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Memristics: Dynamics of Crossbar Systems

Strategies for simplified polycontextural crossbar constructions for memristive computation

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Abstract

Conceptual approaches are presented for crossbar constructions based on monoidal categories for multi-layered crossbar systems and on polycontextural interchangeability for poly-layered crossbar compound systems. The big breakthrough memristors are opening up is not only the possibility of the nano-crossbar construction memristors are enabling but the interactionality and reflectionality between disseminated crossbar arrays. Because of the fundamental complementarity of memristive systems, i.e. the double role of memristors as mutually interacting operators and operands, a new distinction shall be introduced: the distinction of multi-layered and poly-layered memristive crossbar systems. Memristors as crucial switching elements in the designs of existing crossbar arrays are not yet fully realizing their complementarity properties as second-order concepts and devices.

1. Multi-layered systems

1.1. Monoidal categories

1.1.1. System interpretation

"Monoidal categories have recently proven to be an excellent high-level framework for reasoning about quantum information and computation. Features are an intuitive purely diagrammatic calculus, which enables pictorial derivation of several protocols as well as computing the quantum Fourier transform, and comprehension of quantum, classical and mixed data types. The corresponding categorical logic enables automation.
Important operational features, not present in the usual quantum formalism, are types *and* compositionality.*"*
http://www.comlab.ox.ac.uk/projects/NewQuantumFormalism/index.html

Matching conditions for composition and yuxtaposition (Coecke)

Primitive data :

processes / **operations** : f, g, h, ...

which are **typed** as $A \longrightarrow B$, $B \longrightarrow C$, $A \longrightarrow A$, ...

where A, B, C, ... are kinds / **names** of systems.

Primitive connectives

Sequential composition is a primitive connective on processes

$$f \circ g : A \longrightarrow C \text{ for } f : A \longrightarrow \underline{B} \text{ \& } g : \underline{B} \longrightarrow C$$

Parallel composition is a primitive connective both on systems and processes

$$f \otimes g : A \otimes C \longrightarrow B \otimes D \text{ for } f : A \longrightarrow B \text{ \& } g : C \longrightarrow D$$

"There is a very intuitive operational interpretation of monoidal categories. We think of the objects as types of systems. We think of a morphism f : A ⟶ B as a process which takes a system of type A as input and provides a system of type B as output, i.e. given any state ψ of the system of type A, it produces a state f(ψ) of the system of type B. Composition of morphisms is sequential application of processes. The compound type A ⊗ B represents joint systems. We think of I as the trivial system, which can be either 'nothing' or 'unspecified'." (Coecke et al., p. 8)

Abramsky on interaction

"Basic reflection of time: must have sequential *composition of operations. Basic reflection of* space*: must be able to describe compound systems, operations* localized *to part of a compound system, and operations performed independently on different parts of a compound system -* parallel *composition.*
So we want a general setting in which we can describe processes (of whatever kind) closed under sequential and parallel composition.
So we want a general setting in which we can describe processes (of whatever kind) closed under sequential and parallel composition.*"*
(Abramsky, p. 30)

> http://www.comlab.ox.ac.uk/conferences/categorieslogicphysics/clap1/clap1-samsonabramsky.pdf
>
> "Any symmetric monoidal category can be viewed as a setting for describing processes in a resource sensitive way, closed under sequential and parallel composition." (Abramsky, Coecke)
> http://www.indiana.edu/~iulg/qliqc/HQL_fin.pdf

Also the term "*interaction*" is highly prominent in Abramsky's and Coecke's research, such monoidal parallelism is not only lacking super-additivity but also any structural interactivity between parallel systems. Neither in Abramsky's "*interaction categories*" nor in Coecke's contribution to monoidal categories for quantum physicists there is anything like super-additivity and corresponding complementarity thematized. Does that mean, super-additivity is a red herring? As a study of the problems of the project *"Combining Logics"* shows, the fish is rotting somewhere else.

Again, this might be obscure if we know that especially Robin Milner is using *monoidal* categories as a basic conceptual tool for his advanced theory of *informatic* interaction. The riddle is easily resolved: monoidal categories are defined in a single universe and are, therefore, studying interaction that happens 'inside' this universe. Polycontextural categories are studying interactionality between universes.
Kaehr, Double Cross Playing Diamonds, in: Paradoxes of Interactivity, 2008, http://works.bepress.com/thinkartlab/2/

Interactions are described by Abramsky as compositions between strategies, or as interaction between agents and environments of game semantics, hence in a intra-systemic or intra-contextural manner. All the objects involved belong to the same universe. The operators of symmetry or permutations are not yet to be considered as interactional operators between parallel systems.

Multi-layers are functorial additive and junctional, poly-layers are functorial super-additive and transjunctional.
Additive composition is associative. Super-additive combinations are not generally associative but 'dissociative' and tabular.

1.1.2. Bifunctoriality

Bifunctoriality might be applied as a strong tool, defined in monoidal categories, to study inter-dependences between different layers of multi-layer systems. Bifunctoriality is to parallel systems what distributivity is to sequential systems. Hence, bifunctoriality is a important tool to study the inter-relation between serial and parallel processes. Seriality corresponds the rules of composition, parallelity, the rules of yuxtapositions. Both together are framed by the bifunctoriality of their inter-relationality.

This approach is supposing that both concepts, *composition* and *yuxtapositions*, are systematically on the same level. This is not in accordance with the pure abstract mathematical understanding of categories. But it delivers a strong categorical formalism for so called “real world” applications, like quantum physics (Abramsky, Coecke).

In contrast, the interchangeability concept of polycontextural categories is purely mathematical and both, composition and mediation, but also transposition, iteration and replication, are conceptually on the same level. Certainly, the same holds for the distinction of categories and saltatories of the developing diamond category theory.

composition and yuxtaposition

$$\textbf{BIFUNCT}^{(2)}_{\left[\begin{smallmatrix} g_1 & g_2 \\ f_1 & f_2 \end{smallmatrix}\right]} : \begin{pmatrix} f_1 \\ \otimes \\ f_2 \end{pmatrix} \circ \begin{pmatrix} g_1 \\ \otimes \\ g_2 \end{pmatrix} = \begin{pmatrix} (f_1 \circ g_1) \\ \otimes \\ (f_2 \circ g_2) \end{pmatrix}$$

$$\forall f, g \in \text{Universe}\, \mathcal{U}_i,\ i \in \mathcal{N},\ i = 1$$

Juxtaposition of morphisms is additive, i.e. a parallelism of *m* compositions is defining a parallel system of *m-1* levels, $\text{BIF}^{(m,n)}(\circ, \otimes) = < \circ^{(n-1)} \otimes^{(m-1)} >$.

$m, n \in \mathbb{N}$

$$\begin{pmatrix} \left(f_{(1,1)} \circ g_1 \circ \bullet\bullet\bullet \circ h_{(1,n)}\right) \\ \otimes_1 \\ \left(f_{(2,2)} \circ g_2 \circ \bullet\bullet\bullet \circ h_{(2,m)}\right) \\ \otimes \\ \bullet \\ \bullet \\ \bullet \\ \otimes_{m-1} \\ \left(f_{(n,m)} \circ g_m \circ \bullet\bullet\bullet \circ h_{(n,m)}\right) \end{pmatrix} = \begin{pmatrix} f_1 \\ \otimes_1 \\ f_2 \\ \otimes \\ \vdots \\ \otimes_{m-1} \\ f_m \end{pmatrix} \circ \begin{pmatrix} g_1 \\ \otimes_1 \\ g_2 \\ \otimes \\ \bullet \\ \bullet \\ \bullet \\ \otimes_{m-1} \\ g_m \end{pmatrix} \circ \bullet\bullet\bullet \circ \begin{pmatrix} h_1 \\ \otimes_1 \\ h_n \\ \otimes \\ \vdots \\ \otimes_{m-1} \\ h_{(n,m)} \end{pmatrix}$$

$$BIF^{(m,n)}\left(\circ, \otimes\right) = \langle \circ^{(n-1)} \otimes^{(m-1)} \rangle$$

1.1.3. Paths in multi-layered systems

Symmetry and negation

The category PATH (Jochen Pfalzgraf, Ehrig) is defining the paths in mono-contextural multiple layered systems.

Following Jochen Pfalzgraf

"For practical reasons - in order to reach a large area of applications - we extend our CAT modeling approach to *arbitrary relations* (X,R). In such a case, we are not able to associate directly a category to the relation as we did it before since transitivity, reflexivity do not hold, in general.

"But from the categorical perspective again we interpret a relational structure as a certain diagram of arrows “visualizing” the given relations between the objects which form the “nodes” of the diagram. It turns out that we can always “embed” such a diagram in an associated PATH-category having comparable behavior as the category associated to a reflexive, transitive relation, although being a little “bigger” concerning the morphism structure.

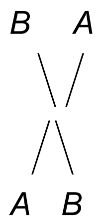
"We point out: The introduction of the associated category **PATH** allows to use and apply all the modeling principles and constructions provided by **CAT** in the corresponding situations."

http://www.cosy.sbg.ac.at/~jpfalz/ACCAT-TutorialSKRIPT.pdf

Symmetry

$\sigma_{A,B} : A \otimes B \longrightarrow B \otimes A$

$\sigma_{B,A} \circ \sigma_{A,B} = 1_{A,B}$



2. Poly-layered systems

2.1. Poly-categories

From the point of view of architectonics of polycontextural systems, the main difference between categorical and contextural systems lies in the different structure of its main operation. For categories, composition is fundamental, for contexture mediation is fundamental. Categorical composition is defined on the base of a homogeneous and linear conceptual space, i.e. uni-verse, while mediation is based on a tabular distribution over the kenomic matrix of a multitude of interacting universes, i.e. a polyverse.
Hence, the idea of poly-categories is different from the known bi- 2- and n-categories as well from double categories. Simply because those systems remain mono-contextural, using objects from one and only one universe.

