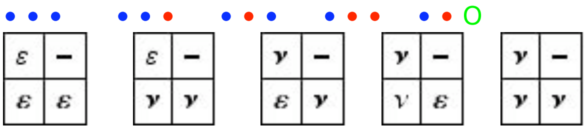
Difference notation with v=non-equal and ε=equal

The fact that the presentation of the morphograms by specific elements is arbitrary has to be considered as crucial. Therefore, not the elements are determining the morphic patterns but the differences between the elements.

This is well depicted for the example [● ■ ●].



A useful notation is given with the *matrix* of the ε/v-structures.



Comparison for systems with 2 elements an 3 places.

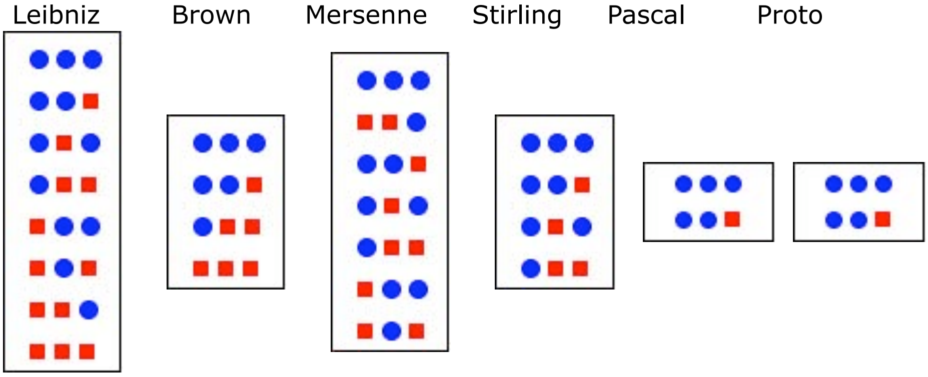
Little task in different worlds

Teacher: Given 2 elements and 3 places, how many different *partitions* of the three different elements on the 3 places are possible?

Child: In which world should the partition happen? In the Leibniz, the Pascal, the Brown, the Mersenne or the Stirling world? Or do you offer some others too?

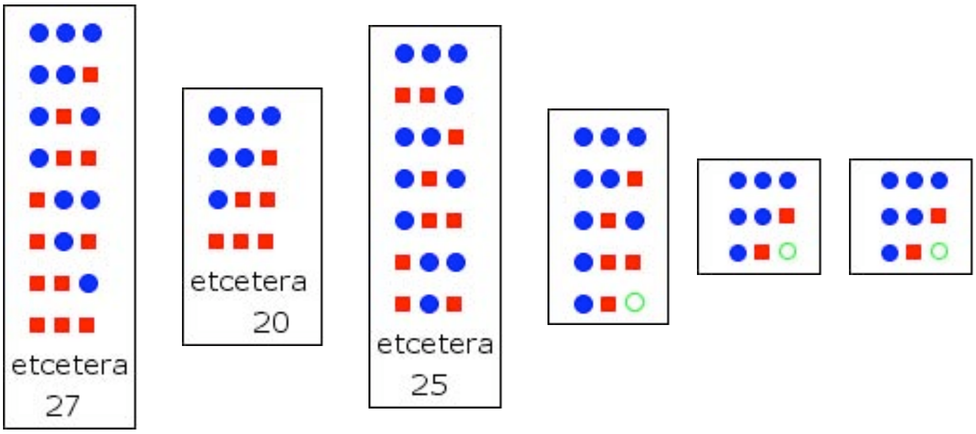
The teachers task can be correctly answered by at least 5 different solutions.

Leibniz Brown Mersenne Stirling Pascal Proto



There is a coincidence in the numbers of Brownian and Stirlingian tables for Sys(3,2). Also Pascal and proto-structures are coinciding on this level, i.e. Sys(3,2) and Sys(3,3).

Leibniz Brown Mersenne Stirling Pascal Proto



Crucial differences

Not just the answers to the little tasks depends on the world model used but also the structure of the simples operations, like succession, addition, reversion, etcetera, differ significantly.

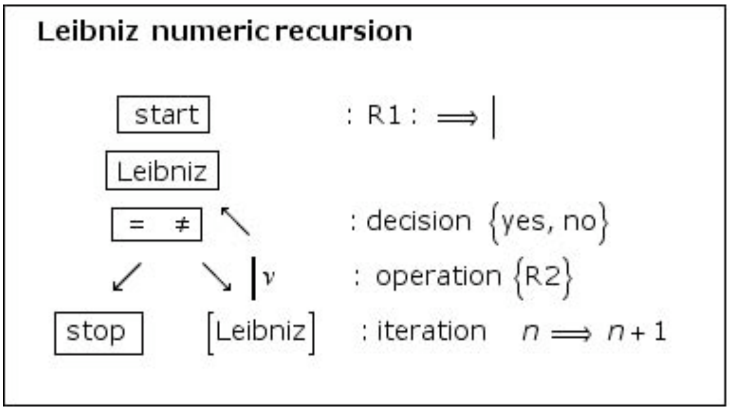
Leibniz world

The main model for succession is given by the Stroke calculus as it is fundamental for the Leibniz world.

Stroke calculus

Rule1. ==> |
Rule2. n ==> n |
Meta-Rule3. n ∊ Var, repetition of Rule2.

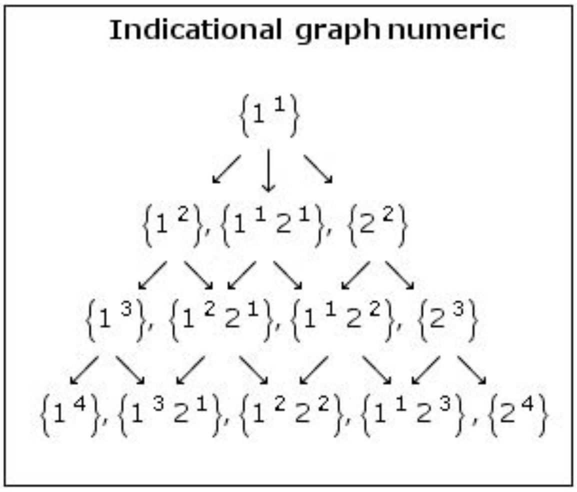
The main feature is the abstract concatenation of an atomic element to the just produced strokes, represented by n. Thus, Rule2. n ==> n |.



