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Morphic Cellular Automata Systems – Part III: Indicational CAs

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### Abstract

As it is reasonable to define "*indicational*" stack machines it is equally reasonable to define the concept of indicational cellular automata. In both cases, the definitions relies on the primary 'semiotic' properties and not on the specific features of the calculus of indication, like the double crossing.

The same hold for all other definitions of CAs. The specific semiotic properties of digital CAs are not considered, what counts are the transition rules on semiotically identical signs. For kenomic CAs, it is the kenomic definition of the 'data' what is relevant. At least all this is correct at a first glance.

This study of "*Indicational CAs*" is focused on the proto-semiotic conditions of George Spencer Brown's *Calculus of Indication* as developed in his work *Laws of Form*, and not on the 'arithmetical' and 'algebraic' rules of the calculus of indication (CI) itself.

Hence, the 'arithmetical' rules of the process of distinctions, the law of *condensation* and the law of *cancellation* are not thematized in this context but their deep-structure of inscription, the '*topological invariance*' of the notation is chosen as the primary topic and studied in the framework of *cellular automata*.

This approach has not yet been studies properly in the literature about Spencer-Brown's calculus of indication.

With this CA-oriented study of the proto-semiotic conditions of the CI, the field of research of the deep-structure of the CI is opened up.

The interesting results of this study are presented as such, and shall not yet be thematized or interpreted by graphematic and grammatical considerations.

Earlier graphematic characterizations of the *Laws of Form* are helpful for an understanding of the presented approach but shall not be involved into the present considerations.

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# Morphic Cellular Automata Systems

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## Part III: Indicational CAs,

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## Part III: Indicational CA

### Motivation

As it is reasonable to define "indicational" stack machines it is equally reasonable to define the concept of indicational cellular automata. In both cases, the definitions relies on the primary 'semiotic' properties and not on the specific features of the calculus of indication, like the double crossing.

The same hold for all other definitions of CAs. The specific semiotic properties of digital CAs are not considered, what counts are the transition rules on semiotically identical signs. For kenomic CAs, it is the kenomic definition of the 'data' what is relevant. At least all this is correct at a first glance.

This study of "Indicational CAs" is focused on the proto-semiotic conditions of George Spencer Brown's Calculus of Indication as developed in his work Laws of Form, and not on the 'arithmetical' and 'algebraic' rules of the calculus of indication (CI) itself.

Hence, the 'arithmetical' rules of the process of distinctions, the law of condensation and the law of cancellation are not thematized in this context but their deep-structure of inscription, the 'topological invariance' of the notation is chosen as the primary topic and studied in the framework of cellular automata.

This approach has not yet been studies properly in the literature about Spencer-Brown's calculus of indication.

With this CA-oriented study of the proto-semiotic conditions of the CI, the field of research of the deep-structure of the CI is opened up.

The interesting results of this study are presented as such, and shall not yet be thematized or interpreted by graphematic and grammatical considerations.

Earlier graphematic characterizations of the Laws of Form are helpful for an understanding of the presented approach but shall not be involved into the present considerations.

### Commutativity of terms for indCA

"A third deviation from classical semiotics is less obvious: the commutativity of the concatenation operation. For any two terms "a" and "b" the terms "ab" and "ba" are identical. That this is indeed a semiotic identity (and not just a logical equality) has been stressed by Varga."

"We call two elements  $x = (x_1, \dots, x_n)$ ,  $y = (y_1, \dots, y_n) \in A^*$  equivalent, in short notation

(1)  $x \approx_1 y$

if and only if there is a permutation  $f: \{1, \dots, n\} \rightarrow \{1, \dots, n\}$  such that

(2)  $y_i = x_{f(i)}$   $i = 1, \dots, n$

We could now say that a "string" in the Brownian sense is an equivalence class of  $A^*$  with respect to  $\approx_1$ , i.e., an element of the quotient set  $A^*/\approx_1$ .

(A) If the two given tokens of strings have different lengths, then they are different. If they have equal lengths, then go to (B').

(B') Check whether each atom appears equally often in both string-tokens. If this is the case, then they are equal,

otherwise they are different." (R. Matzka)

<http://www.rudolf-matzka.de/dharma/semabs.pdf>

<http://www.thinkartlab.com/pkl/media/Diamond%20Calculus/Diamond%20Calculus.html>

<http://memristors.memristics.com/Complementary%20Calculi/Complementary%20Calculi.html>

## Toward indicational CAs

Topological invariance of the 'heads' of CA rules.

### System of indicational CA rules

R1		R2		R3		R4	
R5		R6		R7		R8	

Indicational rules :  $(aa) \neq_{ind} (bb)$ ,  $(ab) =_{ind} (ba)$  ;

$$\text{cardinality } \text{Ind}_{(n, m)} = \binom{n + m - 1}{n},$$

$$m = 2, n = 3 : \binom{3 + 2 - 1}{3} = \binom{4}{3} = 4$$

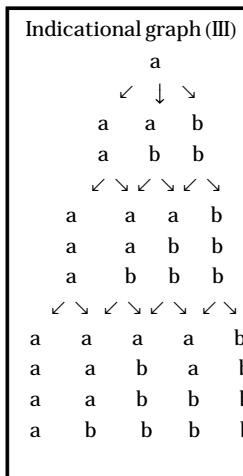
Alphabet = {a, b}

card (Ind(3, 2)) = {aaa, aab, abb, bbb} .

m = 2, n = 4

card (Ind (2, 4)) = 5

{aaaa, bbbb; aaab, abbb, aabb} .



R1		R2		R3		R4	
R5		R6		R7		R8	

Alphabet = {a, b}

card(Ind(3, 2)) = {aaa, aab, abb, bbb} x {a,b}

$R2 \begin{array}{ c c c } \hline \blacksquare & \blacksquare & \square \\ \hline - & \blacksquare & - \\ \hline \end{array}$	$= R5 \begin{array}{ c c c } \hline \blacksquare & \blacksquare & \blacksquare \\ \hline - & \square & - \\ \hline \end{array}$
$R3 \begin{array}{ c c c } \hline \blacksquare & \square & \square \\ \hline - & \blacksquare & - \\ \hline \end{array}$	$= R6 \begin{array}{ c c c } \hline \blacksquare & \blacksquare & \square \\ \hline - & \square & - \\ \hline \end{array}$
$R4 \begin{array}{ c c c } \hline \square & \square & \square \\ \hline - & \blacksquare & - \\ \hline \end{array}$	$= R7 \begin{array}{ c c c } \hline \blacksquare & \square & \square \\ \hline - & \square & - \\ \hline \end{array}$
$R1 \begin{array}{ c c c } \hline \blacksquare & \blacksquare & \blacksquare \\ \hline - & \blacksquare & - \\ \hline \end{array} \quad R2 \begin{array}{ c c c } \hline \blacksquare & \blacksquare & \square \\ \hline - & \blacksquare & - \\ \hline \end{array} \quad R3 \begin{array}{ c c c } \hline \blacksquare & \square & \square \\ \hline - & \blacksquare & - \\ \hline \end{array} \quad R4 \begin{array}{ c c c } \hline \square & \square & \square \\ \hline - & \blacksquare & - \\ \hline \end{array} \quad R8 \begin{array}{ c c c } \hline \square & \square & \square \\ \hline - & \square & - \\ \hline \end{array}$	

## Basic composition scheme

```

ruleCI[{a_, b_, c_, d_}] :=
  Flatten[{rca[{a}], rca[{b}], rca[{c}], rca[{d}]}]

ruleCIR[{a_, b_, c_, d_, e_}] :=
  Flatten[{cir[{a}], cir[{b}], cir[{c}], cir[{d}], cir[{e}]}]

```

## Rule set for indCA<sup>(3,2)</sup>

```

RuleSetCI = {
    rca[{1}] := {1, 1, 1} → 1,
    rca[{5}] := {1, 1, 1} → 0,

    rca[{4}] := {0, 0, 0} → 1,
    rca[{8}] := {0, 0, 0} → 0,

    rca[{3}] :=
    {
        {0, 0, 1} → 1,
        {0, 1, 0} → 1,
        {1, 0, 0} → 1
    },
    rca[{7}] :=
    {
        {0, 0, 1} → 0,
        {0, 1, 0} → 0,
        {1, 0, 0} → 0
    },
    rca[{2}] := {
        {1, 1, 0} → 1,
        {1, 0, 1} → 1,
        {0, 1, 1} → 1
    },
    rca[{6}] :=
    {
        {1, 1, 0} → 0,
        {1, 0, 1} → 0,
        {0, 1, 1} → 0
    }
}

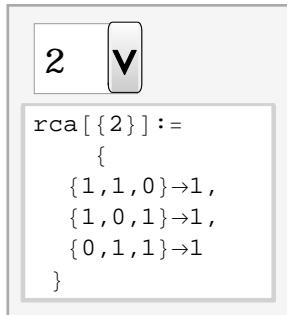
{Null, Null, Null, Null, Null, Null, Null, Null}

```

## TabView for indCA<sup>(3,2)</sup> rules

1	5	4	8	3	7	2	6
<b>rca</b> [{6}] := { {1, 1, 0} → 0, {1, 0, 1} → 0, {0, 1, 1} → 0 }							

## MenuView for indCA<sup>(3,2)</sup> rules



## Rule space for indCA<sup>(3,2)</sup>

$$\{1, 5\} \times \{2, 6\} \times \{3, 7\} \times \{4, 8\} = 16$$

1234	5234
1238	5238
1274	5274
1278	5278
1674	5634
1678	5638
1634	5674
1638	5678

## Program scheme for indicational CA

```
ArrayPlot[CellularAutomaton[
  {
    CI -> rules[i] = perm[i][{a, b, c}] → d, a, b, c ∈ {0, 1}, 1 ≤ i ≤ 4
  },
  init, steps],
Graphics = ColorRules -> {1 -> Red, 0 -> Yellow}]
```

## Indicational normal form (inf)

$$\begin{aligned}\text{inf}([\blacksquare \square \blacksquare]) &= \text{inf}([\square \blacksquare \blacksquare]) = [\blacksquare \blacksquare \square] \\ \text{inf}([\square \blacksquare \square]) &= \text{inf}([\square \square \blacksquare]) = [\blacksquare \square \square] \\ \text{inf}([\blacksquare \blacksquare \blacksquare]) &= [\blacksquare \blacksquare \blacksquare] \\ \text{inf}([\square \square \square]) &= [\square \square \square]\end{aligned}$$

Indicational normal form (inf):

$$\begin{aligned}\text{inf}([\blacksquare \square \blacksquare]) &= \text{inf}([\square \blacksquare \blacksquare]) = [\blacksquare \blacksquare \square] \\ \text{inf}([\square \blacksquare \square]) &= \text{inf}([\square \square \blacksquare]) = [\blacksquare \square \square] \\ \text{inf}([\blacksquare \blacksquare \blacksquare]) &= [\blacksquare \blacksquare \blacksquare] \\ \text{inf}([\square \square \square]) &= [\square \square \square]\end{aligned}$$

Indicational normal forms are not identical with proto-normal forms, pnf, of kenogrammatics.

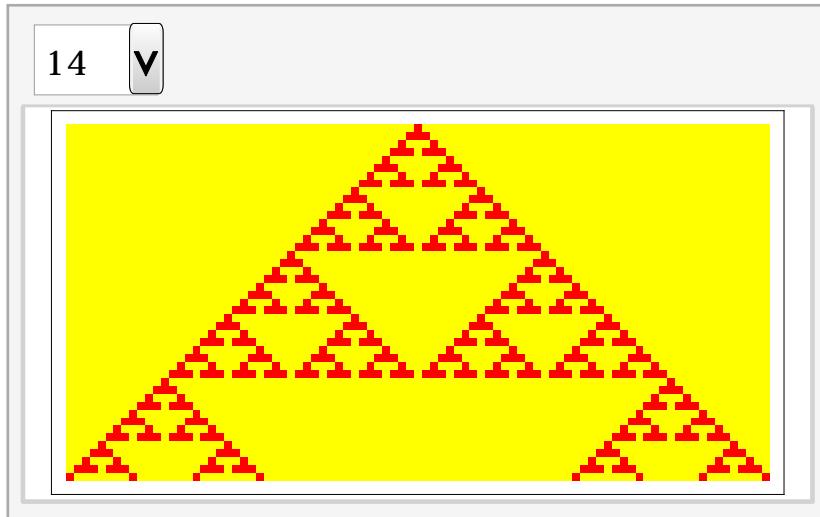
Calculus of indication:  $a=a$ ,  $a \neq b$ ,  $ab=ba$

For 1D indCA<sup>(2,2)</sup>:

$$[aab] = [baa] = [aba]$$

```
[abb] = [baa] = [bab]
[aaa] = [aaa]
[bbb] = [bbb]
pnf[[aaa]] = pnf[[bbb]] but
inf[[aaa]] ≠ inf[[bbb]].
```

### System of indCA<sup>(3,2)</sup>



```
(*  
Manipulate[ListAnimate[Table[ArrayPlot[CellularAutomaton[ruleCI, {{1}, 0}, n],  
ColorRules -> {1 -> Red, 0 -> Yellow, 2 -> Blue, 3 -> Green}, ImageSize -> {400, 200}],  
{ruleCI,  
 {  
 ruleCI[{1, 2, 3, 4}],  
 ruleCI[{1, 2, 3, 8}],  
 ruleCI[{1, 2, 7, 4}],  
 ruleCI[{1, 2, 7, 8}],  
  
 ruleCI[{1, 6, 3, 4}],  
 ruleCI[{1, 6, 3, 8}],  
 ruleCI[{1, 6, 7, 4}],  
 ruleCI[{1, 6, 7, 8}],  
  
 ruleCI[{5, 2, 3, 4}],  
 ruleCI[{5, 2, 3, 8}],  
 ruleCI[{5, 2, 7, 4}],  
 ruleCI[{5, 2, 7, 8}],  
  
 ruleCI[{5, 6, 3, 4}],  
 ruleCI[{5, 6, 3, 8}],  
 ruleCI[{5, 6, 7, 4}],  
 ruleCI[{5, 6, 7, 8}]}  
 }]], {n, 2, 222, 1}]  
*)
```

## Analysis of linear forms

### Groups

1234	5234
1634	5634
5678	5674
1278	1678

1274

### Equalities

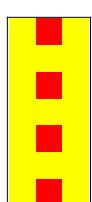
5234 = 5634
1234 = 1634
5678 = 1278

1678 ≈ 5674

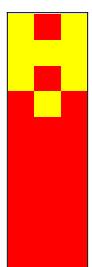
1274

### Examples

rule = 5.6.7.4



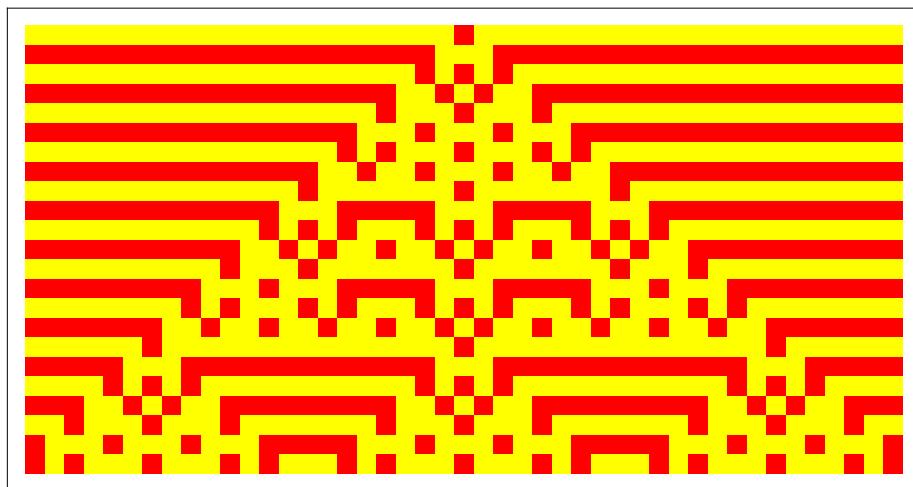
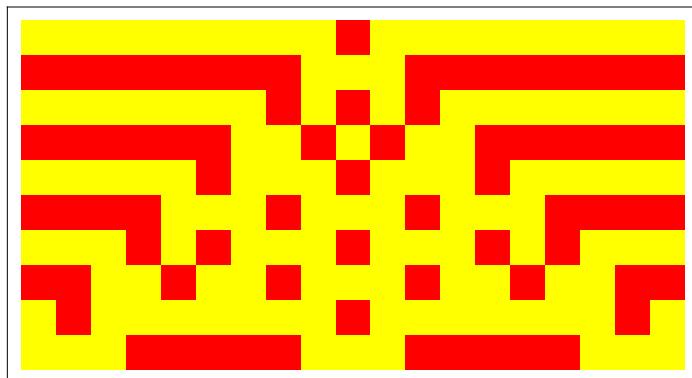
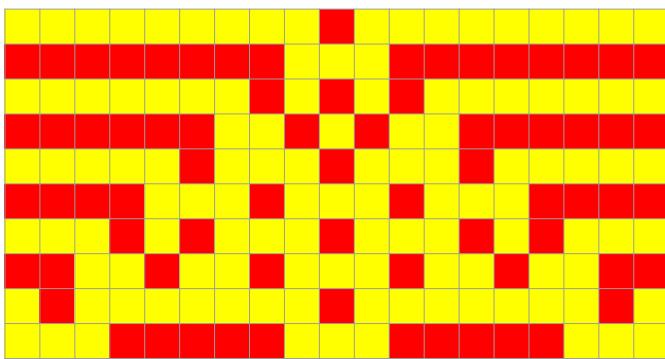
rule = 1274