Standard minimal scheme

$m = n = 2$

$\begin{pmatrix} \mathcal{U}_2 \\ \mathcal{U}_1 \end{pmatrix}, \begin{pmatrix} g_1 & g_2 \\ f_1 & f_2 \end{pmatrix}:$

$$\begin{pmatrix} f_2 \\ \blacksquare \\ f_1 \end{pmatrix} \circ \begin{pmatrix} g_2 \\ \blacksquare \\ g_1 \end{pmatrix} = \begin{pmatrix} f_2 & \circ & g_2 \\ & \blacksquare & \\ f_1 & \circ & g_1 \end{pmatrix}$$

$\amalg$: mediation between contextures

$\circ$: composition of morphisms

$\diamond$: cross - interchange between levels

$\blacksquare \equiv (\amalg \ \diamond)$

$=$: equivalence

$\forall\, f_i, g_i \in \text{Universe}\, \mathcal{U}_i,\ i \in \mathcal{N}$ et

$\forall\, i \neq j : \text{Universe}\, \mathcal{U}_i \bigcap \text{Universe}\, \mathcal{U}_j = \emptyset$

$\forall\, f_i \in \mathcal{U}_i,\ g_j \in \mathcal{U}_j,\ i \neq j = 1, 2$

$\text{cod}(f_2) \simeq \text{dom}(g_1)$

$\text{cod}(f_1) \simeq \text{dom}(g_2)$

$\forall\, f_i, g_i \in \mathcal{U}_i,\ i = 1, 2$

$\text{cod}(f_1) \cong \text{dom}(g_1)$

$\text{cod}(f_2) \cong \text{dom}(g_2)$

Further Metaphors

"As a metaphor, the idea of colored contextures, each containing a full PATH-system, involved in interactions between neighboring contextures, might inspire the understanding of journeys in pluri-labyrinths of JOURN. Such journeys are not safely connected in the spirit of secured transitivity but are challenging by jumps, salti and bridging and transjunctional bifurcations and trans-contextural transitions.
This metaphor of colored categories, logics, arithmetic and set theories gets a scientific implementation with real world systems containing incommensurable and incompatible but interacting domains, like for bio- and social systems."
http://www.thinkartlab.com/pkl/lola/Diamond Relations/Diamond Relations.pdf

Multi-technology hyperintegration

"The idea of a circuit element, which relates the charge q and the magnetic flux φ realizable only at the nanoscale with the ability to remember the past history of charge flow, creates interesting approaches in future CAM-based architectures as we approach the domain of multi-technology hyperintegration *where optimization of disparate technologies becomes the new challenge. The scaling of CMOS technology is challenging below 10 nm and thus nanoscale features of the memristor can be significantly exploited. The memristor is thus a strong candidate for tera-bit memory/compare logic."*
Memristor MOS Content Addressable Memory (MCAM): Hybrid Architecture for Future High Performance Search Engines, http://arxiv.org/pdf/1005.3687

2.1.1. Super-additivity

Super-additivity for polycontextura mediation was first introduced by Gotthard Gunther in his paper "Formal Logic, Totality and The Super-additive Principle", BCL Report # 3,3 (1966); BCL-Microfiche # 36/1, http://www.vordenker.de/ggphilosophy/gg_formal-logic-totality.pdf

The parlance of: "given 3 contextures, the mediation of the first two with the third is the same as the mediation of the first with latter two", seems not working.

Associativity is still conceived in the tradition of the application of *functions*, and functions apply associatively.
Is there any reason to think that the mediation of contextures is functional and therefore associative?
Is the *localization* 'function' of contextures in a kenomic matrix a functions with associativity? Is then the mediation 'function' of contextures a function?

What does it mean to state that polycontextural categories aren't associative but super-additive?
A kind of super-additivity had been connected by Gotthard Gunther to general systems theory (Bertalanffy) and the concept of Gestalt or Ganzheit (Wholeness). Until now I can't see any structural similarity between non-associative mediation in the sense of polycontexturality and associative system composition in the sense of systems theory (and the category of systems).

John Baez, Categories, Quantization, and Much More

> *"In quantum theory one thus learns to like noncommutative, but still associative, algebras.*
> *It is interesting however to note why associativity without commutativity is studied so much more than commutativity without associativity. Basically, because most of our examples of binary operations can be interpreted as composition of functions. For example, if write simply x for the operation of adding x to a real number (where x is a real number), then x + y is just x composed with y. Composition is always associative so the + operation is associative!*
> *If we try to generalize the heck out of the concept of a group, keeping associativity as a sacred property, we get the notion of a category. Categories are some of the most basic structures in mathematics."*
> http://math.ucr.edu/home/baez/categories.html

Categories are not only based on the sacredness of the *associativity* of groups but much more on the insistence on the fundamental taboo of the *identity* of their objects.

Mediation of two universes results super-additively in a compound of three universes.

$$\text{mediation: } \begin{pmatrix} \mathcal{U}_2 \\ \mathcal{U}_1 \end{pmatrix} \xrightarrow{\text{super-additivity}} \begin{pmatrix} \mathcal{U}_2 & - \\ - & \mathcal{U}_3 \\ \mathcal{U}_1 & - \end{pmatrix}$$

$$\text{mediation: } \begin{bmatrix} g_1 & - \\ f_1 & g_2 \\ - & f_2 \end{bmatrix} \xrightarrow{\text{super-additivity}} \begin{bmatrix} g_1 & - & g_3 \\ f_1 & g_2 & - \\ - & f_2 & f_3 \end{bmatrix}$$

Super-additivity is not the same as commutativity for categorical composition and yuxtaposition.

$f \circ g : A \longrightarrow C$ for $f : A \longrightarrow \underline{B}$ & $g : \underline{B} \longrightarrow C$

$f \otimes g : A \otimes C \longrightarrow B \otimes D$ for $f : A \longrightarrow B$ & $g : C \longrightarrow D$

Hence, the composition of the morphisms f and g, $f \circ g$, results in the morphism $A \longrightarrow C$.

But the mediation of the morphism f_1 and f_2, $f_1 \amalg f_2$, results in the super-additive compound $(f_1 \amalg f_2) \amalg f_3$.

$$A_1 \longrightarrow B_1 \amalg B_2 \longrightarrow C_2, \text{ in general}: \left(\begin{pmatrix} A_1 \longrightarrow B_1 \\ \amalg \\ A_2 \longrightarrow B_2 \end{pmatrix} \\ \amalg \\ A_3 \longrightarrow B_3\right), \text{ i.e.} \left(\begin{pmatrix} f_1 \\ \amalg_{1.2\times.0} \\ f_2 \end{pmatrix} \\ \amalg_{0.2\times.3} \\ f_3\right).$$

Additivity and super – additivity

$$\text{BIFUNCT}^{(m,n)}\left(\circ, \bigotimes\right) = \left[\circ^{(n-1)}, \bigotimes^{(m-1)}\right]$$

$$\text{INTERCH}_{\text{diag}}{}^{(m,n)}\left(\circ, \amalg\right) = \left[\amalg^{s(m)}, \circ^{(n-1)}\right]$$

$$\text{INTERCH}_{\text{matrix}}{}^{(m,n)}\left(\circ, \amalg\right) = \left[\amalg^{m^n}, \circ^{(n-1)}\right]$$

Nummeration of subsystems for $\text{INTERCH}_{\text{diag}}$

The truth values i, j of L_k are given by : $i = j(j-1)/2 - k + 1$,

and $j = \left[3/2 + \sqrt{2k - 7/4}\right]$ (The integer part)

Example

$O_{num}: \left[1\ 2\ 3\ 4\ 5\right] \Longrightarrow$

$$\left[\begin{matrix} (f \circ g)_{10} \\ (f \circ g)_6 (f \circ g)_9 \\ (f \circ g)_3 (f \circ g)_5 (f \circ g)_8 \\ (f \circ g)_1 (f \circ g)_2 (f \circ g)_4 (f \circ g)_7 \end{matrix}\right]$$

Matching conditions

$O_{num}: \ [1\ 2\ 3\ 4\ 5\ 6] \Rightarrow$

$(\text{cod}(f_i) \equiv \text{dom}(g_i)),\ i = s(6)$

$\text{cod}(g_1) \equiv \text{dom}(f_2),\ \text{cod}(g_2) \equiv \text{dom}(f_4),\ \text{cod}(g_4) \equiv \text{dom}(f_7)$

$\text{cod}(g_3) \equiv \text{dom}(f_5),\ \text{cod}(g_5) \equiv \text{dom}(f_8)$

$\text{cod}(g_6) \equiv \text{dom}(f_9)$

$\text{dom}(f_1) \equiv \text{dom}(f_3) \equiv \text{dom}(f_6) \equiv \text{dom}(f_{10})$

$\text{dom}(f_2) \equiv \text{dom}(f_5) \equiv \text{dom}(f_9)$

$\text{dom}(f_4) \equiv \text{dom}(f_8)$

$\text{cod}(f_7) \equiv \text{cod}(f_8) \equiv \text{cod}(f_9) \equiv \text{cod}(f_{10})$
$\text{cod}(f_2) \equiv \text{cod}(f_5) \equiv \text{cod}(f_9)$
$\text{cod}(f_4) \equiv \text{cod}(f_8)$

2.1.2. Interchangeability

Super - additivity of a 3 - contextural category

$m = 3,\ n = 2$

$$\begin{bmatrix} g_1 & - & g_3 \\ f_1 & g_2 & - \\ - & f_2 & f_3 \end{bmatrix}:$$

$$\left(\begin{array}{c} \left(\begin{array}{c} \left(f_1 \circ_{1.0\times.0} g_1\right) \\ \amalg_{1.2\times.0} \\ \left(f_2 \circ_{0.2\times.0} g_2\right) \end{array}\right) \\ \amalg_{0.2\times.3} \\ \left(f_3 \circ_{0.0\times.3} g_3\right) \end{array}\right) = \left(\begin{array}{c} \left(\begin{array}{c} f_1 \\ \amalg_{1.2\times.0} \\ f_2 \end{array}\right) \\ \amalg_{0.2\times.3} \\ f_3 \end{array}\right) \circ_1 \circ_2 \circ_3 \left(\begin{array}{c} \left(\begin{array}{c} g_1 \\ \amalg_{1.2\times.0} \\ g_2 \end{array}\right) \\ \amalg_{0.2\times.3} \\ g_3 \end{array}\right)$$

Interchangeability is, as a generalization of categorical bifunctoriality, a highly abstract concept to expose complex situations of polycategorical constructions. Bifunctoriality, as it is promoted by Samson Abramsky and Bob Coecke, is defined as a proportional relationship between categorical composition and yuxtaposition, and is part of the axiomatics of monoidal categories.

Interchangeability is part of a new axiomatics of poly-categorical diamond systems still to be developed. Interchangeability is defined *intra*-contextural for composition and yuxtaposition, and *trans*-contextural for interactions, like mediation, replication, iteration and transposition.

Interchangeability between intra- and trans-categorical constructs is not necessarily excluding forms of temporal and processual *directionality* of formal systems, important for a theory of living systems implementing aspects of ‘lived’ time.

The presentations proposed in this paper of interchangeability is not yet reflecting on the role of the equality or equivalence between the interchangeable parts of the formula. A closer look at the interchange will discover some sorts of asymmetries, disturbing the probably desired harmony. But such a dynamics is just what we need in the situation of a theory of living systems. In contrast, bifunctoriality is conceptually much too narrow to cover such a dynamics between co-creative harmony and pre-established regulations.