This simple feature differs depending on the world model.

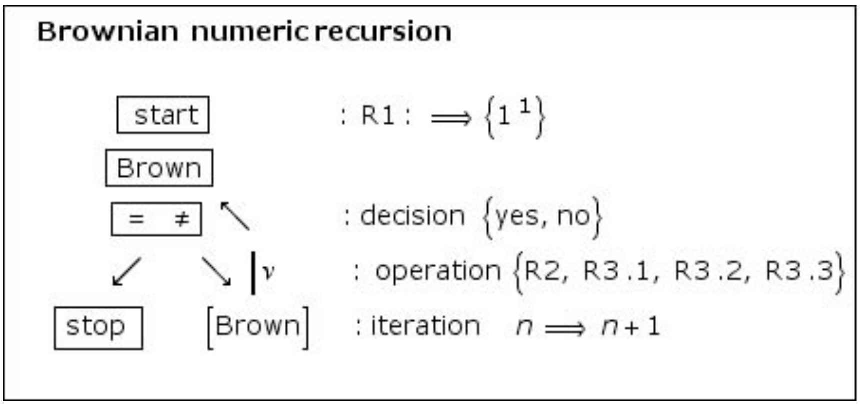
Brownian world

For the Brownian model, the successor operation is still in the spirit of the Leibnizian successor but modified by the specific Brownian features of commutativity.



Numeric indicational rules

$$\begin{aligned}
&R1: && \Longrightarrow \{1^1\}\\
&R2: \ \{1^1\} && \Longrightarrow \{1^2\} \mid \{1^1 2^1\} \mid \{2^2\}\\
&R3.1: \{1^n\} && \Longrightarrow \{1^{n+1}\} \mid \{1^n 2^1\}\\
&R3.2: \{2^n\} && \Longrightarrow \{2^{n+1}\} \mid \{1^1 2^n\}\\
&R3.3: \{1^n 2^n\} && \Longrightarrow \{1^{n+1} 2^n\} \mid \{1^n 2^{n+1}\}
\end{aligned}$$

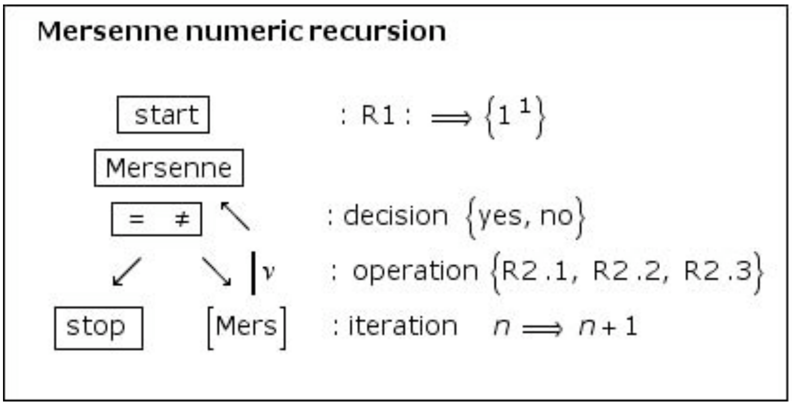


Mersennian world

For the Mersennian model, the successor operation is still in the spirit of the Leibnizian successor but modified by the specific Mersennian features of non-commutativity.

Numeric Mersenne rules

$$\begin{aligned}
&R1: && \Longrightarrow \{1^1\}\\
&R2.1: \{1^n\} && \Longrightarrow \{1^{n+1}\} \mid \{2^n 1^1\} \mid \{1^n 2^1\}\\
&R2.2: \{1^n 2^n\} && \Longrightarrow \{1^n 2^n 1^1\} \mid \{1^n 2^{n+1}\}\\
&R2.3: \{2^n 1^n\} && \Longrightarrow \{2^n 1^{n+1}\} \mid \{2^n 1^n 2^1\}
\end{aligned}$$