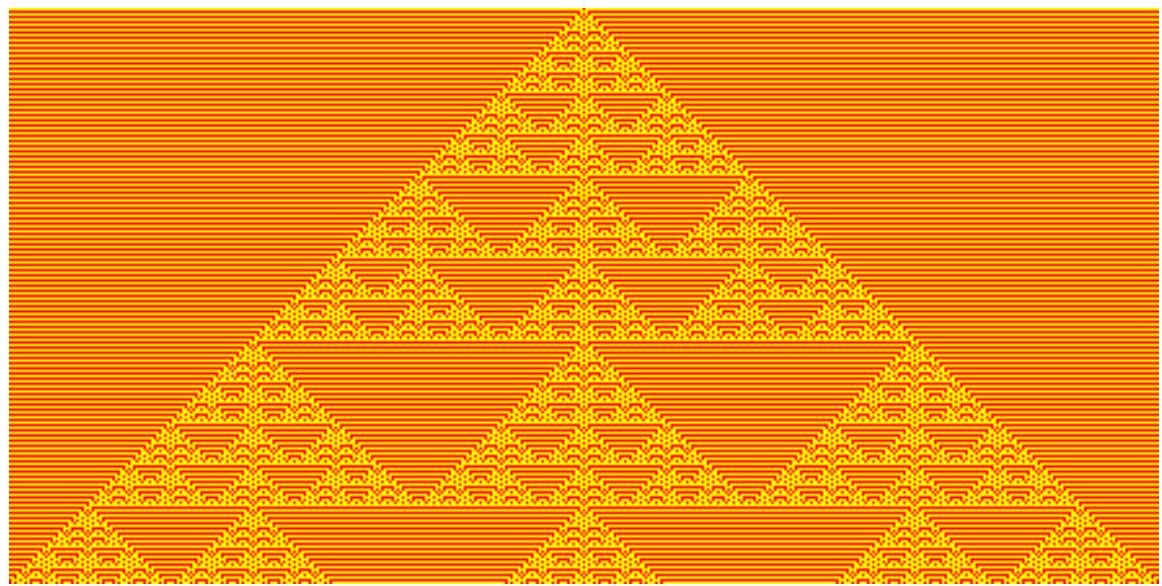


## Analysis of developping indCA forms

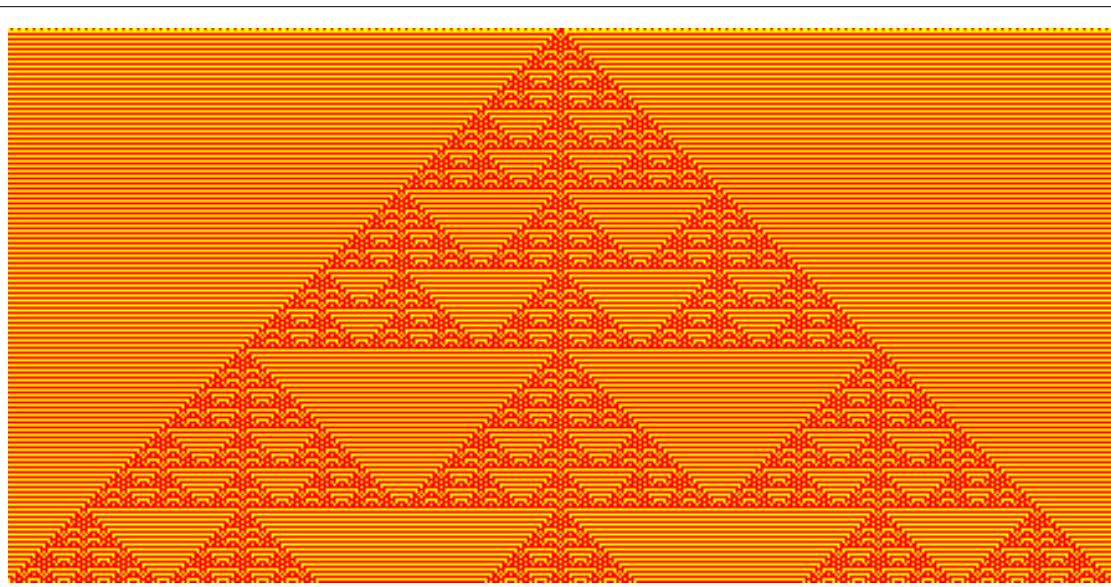
1634	5638
1638	5238

rule = 1634

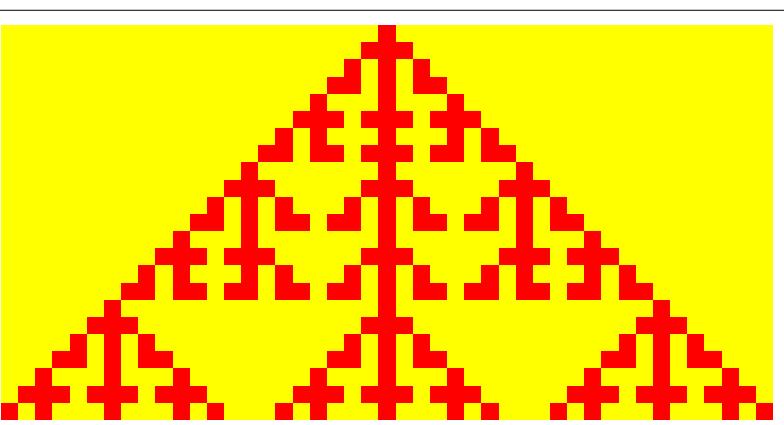


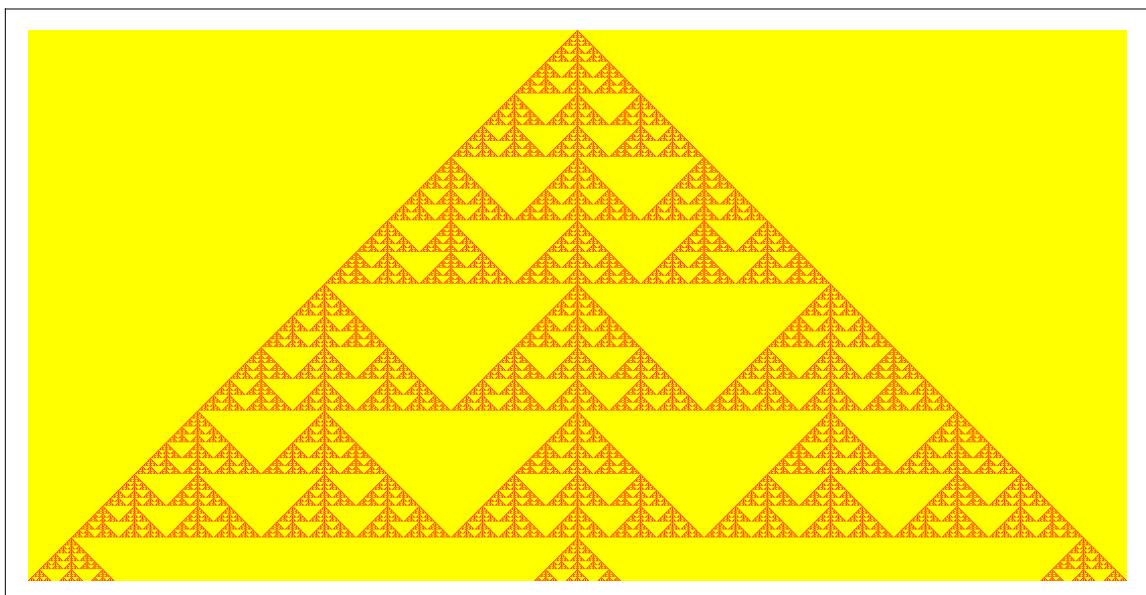
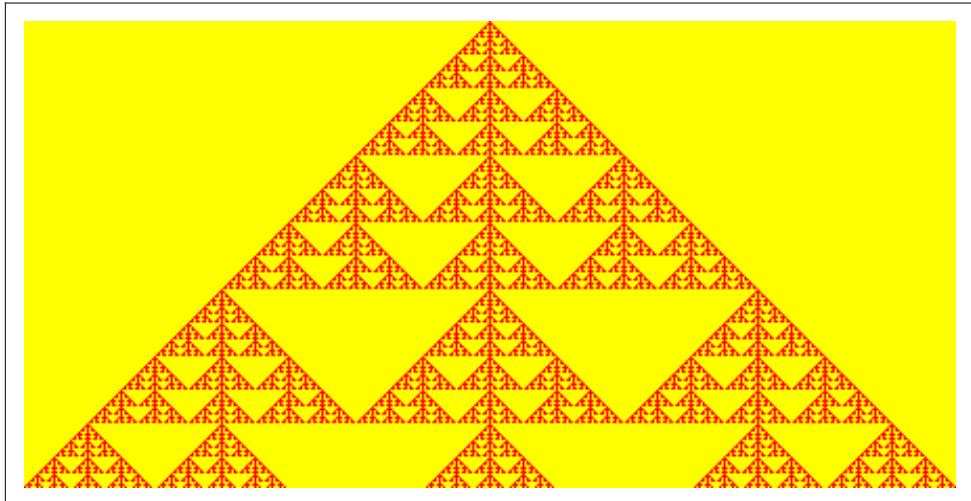


**seed = {0, 1, 0}**

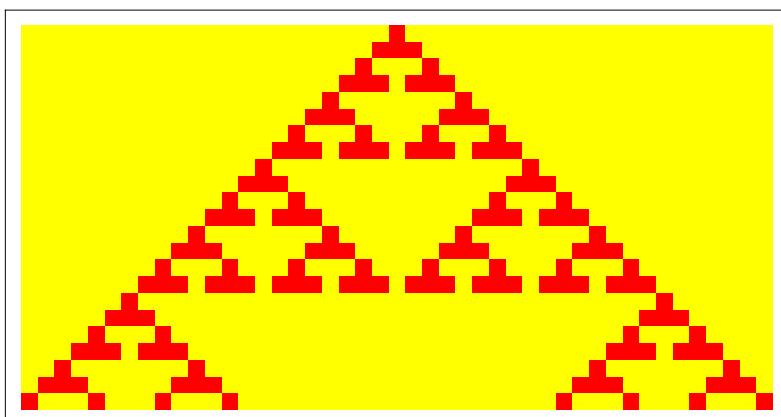


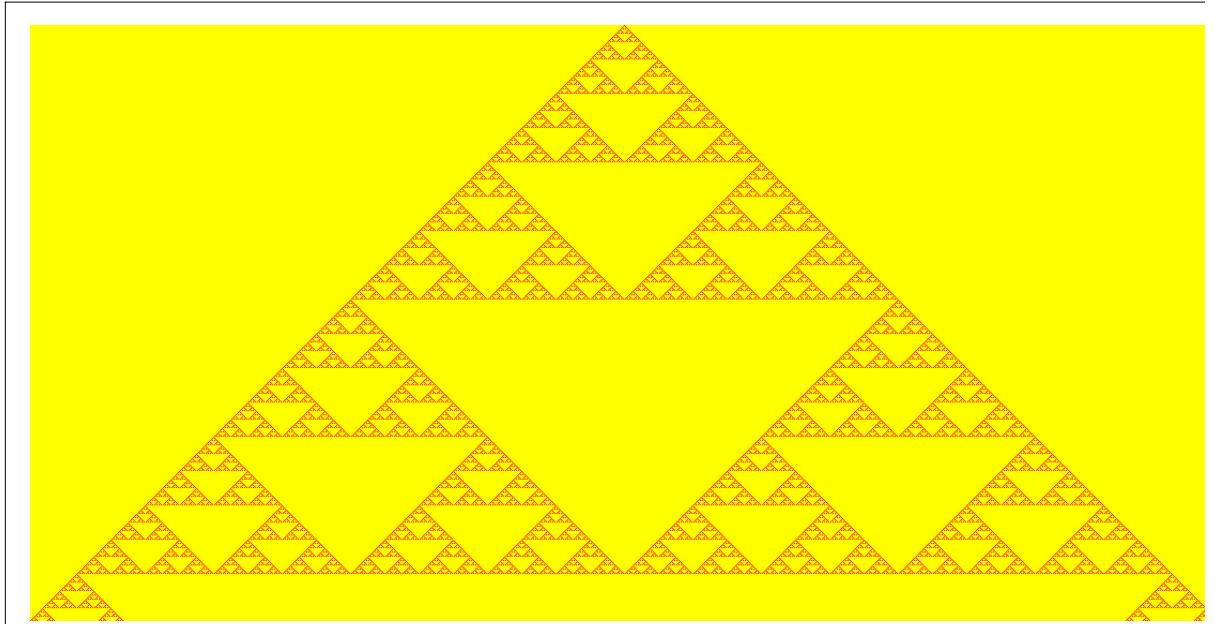
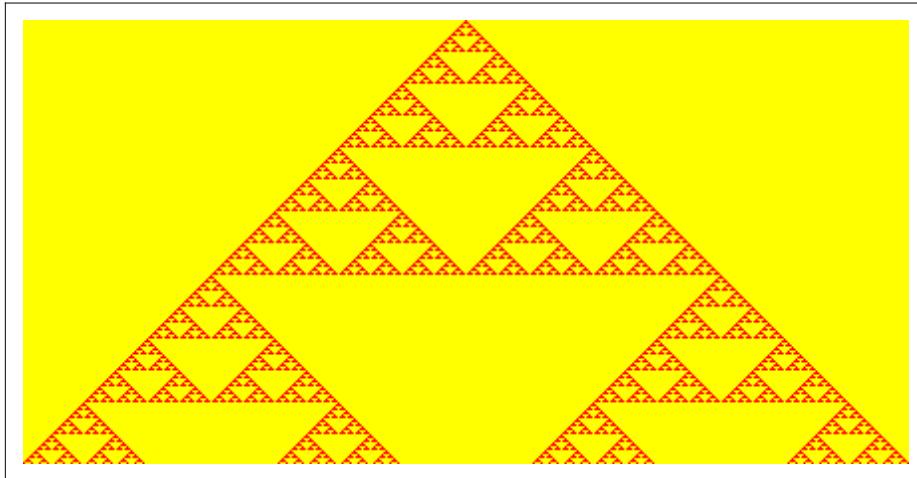
**rule = 1638**