2.1.3. Double play of layers

Interpretation

In strict contrast to multi-layered compositions, poly-layered compound systems are not only super-additive but are also involved in mutual interchange of functionalities, especially in to the interplay of memory and computing activities. This mutual interplay is different from the feature of mediation between levels of memristive systems.

In a polycontextural compound systems some levels might play the memorizing part downwards the levels and simultaneously, in an other interplay, the computing for upwards parts.

$$m = 4,\ n = 2$$

$$\begin{pmatrix} g_1 & - & g_3 & - & - & g_6 \\ f_1 & g_2 & - & - & g_5 & - \\ - & f_2 & f_3 & g_4 & - & - \\ - & - & - & f_4 & f_5 & f_6 \end{pmatrix}:$$

$$\left[\begin{array}{c} (f \circ g)_6 \\ \amalg \\ (f \circ g)_3 \amalg (f \circ g)_5 \\ \amalg \\ (f \circ g)_1 \amalg (f \circ g)_2 \amalg (f \circ g)_4 \end{array}\right] = \left[\begin{array}{c} (f)_6 \\ \amalg \\ (f)_3 \amalg (f)_5 \\ \amalg \\ (f)_1 \amalg (f)_2 \amalg (f)_4 \end{array}\right] \circ^{(4)} \left[\begin{array}{c} (g)_6 \\ \amalg \\ (g)_3 \amalg (g)_5 \\ \amalg \\ (g)_1 \amalg (g)_2 \amalg (g)_4 \end{array}\right]$$

Linear distribution of three mediated crossbar systems, $Sys_{1,2.4}$, with two regulating (monitoring) systems, $Sys_{3,5}$, regulating the mediation of $Sys_{1,2}$and $Sys_{2,4}$, and a system Sys_6, regulating the systems $Sys_{3,5}$.

It might be stipulated that the systems $Sys_{3,5}$are in a double role as *computational* systems (operators) for $Sys_{1,2,4}$and in a *storage* (operands) role for the regulating system Sys_6.

$$m = n = 3$$

$$\begin{pmatrix} h_1 & - & h_3 \\ g_1 & h_2 & g_3 \\ f_1 & g_2 & - \\ - & f_2 & f_3 \end{pmatrix}:$$

$$\left(\begin{array}{c} \left(\begin{array}{c} \left(f_1 \circ_{1.0\times.0} g_1 \circ_{1.0\times.0} h_1\right) \\ \text{Ш}_{1.2\times.0} \\ \left(f_2 \circ_{0.2\times.0} g_2 \circ_{0.2\times.0} h_2\right) \end{array}\right) \\ \text{Ш}_{1.2\times.3} \\ \left(f_3 \circ_{0.0\times.3} g_3 \circ_{0.0\times.3} h_3\right) \end{array}\right)$$

$$\left(\begin{array}{c} \left(\begin{array}{c} f_1 \\ \text{Ш}_{1.2\times.0} \\ f_2 \end{array}\right) \\ \text{Ш}_{1.2\times.3} \\ f_3 \end{array}\right) \circ_1 \circ_2 \circ_3 \left(\begin{array}{c} \left(\begin{array}{c} g_1 \\ \text{Ш}_{1.2\times.0} \\ g_2 \end{array}\right) \\ \text{Ш}_{1.2\times.3} \\ g_3 \end{array}\right) \circ_1 \circ_2 \circ_3 \left(\begin{array}{c} \left(\begin{array}{c} h_1 \\ \text{Ш}_{1.2\times.0} \\ h_2 \end{array}\right) \\ \text{Ш}_{1.2\times.3} \\ h_3 \end{array}\right)$$

$m = 4,\ n = 3$

$$\begin{pmatrix} h_1 & - & h_3 & - & - & h_6 \\ g_1 & h_2 & - & h_4 & h_5 & - \\ f_1 & g_2 & g_3 & g_4 & g_5 & g_6 \\ - & f_2 & f_3 & - & - & - \\ - & - & - & f_4 & f_5 & f_6 \end{pmatrix} :$$

$$\left[\begin{array}{c} (f \circ g \circ h)_6 \\ \amalg \\ (f \circ g \circ h)_3 \amalg (f \circ g \circ h)_5 \\ \amalg \\ (f \circ g \circ h)_1 \amalg (f \circ g \circ h)_2 \amalg (f \circ g \circ h)_4 \end{array}\right] =$$

$$\left[\begin{array}{c} (f)_6 \\ \amalg \\ (f)_3 \amalg (f)_5 \\ \amalg \\ (f)_1 \amalg (f)_2 \amalg (f)_4 \end{array}\right] \circ^{(4)}$$

$$\left[\begin{array}{c} (g)_6 \\ \amalg \\ (g)_3 \amalg (g)_5 \\ \amalg \\ (g)_1 \amalg (g)_2 \amalg (g)_4 \end{array}\right] \circ^{(4)} \left[\begin{array}{c} (h)_6 \\ \amalg \\ (h)_3 \amalg (h)_5 \\ \amalg \\ (h)_1 \amalg (h)_2 \amalg (h)_4 \end{array}\right]$$

2.1.4. Architectonics of poly-categories

Interchangeability in polycontextural categories has many faces, depending on the kind of dissemination of categories over the kenomic matrix.
This presentation is focused on categories and is omitting saltatories and diamond categories. The interchangeability formulas are omitted if they had been mentioned in this paper already.
This architectonics had been presented, without category-theoretic constructions, in the years 2003/05, now at:
http://works.bepress.com/thinkartlab/20

Interactionality (◇)

As shown with the examples for bifurcation, interactionality is structurally well covered with transjunctional operators or logical systems or transpositions in general. Interactionality is interpreted as a kind of a transposition between different loci O.

[bif, id, id]	O_1	O_2	O_3
M_1	$id_{1.1}$	$trans_{2.1}$	$trans_{3.1}$
M_2	–	$id_{2.2}$	–
M_3	–	–	$id_{3.3}$

Reflectionality (▫)

Reflectionality is interpreted as a kind of replication of a system at the same locus O.

PM	O_1	O_2	O_3
M_1	$id_{comp\ 1.1}$	–	–
M_2	$repl_{1.2}$	$id_{comp\ 2.2}$	–
M_3	$repl_{1.3}$	–	$id_{comp3.3}$

Iterativity (▸)

Iterativity happens at the same locus and at the same system place. It corresponds to a kind of self-inspection of a reflectional system, hence reflection or replication on itself.

PM	O_1	O_2	O_3
M_1	$iter_{1.1\times.1\times.1}$	–	–
M_2	–	$id_{2.2}$	–
M_3	–	–	$id_{3.3}$

Mixed interactions (◦, ∐ ▫, ▸)

PM	O_1	O_2	O_3
M_1	$iter_{1.1\times.1\times.1}$	$repl_{2.1}$	–
M_2	–	$id_{2.2}$	$repl_{3.2}$
M_3	–	$repl_{2.3}$	$id_{3.3}$

ops = $[\circ, \amalg\ \square, \blacktriangleright]$, i.e. composition, mediation, replication and iteration

$$\begin{pmatrix} O_1 \\ \amalg \\ O_2 \\ \amalg \\ O_3 \end{pmatrix} \begin{pmatrix} \circ - - \\ \amalg \\ - \circ - \\ \amalg \\ - - \circ \end{pmatrix} \begin{pmatrix} M_1 \square M_2 \blacktriangleright M_2 \blacktriangleright M_2 \\ \amalg \\ M_2 \square_{2.1} M_1 \square_{2.3} M_3 \\ \amalg \\ M_3 \square_{3.2} M_2 \end{pmatrix} =$$

$$\begin{pmatrix} (O_1 \circ M_1) \square (O_1 \circ M_2) \blacktriangleright (O_1 \circ M_2) \blacktriangleright (O_1 \circ M_2) \\ \amalg \\ (O_2 \circ M_2) \square_{2.1} (O_2 \circ M_1) \square_{2.3} (O_2 \circ M_3) \\ \amalg \\ (O_3 \circ M_3) \square_{3.2} (O_3 \circ M_2) \end{pmatrix}$$

Metamorphosis ($\approx$, $\diamond$)

Metamorphosis is a new kind of interactivity. Its terms are defined in the as-mode, in contrast to the other examples, where the terms are defined in the is-mode of the is-abstraction.

The different modi of abstraction are crucial in the theory of polycontextural systems.

As-mode: X as Y is Z, or: X as Y is Z as V.

Is-mode: X as X is X, hence X is X.

```
   | chiasm of type & term |
[σ1 ≈ σ1']  [σ2' ≈ σ2]
  |        \ /        |
  |        / \        |
[M1 ≈ M'1]   [M'2 ≈ M2]
  |                   |
```

Metamorphic chiasm of type and term $[(M, \sigma), \ \approx, \ \diamond, \ \circ, \ \amalg]$

$$\begin{pmatrix} ((M_1 \approx M'_1) & \circ & (\sigma_1 \approx \sigma'_1)) \\ \diamond & \amalg & \diamond \\ ((M_2 \approx M'_2) & \circ & (\sigma_2 \approx \sigma'_2)) \end{pmatrix}$$

$$\begin{bmatrix} (M_1 \approx & M'_1) \\ \amalg & \diamond \\ (M_2 \approx & M'_2) \end{bmatrix} \circ \begin{bmatrix} (\sigma_1 & \approx & \sigma'_1) \\ & \amalg & \diamond \\ (\sigma_2 & \approx & \sigma'_2) \end{bmatrix} = \begin{bmatrix} (M_1 \circ \sigma_1) \\ \amalg \\ (M_2 \circ \sigma_2) \end{bmatrix} \approx \begin{bmatrix} (M'_1 \circ \sigma'_1) \\ \diamond \\ (\sigma'_2 \circ M'_2) \end{bmatrix}$$

Mixed forms ($\circ$, $\amalg$, ▫, $\otimes$)

Multi-layer systems in poly-layered configurations. In the example, the third layer of the polycontextural system is intra-contexturally a multi-layered system ruled by the parallel/serial process of yuxtaposition and internal composition.