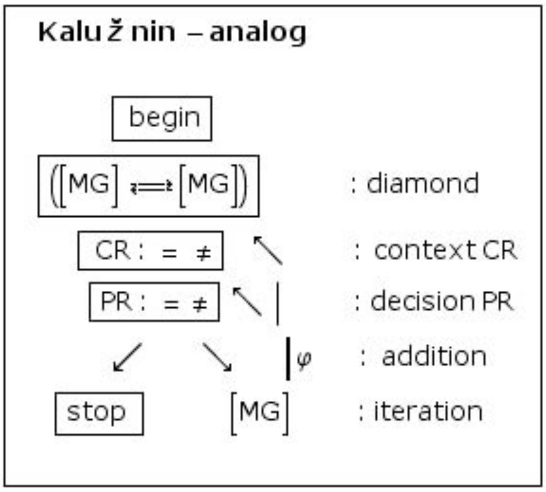


Stirlingian world

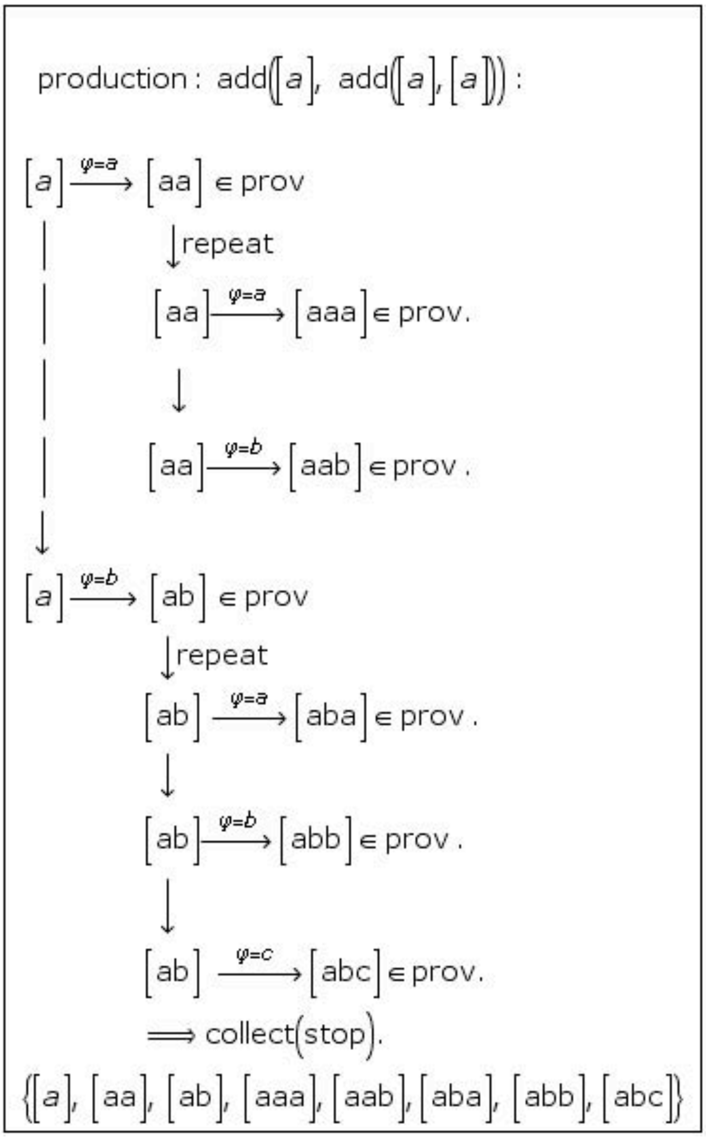
For the Stirlingian model, the successor operation is not anymore in the spirit of the Leibnizian successor.

The Stirlingian successor operation is defined by the feature of *"retro-grade" recursivity.*

Hence, the retrograde recursion gets a meta-rule which controls the successor-procedure in respect of the structure of the added morphogram.



Retrograde recursion example of *a* Stirling addition



1.2. Palindromes all over the worlds

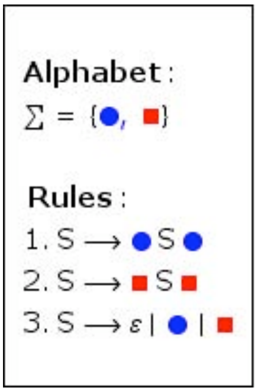
1.2.1. Leibniz' palindromes

The rules for building classical palindromes are easy to understand. If we add to a given element the same additional element on the *right* and on the *left* side, we get a palindrome:

For an alphabet Σ = {•, •} we get:
• ==> • • •, • • •
• ==> • • •, • • •.

Leibniz palindromes are standard and well studied.

Instead of demonstrating many examples, the *production* rules are defining how to produce palindromes. They are given with this little grammar, consiting on an alphabet and a set of production rules.



Odd and even palindromes

Odd palindrome

With rule 3 we introduce a start token, say ●. Now S is ●, and ● is palindrome.
Apply rule1 to S: S --> ● S ●. Now S is ● ● ●, is palindrome,
Apply rule2 to S: S --> ■(● S ●) ■. Now S is ■ ● ● ● ■, is palindrome.
Apply rule1 to S: S --> ● S ●. Now S is ● ■ ● ● ● ■ ●, is palindrome.
And so on.

The order of the application of the rules rule1 and rule2 is free. The result is always symmetric, and therefore a palindrome. There are no surprises included in this parcel.

Even palindrome

A more interesting example is given with (O • • ▲ ▲ • • O).
The alphabet is: ∑ = {●, ■, ○, ▲} and a new
rule4: S --> ▲ S ▲,
rule5: S --> O S O.

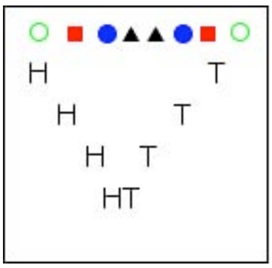
With rule 3 we introduce a start with the empty token ε. Now S is ε, ε is a nil-palindrome.
rule4: S --> ▲ S ▲ ,
rule1: ▲ S ▲ --> ● (▲ S ▲) ●,
rule2: ● ▲ S ▲ ● --> • (● ▲ S ▲ ●) • ,
rule5: • (● ▲ S ▲ ●) • --> O (• (● ▲ S ▲ ●) •) O.

With rule3 we replace S by ε : thus we get the palindrome: O • • ▲ ▲ • • O.
The same holds here. Free application of the rules, and no surprise in the box.

A simple palindromy checker for Leibniz palindromes

The little palindrome production rules are producing palindromes. The same rules applied backwards to an arbitray string lets easily decide if the string is a palindrome or not.

This is easily realized with two palyers, the *head* (H) and the *tail* (T) manager, deciding the equality or non-equality of their states in respect to the positions.



1.2.2. George Spencer Brown's palindromes

Partition based palindromes

Brownian constellations are order-free, i.e. their elements are commutative, and are allowed

to change position. Hence, Brownian palindromes are free under permutation.
This commutativity poses a problem for a proper definition of Brownian palindromes.
Brownian constellations are mathematically heap, i.e. multi-sets, and not linear sequences.

Therefore, can a multi-set be palindromic?

A solution is given with the application of the Brownian standard normal form, *bnf*, of constellations.

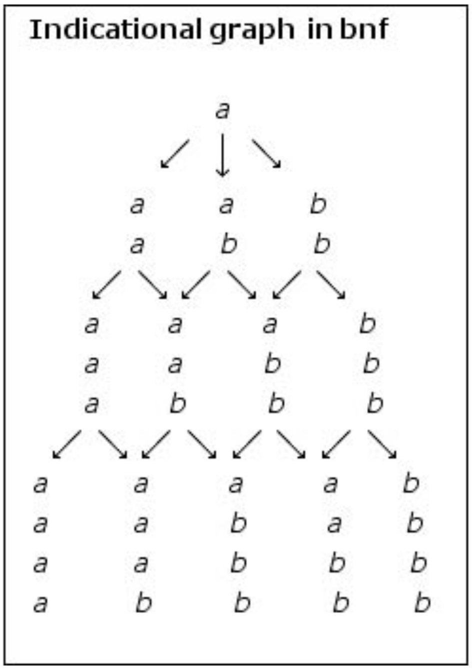
rev(●■) = (■●) and (●■) = (■●).

The constellation (●■) is a palindrome because of its commutatiity.