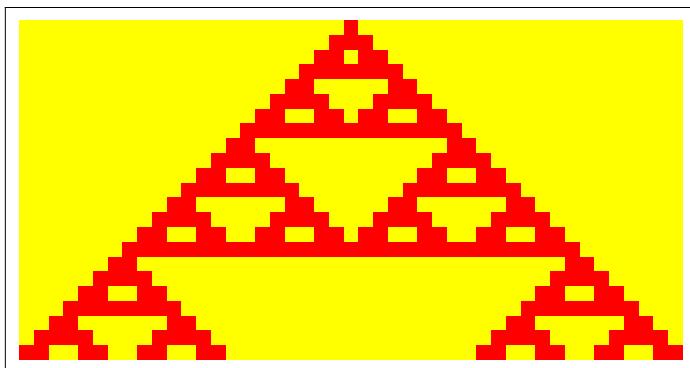


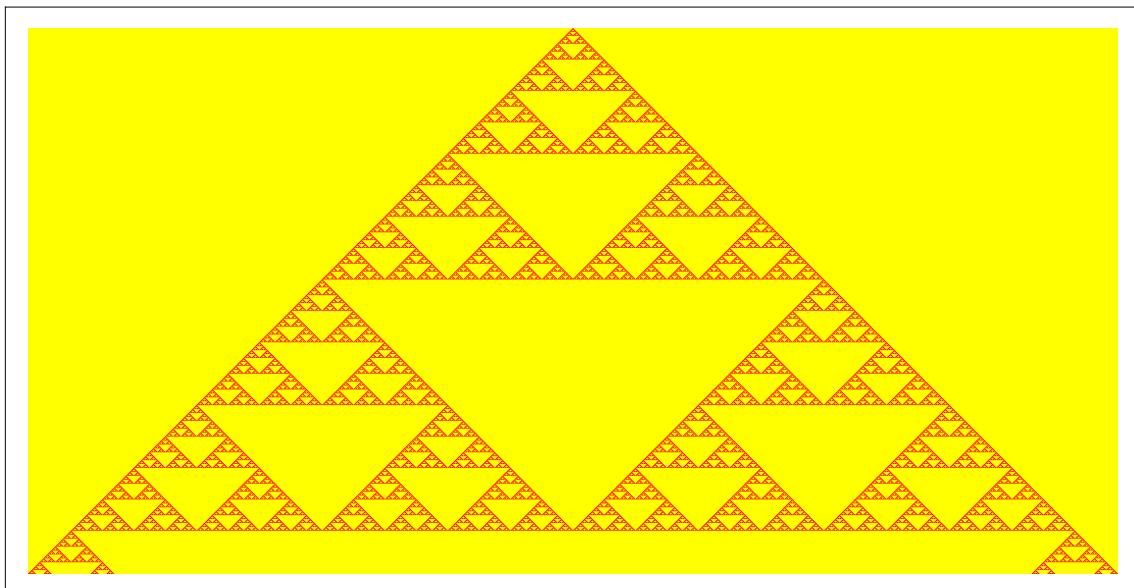
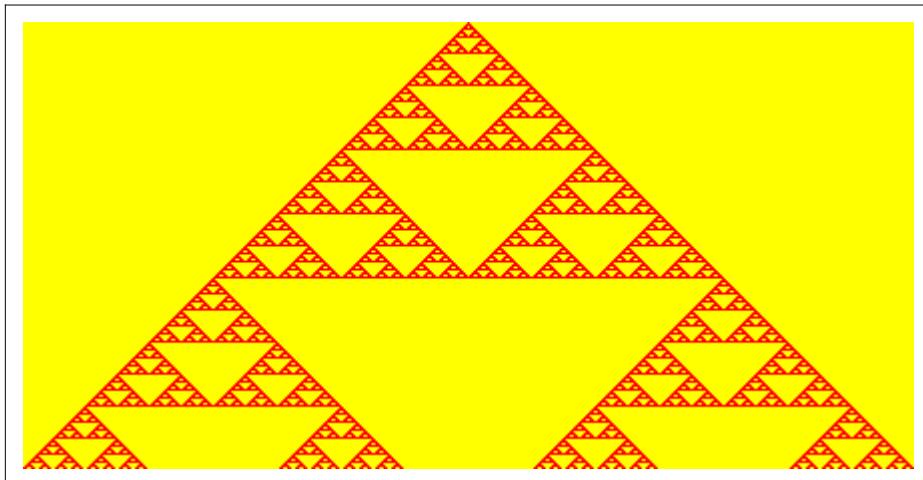
Cl - rule = 5.6 .3 .8





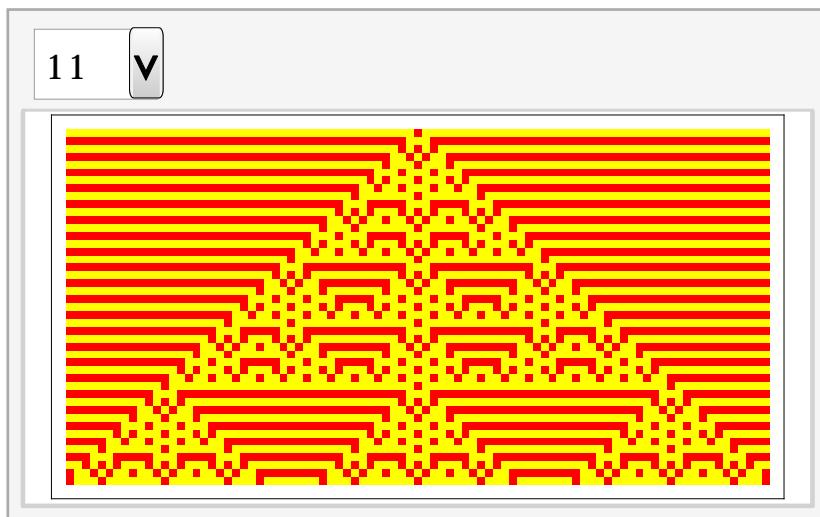
CI - rule = 5.2 .3.8





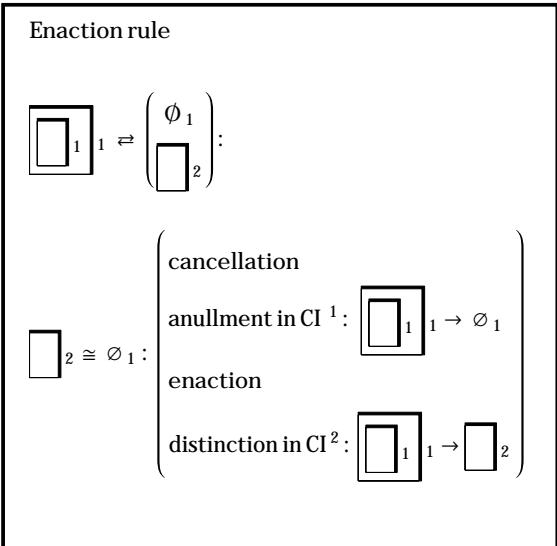
## System of indCA<sup>(4,2)</sup> rules

```
(*  
Manipulate[MenuView[Table[ArrayPlot[CellularAutomaton[ruleCI, {{1}, 0}, n],  
ColorRules -> {1 -> Red, 0 -> Yellow, 2 -> Blue, 3 -> Green}, ImageSize -> {400, 200}],  
{ruleCI,  
 {  
 ruleCI[{1, 2, 3, 4}],  
 ruleCI[{1, 2, 3, 8}],  
 ruleCI[{1, 2, 7, 4}],  
 ruleCI[{1, 2, 7, 8}],  
  
 ruleCI[{1, 6, 3, 4}],  
 ruleCI[{1, 6, 3, 8}],  
 ruleCI[{1, 6, 7, 4}],  
 ruleCI[{1, 6, 7, 8}],  
  
 ruleCI[{5, 2, 3, 4}],  
 ruleCI[{5, 2, 3, 8}],  
 ruleCI[{5, 2, 7, 4}],  
 ruleCI[{5, 2, 7, 8}],  
  
 ruleCI[{5, 6, 3, 4}],  
 ruleCI[{5, 6, 3, 8}],  
 ruleCI[{5, 6, 7, 4}],  
 ruleCI[{5, 6, 7, 8}]  
 }]], {n, 2, 222, 1}]  
*)
```




---

## Reflectional enactment for indCA<sup>(3,3)</sup> rules

**Example for indCA enaction**

$$\begin{array}{ccc} \boxed{\phi \ \square \ \phi} & \Rightarrow & \boxed{\phi \ \boxed{\square}_{i.j} \ \phi} \\ & & = \begin{pmatrix} \emptyset \ \boxed{\square}_{i.j} \ \emptyset \\ - \ \ \phi \ \ - \end{pmatrix} \\ & & \boxed{\phi \ \boxed{\square}_{i.j} \ \phi} \\ & & = \begin{pmatrix} \emptyset \ \boxed{\square}_{i.j} \ \emptyset \\ - \ \boxed{\square}_{i.j+1} \ \ - \end{pmatrix} \end{array}$$
  

$$\text{indCA : } \boxed{R3 \ \square \ \blacksquare \ \square} \ \Rightarrow \ \begin{pmatrix} R3 .1 \ \square \ \blacksquare \ \square \\ - \ \ \square \ \ - \end{pmatrix}$$

$$\begin{pmatrix} R3 .2 \ \square \ \blacksquare \ \square \\ - \ \ \textcolor{red}{\blacksquare} \ \ - \end{pmatrix}$$

with the function  $\blacksquare (\blacksquare) = \begin{pmatrix} \square \\ \blacksquare \end{pmatrix}$ .

Hence, the set of indCA rules might be extended by its interactional enaction rules which are relating to CA systems of neighbor contexts. This will be studied in another paper.

**Combinatorics**

$$m = n = 3$$

$$\text{card(Ind}(3, 3)\text{)} = 10$$

$$\text{Ind}(3, 3) = \{\text{aaa, bbb, ccc, aab, aac, abb, acc, bcc, bbc, abc}\}$$

$$\text{card (Ind (4, 3))} = \text{card (Ind}(3, 3)\text{)} \times \{3\} = 30$$

$$m = 4, n = 3 : \binom{4 + 3 - 1}{4} = \binom{6}{4} = 30$$

**Set of combinations for indCA<sup>(4,3)</sup> :**

$$\{\text{aaa, bbb, ccc, aab, aac, abb, acc, bcc, bbc, abc}\} \times \{a, b, c\} =$$

## Example

R11		R12		R13	
R21		R22		R23	
R31		R32		R33	
R41		R42		R43	
R51		R52		R53	
R61		R62		R63	
R71		R72		R73	
R81		R82		R83	
R91		R92		R93	
R101		R102		R103	

## Rule scheme

```
ruleCIR[{a_, b_, c_, d_, e_, f_, g_, h_, i_, k_}] :=
  Flatten[
  {cir[{a}], cir[{b}], cir[{c}], cir[{d}],
   cir[{e}], cir[{f}], cir[{g}],
   cir[{h}], cir[{i}], cir[{k}]}]
```

## Rule set for indCA<sup>(3,3)</sup>

```

ruleSetCIR =
{
  cir[{}{11}] := {1, 1, 1} → 1,
  cir[{}{21}] := {0, 0, 0} → 1,
  cir[{}{31}] := {2, 2, 2} → 1,

  cir[{}{12}] := {1, 1, 1} → 0,
  cir[{}{22}] := {0, 0, 0} → 0,
  cir[{}{32}] := {2, 2, 2} → 0,

  cir[{}{13}] := {1, 1, 1} → 2,
  cir[{}{23}] := {0, 0, 0} → 2,
  cir[{}{33}] := {2, 2, 2} → 2,

  cir[{}{41}] := {{1, 1, 0} → 1, {1, 0, 1} → 1, {0, 1, 1} → 1},
  cir[{}{51}] := {{1, 1, 2} → 1, {1, 2, 1} → 1, {2, 1, 1} → 1},
  cir[{}{61}] := {{1, 0, 0} → 1, {0, 1, 0} → 1, {0, 0, 1} → 1},

  cir[{}{42}] := {{1, 1, 0} → 0, {1, 0, 1} → 0, {0, 1, 1} → 0},
  cir[{}{52}] := {{1, 1, 2} → 0, {1, 2, 1} → 0, {2, 1, 1} → 0},
  cir[{}{62}] := {{1, 0, 0} → 0, {0, 1, 0} → 0, {0, 0, 1} → 0},

  cir[{}{43}] := {{1, 1, 0} → 2, {1, 0, 1} → 2, {0, 1, 1} → 2},
  cir[{}{53}] := {{1, 1, 2} → 2, {1, 2, 1} → 2, {2, 1, 1} → 2},
  cir[{}{63}] := {{1, 0, 0} → 2, {0, 1, 0} → 2, {0, 0, 1} → 2},

  cir[{}{71}] := {{1, 2, 2} → 1, {2, 1, 2} → 1, {2, 2, 1} → 1},
  cir[{}{81}] := {{0, 2, 2} → 1, {2, 0, 2} → 1, {2, 2, 0} → 1},
  cir[{}{91}] := {{0, 0, 2} → 1, {0, 2, 0} → 1, {2, 0, 0} → 1},

  cir[{}{72}] := {{1, 2, 2} → 0, {2, 1, 2} → 0, {2, 2, 1} → 0},
  cir[{}{82}] := {{0, 2, 2} → 0, {2, 0, 2} → 0, {2, 2, 0} → 0},
  cir[{}{92}] := {{0, 0, 2} → 0, {0, 2, 0} → 0, {2, 0, 0} → 0},

  cir[{}{73}] := {{1, 2, 2} → 2, {2, 1, 2} → 2, {2, 2, 1} → 1},
  cir[{}{83}] := {{0, 2, 2} → 2, {2, 0, 2} → 2, {2, 2, 0} → 2},
  cir[{}{93}] := {{0, 0, 2} → 2, {0, 2, 0} → 2, {2, 0, 0} → 2},

  cir[{}{101}] := {{1, 0, 2} → 1, {1, 2, 0} → 1, {2, 0, 1} → 1,
                  {2, 1, 0} → 1, {0, 1, 2} → 1, {0, 2, 1} → 1},
  cir[{}{102}] := {{1, 0, 2} → 0, {1, 2, 0} → 0, {2, 0, 1} → 0,
                  {2, 1, 0} → 0, {0, 1, 2} → 0, {0, 2, 1} → 0},
  cir[{}{103}] := {{1, 0, 2} → 2, {1, 2, 0} → 2, {2, 0, 1} → 2,
                  {2, 1, 0} → 2, {0, 1, 2} → 2, {0, 2, 1} → 2}
}

```

72

▼

`cir[{}{72}] := {{1, 2, 2} → 0, {2, 1, 2} → 0, {2, 2, 1} → 0}`

### indCA<sup>(3,3)</sup> Rule scheme ruleCIR, 10x3

$\{0, 1, 1\} \rightarrow 2,$	$\{1, 2, 1\} \rightarrow 2,$	$\{1, 1, 1\} \rightarrow 1,$
$\{1, 0, 1\} \rightarrow 2,$	$\{1, 1, 2\} \rightarrow 2,$	$\{2, 2, 2\} \rightarrow 2,$
$\{1, 1, 0\} \rightarrow 2,$	$\{2, 1, 1\} \rightarrow 2,$	$\{0, 0, 0\} \rightarrow 0,$
$\{0, 0, 1\} \rightarrow 0,$	$\{2, 1, 2\} \rightarrow 0,$	$\{2, 0, 1\} \rightarrow 1,$
$\{1, 0, 0\} \rightarrow 0,$	$\{2, 2, 1\} \rightarrow 0,$	$\{2, 1, 0\} \rightarrow 1,$
$\{0, 1, 0\} \rightarrow 0,$	$\{1, 2, 2\} \rightarrow 0,$	$\{1, 0, 2\} \rightarrow 1,$
$\{0, 0, 2\} \rightarrow 2,$	$\{2, 0, 2\} \rightarrow 0,$	$\{1, 2, 0\} \rightarrow 1,$
$\{0, 2, 0\} \rightarrow 2,$	$\{0, 2, 2\} \rightarrow 0,$	$\{0, 1, 2\} \rightarrow 1,$
$\{2, 0, 0\} \rightarrow 2,$	$\{2, 2, 0\} \rightarrow 0$	$\{0, 2, 1\} \rightarrow 1$

### CA Program scheme

```
ArrayPlot[CellularAutomaton[
 {
   {CIR - Rules}
 },
 init, steps],
 ColorRules -> {2 -> Green, 1 -> Red, 0 -> Yellow}]
```