Mixed forms for replication, transposition, yuxtaposition, composition, mediation

$$\begin{pmatrix} f_1 \square_{1.2} f_1 \square_{1.3} f_1 \\ \amalg_{1.2} \\ f_2 \diamond_{2.1} f_1 \\ \amalg_{2.3} \\ \begin{pmatrix} f_1 \\ \otimes_3 \\ f_2 \end{pmatrix} \diamond_{3.1} f_1 \end{pmatrix} \begin{matrix} \left[\circ_{1.1} \square_{1.2} \square_{1.3}\right] -- \\ \left[\circ_{2.1} \circ_{2.2} - \right] \\ \circ_{3.1} - \circ_{3.3} \end{matrix} \begin{pmatrix} g_1 \square_{1.2} g_1 \square_{1.3} g_1 \\ \amalg_{1.2} \\ g_2 \diamond g_1 \\ \amalg_{2.3} \\ \begin{pmatrix} g_1 \\ \otimes_3 \\ g_2 \end{pmatrix} \diamond g_1 \end{pmatrix} =$$

$$\begin{pmatrix} \left(\left(f_1 \circ_{1.1} g_1\right) \square_{1.2} \left(f_1 \circ_{1.2} g_1\right)\right) \square_{1.3} \left(f_1 \circ_{1.3} g_1\right) \\ \amalg_{1.2} \\ \left(f_2 \circ_{2.2} g_2\right) \diamond_{2.1} \left(f_1 \circ_{2.1} g_1\right) \\ \amalg_{2.3} \\ \begin{pmatrix} \left(f_1 \circ_3 g_1\right) \\ \otimes_3 \\ \left(f_2 \circ_3 g_2\right) \end{pmatrix} \diamond_{3.1} \left(f_1 \circ_{3.1} g_1\right) \end{pmatrix}$$

2.1.5. Journeys JOURN in poly-layered systems

In contrast to the homogeneous structure of the category PATH, journeys, JOURN, in the polycontextural system are involved with jumps over gaps between contextural systems.

JOURN's catalogue of journeys

"There are structurally different kinds of journeys on offer.

1. PATH is a very special type of journey. It is an intra-contextural journey in a single contexture without structural environment. Hence, properly formalized as a category.
2. This situation might be distributed. Journeys in different but mediated contextures are possible. Still isolated and each thus intra-contextural.
3. A new kind appears with possible switches (permutation) and transjunctional splitting (bifurcation) simultaneously into paths of different contextures. Still without complementary environment in the sense of diamond theory.
4. Now, each contexture, even an isolated mono-contexture, might be involved into itself and its environment. This happens for diamonds, which are containing antidromically oriented path in categorical and saltatorial systems. Such journeys are group-journeys with running into opposite directions.
5. Here, a new and risky journey is offered by the travel agency by inviting to

use the bridging rules between complementary acceptional and rejectional domains of categories and saltatories of a diamonds. All that happens intra-contexturally, i.e. diamonds are defined as the complementarity of an elementary contexture.
6. Obviously, diamond journeys might be organized for advanced travellers into polycontextural constellations. Hence, there are transcontextural transitions between diamonds to risk. Interestingly, such journeys might be involved into metamorphic changes between acceptional and rejectional domains of different contextures of the polycontextural scenario."
http://www.thinkartlab.com/pkl/lola/Diamond%20Relations/Diamond%20Relations.pdf

3. Crossbars in comparison

3.1. Multi-layered crossbar arrays

The homogeneous mono-contextural design of multiple layered crossbar systems are distributing their operators, especially memristors, in a uniform manner. Therefore, memristive devices, interpreted as material implications, IMP, are not characterized, additionally to their logical functionality, by a place-designator, implementing the place (locus) of an operator in the multiple layered system. All operators are uniquely the same, wherever they are acting.

It comes as a big myth that the brain is build homogeneously of synapses, now modeled by memristors.
Locally, brain functions might be described in terms of synapses/memristors but the interactions between domains, cell assemblies (von der Malsburg) are not covered by the concept of synapses (binding problem). Hence, memristors might play an important part, but the aim to build a brain (of a cat) on the base of a homogeneous system of memristors is, again, a misleading metaphor.

> *"The critical importance of multidisciplinary contributions is evident in this figure, as no one faculty member's expertise spans such a wide range of scientific and technological approaches, including experience with the capabilities of the human visual system (including access to and protocols for human subjects); theoretical capabilities; system analysis and modeling skills; breadth of simulation tools and computer-aided design tools; experience with device design, characterization, and testing; and system-level (or sub-system-level) experimental facilities."*
> Adaptive Optoelectronic Eyes: Hybrid Sensor/Processor Architectures,

Final Progress Report (1 June, 1998 - 31 May, 2004) ,
http://www.dtic.mil/cgi-bin/

"However, the expected paradigm change has not yet taken place because the general problem of selecting a designated cell within a passive crossbar array without interference from sneak-path currents through neighbouring cells has not yet been solved satisfactorily. Here we introduce a complementary *resistive switch. It consists of two* antiserial *memristive elements and allows for the construction of large passive crossbar arrays by solving the sneak path problem in combination with a drastic reduction of the power consumption."*
Eike Linn, Roland Rosezin, Carsten Kügeler & Rainer Waser, Complementary resistive switches for passive nanocrossbar memories, http://www.nature.com/nmat/journal/v9/n5/abs/nmat2748.html

3.2. Poly-layered crossbar systems

3.2.1. Architectonics of poly-layered systems

Architectonics of polycontextural stsems had first been developed recently in papers like "Contextures".
It is not the place to go into this matter now. To show the principle, the simplest architectonics, the linear mediation of contextures, will do the job.

$$\begin{pmatrix} CR_1 & - \\ - & CR_3 \\ CR_2 & - \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} on_1 \\ off_1 \end{pmatrix} \\ switch_3 \\ \begin{pmatrix} on_2 \\ off_2 \end{pmatrix} \end{pmatrix}$$

$switch_3$ is represented by the mediation of a CR^1 and CR^2,
hence between " $on_1 \equiv on_3$ " and " $off_2 \equiv off_3$ ".
$switch_3$ is enabling, i.e. computing and regulating,
the interplay of CR^1 and CR^2.

Are systems Sys_1 and Sys_3 mediated? Answer is in $Sys_3 = \begin{pmatrix} yes_3 \\ no_3 \end{pmatrix}$.

$\left(CR^1 \amalg CR^2\right) = (ABC)_1 \, B' \, (ABC)_2$

Double functionality of B as B in
(ABC) and as B' between $(ABC)_1$ and $(ABC)_2$.

This double functionality of B might be realized as a memory and as a computing function of B and B'.

$$\begin{pmatrix} CR_1 & - \\ - & CR_3 \\ CR_2 & - \end{pmatrix} = \begin{pmatrix} (ABC)_1 & - \\ - & B' \\ (ABC)_2 & - \end{pmatrix} \Longrightarrow \begin{pmatrix} (ABC)_1 & - \\ - & (ABC)_3 \\ (ABC)_2 & - \end{pmatrix}$$

Interpretation

$$\begin{pmatrix} \begin{pmatrix} on_1 \\ off_1 \end{pmatrix} \\ switch_3 \\ \begin{pmatrix} on_2 \\ off_2 \end{pmatrix} \end{pmatrix} \Longrightarrow \begin{pmatrix} \begin{pmatrix} on_1 \\ off_1 \end{pmatrix} & - \\ - & \begin{pmatrix} yes_3 \\ no_3 \end{pmatrix} \\ \begin{pmatrix} on_2 \\ off_2 \end{pmatrix} & - \end{pmatrix}$$

The formal and the physical definition and measurement of R_{mem} has to be distributed and mediated according to the structure of the polycontextural grid.
Memristics: Memristors, again, Part II,
http://works.bepress.com/thinkartlab/38/

3.2.2. Logical functions in compound systems

$$\left(CR^1 \amalg CR^2\right) = \begin{pmatrix} \begin{pmatrix} on_1 \\ off_1 \end{pmatrix} \\ switch_3 \\ \begin{pmatrix} on_2 \\ off_2 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} CR_1 & - \\ - & CR_3 \\ CR_2 & - \end{pmatrix}$$

To run the game properly, each contexture and therefore, each crossbar in a crossbar compound, is entitled to get its own logic and therefore its own logical implication.

Hence, for $\left(CR^1 \amalg CR^2\right)$, two logical implications have to be implemented, $IMP_1 \in CR_1$ and $IMP_2 \in CR_2$.
This again, is not the same as an application of the same implication IMP_0 at different places.

What is the role of the third contexture? If interpreted as $switch_3$, how does it work?

The $switch_3$device is mediating the two crossbars. This might function as a computational or controlling action. Thus, the functionality of the $switch_3$ device corresponds to a computing activity and, in this context, not to a memory activity.

The logical range for switch
$_3$is defined by the definition of the two logical function which are mediated, i.e. here the logical implications at two different systematic places in the crossbar compound.

How to distinguish different logical implications?

$$\begin{pmatrix} CR_1 & - \\ - & CR_3 \\ CR_2 & - \end{pmatrix} \longrightarrow$$

$$\left\{ \begin{pmatrix} (1,2)-IMP_1 & - \\ - & (1,2)-IMP_3 \\ (1,2)-MP_2 & - \end{pmatrix}, \begin{pmatrix} (1,2)-IMP_1 & - \\ - & (1,3)-IMP_3 \\ (1,2)-MP_2 & - \end{pmatrix}, \begin{pmatrix} (1,2)-IMP_1 & - \\ - & (1,3)-IMP_3 \\ (1,3)-MP_2 & - \end{pmatrix} \right\}$$

Implication $IMP^{1.1\times.1}$

$$IMP^{1.1\times.1} = \begin{pmatrix} (1,2)-IMP_1 & - \\ - & (1,2)-IMP_3 \\ (1,2)-MP_2 & - \end{pmatrix} \Rightarrow$$

[impl]	O_1	O_2	O_3
M_1	12 11	–	–
M_2	12 11	–	–
M_3	12 11	–	–

Interpretation of $IMP^{1.1\times.1}$

Identification of the implication IMP^1at the locus O_1 as $IMP^{1.1}$, replication of the implication IMP^1as $IMP^{1.2}$ and at the locus O_1 as $IMP^{1.3}$.

Implication IMP $^{1.3 \times .3}$

$$\begin{pmatrix} (1,2)-\text{IMP}_1 & - \\ - & (1,3)-\text{IMP}_2 \\ - & (1,3)-\text{MP}_3 \end{pmatrix} \Longrightarrow$$

[impl]	O_1	O_2	O_3
M_1	12 11	–	–
M_2	–	–	13 11
M_3	–	–	13 11

Interpretation of IMP $^{1.3 \times .3}$

Identification of the implication IMP ^{1} at the locus O_1 as IMP $^{1.1}$,
replication of IMP ^{2} as IMP$^{3.2}$ at the locus O_3 and
identification of the implication IMP ^{3} at the locus O_3 as IMP $^{3.3}$.

3.2.3. Discussion of the realization of the located implication IMP$^{1.1 \times .3}$

This discussion gives a sketch of a implementation of distributed and mediated implications, following the example for an implementation of material implication by Mika Laiho and Eero Lehtonen.

Implication Logic with Memristors

"The processing cell uses memristors as ON-OFF programmable synapses, local logic and memory. Local logic is based on memristor computations using material implication."