Is the accepted Brownian constellation [• • •] a palindrome?

Because of the standard normal form convention for Brownian constellations, *bnf*, we know that [• • •] $=_{\text{Brown}}$ [• • •]. But, the pattern [• • •] is palindromic.

That is, the standard form constellation [• • •] represents the set of equivalent constellations
{[• • •], [• • •]}.

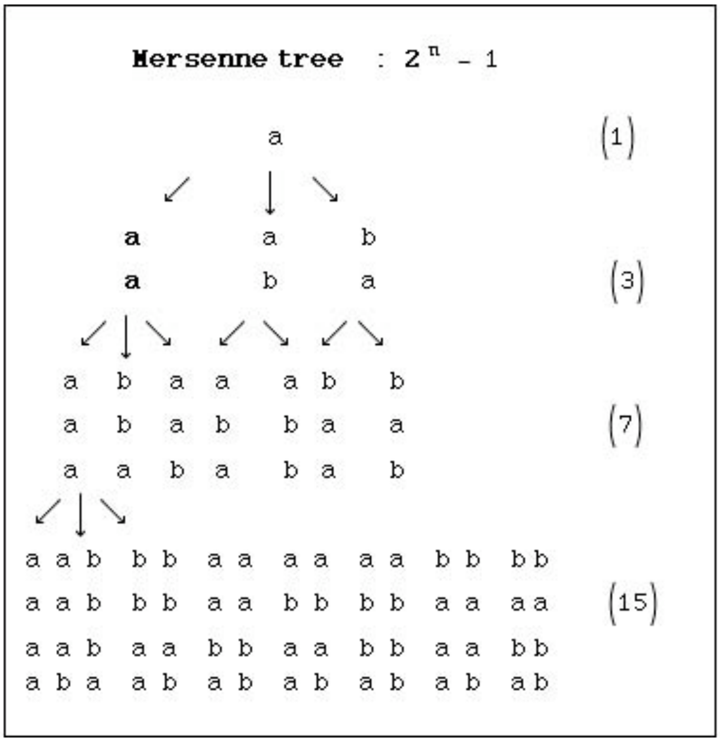


Brownian palindromes

$\text{Pal}_{\text{Brown}}$ (4) = {(aa), (bb); (aaa), (bbb); (aaaa), (bbbb)}.

1.2.3. Mersenne's palindromes

Mersenne situations are ordered sequences. They can be read forwards and backwards, hence they might be palindromic. Mersenne's palindromes are similar to Leibniz palindromes with the crucial difference that homogeneous sequences are equal, i.e. • • = ■ ■.
rev(●■) = (■●) and (●■) != (■●).



Mersennian palindromes

Pal_{Mers} (4,2) = {(aa); (aaa), (aba), (bab); (aaaa), (abba), (baab)}

1.2.4. Stirling's palindromes

Stirling palindromes had been well studied recently.

Palindrome grammar – production

Rule1 : $[P] \Longrightarrow [w1Pw2]$

Rule2 : $[P] \Longrightarrow [w2Pw1]$

Rule3 : $[P] \Longrightarrow [w3Pw3]$

Rule4 : $[P] \Longrightarrow [w3Pw4]$.

Rule5 : [if length w odd]

$[P] \Longrightarrow wM[P]wM$

Defs

$P = [w] = [w1\ w2]$

$w3 = add(|w1|, 1)$

$w4 = add(|w3|, 1) = add(add(|w2|, 1), 1)$.

$wM = middleElement(w)$

Production examples for even palindromes

P: w1!=w2: [w1=●, w2=■]: P = [●,■]
P: w1=w2: [w1=●, w2=●]: P = [●,●].

$Pal_{Stirling}(4)$:

	rules	results	
P = [●,●]:	w1Pw2	: [●,●,●,●]	; rule1(=rule2)
	w3Pw3	: [■,●,●,■]	; rule3
	w3Pw4	: [■,●,●,○]	; rule4
P = [●,■]:	w1Pw2	: [●,●,■,■]	; rule1
	w2Pw1	: [■,●,■,●]	; rule2
	w3Pw3	: [○,●,■,○]	; rule3
	w3Pw4	: [○,●,■, ◆]	; rule4

Quite obviously, a pattern like [○ ● ■ ◆] doesn't read forwards and backwards the same in a Leibniz world. But read as a deep-structural pattern of differences it does. Hence, the pattern is a Stirlingian palindrome.

Test

The difference-structure of [○ ● ■ ◆] is:

v	–	–
v	v	–
v	v	v

This matrix is obviously symmetric. Thus, it represents a palindrome.
The same holds for the next example [○ ● ■ ○]:

v	–	–
v	v	–
e	v	v

.

Test

- ENstructureEN[1,2,3,1];
val it = [[],[N],[N,N],[E,N,N]] : EN list list
- ENstructureEN[1,3,2,1];
val it = [[],[N],[N,N],[E,N,N]] : EN list list

Matrix comparison

[1,2,3,1]

v	–	–
v	v	–
e	v	v

[1,3,2,1]

v	–	–
v	v	–
e	v	v

Therefore, the pattern (● ■ ○ ●) is palindromic.

Bisymmetric examples

Bisymmetric examples

Is the pattern [• • • O • O] a palindrome?

Reversion method

The naive method deals with the pattern as they are perceived and not with the differences that are not perceived but recognized by analysis. Hence the inversion of the pattern [• • • O • O] is the pattern [O • O • • •]. Both are differential symmetric and the matrix of the differences are equal. Hence the patterns are palindromic.

ENstructureEN([• • • O • O]) = ENstructureEN(rev([• • • O • O])).

But with this approach we are not dealing with the differences as our primary objects but with the patterns with their arbitrary elements.