### Rule space for indCA<sup>(3,2)</sup>

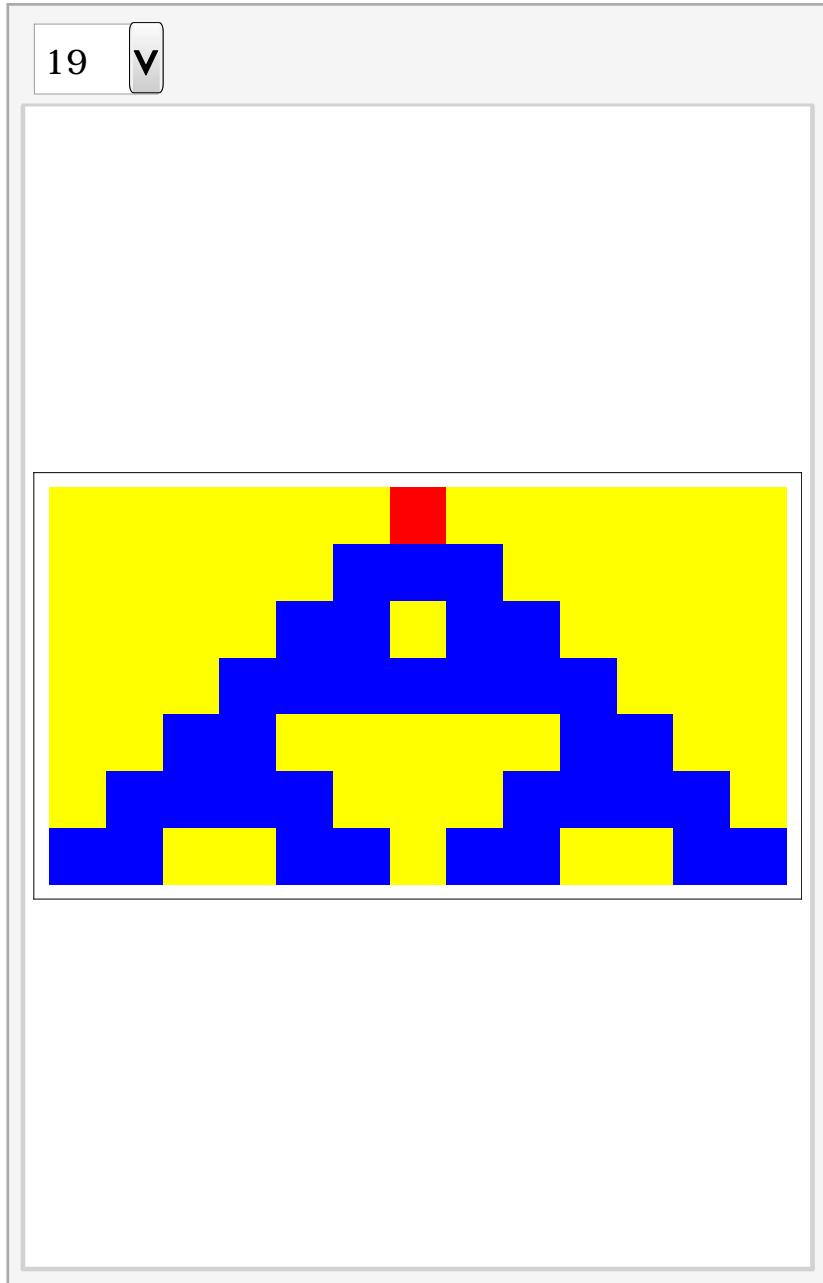
$$\{1, 5\} \times \{2, 6\} \times \{3, 7\} \times \{4, 8\} = 16$$

1234 – 8'	5234
1238	5238
1274	5274
1278	5278
1674	5634
1678	5638
1634	5674
1638	5678 – 8'

### Indicational normal form (inf)

```
inf([\square \blacksquare \square]) = inf([\square \square \blacksquare]) = [\blacksquare \square \square]
inf([\square \blacksquare \square]) = inf([\square \square \blacksquare]) = [\blacksquare \square \square]
inf([\blacksquare \blacksquare \blacksquare]) = inf([\blacksquare \blacksquare \blacksquare]) = [\blacksquare \blacksquare \blacksquare]
inf([\blacksquare \square \blacksquare]) = inf([\square \blacksquare \blacksquare]) = [\blacksquare \blacksquare \square]
inf([\blacksquare \blacksquare \square]) = inf([\blacksquare \blacksquare \square]) = [\blacksquare \blacksquare \square]
inf([\blacksquare \blacksquare \blacksquare]) = inf([\blacksquare \blacksquare \blacksquare]) = [\blacksquare \blacksquare \blacksquare]
inf([\blacksquare \blacksquare \blacksquare]) = [\blacksquare \blacksquare \blacksquare]
inf([\square \square \square]) = [\square \square \square]
inf([\blacksquare \square \square]) = [\square \blacksquare \square]
```

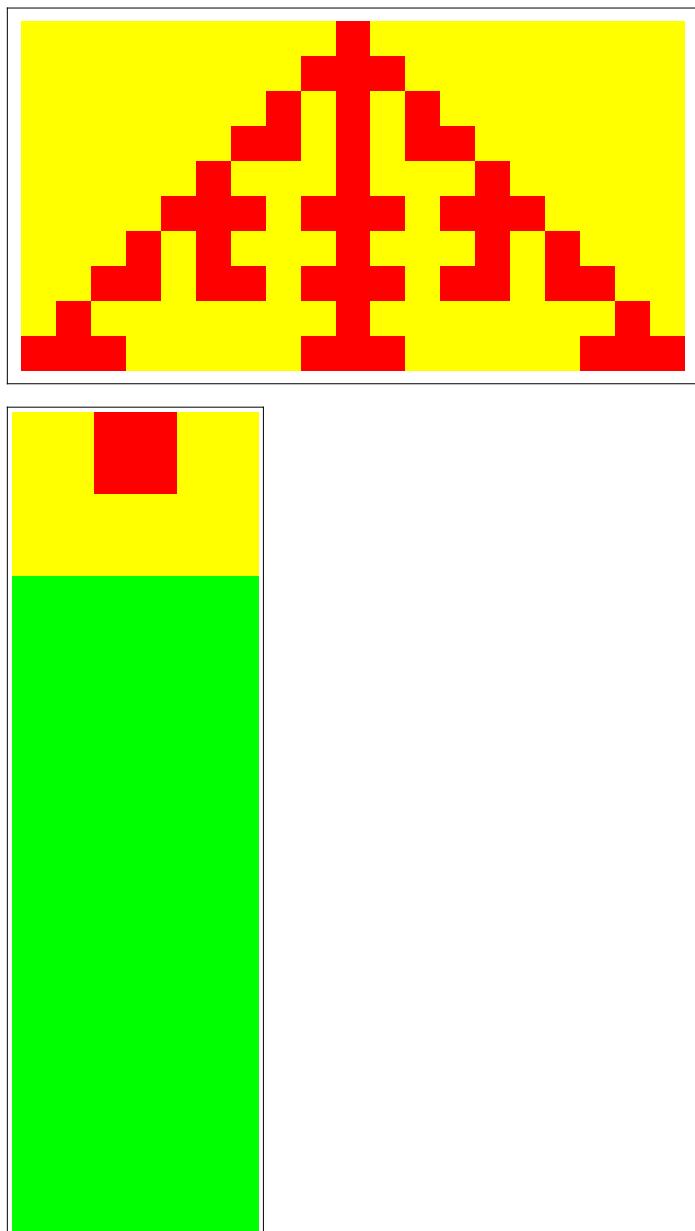
perm [{\blacksquare, \square, \blacksquare}]

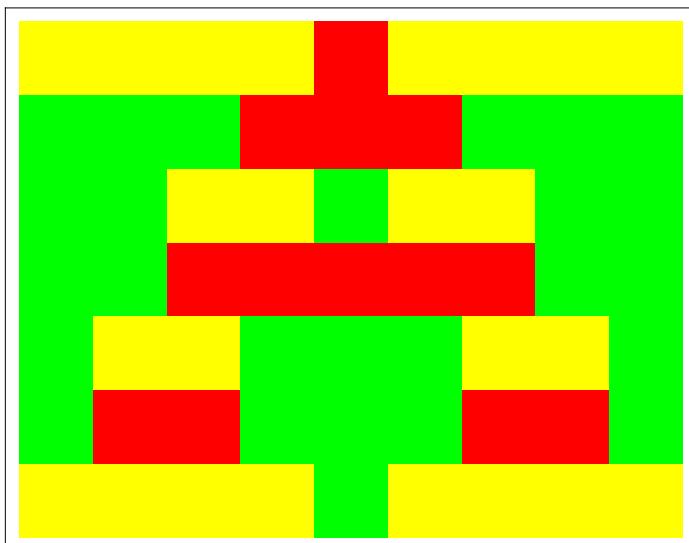
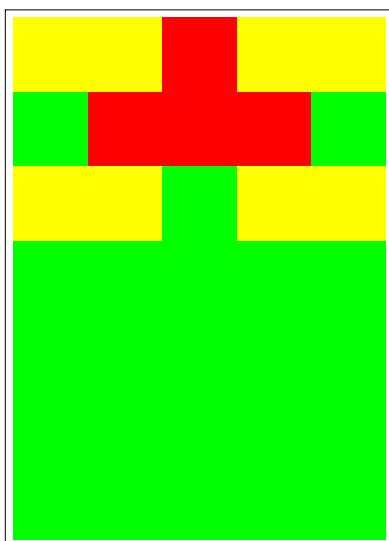
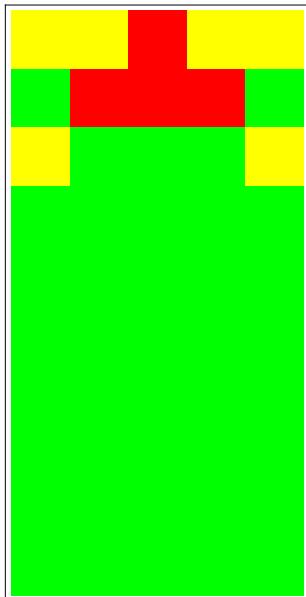
IndCA<sup>(3,3)</sup>

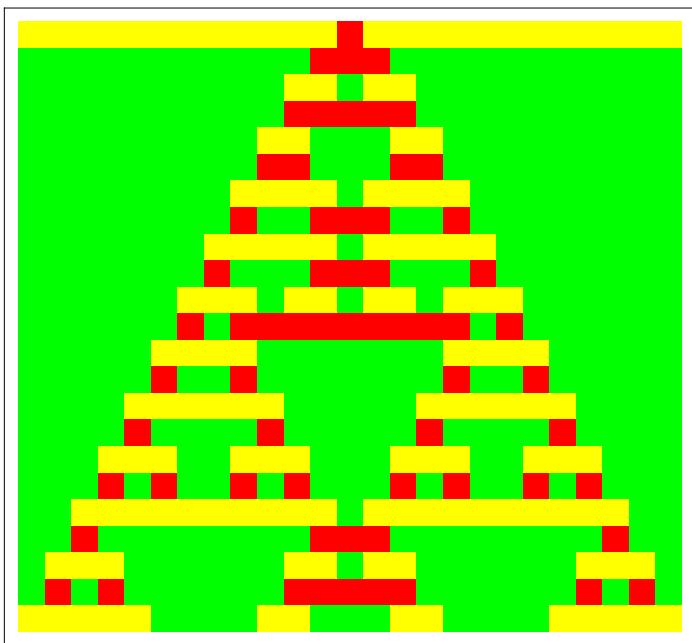
```
ruleCIR[{11, 22, 32, 42, 51, 63, 73, 83, 93, 102 }]
```

```
{ {1, 1, 1} → 1, {0, 0, 0} → 0, {2, 2, 2} → 0, {1, 1, 0} → 0, {1, 0, 1} → 0, {0, 1, 1} → 0,
{1, 1, 2} → 1, {1, 2, 1} → 1, {2, 1, 1} → 1, {1, 0, 0} → 2, {0, 1, 0} → 2, {0, 0, 1} → 2,
{1, 2, 2} → 2, {2, 1, 2} → 2, {2, 2, 1} → 1, {0, 2, 2} → 2, {2, 0, 2} → 2,
{2, 2, 0} → 2, {0, 0, 2} → 2, {0, 2, 0} → 2, {2, 0, 0} → 2, {1, 0, 2} → 0,
{1, 2, 0} → 0, {2, 0, 1} → 0, {2, 1, 0} → 0, {0, 1, 2} → 0, {0, 2, 1} → 0 }
```

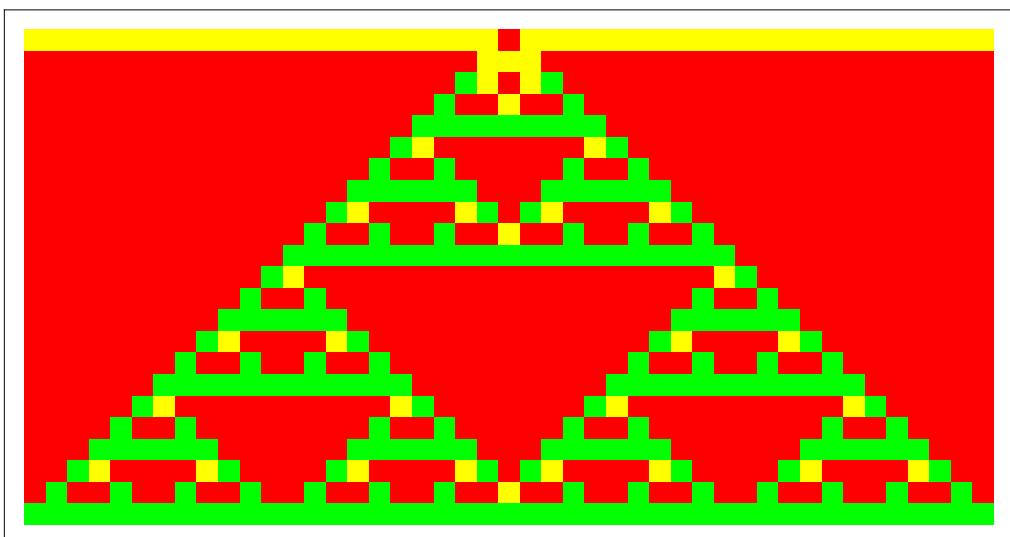
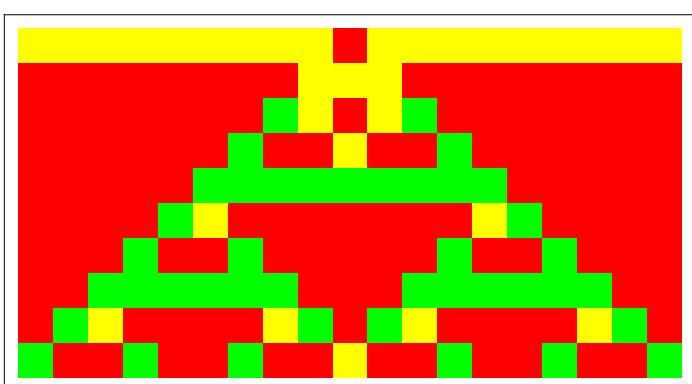
```
ruleCIR[{11, 22, 32, 42, 53, 61, 72, 83, 92, 102 }]
```