"Material implication $p \rightarrow q$ is a type of logic that was in [11] shown to be naturally suited for computing with memristors, assuming that a memristor has a programming threshold. Figure 2 shows two memristors and a resistor arranged for performing the implication operation, and the corresponding truth table.

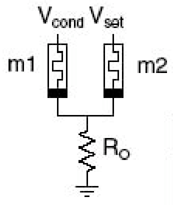
Numbers 0 and 1 in the table correspond to the memristor being in OFF (nonconducting) and ON (conducting) state, respectively. It is assumed that R_O be much larger than m_1 or m_2 in ON state and $V_{cond} < V_{set}$.

The table shows that only when memristor m_1 is ON and m_2 is OFF, the result $m_2 = m_1 \rightarrow m_2$ will be OFF. In this case voltage over m_2 is V_{set} - V_{cond} which is designed to be below the programming threshold of a memristor.

It is well known that together with the false condition ($\perp$), implication forms a functionally complete set $H = \{\rightarrow, \perp\}$"

Mika Laiho, Eero Lehtonen, Microelectronics Laboratory, University of Turku

Cellular Nanoscale Network Cell with Memristors for Local Implication Logic and Synapses



m_1	m_2	$m_2 = m_1 \rightarrow m_2$
0	0	1
0	1	1
1	0	0
1	1	1

Figure 2. Implication logic with memristors.

Distribution of the implication IMP over three loci

$$IMP^{1.1\times.3} = \begin{pmatrix} (1,2)-IMP_1 & - \\ - & (1,3)-IMP_3 \\ (1,2)-MP_2 & - \end{pmatrix} \Longrightarrow$$

[impl]	O_1	O_2	O_3
M_1	12 11	–	–
M_2	12 11	–	–
M_3	–	–	13 11

Short notations for $IMP^{1.1\times.3}$

$$IMP^{(3)}_{\rightarrow\ 1\rightarrow\ 1\rightarrow 3} = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix} \Longrightarrow \begin{pmatrix} ON_{1.3} & \mathbf{OFF_1} & \mathbf{OFF_3} \\ ON_1 & ON1 & \mathbf{OFF_1} \\ ON_3 & ON_1 & ON_{1.3} \end{pmatrix}$$

Interpretation

Following the approach, mentioned by Mika Laiho and Eero Lehtonen, a speculative construction of a dissemination of the voltage use shall be sketched.

Voltage V^i, V^j:

$\forall i \neq j$: i, $j \in s(m)$: $V^i \frown V^j = \emptyset$

s(m) = 1/2m(m-1)

Voltages of domains are disjunct. How this might be realized is a question for engineers but it seems not to be something strange. Similar disjunctive sets are accessible on the base of opto-electronics. What has to be considered for the mediated case are the matching conditions for the voltage at the places of mediation. That is, the voltage range is not simply to divide into a chain of disjunct parts in an additive way, but the fact of super-additivity has to be considered too.

$\forall i \neq j$: i, $j \in s(m)$: $V^{i}_{set} \frown V^{j}_{set} = \emptyset$

$\forall i \neq j$: i, $j \in s(m)$: $V^{i}_{cond} \frown V^{j}_{cond} = \emptyset$

The table IMPshows that only when memristor $m^{i}_{1}$is ON^{i} and m^{i}_{2} is OFF^{i}, the result $m^{i}_{2} = m^{i}_{1} \rightarrow m^{i}_{2}$ is OFF^{i}, i.e.
$\forall i$, $i \in s(m)$: $m^{i}_{1}$is ON^{i} and m^{i}_{2} is OFF^{i}, then $m^{i}_{2} = m^{i}_{1} \rightarrow m^{i}_{2}$ is OFF^{i}.

It is assumed that R^{i}_{0} be much larger than m^{i}_{1} or m^{i}_{2} in ON^{i} state and $V^{i}_{cond} < V^{i}_{set}$.

In this case voltage over m^{i}_{2} is $V^{i}_{set} - V^{i}_{cond}$ which is designed to be below the programming threshold of a memristor MEM^{i}.

Mediation

$IMP^{(3)}_{\rightarrow\ 1\rightarrow\ 1\rightarrow 3} =$

1. $\left(IMP^{1} \amalg_{1.2} IMP^{1}\right) \amalg_{2.3} IMP^{3}$:

conceptual mediation ($\amalg$) of the distributed implications IMP.

2. $\left(\left(MEM^{1}_{1} \circ MEM^{1}_{2}\right) \amalg_{1.2} \left(MEM^{1}_{1} \circ MEM^{1}_{2}\right)\right) \amalg_{2.3} \left(MEM^{3}_{1} \circ MEM^{3}_{2}\right)$:

conceptual mediation of implications and realization of implication IMP by two memristors per contexture.
R_0, respectively R^{i}_{0},is omitted.

3. $\left(\left(MEM^{1}_{1} \circ MEM^{1}_{2}\right) procm_{1.2} \left(MEM^{1}_{1} \circ MEM^{1}_{2}\right)\right) procm_{2.3} \left(MEM^{3}_{1} \circ MEM^{3}_{2}\right)$

realization of implication IMP and mediation by two memristors per contexture and
one memristive processor between contextures (procm).

4. $\left(\left(MEM_1^1 \circ MEM_2^1\right) \text{procm} - \text{repl}_{1.2}\right.$.

$$\left(MEM_1^1 \circ MEM_2^1\right)\right) \text{procm} - \text{med}_{2.3} \left(MEM_1^3 \circ MEM_2^3\right)$$

More concretely, the interaction of $IMP^{1.0 \times .0}$ and $IMP^{0.1 \times .0}$ is ruled by the operator of *replication* "▫", which is a 'horizontal' mediation in contrast to the 'vertical' mediation "Ш" of the planar kenomic matrix.

This together constitutes a memristive system of distributed and mediated memristors in their double role as memory for the realization of the implications *IMP* and in the role as processors for the mediation "Ш" of the contextures of the distributed implications.

Coincidence relation

For $ON_{1.3}$ a coincidence relation between $v(ON_1)$ and $v(ON_3)$ holds.

Also V^1 and V^3 are disjunct, $V^1 \frown V^3 = \emptyset$, they *coincede* at $ON_{1.3}$ with $v(ON_1) \equiv v(ON_3)$ and at $OFF_{2.3}$ with
$v(OFF_2) \equiv v(OFF_3)$.

Because ON_1 and ON_3 are mediated by the operator Ш, which is realized by a processing memristor, there is no logical or electronic contradiction or conflict involved in this mediating mechanism. The same holds for OFF_2 and OFF_3.

Formal modeling

A simplified categorical formulation for distributed memristors and implications based on memristors is shown with the following two formulas.

Interchangeability for $MEM^{(1.1 \times .3)}$ and $IMP^{1.1 \times .3}$

Distribution of memristive implication scheme

$m=3,\ n=2,$ with $\amalg_{1.2\times.0} \equiv$ replicator " $\square_{1.2\times.0}$ "

$$\begin{bmatrix} & \mathrm{MEM}_2^1 & - & \mathrm{MEM}_2^3 \\ \mathrm{MEM}_1^1 & \mathrm{MEM}_2^1 & - & \\ - & \mathrm{MEM}_1^1 & \mathrm{MEM}_1^3 & \end{bmatrix}:$$

$$\begin{pmatrix} \left(\mathrm{MEM}_1^1 \square_{1.2\times.0} \mathrm{MEM}_1^1\right) \\ \amalg_{1.0\times.3} \\ \mathrm{MEM}_1^3 \end{pmatrix} \begin{bmatrix} \circ_{1.1} \circ_{1.2} \\ \\ \circ_{3.3} \end{bmatrix} \begin{pmatrix} \left(\mathrm{MEM}_2^1 \square_{1.2\times.0} \mathrm{MEM}_2^1\right) \\ \amalg_{1.0\times.3} \\ \mathrm{MEM}_2^3 \end{pmatrix} =$$

$$\begin{pmatrix} \left(\mathrm{MEM}_1^1 \circ_{1.0\times.0} \mathrm{MEM}_2^1\right) \square_{1.2\times.0} \left(\mathrm{MEM}_1^1 \circ_{0.2\times.0} \mathrm{MEM}_2^1\right) \\ \amalg_{1.0\times.3} \\ \left(\mathrm{MEM}_1^3 \circ_{0.0\times.3} \mathrm{MEM}_2^3\right) \end{pmatrix}$$

Distribution of logical implication

$m=3,\ n=2,$ with $\amalg_{1.2\times.0} \equiv$ replicator " $\square_{1.2\times.0}$ "

$$\begin{bmatrix} & Y_2^1 & - & Y_2^3 \\ X_1^1 & Y_2^1 & - & \\ - & X_1^1 & X_1^3 & \end{bmatrix}:$$

$$\begin{pmatrix} \left(X_1^1 \square_{1.2\times.0} X_1^1\right) \\ \amalg_{1.0\times.3} \\ X_1^3 \end{pmatrix} \begin{bmatrix} \xrightarrow{1.1} \xrightarrow{1.2} \\ \xrightarrow{3.3} \end{bmatrix} \begin{pmatrix} \left(Y_2^1 \square_{1.2\times.0} Y_2^1\right) \\ \amalg_{1.0\times.3} \\ Y_2^3 \end{pmatrix} =$$

$$\begin{pmatrix} \left(X_1^1 \xrightarrow{1.0\times.0} Y_2^1\right) \square_{1.2\times.0} \left(X_1^1 \xrightarrow{0.2\times.0} Y_2^1\right) \\ \amalg_{1.0\times.3} \\ \left(X_1^3 \xrightarrow{0.0\times.3} Y_2^3\right) \end{pmatrix}$$

Fazit

There are no logical connectives, like implications and their memristive implementations, which are not *localized* in the contextural grid. Classical implication is obscuring its own singular locus, therefore it doesn't appear in the game of logics.

3.3. An interplay between multi- and poly-layered systems

Following the drive of dissemination and interchangeability, the distinction of multi- and poly-layered crossbar systems gets easily involved into its own dynamization.

A first step is the stable *dissemination* of poly- and multi-layered systems. A second step would have to install a *metamorphic* interplay between multi- and poly-layered crossbar systems. This is for importance if the mode of addressing for a layer of a compound systems is changing between multi- and poly-layer status.