(a) =[• • • O • O] rev(a) = [O • O • • •]

v	–	–	–	–
e	v	–	–	–
v	v	v	–	–
v	**e**	v	v	–
v	v	v	**e**	v

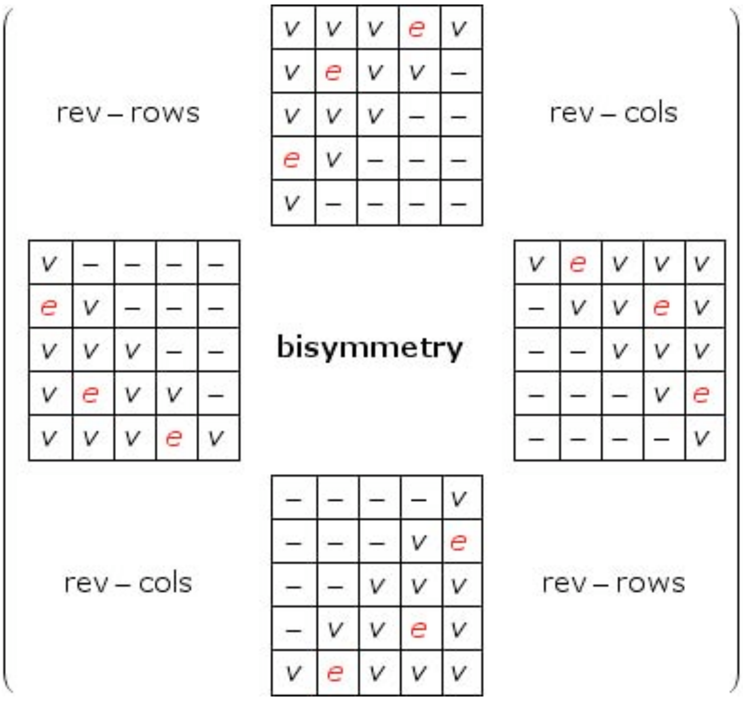
v	–	–	–	–
e	v	–	–	–
v	v	v	–	–
v	**e**	v	v	–
v	v	v	**e**	v

Bisymmetry method

Morphograms are not dealing with the identity of their elements, but with the pattern defined by the differences between the elements only. Therefore we have to apply a different method. This method is focusing on the differences they are notified with the matrix only.

The method is called bilateral symmetry, in short: **bisymmetry**.

ENstructureEN([• • • O • O]) $=_{bisym}$ rev(ENstructureEN([• • • O • O])).



Examples for Stirling palindromes

Palindromes pal(7,7):

[1,1,1,1,1,1,1],[1,1,1,2,1,1,1],[1,1,1,2,3,3,3],[1,1,2,1,2,1,1],
[1,1,2,1,3,1,1],[1,1,2,2,2,1,1],[1,1,2,2,2,3,3],[1,1,2,3,1,2,2],
[1,1,2,3,2,1,1],[1,1,2,3,2,4,4],[1,1,2,3,4,1,1],[1,1,2,3,4,5,5],
[1,2,1,1,1,2,1],[1,2,1,1,1,3,1],[1,2,1,2,1,2,1],[1,2,1,2,3,2,3],
[1,2,1,3,1,2,1],[1,2,1,3,1,4,1],[1,2,1,3,2,1,2],[1,2,1,3,4,2,4],
[1,2,1,3,4,5,4],[1,2,2,1,2,2,1],[1,2,2,1,3,3,1],[1,2,2,2,2,2,1],
[1,2,2,2,2,2,3],[1,2,2,3,1,1,2],[1,2,2,3,2,2,1],[1,2,2,3,2,2,4],
[1,2,2,3,4,4,1],[1,2,2,3,4,4,5],[1,2,3,1,2,3,1],[1,2,3,1,3,2,1],
[1,2,3,1,3,4,1],[1,2,3,1,4,2,1],[1,2,3,1,4,5,1],[1,2,3,2,1,2,3],
[1,2,3,2,3,2,1],[1,2,3,2,3,2,4],[1,2,3,2,4,2,1],[1,2,3,2,4,2,5],
[1,2,3,3,3,1,2],[1,2,3,3,3,2,1],[1,2,3,3,3,2,4],[1,2,3,3,3,4,1],
[1,2,3,3,3,4,5],[1,2,3,4,1,2,3],[1,2,3,4,1,5,3],[1,2,3,4,2,3,1],
[1,2,3,4,2,3,5],[1,2,3,4,3,1,2],[1,2,3,4,3,2,1],[1,2,3,4,3,2,5],
[1,2,3,4,3,5,1],[1,2,3,4,3,5,6],[1,2,3,4,5,1,2],[1,2,3,4,5,2,1],
[1,2,3,4,5,2,6],[1,2,3,4,5,6,1],[1,2,3,4,5,6,**7**].

1.3. Logics in different worlds

1.3.1. Logic in a Brownian world

Boolean domain : B = {true, false}.

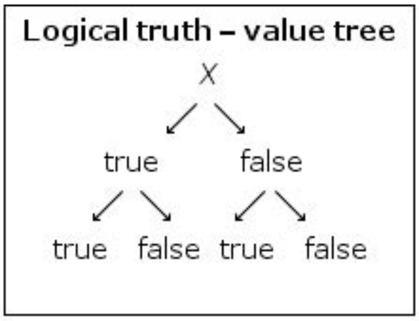
Brownian domain : CI = {⏋, ⏋⏋}.

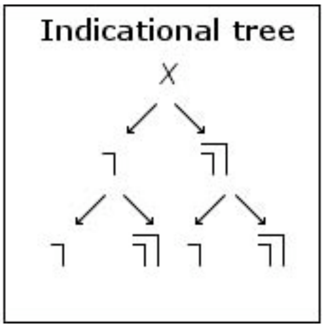
Classical interpretation:

Log(CI) = {true ≡⏋, false ≡ ⏋⏋}

"Here is how we shall model elementary logic using Laws of Form. We shall take the marked state for the value T *(true) and the unmarked state for the value* F*(false). We take* NOT *as the operation of enclosure by the mark."* (I. Kauffman)