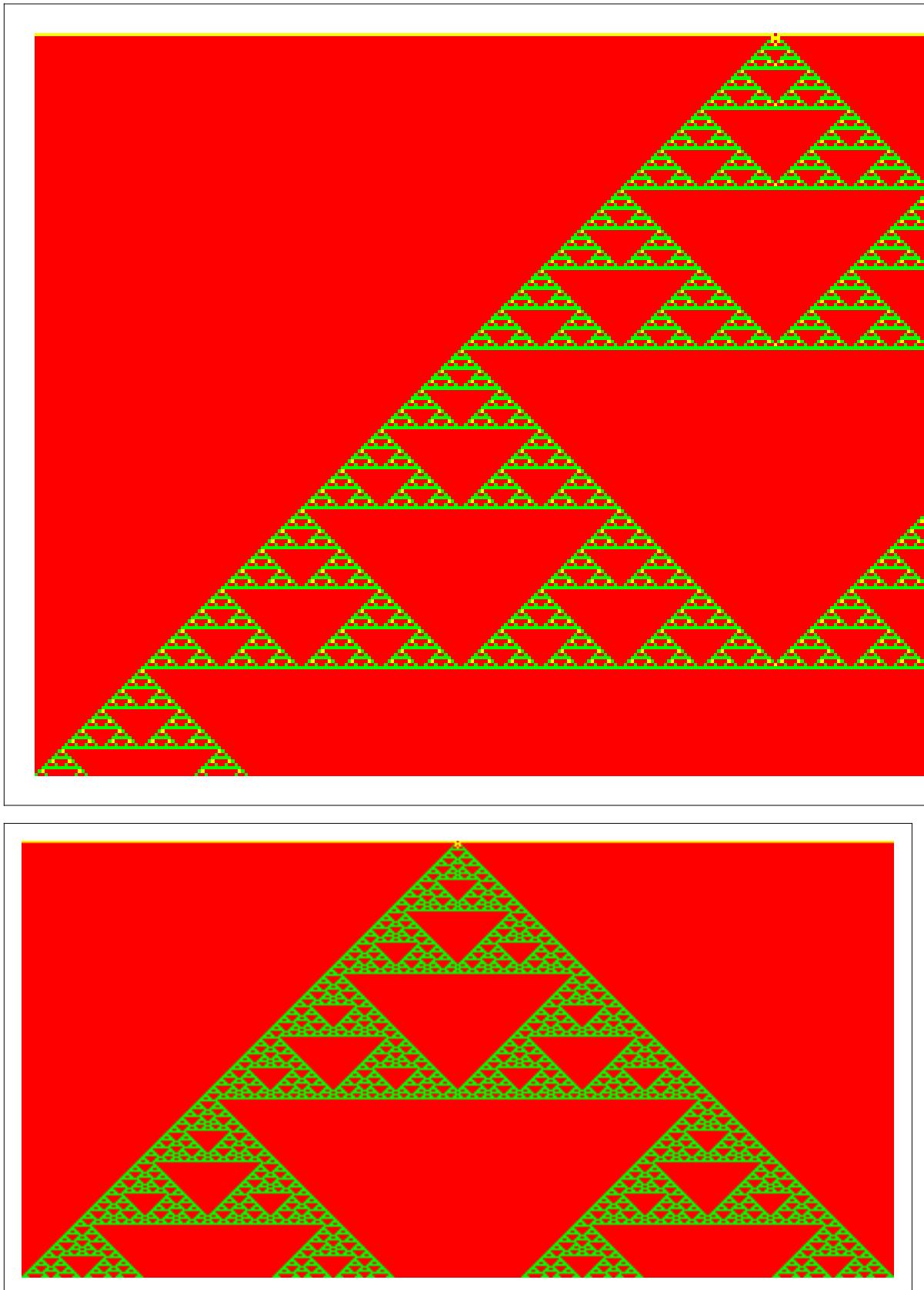




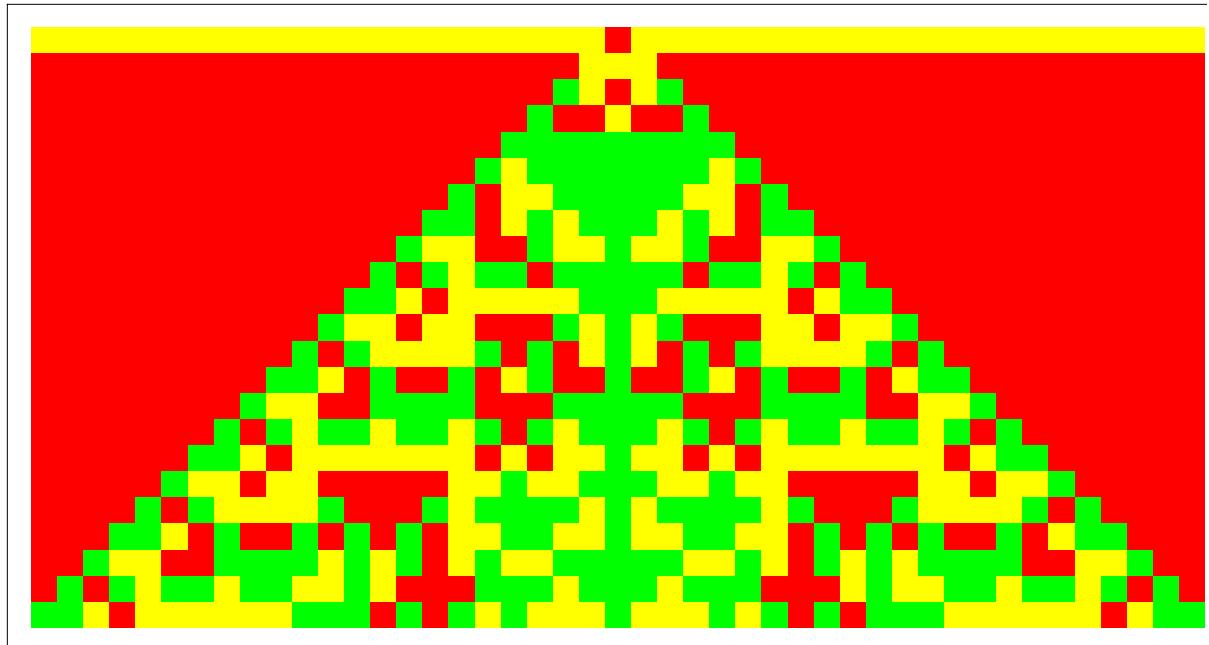
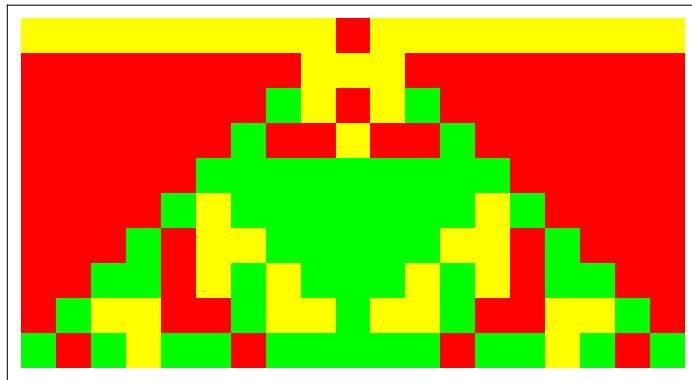
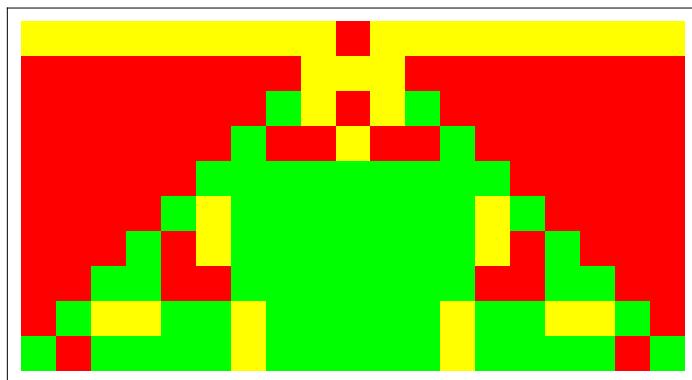


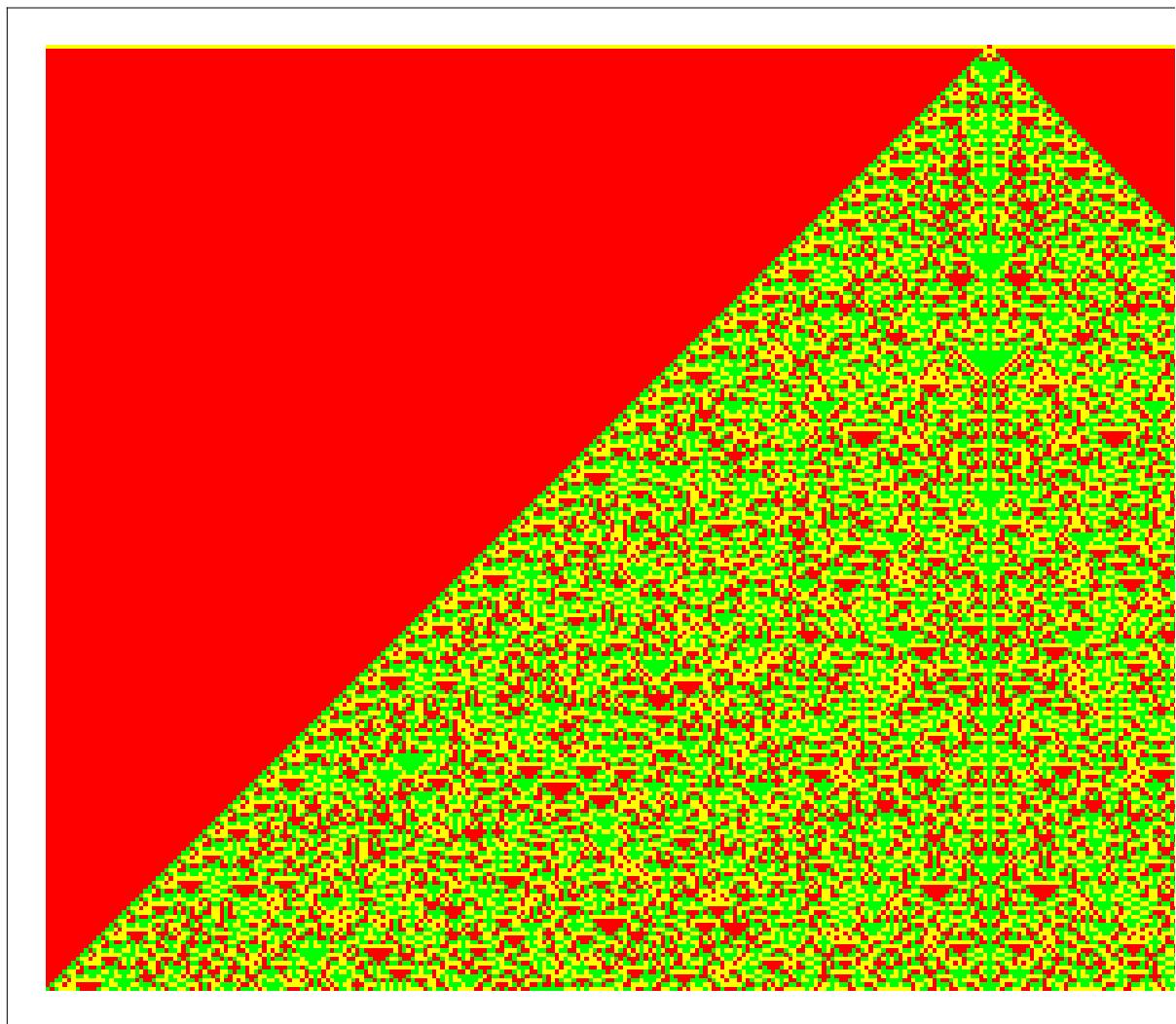
rule = 1.22.33 .4

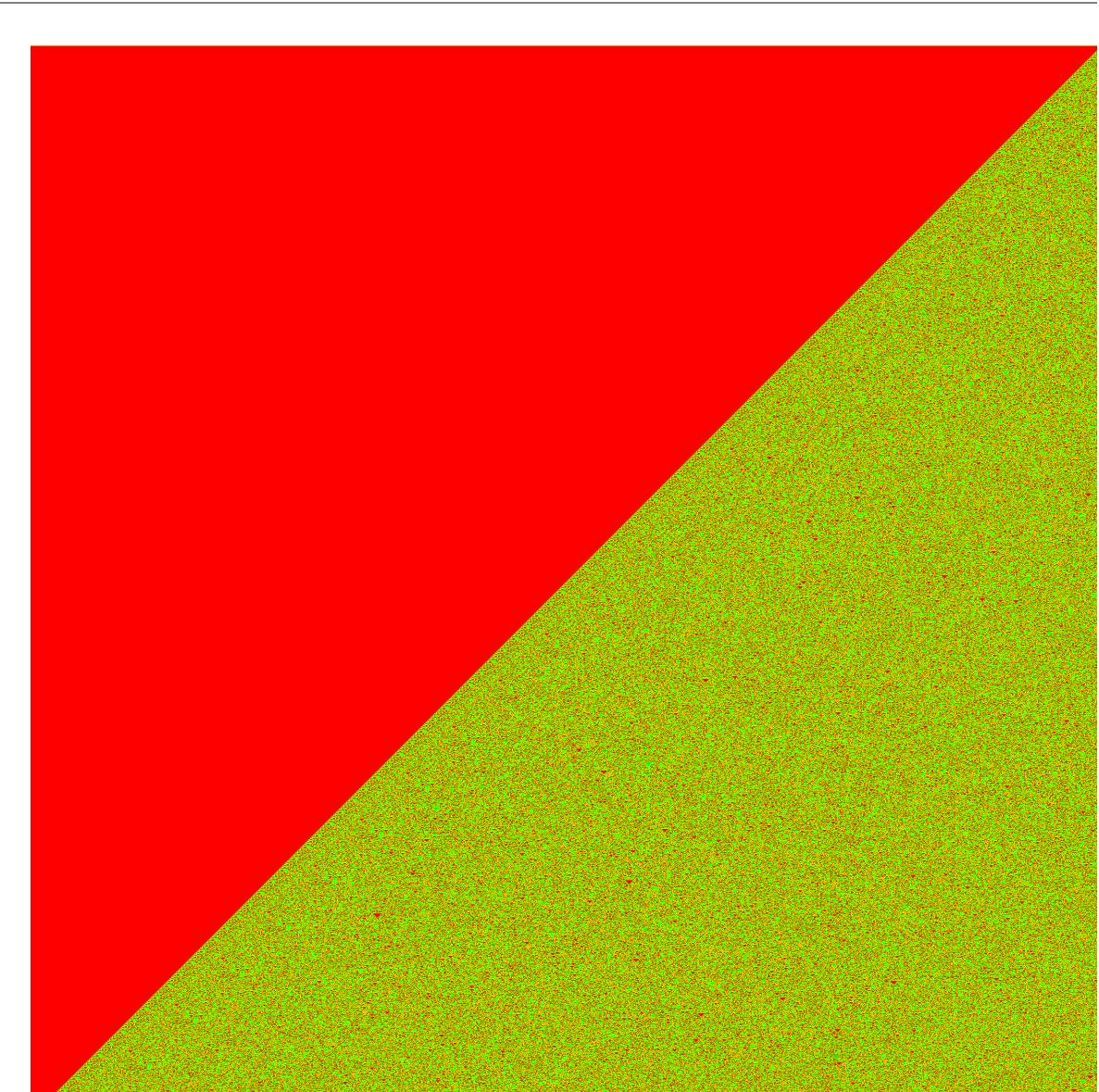


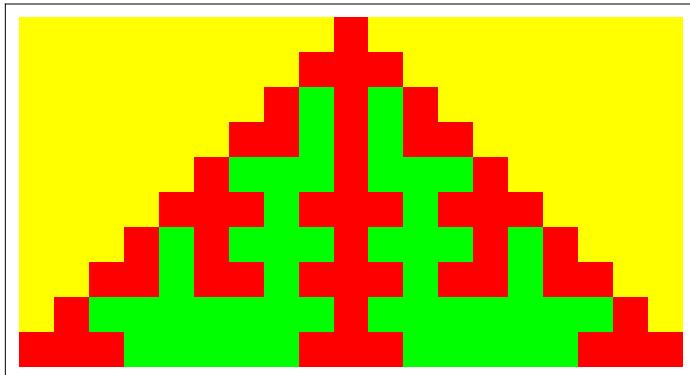
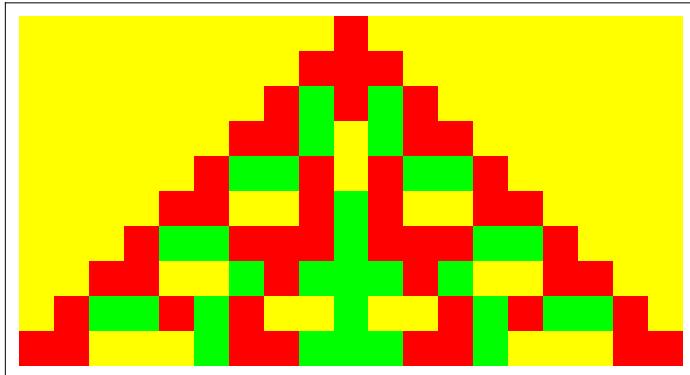