For the first case, level three, $(..)_{3.3}$ is defined as a multi-layer system of two layers, M_1, $N_1 \in P_3$ and $M_2, N_2 \in Q_3$ at $L_{(3.3)}$, their parallelism is ruled by yuxtaposition ($\otimes$), and the serial composition is ($\circ_3$). In this case, the composition ($\circ_3$) holds for both types of layers, the multi-layered with $(M_1 \circ N_1)$, $(M_2 \circ N_2)$ and for the poly-layered with $(M_1 \otimes M_2) \circ (N_1 \otimes N_2)$. Obviously, mediation ($\amalg$) and yuxtaposition ($\otimes$) are different kinds of combinations.

$$\begin{pmatrix} \mathcal{U}_2 & - \\ - & \mathcal{U}_3 \\ \mathcal{U}_1 & - \end{pmatrix} : P_i,\ Q\, i_1 \in \mathcal{U}_i,\ i=1,2 \text{ and } M, N \in \mathcal{U}_3$$

Hence, $(M_1 \otimes M_2) \circ (N_1 \otimes N_2) = (M_1 \circ N_1) \otimes (M_2 \circ N_2)$ at position $Pos_{3.3}$

Interchangeability for mixed layered systems

$$\begin{pmatrix} P_1 \\ \amalg_{1.2} \\ P_2 \\ \amalg_{2.3} \\ \left(M_1 \otimes M_2\right)_{3.3} \end{pmatrix} \begin{matrix} \circ_{1.1} \\ \left[\circ_{2.2}\right] \\ \circ_{3.3} \end{matrix} \begin{pmatrix} Q_1 \\ \amalg_{1.2} \\ Q_2 \\ \amalg_{2.3} \\ \left(N_1 \otimes N_2\right)_{3.3} \end{pmatrix} = \begin{pmatrix} \left(P_1 \circ_{1.1} Q_1\right) \\ \amalg_{1.2} \\ \left(P_2 \circ_{2.2} Q_2\right) \\ \amalg_{2.3} \\ \begin{pmatrix} \left(M_1 \circ_3 N_1\right) \\ \otimes_3 \\ \left(M_2 \circ_3 N_2\right) \end{pmatrix}_{3.3} \end{pmatrix}$$

3.4. HP's construction of NAND from material implication IMP and False

3.4.1. HP's construction

Material vs. logical implication

"*'Material implication'*: a forgotten logic building block is a natural fit to nano-crossbars.
A 'conditional copy with inversion'" (Dmitri Strukov, Memristors & Their Applications, 2008).

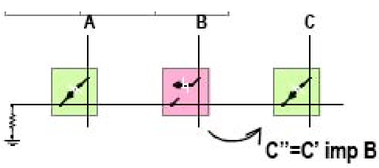
HP'S CONSTRUCTION

Boolean implication:

C' = C IMP A

A	*C*	C'
0	0	1
1	0	0
0	1	1
1	1	1

[Valuation: 1 = true, 0 = false]



Procedures to build NAND from IMP.
(1) CLEAR C; C=0
(2) C’ = C IMP A
(3) C” = C’ imp B

–	–	(1)	(2)	(3)
A	*B*	*C*	*C*'	*C*"
0	0	0	1	1
0	1	0	1	1
1	0	0	0	1
1	1	0	0	0

[Valuation: 0=false, 1 = true; contrary to the implication truth-table!?]

Discussion of HP’s NAND-construction

The function C in table two is logically quite magic. Electronically it simply says, “clear C” (HP).
But logically it is
$A \wedge \neg A$ (or dually: $A \vee \neg A$), *i.e. a* contradiction (or dually: a tautology), also marked as **F** (**T**) or $\perp$ ($\top$).
Thus a lot, i.e. negation and conjunction, is presumed that has not yet been constructed. The use of a constant **F** is logically slightly ‘ad hoc’ but common.

The HP truth table takes the contrary interpretation with 0=true, in contrast to 1=true, hence C=TRUE, T or $\top$.
And C” = (1110) = (false false flase true), table for ¬(A ∧ B), i.e. notAND, A NAND B. (NOR??)

This step might go together with the special definition of *‘material implication’*. It is truth-functionally equivalent to the logical implication but is supposing, to complete the functionality of the calculus, some special conditions, i.e. the constant **F** for false.

> "The modifier *material* in material conditional makes the distinction from linguistic conditionals explicit. It isolates the underlying, unambiguous truth functional relationship.
> Therefore, exact natural language encapsulation of the material conditional X → Y, in isolation, is seen to be *"it's false that X be true while Y false"* — i.e. in symbols, ¬ (X ∧ ¬ *Y*).
> Arguably this is more intuitive than its logically equivalent disjunction ¬ X ∨ *Y*.” (WiKi)

Another argument for the choice of material implication instead of conjunction or disjunction, hence NAND and NOR, might be seen in the *directionality* of material implication in contrast to the *commutativity* of conjunction and disjunction.
This again, is based on *intensional* arguments, and disapears formally with the extensional definition of “X IMP Y” as “nonX or Y”, i.e. $X \rightarrow Y \equiv \neg X \vee Y$.

Trivially, NAND and NOR are commutative, i.e. X NAND Y = Y NAND X, and X NOR Y = Y NOR X.
$X \wedge Y = Y \wedge X \Longrightarrow \neg X \wedge \neg Y = \neg Y \wedge \neg X$. But IMP is not commutative, X IMP Y = Y REP X, i.e.
$X \longrightarrow Y = Y \longleftarrow X$.

Transcription

Logically, HP’ s construction is transcribed as:

Statement :

$$\text{C eq False} \Longrightarrow \text{C IMP A} \wedge \text{C' IMP B} \Longleftrightarrow \text{A NAND B}.$$

Proof :

$$\text{C eq False} \Longrightarrow \text{C IMP A} \wedge \text{C' IMP B}$$

$$\text{C eq False} \Longrightarrow \text{C IMP A} \wedge \left(\text{C IMP A}\right)\text{IMP B} \Longrightarrow \left(\text{C IMP A}\right)\text{IMP B}$$

$$\Longleftrightarrow \text{A NAND B}.$$

Formal logically :

$$A \longrightarrow B \equiv \neg A \vee B \equiv \neg\left(\neg\neg A \wedge \neg B\right) \equiv \neg\left(A \wedge \neg B\right)$$

$$\text{NAND} : \neg\left(A \wedge B\right)$$

Truth – valuation

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}, \; B = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix},$$

$$C = A \wedge \neg A = B \wedge \neg B = 1\left(1 = \text{false}!?\right)$$

$$C' = \text{C IMP A} : \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \text{IMP} \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} \left(= \neg A\right)$$

$$C'' = \text{C' IMP B} : \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} \text{IMP} \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \left(= \text{NAND}\right)$$

$$\neg \text{ A IMP B} \Longleftrightarrow \text{A NAND B}$$

$$\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} \text{IMP} \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}.$$

Implicational propositional calculus

By adding the proposition **F**, which is known to be false, and which is not deducible from from the axioms of the calculus of pure implication, a complete axiomatisation of propositional calculus, based on F or (⊥) and implication only, is achieved. Hence, the set {→, ⊥} is truth-functionally complete.

Recall the *definitions*:

* ¬P is equivalent to P→F
* P∧Q is equivalent to (P→(Q→F))→F
* P∨Q is equivalent to (P→F)→Q
* P↔Q is equivalent to ((P→Q)→((Q→P)→F))→F.

Axiom system for implicational logic

* Axiom schema 1 is P→(Q→P).
* Axiom schema 2 is (P→(Q→R))→((P→Q)→(P→R)).
* Axiom schema 3 (Peirce's law) is ((P→Q)→P)→P.
* The one rule of inference (modus ponens) is: from P and P→Q infer Q.

Where in each case, P, Q, R may be replaced by any proposition which contains only "→" as a connective. (WiKi)

As usual in propositional logic, the *dual* system holds too.

Hence, instead of the constant F, the constant T might be used for the dual axiom system of implication, which then is an axiom system for *replication*.

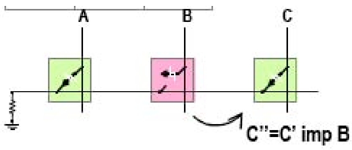
* ¬P eq P→F .dual. P eq ¬P ← T.

That is, dual(*) : ¬¬ P eq (¬P→¬F) eq (¬ P ← T).

3.4.2. Dissemination of logical implications

Physical conditions

At all levels of a poly-crossbar construction, the pysical conditions hold to emulate memristic realizations of the material implication logic.
The ABC-structure, enabling or realizing material implication, might be distrubuted over different loci of the contextural grid.



Therefore, a purely formal approach with logical NAND od NOR is missing the *intention* of material realization, especially the *temporal* aspect of a *'conditional copy with inversion'* (Strukov) *w*ithin memristic systems.

Also both concepts, the NAND and the IMP, are logically very close, it is reasonable to understand classical implementations with NAND as programming or inscribing logical structures into silicon, while the material implication, IMP, approach is following more the emulation and realization character of memristic systems. Hence, the physical system is behaving or acting in an implicative way with the material implication approach, while the abstract approach with NAND is designing and programming the system from the point of view of an external designer.

A more adequate logical modeling than the use of classical logic would be enabled by the family of constructive logics, like Jean-Yves Girard's *linear* logic.

The formal definition of logical implication is not concerning adequacy to ordinary language. It is defined an the base of logical values and functions between values, or on formal game rules, and not on contents.
Material implication tries to reflect on meaningful content as a contextual environment for the 'material' aspect of material implication.

Localizations of implications

Localizations of logical functors in memristive systems is a direct consequence of the *materiality* of their realizations in a memristive grid.

The principle of *localization* might be omitted in a homogenous system, i.e., a multi-layered crossbar system might be conceived and designed as a homogeneous array where all operations are indexed with the same place-value. Such a place-value, which is defined in general by the physical crossbar structure, might be omitted if there is one and only one systemic or architectonic value necessary.