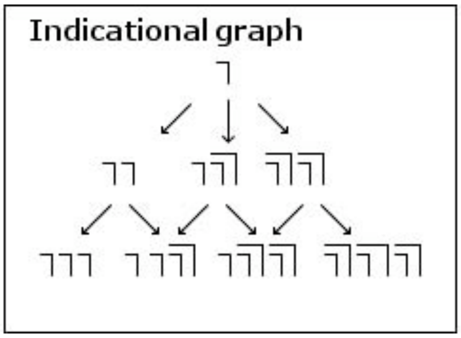
Log(CI) = {true ≡⏋, false ≡ ⏋⏋}

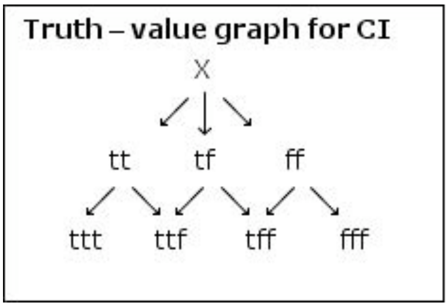


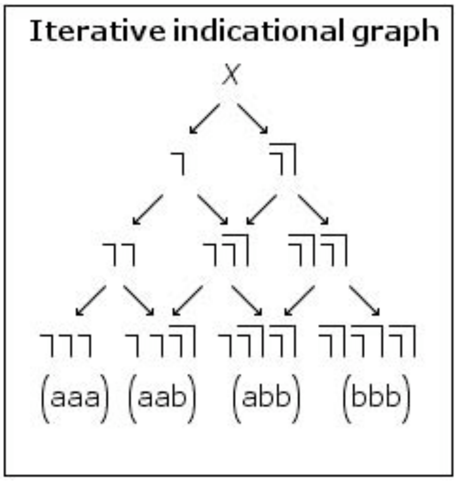


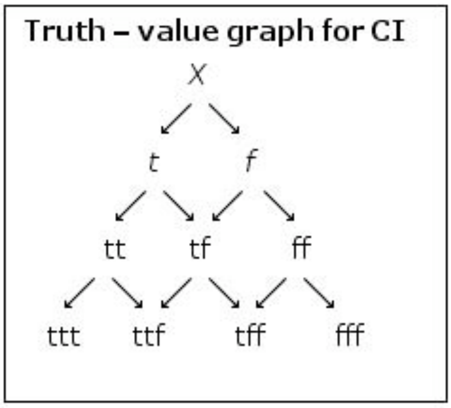
Graphematical interpretation

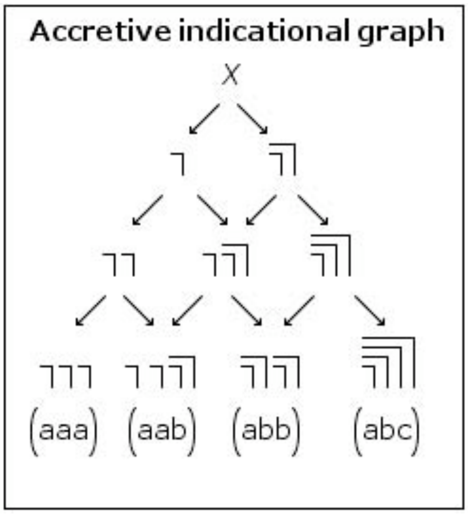
The graphematical interpretation of the Brownian calculus is emphasizing the crucial fact of its intrinsic commutativity.











Semantics of the indicational domain

val({aa, ab, bb}) = {tt, tf, ff}

val(aa) = (tt)
val(ab) = (tf)
val(bb) = (ff).

Negation

non(tt) = ff
non(tf) = tf, because (ab) $=_{\mathbf{Ind}}$(ba)

X	neg
tt	ff
tf	tf
ff	tt

Statement: Negation in Brown is inversion (negation).

Hence, the conjunction of a true Brownian statement and its negation is a contradictional statement.

Numerical truth-values

num(tt) = (1)
num(tf) = (2)
num(ff) = (3).

non(1, 2, 3) = (3, 2, 1)

Conjunction

(tt) (tt) --> (tt)

```
(tt)  (tf)  -->  (tf)
(tt)  (ff)  -->  (ff)

(tf)  (tt)  -->  (tf)
(tf)  (tf)  -->  (tf)
(tf)  (ff)  -->  (ff)

(ff)  (tt)  -->  (ff)
(ff)  (tf)  -->  (ff)
(ff)  (ff)  -->  (ff)
```

Truth-tables:

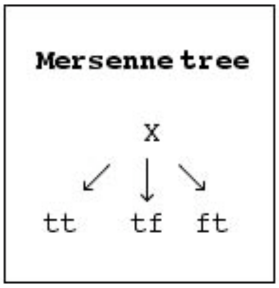
conj	tt	tf	ff
tt	tt	tf	ff
tf	tf	tf	ff
ff	ff	ff	ff

conj	1	2	3
1	1	2	3
2	2	2	3
3	3	3	3

1.3.2. Logic in a Mersennian world

The *distinctional* domain of the Mersenne calculus is trichotomic too:

Mers = {tt, tf, ft}.



Negation

X	neg
tt	tt
tf	ft
ft	tf

Statement: Negation in Mers is permutation.

Hence, the conjunction of a true Mersenne statement and its negation remains a true statement.

Conjunctiom

num({tt, tf, ft}) = (1,2,3)

```
non(1,2,3) = (1, 3, 2)
```

conj	1	2	3
1	1	2	3
2	2	2	3
3	3	3	3

conj	tt	tf	ft
tt	tt	tf	ft
tf	tf	tf	ft
ft	ft	ft	ft

1.3.3. Comparisons

Blending matrices together

conj Brown/Mers	1	2	3	4
1	1	2	3	4
2	2	2	3	4
3	3	3	3	–
4	4	4	–	4

red: Brown ∩ Mersenne
blue: Mersenne \ {red, green}
green: Brown \ {red, blue}
Semiotics: Brown ∪ Mersenne

Comparision of truth-tables

conj Brown	1	2	–	4
1	1	2	–	4
2	2	2	–	4
–	–	–	–	–
4	4	4	–	4

conj Mers	1	2	3	–
1	1	2	3	–
2	2	2	3	–
3	3	3	3	–
–	–	–	–	–

standardized:

conj Brown	1	2	3
1	1	2	3
2	2	2	3
3	3	3	3

conj Mers	1	2	3
1	1	2	3
2	2	2	3
3	3	3	3

Proof-theory and tableaux calculi

For *Mersenne* calculi a branch of the tableau terminates if the same formulas contains the signatures (tf) and (ft).
For *Brown* calculi a branch of the tableau terminates if the same formula contains the signatures (tt) and (ff).

1.4. Cellular automata in different worlds

1.4.1. CAs in Brownian worlds

A further understanding of the indicational calculus is offered by the study of its dynamism, sketched as cellular automata.