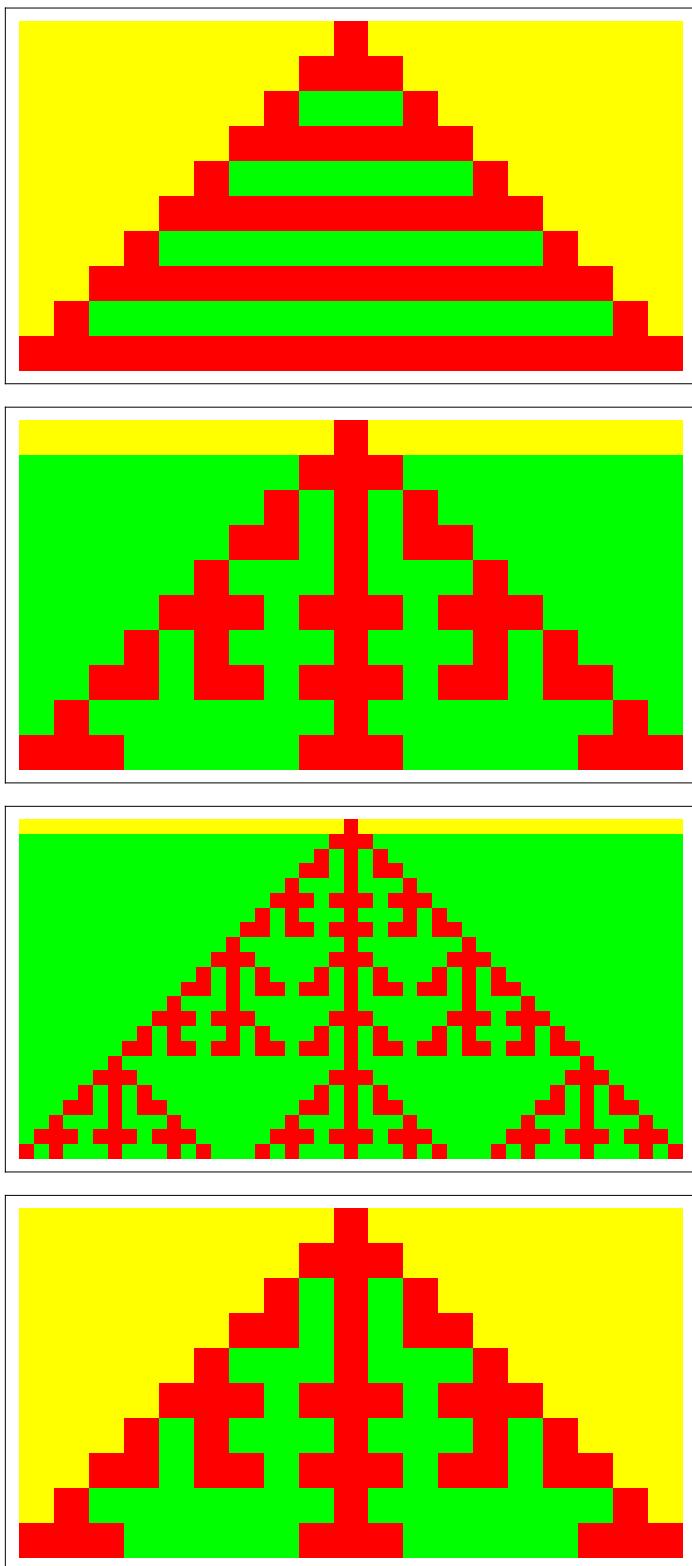
**ruleCIR = 1.4.x .22 .32.**

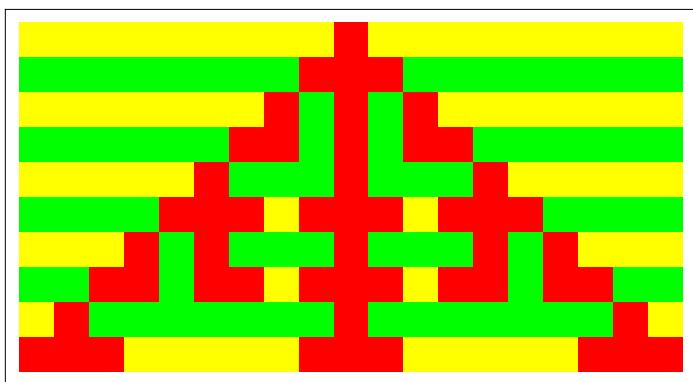
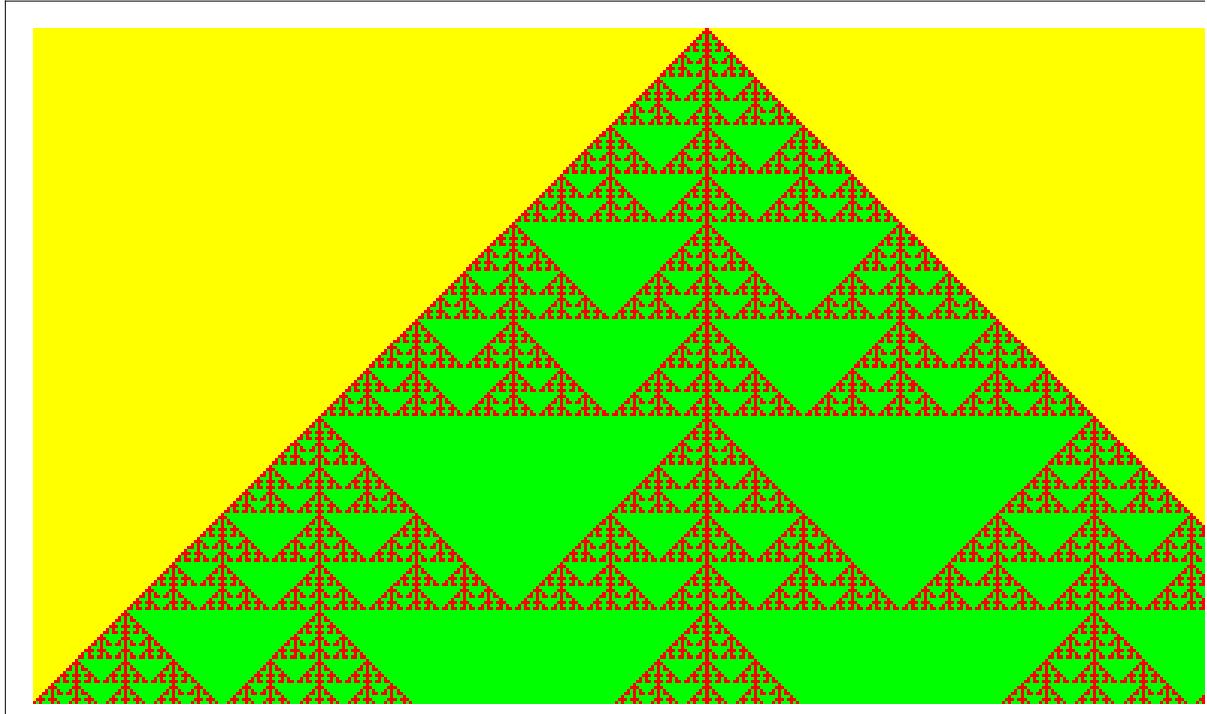
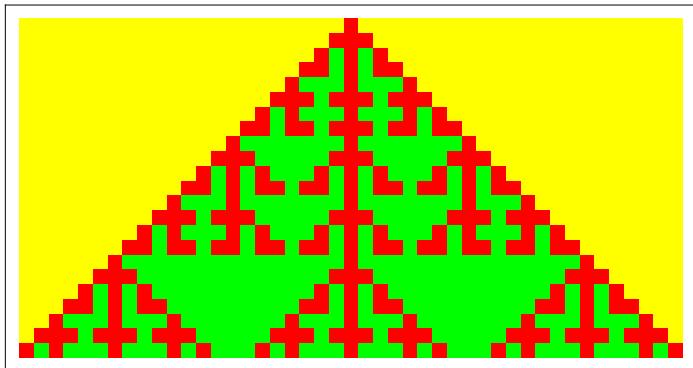


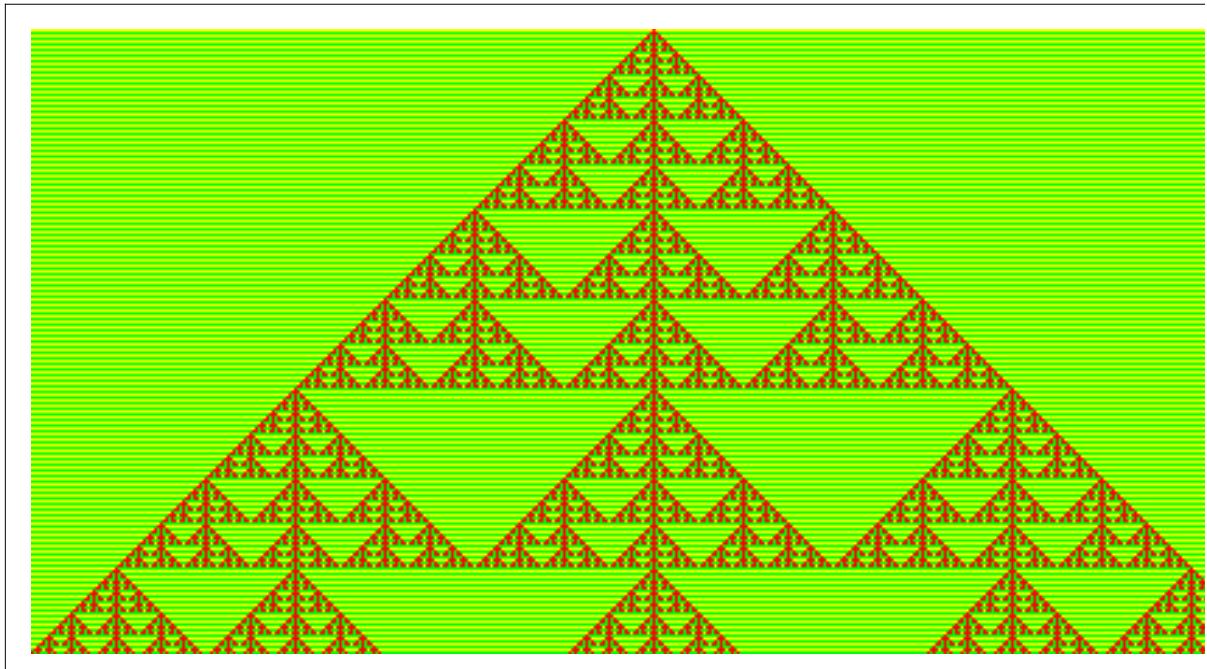
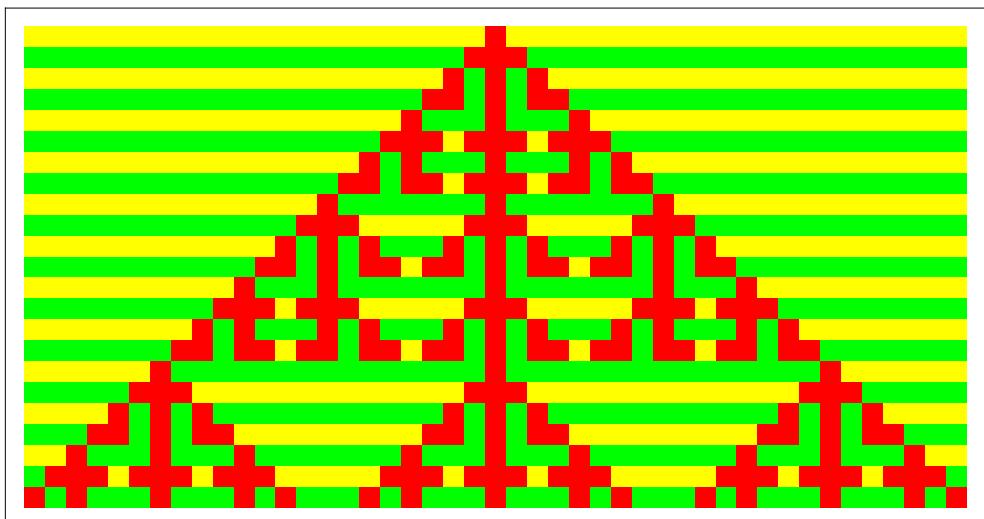
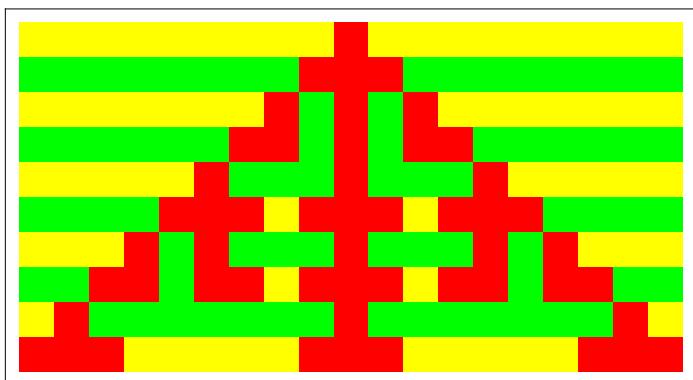


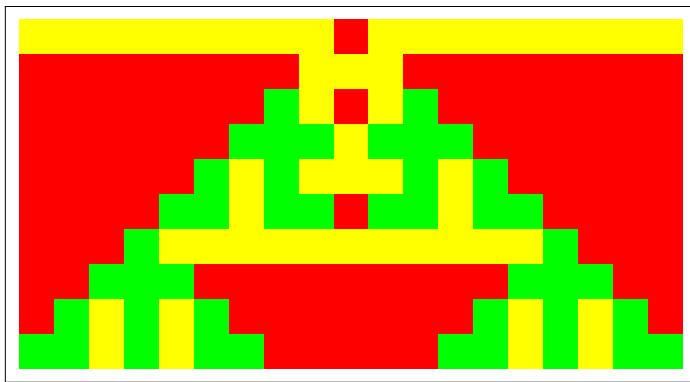
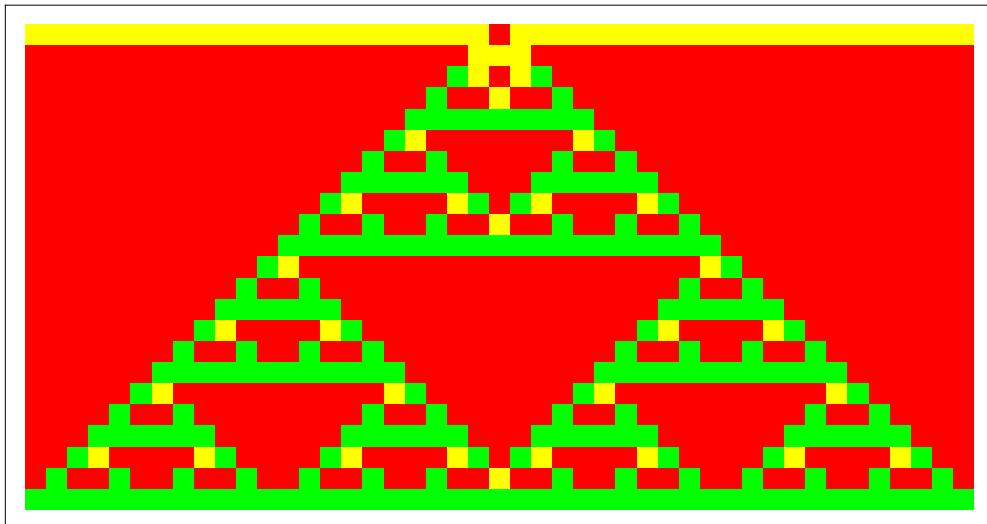
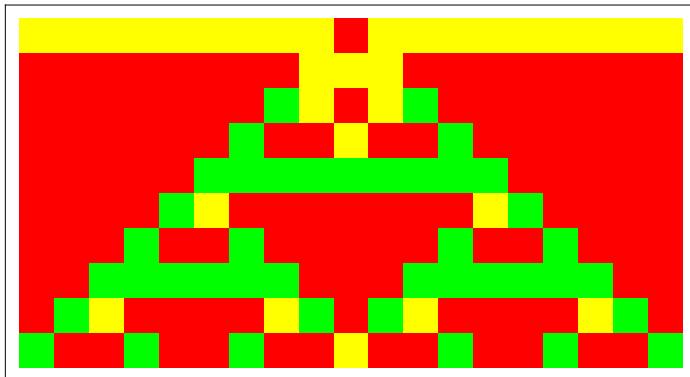