Different Boolean implications:

$C_1' = C \; IMP_1 \; A$

Sys_{IMP_1}	–	–	Sys_{IMP_2}	–	–	Sys_{IMP_3}	–	–
A	***C***	***C₁'***	***A***	***C***	***C₂'***	***A***	***C***	***C₂'***
0_1	0_1	1_1	0_2	0_2	1_2	0_3	0_3	1_3
1_1	0_1	0_1	1_2	0_2	0_2	1_3	0_3	0_3
0_1	1_1	1_1	0_2	1_2	1_2	0_3	1_3	1_3
1_1	1_1	1_1	1_2	1_2	1_2	1_3	1_3	1_3

Sys_1	–	(1)	(2)	(3)
A	*B*	*C*	*C'*	*C"*
0_1	0_1	0_1	1_1	1_1
0_1	1_1	0_1	1_1	1_1
1_1	0_1	0_1	0_1	1_1
1_1	1_1	0_1	0_1	0_1

Sys_2	–	(1)	(2)	(3)
A	*B*	*C*	*C'*	*C"*
0_2	0_2	0_2	1_2	1_2
0_2	1_2	0_2	1_2	1_2
1_2	0_2	0_2	0_2	1_2
1_2	1_2	0_2	0_2	0_2

Sys_3	–	(1)	(2)	(3)
A	*B*	*C*	*C'*	*C"*
0_3	0_3	0_3	1_3	1_3
0_3	1_3	0_3	1_3	1_3
1_3	0_3	0_3	0_3	1_3
1_3	1_3	0_3	0_3	0_3

Simplified notation for $IMP^{(3)}_{\rightarrow\ 1\rightarrow\ 1\rightarrow 3}$

$$IMP^{(3)}_{\rightarrow\ 1\rightarrow\ 1\rightarrow 3} = \left(Sys_{IMP_1} \amalg Sys_{IMP_2}\right) \amalg Sys_{IMP_3} \Longrightarrow \left(Sys_{IMP_{1.1}} \amalg Sys_{IMP_{2.1}}\right) \amalg Sys_{IMP_{3.1}}$$

$$IMP^{(3)}_{\rightarrow\ 1\rightarrow\ 1\rightarrow 3} = \begin{pmatrix} 1_{1.3} & \mathbf{0_1} & \mathbf{0_3} \\ 1_1 & 1_1 & \mathbf{0_1} \\ 1_3 & 1_1 & 1_{1.3} \end{pmatrix} \Longrightarrow \begin{pmatrix} ON_{1.3} & \mathbf{OFF_1} & \mathbf{OFF_3} \\ ON_1 & ON1 & \mathbf{OFF_1} \\ ON_3 & ON_1 & ON_{1.3} \end{pmatrix} \Longrightarrow \begin{pmatrix} \mathbf{IMP}_1^1 & \mathbf{IMP}_2^1 \\ \mathbf{IMP}_3^1 & - \end{pmatrix}.$$

3.4.3. Junctional actions

Not everything is mediated directly

To go on with this descriptive approach, it becomes immediately clear that mediation is not an arbitrary combination of subsystems.

These rather trivial formal aspects might be of interest in the context of concrete poly-layered distributions of functors, like NAND, NOR or IMP. The formal aspects, especially mediation concerning transjunctions, had been elaborated in detail at different occasions.
Mahler, Kaehr: Morphogrammatik, 1992:
http://www.thinkartlab.com/pkl/tm/MG-Buch.pdf
Pfalzgraf, Fibered Logics:
http://jigpal.oxfordjournals.org/cgi/reprint/4/3/445.pdf
Kaehr, PolyLogics: http://works.bepress.com/thinkartlab/25/

Examples for NAND

$$\text{Sys}^{(4,2)} = \begin{pmatrix} \mathbf{2} & \mathbf{2} & x & \mathbf{2} \\ \mathbf{2} & \mathbf{1} & x & x \\ x & x & \mathbf{2} & \mathbf{2} \\ \mathbf{2} & x & \mathbf{2} & \mathbf{1} \end{pmatrix} \Longrightarrow \begin{pmatrix} \mathbf{NAND}_1^1 & -_2 & \mathbf{NAND}_4^1 \\ -_3 & -_5 & - \\ \mathbf{NAND}_6^1 & - & - \end{pmatrix}$$

Some possible full interpretations of $\text{Sys}^{(4,2)}$:

$$\text{Sys}^{(4,2)} = \left\{ \begin{pmatrix} \mathbf{NAND}_1^1 & \text{AND}_2^1 & \mathbf{NAND}_4^1 \\ \text{NEQ}_3^1 & \text{EQ}_5^1 & - \\ \mathbf{NAND}_6^1 & - & - \end{pmatrix}, \begin{pmatrix} \mathbf{NAND}_1^1 & \text{AND}_2^1 & \mathbf{NAND}_4^1 \\ F_3^1 & T_5^1 & - \\ \mathbf{NAND}_6^1 & - & - \end{pmatrix}, \begin{pmatrix} \mathbf{NAND}_1^1 & \text{AND}_2^1 & \mathbf{NAND}_4^1 \\ \text{NEQ}_3^1 & T_5^1 & - \\ \mathbf{NAND}_6^1 & - & - \end{pmatrix} \right\} \cup$$

$$\left\{ \begin{pmatrix} \mathbf{NAND}_1^1 & \text{OR}_2^1 & \mathbf{NAND}_4^1 \\ \text{NIMP}_3^1 & \text{IMP}_5^1 & - \\ \mathbf{NAND}_6^1 & - & - \end{pmatrix}, \begin{pmatrix} \mathbf{NAND}_1^1 & \text{OR}_2^1 & \mathbf{NAND}_4^1 \\ \text{IMP}_3^2 & \text{IMP}_5^1 & - \\ \mathbf{NAND}_6^1 & - & - \end{pmatrix}, \cup \right.$$

$$\left. \begin{pmatrix} \mathbf{NAND}_1^1 & \text{OR}_2^1 & \mathbf{NAND}_4^1 \\ \text{NIMP}_3^1 & \text{IMP}_5^3 & - \\ \mathbf{NAND}_6^1 & - & - \end{pmatrix}, \begin{pmatrix} \mathbf{NAND}_1^1 & \text{OR}_2^1 & \mathbf{NAND}_4^1 \\ \text{IMP}_3^2 & \text{IMP}_5^3 & - \\ \mathbf{NAND}_6^1 & - & - \end{pmatrix} \right\}$$

$$\left\{\begin{pmatrix} \mathbf{NAND}_1^1 & \mathrm{TRANS}_2^1 & \mathbf{NAND}_4^1 \\ \mathrm{NEQ}_3^1 & T_5^1 & - \\ \mathbf{NAND}_6^1 & - & - \end{pmatrix}\right\}$$

Modularity for Sys $^{(6,2)}$

$$\mathrm{Sys}^{(6,2)} = \begin{pmatrix} 2 & 2 & x & 2 & 2 & 2 \\ 2 & 1 & 2 & x & 1 & 1 \\ x & 2 & 2 & 2 & 2 & 2 \\ 2 & x & 2 & 1 & 6 & 1 \\ 2 & 1 & 2 & 6 & 6 & 6 \\ 2 & 1 & 2 & 1 & 6 & 5 \end{pmatrix}$$

$$\mathrm{Sys}^{(6,2)} = \begin{pmatrix} \mathbf{NAND}_1^1 & \mathrm{AND}_2^1 & \mathbf{NAND}_4^1 & \mathrm{AND}_7^{15} & \mathbf{NAND}_{11}^{11} \\ -_3 & -_5 & \mathrm{AND}_8^{14} & \mathrm{AND}_{12}^{10} & - \\ \mathbf{NAND}_6^1 & \mathrm{AND}_9^{14} & \mathrm{AND}_{13}^{9} & - & - \\ \mathrm{AND}_{10}^{14} & \mathrm{AND}_{14}^{10} & - & - & - \\ \mathrm{AND}_{15}^{9} & - & - & - & - \end{pmatrix},$$

$$\begin{pmatrix} \mathbf{NAND}_1^1 & \mathrm{OR}_2^1 & \mathbf{NAND}_4^1 & \mathrm{OR}_7^{15} & \mathbf{NAND}_{11}^{11} \\ \mathrm{N\,IMP}_3^1 & \mathrm{IMP}_5^3 & \mathrm{OR}_8^{14} & \mathrm{OR}_{12}^{10} & - \\ \mathbf{NAND}_6^1 & \mathrm{OR}_9^{14} & \mathrm{OR}_{13}^{9} & - & - \\ \mathrm{AND}_{10}^{14} & \mathrm{OR}_{14}^{10} & - & - & - \\ \mathrm{AND}_{15}^{9} & - & - & - & - \end{pmatrix}$$

It is easy to see that there is some *modularity* involved, i.e. systems of higher complexity might be constructed out of patterns of systems of lower complexity.

3.4.4. Transjunctional interactions between poly-layers

Transjunctional operators are operating as operators at once at different places of a compound system. Hence, applied to poly-layered crossbar systems, transjunctions are operating at once at differend layers of the poly-layered system. In contrast, junctional operators are operating intra-contexturally at the places of the compound system. They are mediated in different ways but are not intrinsically interacting together.
Transjunctional operators had first been introduced by Gotthard Gunther in the early 1960s at the Biological Computer Laboratory, Urbana, Ill, with the paper: “Cybernetic ontology and transjunctional operators”.
www.vordenker.de/ggphilosophy/gg_cyb_ontology.pdf

Transjunctions in semiotics:
http://www.thinkartlab.com/pkl/lola/Transjunctional%20Semiotics/Transjunctional%20Semiotics.html

Tiny example

A tiny descriptive example shall gives a first hint about the interactional behavior of a transjunction in a 3-contextural compound system with the junctions AND and OR.

Example: (transjunction, conjunction, disjunction)

$$\mathrm{Sys}^{(3,2)} = \begin{pmatrix} \mathbf{1} & \mathbf{3} & 1 \\ \mathbf{3} & \mathbf{2} & 3 \\ 1 & 3 & \mathbf{3} \end{pmatrix} \Longrightarrow \begin{pmatrix} \mathbf{TRANS}_1^{1.2\times.3} & \mathrm{AND}_2^1 \\ \mathrm{OR}_3^3 & - \end{pmatrix}$$

General scheme for (inter, act, act)

$$\mathrm{Sem}^{(3,2)}_{(\mathrm{inter,\,act,\,act})} = \begin{pmatrix} [\blacklozenge, \circ, \circ] & 1 & 2 & 3 \\ 1 & \mathbf{1.1}_{1.3} & \mathbf{2.3}_{2.3} & \mathbf{1.3}_{3} \\ 2 & \mathbf{3.2}_{2.3} & \mathbf{2.2}_{1.2} & \mathbf{2.3}_{2} \\ 3 & \mathbf{3.1}_{3} & \mathbf{3.2}_{2} & \mathbf{3.3}_{2.3} \end{pmatrix}$$

General distribution table for [inter, act, act]

[◆, ○, ○]	O_1	O_2	O_3
M_1	$\mathrm{trans}_{1.1}$	$\mathrm{trans}_{2.1}$	$\mathrm{trans}_{3.1}$
M_2	–	$\mathrm{junct}_{2.2}$	*x*
M_3	–	–	$\mathrm{junct}_{3.3}$

The transjunction $\mathbf{TRANS}_1^{1.2\times.3}$ of place O_1 acts simultaneously at the places $O_2$and O_3.

$\mathbf{TRANS}_1^{1.2\times.3}$ is localized at the the place of subsystem sys_1, and is "penetrating" logically as an interaction into the subsystems sys_2 and sys_3.

3.4.5. Interchangeability for transjunctions

From a more formal point of view, transjunctions are bifurcations of operators in polycontextural formal systems.