For 1 D indCA :

Indicational normal form (inf) :

inf([■ □ ■]) = inf([□ ■ ■]) = [■ ■ □]

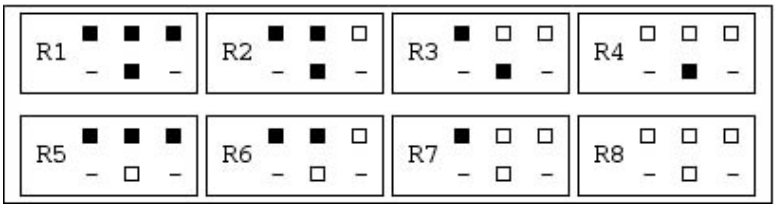
inf([□ ■ □]) = inf([□ □ ■]) = [■ □ □)]

inf([■ ■ ■]) = [■ ■ ■]

inf([□ □ □]) = [□ □ □].

perm(Head(rule)) = Head(rule).

System of indicational 1 D CA rules



Example

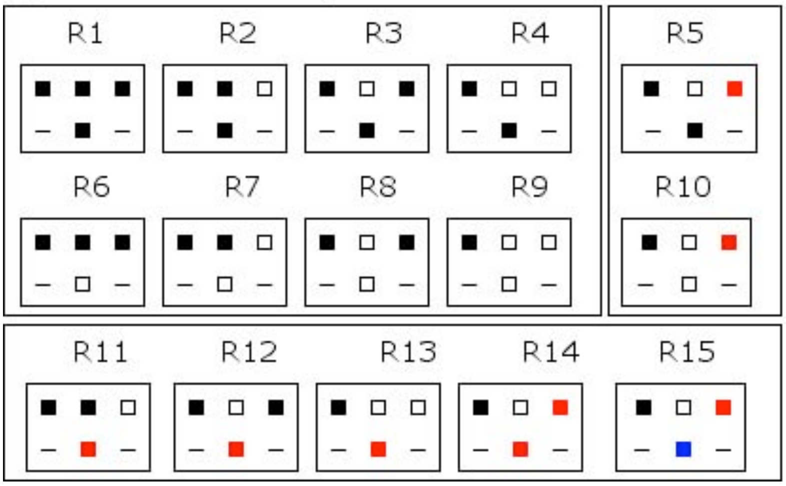
indCA, r = {5, 6, 3, 8}

Nr.I	1	2	3	4	5	6	7	8	9	rule = {5, 6, 3, 8}
1	□	□	□	□	■	□	□	□	□	3, 3, 3
2	□	□	□	■	■	■	□	□	□	3, 6, 5, 6, 3
3	□	□	■	x	x	x	■	□	□	3, 3, 3, 8, 3, 3, 3
4	□	■	■	■	x	■	■	■	□	6, 5, 5, 6, 6, 6, 5, 5, 6
5	x	x	x	x	x	x	x	x	x	stop

1.4.2. CAs in Mersenne worlds

1.4.3. CAs in Stirling worlds

System of elementary kenomic cellular rules



Homogeneous kenoCA$^{(4,2)}$ applications

Nr.I	1	2	3	4	5	6	7	8	9	rule = rule1 .2 .3 .9 : [1110]
1	□	□	□	□	■	□	□	□	□	2, 3, 9
2	□	□	□	■	■	x	□	□	□	2, 9, 2, 9, 1 (9 = x)
3	□	□	■	x	■	x	■	□	□	2, 3, 3, 3, 3, 3, 9
4	□	■	■	■	■	■	■	x	□	2, 1, 1, 1, 1, 2, 9, 1
5	■	x	■	■	■	■	■	x	■	stop

Nr.I	1	2	3	4	5	6	7	8	9	rule = rule1 .2 .8 .9[1100]
1	□	□	□	□	■	□	□	□	□	2, 8, 9
2	□	□	□	■	x	x	□	□	□	2, 8, 9, 1, 1
3	□	□	■	x	x	■	■	□	□	2, 8, 9, 2, 9, 2, 9
4	□	■	x	x	■	x	■	x	□	2, 8, 9, 2, 8, 8, 8, 9, 1
5	■	x	x	■	x	x	x	x	■	stop

Heterogeneous kenoCA$^{(4,3)}$ compositions

keno = r1.8.11.13.14 = [10222]

R1			R8			R11			R13			R14		
■	■	■	■	□	■	■	■	□	■	□	□	■	□	■
–	■	–	–	□	–	–	■	–	–	■	–	–	■	–

Application

Nr.l	1	2	3	4	**5**	6	7	8	9	rule ={1.8.11.13.14}
1	□	□	□	□	■	□	□	□	□	11, 8, 13
2	□	□	□	■	x	■	□	□	□	11, 8, 8, 8, 13
3	□	□	■	x	x	x	■	□	□	11, 8, 8, 8, 13
4	□	■	x	■	■	■	x	■	□	11, 8, 13, 1, 11, 8
5	■	x	x	■	x	■	x	x	■	11, 8, 8, 14, 8, 14, 8, 8, 13

S_1^1 = {R1, R8} = {0, 1}, $S_1^{1.2.3}$ = {R11, R13, R14} = {0, 1, 2}

Formal interpretation

⊕	S_1^1	S_1^2	S_1^3
1	■	–	■
2	–	■	■
3	–	■	■
4	x	x	–

$S_1^{1.2.3}$	1	2	3	4	**5**	6	7	8	9
1	–	–	–	–	■	–	–	–	–
2	–	–	–	■	x	■	–	–	–
3	–	–	■	x	x	x	■	–	–
4	–	■	x	■	■	■	x	■	–
5	■	x	x	■	x	■	x	x	■

⇒

S_1^1	1	2	3	4	**5**	6	7	8
1	–	–	–	–	■	–	–	–
2	–	–	–	–	x	–	–	–
3	–	–	–	x	x	x	–	–
4	–	–	x	–	■	–	x	–