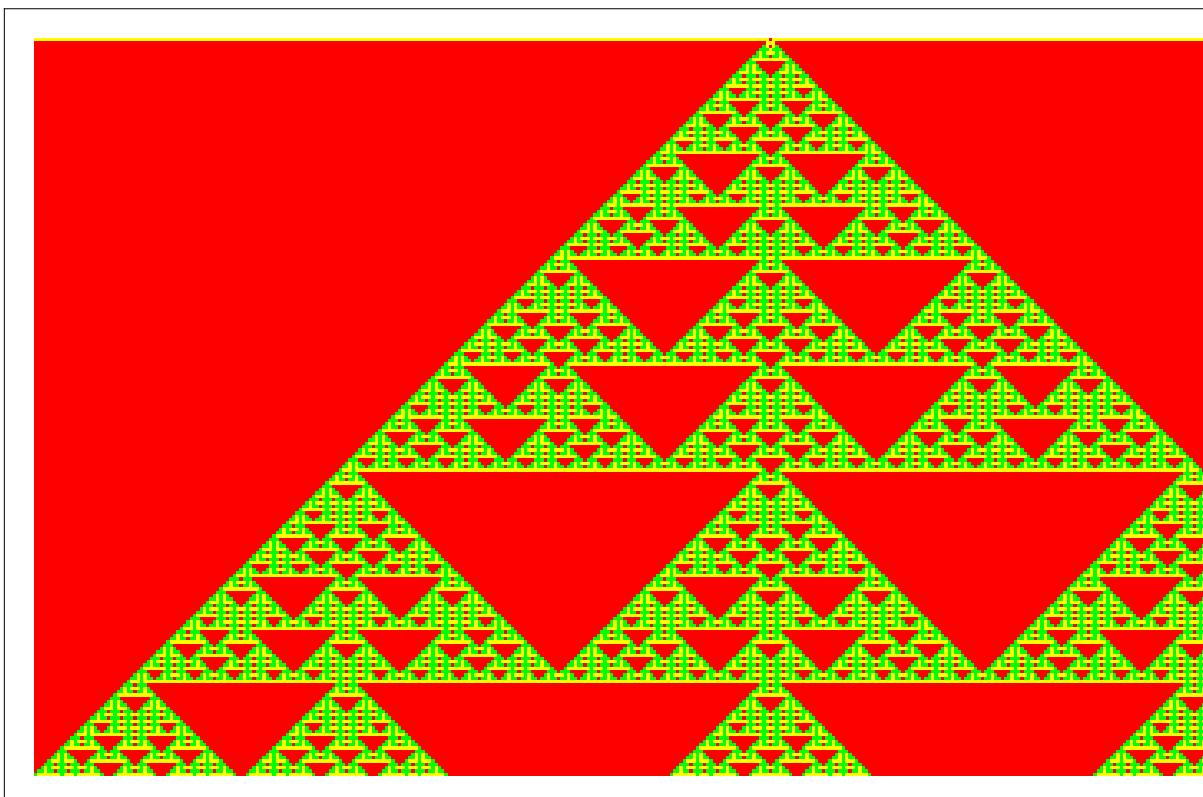
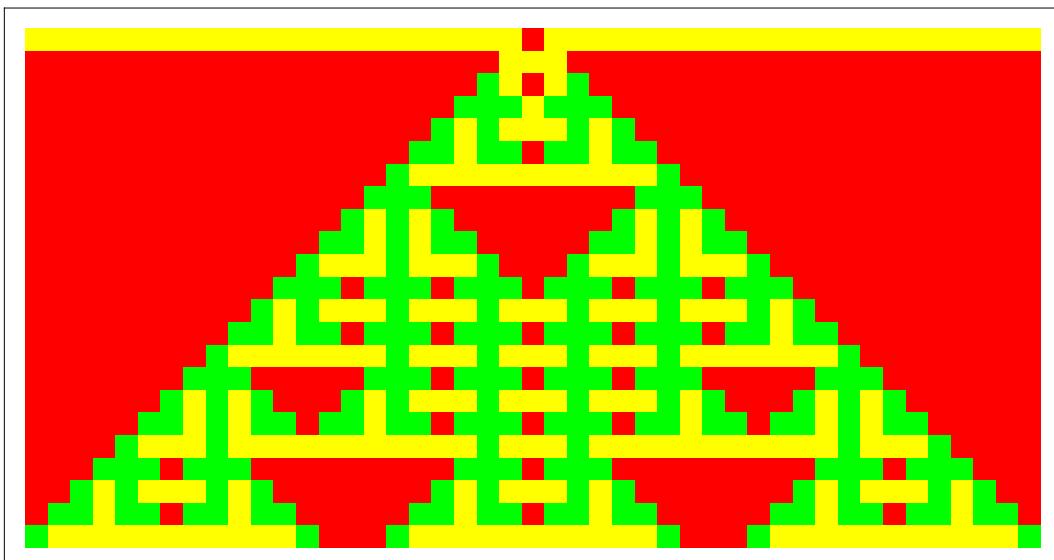


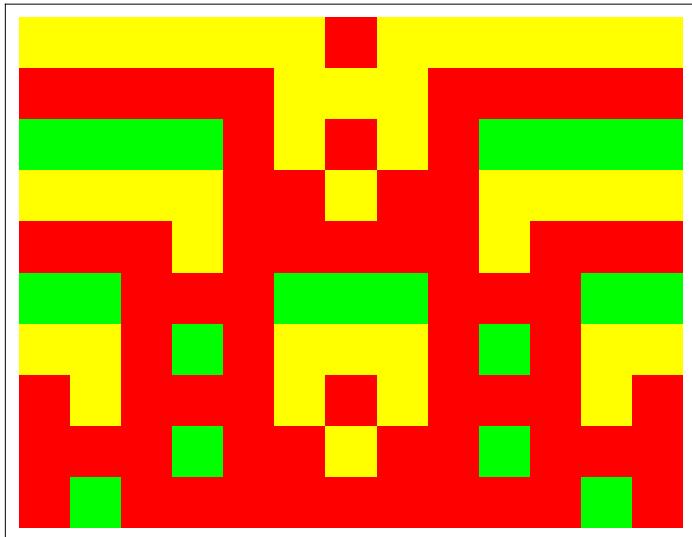
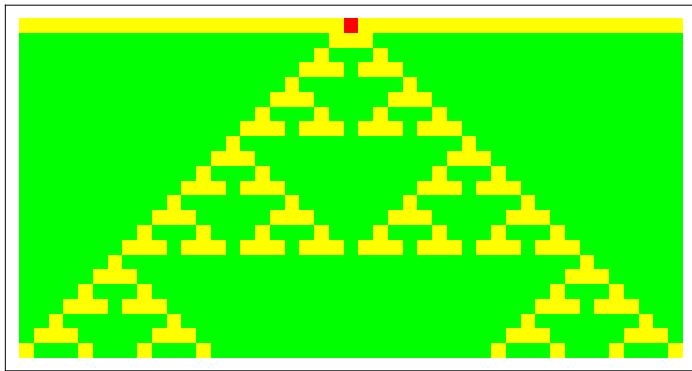
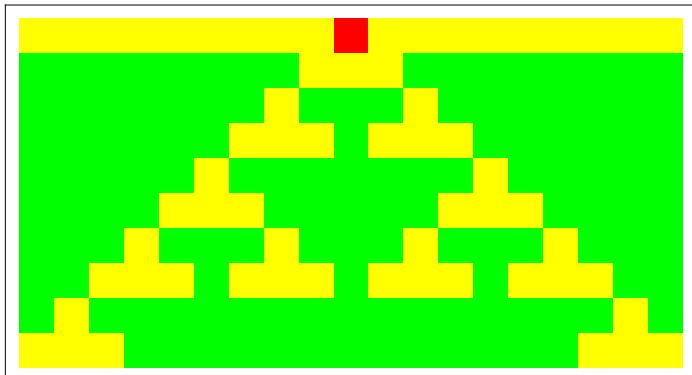