[bif, id, id]	O_1	O_2	O_3
M_1	$S_{1.1}$	$S_{2.1}$	$S_{3.1}$
M_2	–	$S_{2.2}$	x
M_3	–	x	$S_{3.3}$

Bifunctoriality for the pattern : $[(\text{bif, bif, bif}), (-, \text{id}, -), (-, -, \text{id})]$

$$\begin{pmatrix} O_1 \\ \amalg \\ O_2 \\ \amalg \\ O_3 \end{pmatrix} \begin{pmatrix} \circ_{1.1} \; - - \\ \amalg \\ - \circ_{2.2} - \\ \amalg \\ - - \circ_{3.3} \end{pmatrix} \begin{pmatrix} M_1 \\ \amalg \\ M_2 \, \square_{2.1} \, M_1 \\ \amalg \\ M_3 \, \square_{3.1} \, M_1 \end{pmatrix} =$$

$$\begin{pmatrix} (O_1 \circ_{1.1} M_1) \\ \amalg \\ (O_2 \circ_{2.2} M_2) \square_{2.1} (O_2 \circ_{2.1} M_1) \\ \amalg \\ (O_3 \circ_{3.3} M_3) \square_{3.1} (O_3 \circ_{3.1} M_1) \end{pmatrix}$$

This formal pattern for bifunctioriality (interchangeability) gets a natural interpretation by a mapping of logics onto the scheme. Other mappings of other formal systems are reasonable as well. Especially, formal definitions of programming languages might be mapped onto this and other schemes.

Interchangeability for $\left[\text{bif, id, id}\right]$

$$\begin{pmatrix} f_1 \\ \amalg_{1.2} \\ f_2 \diamond_{2.1} f_1 \\ \amalg_{2.3} \\ f_3 \diamond_{3.1} f_1 \end{pmatrix} \begin{matrix} \circ_{1.1} -- \\ \\ \left[\circ_{2.1} \circ_{2.2} - \right] \\ \square \\ \circ_{3.1} - \circ_{3.3} \end{matrix} \begin{pmatrix} g_1 \\ \amalg_{1.2} \\ g_2 \diamond_{2.1} g_1 \\ \amalg_{2.3} \\ g_3 \diamond_{3.1} g_1 \end{pmatrix} =$$

$$\begin{pmatrix} \left(f_1 \circ_{1.1} g_1\right) \\ \amalg_{1.2} \\ \left(f_2 \circ_{2.2} g_2\right) \diamond_{2.1} \left(f_1 \circ_{2.1} g_1\right) \\ \amalg_{2.3} \\ \left(f_3 \circ_{3.3} g_3\right) \diamond_{3.1} \left(f_1 \circ_{3.1} g_1\right) \end{pmatrix}$$

$\amalg$: mediation between contextures
$\circ$: composition of morphisms
$\diamond$: bifurcational transposition
$=$: equivalence

3.4.6. Disseminated formal systems

A more formal approach to a modeling of transjunctional logical systems might be achieved with a dissemination of formal entailment systems, institutions and logics in the sense of Joseph Goguen's general framework for programming languages.

The example for transjunctional, with $sys_{2.1}$ and $sys_{3.1}$ and replicational, with $sys_{1.2}$ and $sys_{3.1}$ dissemination shows clearly the scheme of the interactivity of formal systems as a structural base for any programming languages, and programming of poly-layered memristive systems.

General [repl, transp] – scheme for a formal system $\mathcal{F}^{(3)}$

$$
\begin{pmatrix} O_1 \\ \coprod \\ O_2 \diamond_{2.1} O_1 \\ \coprod \\ O_3 \diamond_{3.1} O_1 \end{pmatrix}
\begin{pmatrix} \circ\ -- \\ \coprod \\ -\circ- \\ \coprod \\ --\ \circ \end{pmatrix}
\begin{pmatrix} \mathcal{F}_1 \square_{1.2} \mathcal{F}_2 \square_{1.3} \mathcal{F}_3 \\ \coprod \\ \mathcal{F}_2 \diamond_{2.1} \mathcal{F}_1 \\ \coprod \\ \mathcal{F}_3 \diamond_{3.1} \mathcal{F}_1 \end{pmatrix}
$$

$$
\begin{pmatrix} (O_1 \circ \mathcal{F}_1) \square_{1.2} (O_1 \circ \mathcal{F}_2) \square_{1.3} (O_1 \circ \mathcal{F}_3) \\ \coprod \\ (O_2 \circ \mathcal{F}_2) \diamond_{2.1} (O_1 \circ \mathcal{F}_1) \\ \coprod \\ (O_3 \circ \mathcal{F}_3) \diamond_{3.1} (O_1 \circ \mathcal{F}_1) \end{pmatrix} =
$$

$$
\begin{pmatrix} (\mathcal{F}_{1.1}) \square_{1.2} (\mathcal{F}_{2.1}) \square_{1.3} (\mathcal{F}_{3.1}) \\ \coprod \\ (\mathcal{F}_{2.2}) \diamond_{2.1} (\mathcal{F}_{2.1}) \\ \coprod \\ (\mathcal{F}_{3.3}) \diamond_{3.1} (\mathcal{F}_{3.1}) \end{pmatrix}
$$

Dissemination of an entailment system $\mathcal{E}^{(3)} = (\mathbf{Sign},\ \mathrm{sen},\ \vdash)$ **for** M

$$\begin{pmatrix} (\mathcal{E}_{1.1}) \square (\mathcal{E}_{2.1}) \square (\mathcal{E}_{3.1}) \\ \amalg \\ (\mathcal{E}_2) \diamond_{2.1} (\mathcal{E}_1) \\ \amalg \\ (\mathcal{E}_3) \diamond_{3.1} (\mathcal{E}_1) \end{pmatrix} =$$

$$\begin{pmatrix} (\mathbf{Sign},\ \mathrm{sen},\ \vdash)_{1.1} \square (\mathbf{Sign},\ \mathrm{sen},\ \vdash)_{1.2} \square (\mathbf{Sign},\ \mathrm{sen},\ \vdash)_{1.3} \\ \amalg \\ (\mathbf{Sign},\ \mathrm{sen},\ \vdash)_{2.2} \diamond_{2.1} (\mathbf{Sign},\ \mathrm{sen},\ \vdash)_{2.1} \\ \amalg \\ (\mathbf{Sign},\ \mathrm{sen},\ \vdash)_{3.3} \diamond_{3.1} (\mathbf{Sign},\ \mathrm{sen},\ \vdash)_{3.1} \end{pmatrix}$$

Dissemination of institutions $\mathcal{I}^{(3)} = (\mathbf{Sign},\ \mathrm{sen},\ \mathbf{Mod},\ \models)$ is *a* 4 – tuple

$$\begin{pmatrix} (\mathcal{I}_{1.1}) \square (\mathcal{I}_{2.1}) \square (\mathcal{I}_{3.1}) \\ \amalg \\ (\mathcal{I}_2) \diamond_{2.1} (\mathcal{I}_1) \\ \amalg \\ (\mathcal{I}_3) \diamond_{3.1} (\mathcal{I}_1) \end{pmatrix} =$$

$$\begin{pmatrix} (\mathbf{Sign},\ \mathrm{sen},\ \mathbf{Mod},\ \models)_{1.1} \square (\mathbf{Sign},\ \mathrm{sen},\ \mathbf{Mod},\ \models)_{1.2} \square (\mathbf{Sign},\ \mathrm{sen},\ \mathbf{Mod},\ \models)_{1.3} \\ \amalg \\ (\mathbf{Sign},\ \mathrm{sen},\ \mathbf{Mod},\ \models)_{2.2} \diamond_{2.1} (\mathbf{Sign},\ \mathrm{sen},\ \mathbf{Mod},\ \models)_{2.1} \\ \amalg \\ (\mathbf{Sign},\ \mathrm{sen},\ \mathbf{Mod},\ \models)_{3.3} \diamond_{3.1} (\mathbf{Sign},\ \mathrm{sen},\ \mathbf{Mod},\ \models)_{3.1} \end{pmatrix}$$

Dissemination of logics $\mathcal{L}$: *A* logic is *a* 5 – tuple $\mathcal{L}^{(3)}$ =
(**Sign**, sen, **Mod**, ⊢ , ⊨)

$$\begin{pmatrix} (\mathcal{L}_{1.1}) \square (\mathcal{L}_{2.1}) \square (\mathcal{L}_{3.1}) \\ \amalg \\ (\mathcal{L}_2) \diamond_{2.1} (\mathcal{L}_1) \\ \amalg \\ (\mathcal{L}_3) \diamond_{3.1} (\mathcal{L}_1) \end{pmatrix} =$$

$$\begin{pmatrix} (\mathbf{Sign},\ \mathrm{sen},\ \mathbf{Mod},\ \vdash,\ \models)_{1.1} \square (\mathbf{Sign},\ \mathrm{sen},\ \mathbf{Mod},\ \vdash,\ \models)_{1.2} \square (\mathbf{Sign},\ \mathrm{sen},\ \mathbf{Moc} \\ \amalg \\ (\mathbf{Sign},\ \mathrm{sen},\ \mathbf{Mod},\ \vdash,\ \models)_{2.2} \diamond_{2.1} (\mathbf{Sign},\ \mathrm{sen},\ \mathbf{Mod},\ \vdash,\ \models)_{2.1} \\ \amalg \\ (\mathbf{Sign},\ \mathrm{sen},\ \mathbf{Mod},\ \vdash,\ \models)_{3.3} \diamond_{3.1} (\mathbf{Sign},\ \mathrm{sen},\ \mathbf{Mod},\ \vdash,\ \models)_{3.1} \end{pmatrix}$$

Soundness conditions for $\mathcal{L}$

$\mathcal{E}^{(3,3)}$ is a complex entailment system
$\mathcal{I}^{(3,3)}$ is a complex institution,
$\forall\ \Sigma \in |\mathrm{sign}|,\ \Gamma \subseteq \mathrm{sen}(\Gamma),$ and $\varphi \in \mathrm{sen}(\Gamma),$
$\boldsymbol{\Gamma \vdash_\Sigma \varphi \Longrightarrow \Gamma \models_\Sigma \varphi}$

All that gives a general dissemination *scheme* only. What has to be elaborated are the corresponding matching and mediating conditions for the constituents of the disseminated general entailment systems $\mathcal{E}$, institutions $\mathcal{I}$ and logics $\mathcal{L}$.

Dissemination has a vague connection to *parametrization* (Goguen, Burstall) and *fibering* (Gabbay, Pfalzgraf) of theories. This concretization might be realized step-wise for entailment systems, institutions and logics. The mediation conditions for concrete logics, then, are delivering the concrete matching conditions for the whole construction, i.e. for entailment (provability) systems, institutions (models) and polycontextural logics.

Soundness is defined between an entailment system and an institution for each distribution. A kind of a harmony is defined between disseminated sound systems.