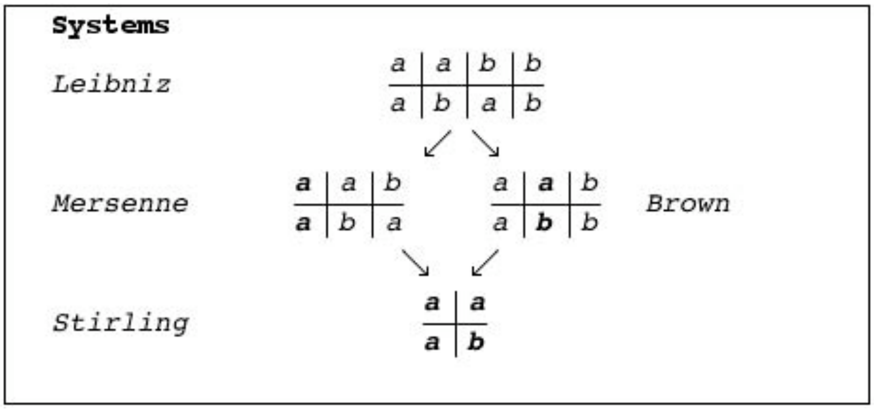
S_1^2	1	2	3	4	**5**	6	7	8	9
1	–	–	–	–	■	–	–	–	–
2	–	–	–	■	x	■	–	–	–
3	–	–	■	x	x	x	■	–	–
4	–	■	x	■	–	■	x	■	–

S_1^3	1	2	3	4	**5**	6	7	8	9
1	–	–	–	–	■	–	–	–	–
2	–	–	–	■	–	■	–	–	–
3	–	–	■	–	–	–	■	–	–
4	–	■	–	■	■	■	–	■	–

S_1^1 = {R1, R8} = {■, x} , S_1^2 = {R11, R13, R14} = {0, x, ■}, S_1^3 = {R11, R13, R14} = {■, 1, ■}

2. How are the 4 games inter-related?

2.1. Systematic Diagrams



types \ values	aa	ab	ba	bb	combinatorics
Leibniz	aa	ab	ba	bb	m^n
Mersenne	aa	ab	ba	–	$2^n - 1$
Brown	aa	ab	–	bb	$\binom{n+m-1}{n}$
Stirling	aa	ab	–	–	$\sum_{k=1}^{M} S(n, k)$

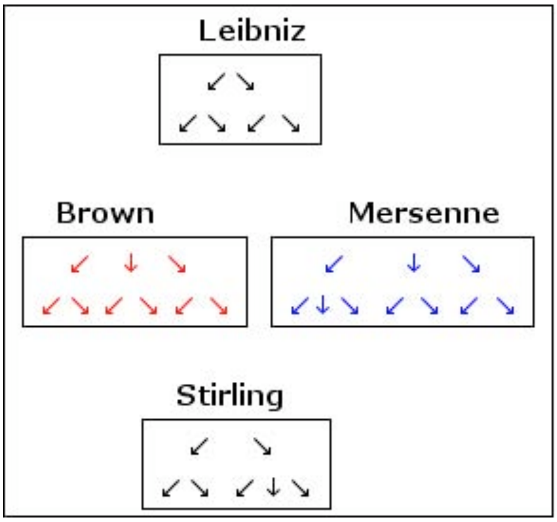
Connection table in normal form for sys(2,3)

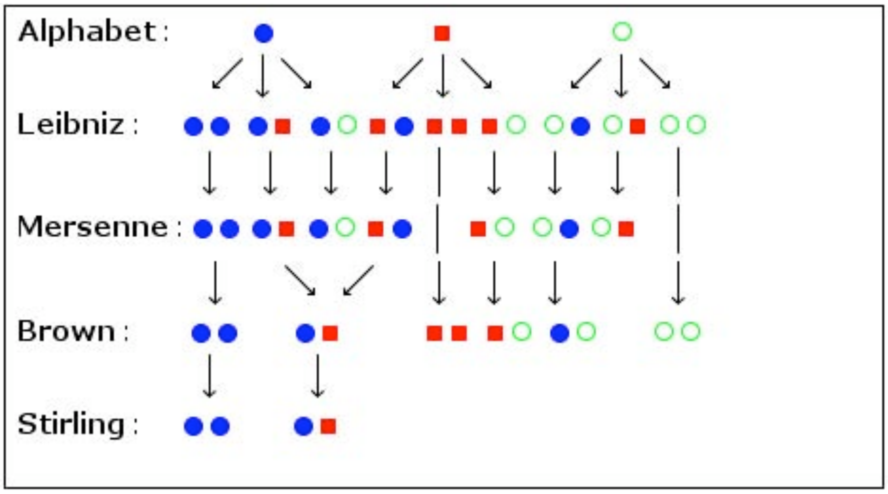
elem	●	■	○
●	L, M, B, S	L, M, B, S	L, M, B
■	L, M	L, B	L, M, B
○	L, M	L, M	L, B

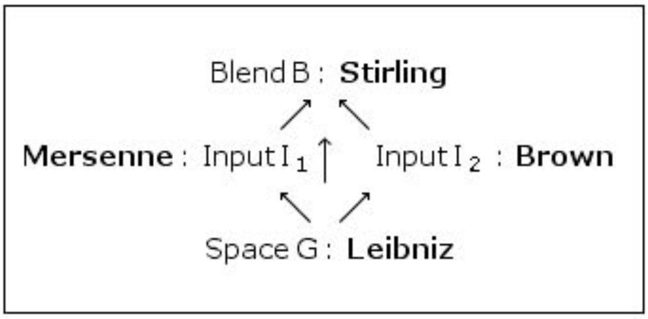
Connection table in normal form for sys(2,2)

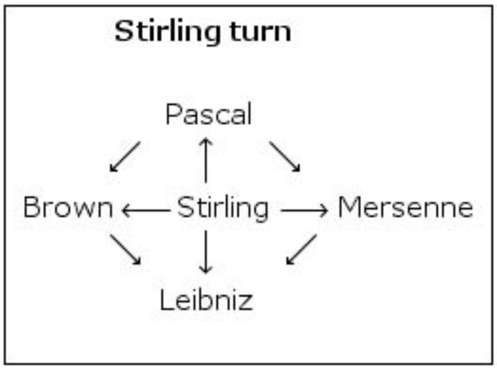
elem	●	■
●	L, M, B, S	L, M, B, S
■	L, M	L, B

2.2. Systematic graphs









2.3. Polyfunctorial mediations