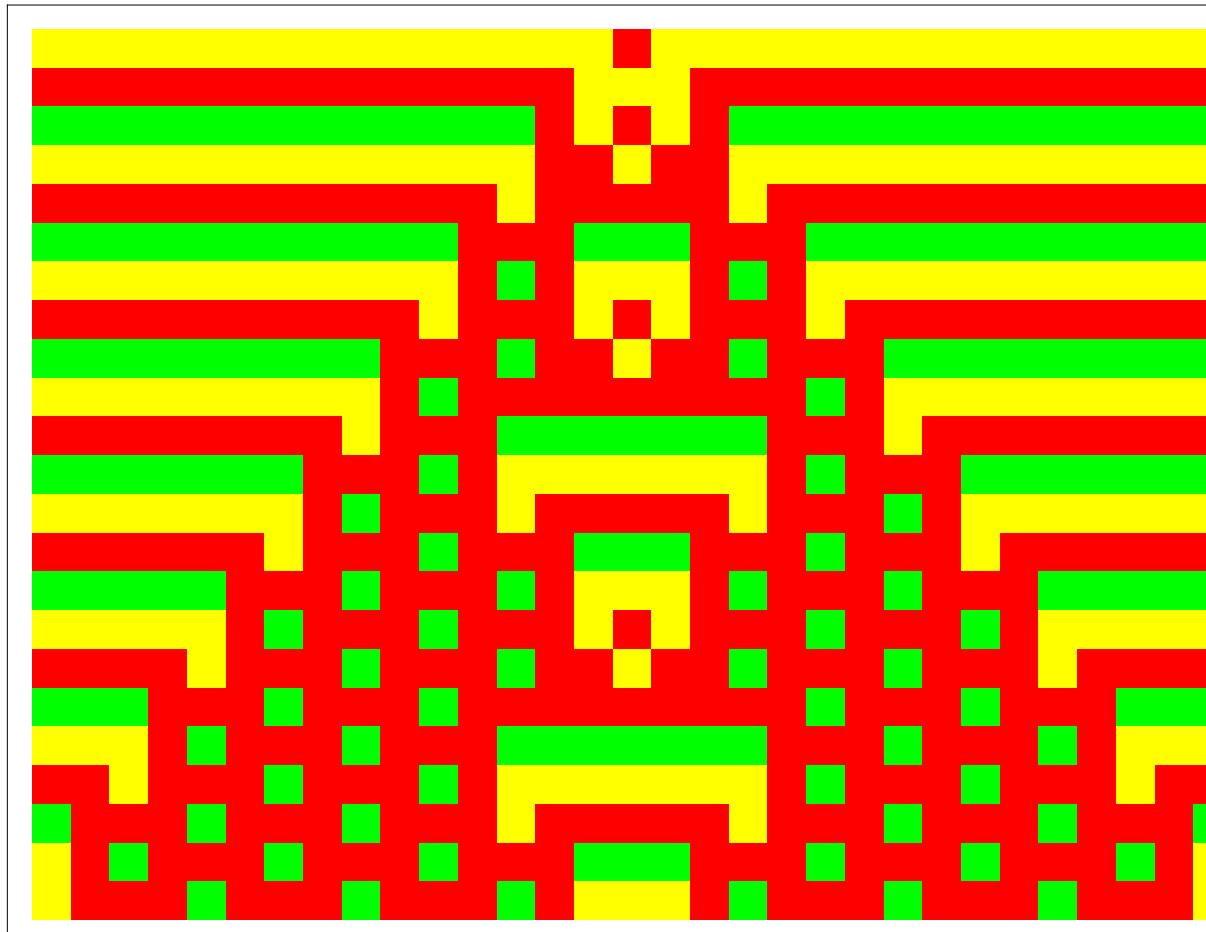


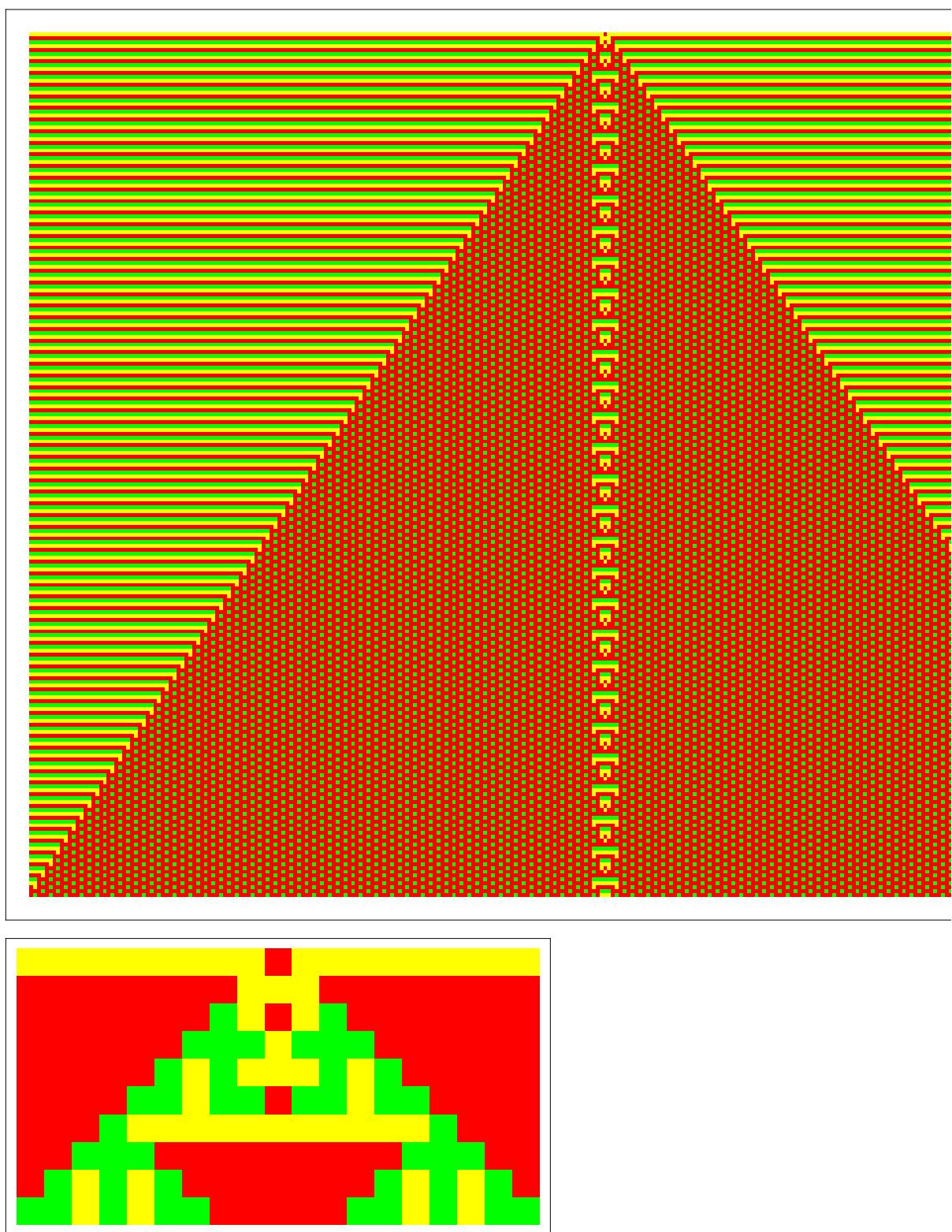


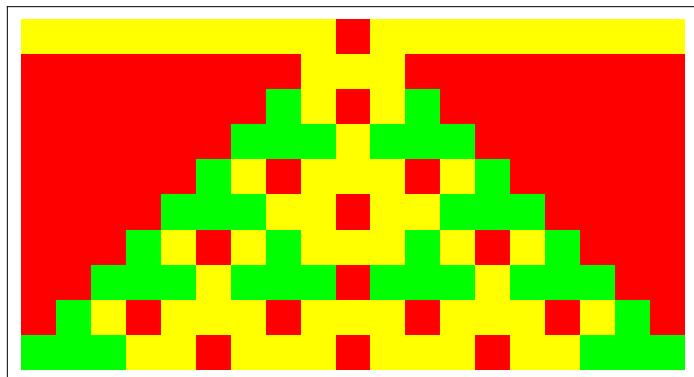
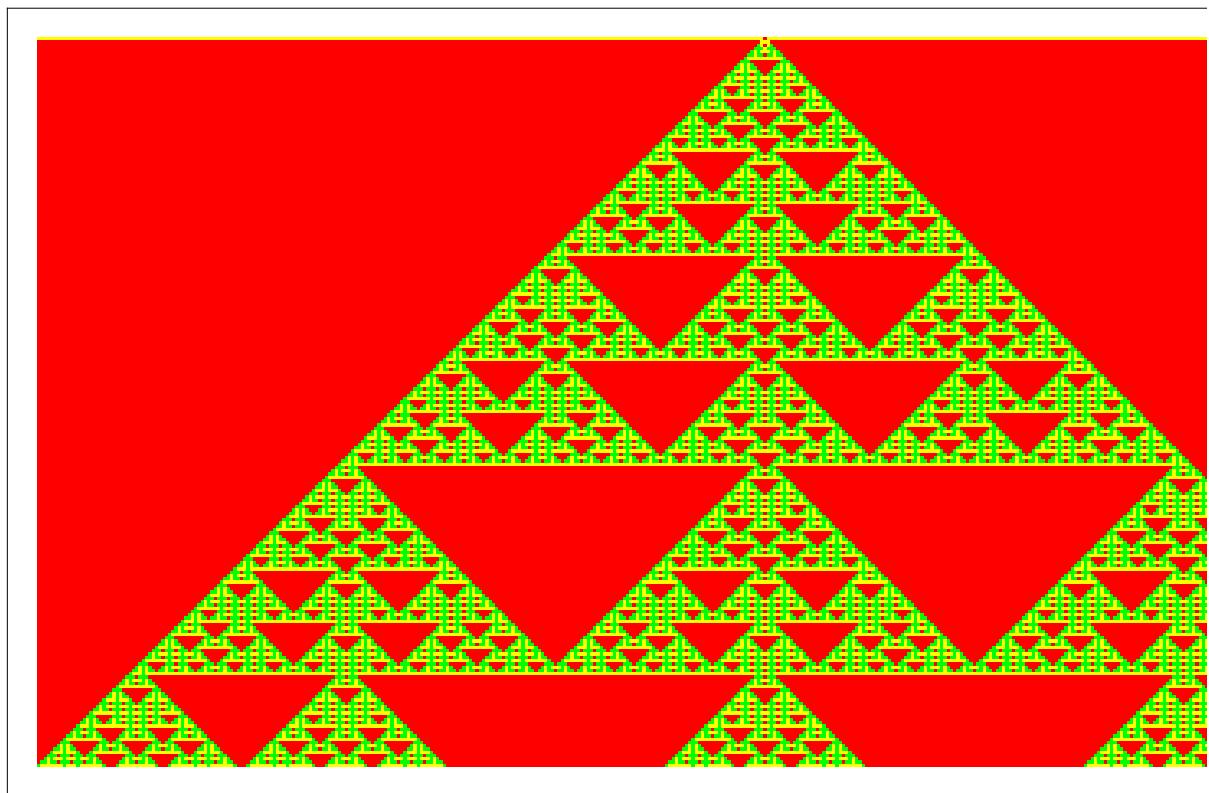
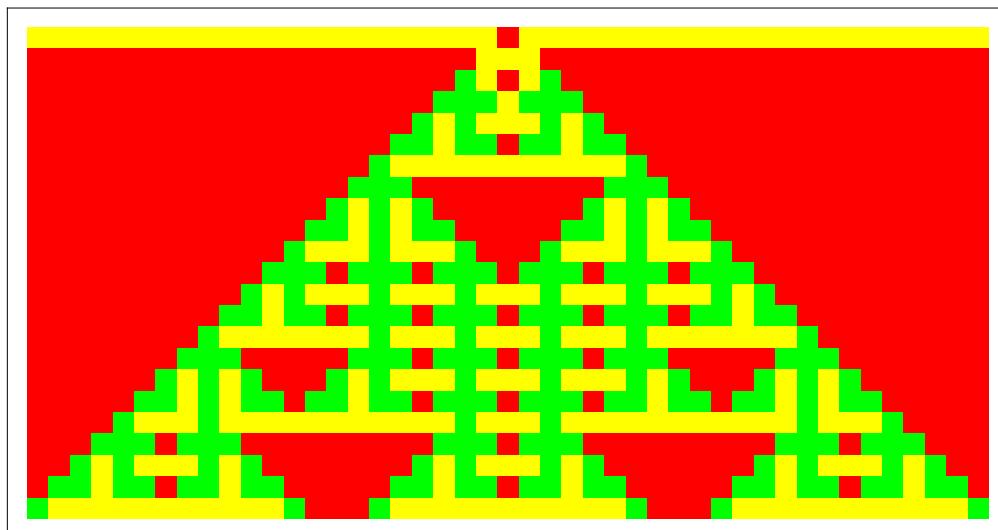


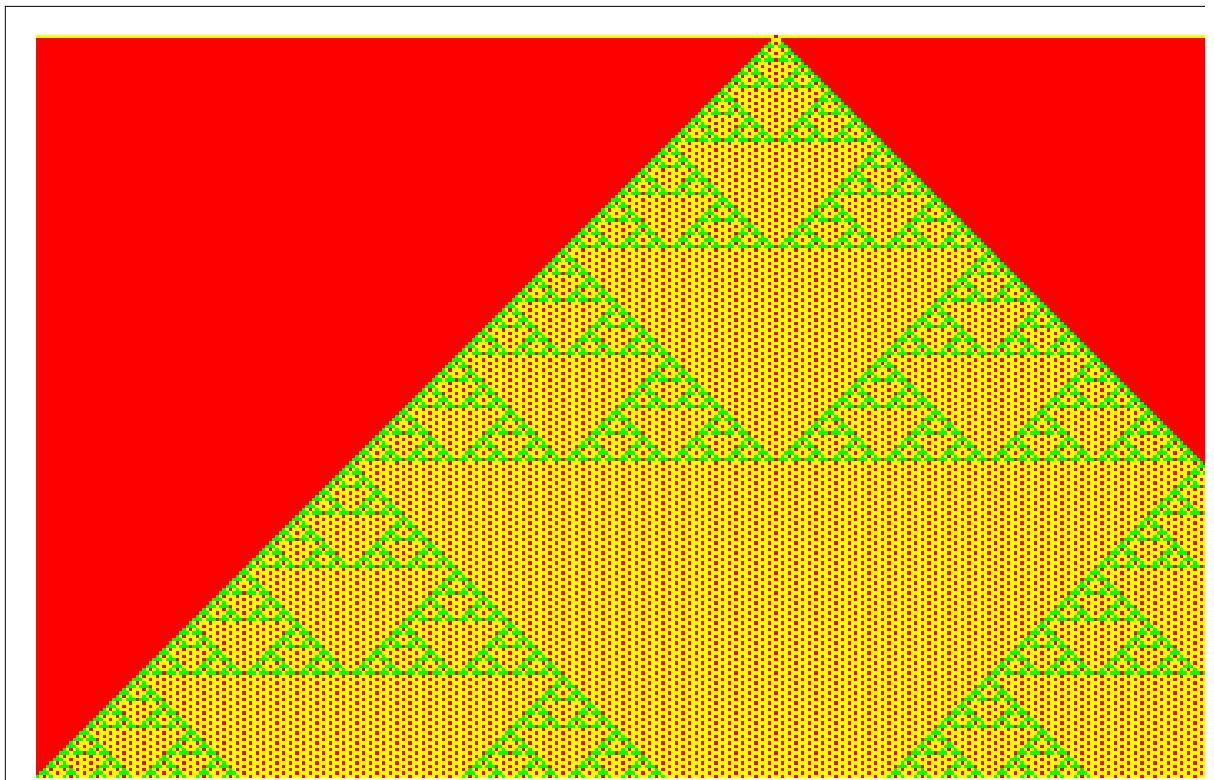
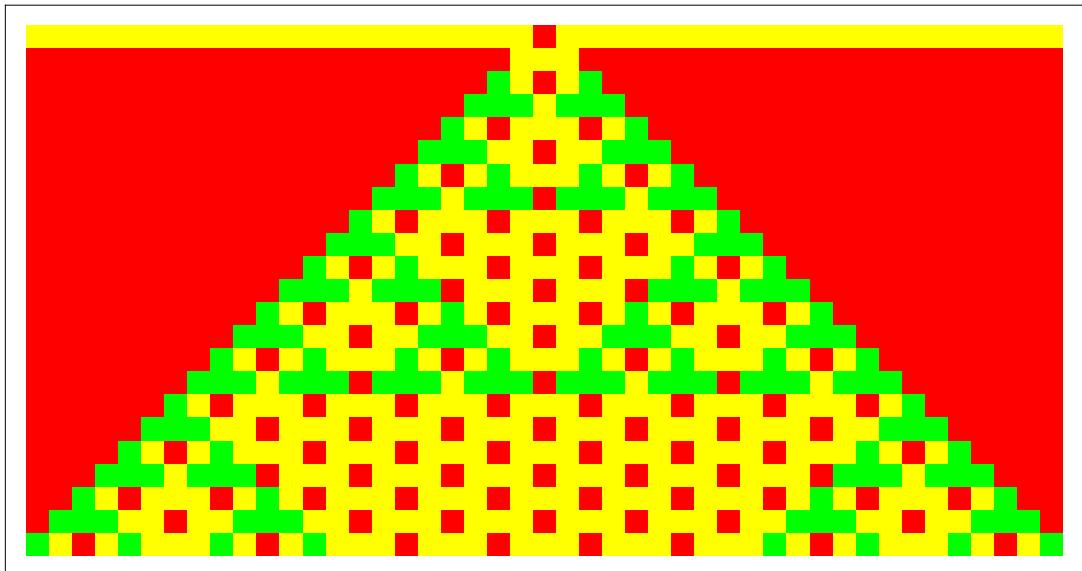
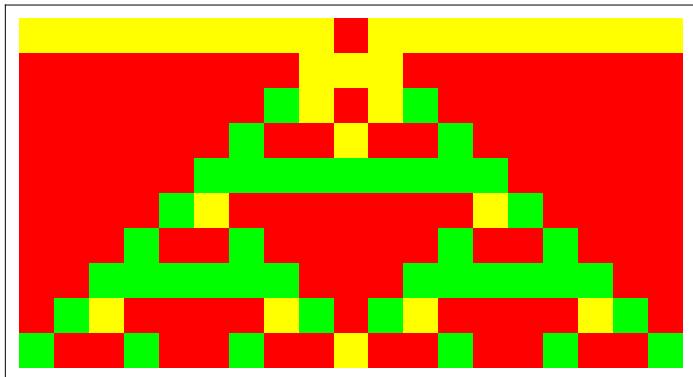


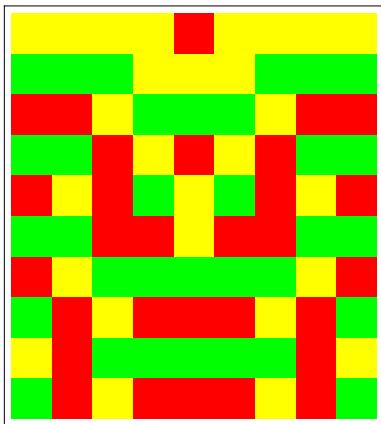




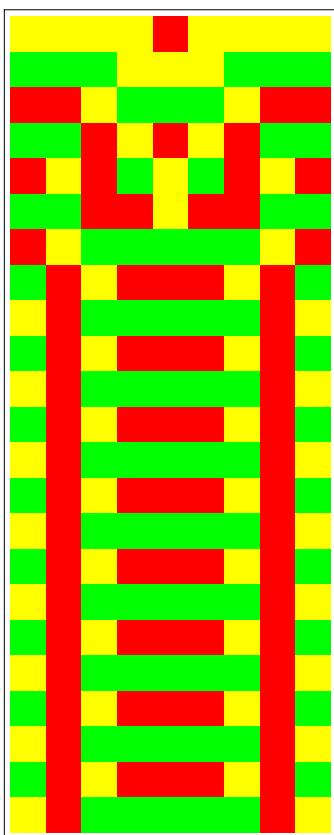






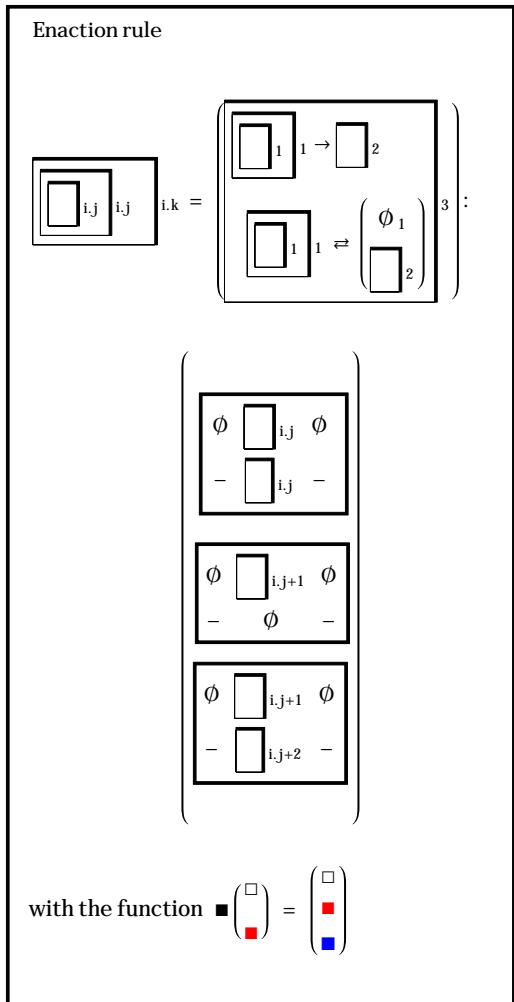


corrected (no difference)



---

Double reflectional indCI<sup>(3,4)</sup>



### Some examples

R11	R12	R13	R14

R61		R62		R63		R64	
R71		R72		R73		R74	
R81		R82		R83		R84	
R91		R92		R93		R94	
R101		R102		R103		R104	

### indCR<sup>(3,4)</sup> Rule table: perm{i,j,k}

Number of rules: 80 = (12+4+4) x 4

$\{1, 1, 0\} \rightarrow 0,$ $\{1, 0, 1\} \rightarrow 0,$ $\{0, 1, 1\} \rightarrow 0,$  $\{1, 0, 0\} \rightarrow 2,$ $\{0, 0, 1\} \rightarrow 2,$ $\{0, 1, 0\} \rightarrow 2,$	$\{0, 0, 2\} \rightarrow 2,$ $\{0, 2, 0\} \rightarrow 2,$ $\{2, 0, 0\} \rightarrow 2,$  $\{0, 2, 2\} \rightarrow 2,$ $\{2, 0, 2\} \rightarrow 2,$ $\{2, 2, 0\} \rightarrow 2,$	$\{2, 1, 1\} \rightarrow 1,$ $\{1, 2, 1\} \rightarrow 1,$ $\{1, 1, 2\} \rightarrow 1,$  $\{1, 2, 2\} \rightarrow 2,$ $\{2, 1, 2\} \rightarrow 2,$ $\{2, 2, 1\} \rightarrow 2,$
$\{0, 3, 3\} \rightarrow 3,$ $\{3, 0, 3\} \rightarrow 3,$ $\{3, 3, 0\} \rightarrow 3,$  $\{0, 0, 3\} \rightarrow 3,$ $\{0, 3, 0\} \rightarrow 3,$ $\{3, 0, 0\} \rightarrow 3,$	$\{1, 1, 3\} \rightarrow 1,$ $\{1, 3, 1\} \rightarrow 1,$ $\{3, 1, 1\} \rightarrow 1,$  $\{1, 3, 3\} \rightarrow 3,$ $\{3, 3, 1\} \rightarrow 2,$ $\{3, 1, 3\} \rightarrow 3,$	$\{2, 2, 3\} \rightarrow 2,$ $\{2, 3, 2\} \rightarrow 2,$ $\{3, 2, 2\} \rightarrow 2,$  $\{2, 3, 3\} \rightarrow 3,$ $\{3, 2, 3\} \rightarrow 3,$ $\{3, 3, 2\} \rightarrow 3,$
$\{2, 0, 1\} \rightarrow 1,$ $\{2, 1, 0\} \rightarrow 1,$ $\{1, 0, 2\} \rightarrow 1,$ $\{1, 2, 0\} \rightarrow 1,$ $\{0, 1, 2\} \rightarrow 1,$ $\{0, 2, 1\} \rightarrow 1,$	$\{3, 0, 1\} \rightarrow 0,$ $\{3, 1, 0\} \rightarrow 0,$ $\{1, 3, 0\} \rightarrow 0,$  $\{0, 1, 3\} \rightarrow 0,$ $\{0, 3, 1\} \rightarrow 0,$ $\{1, 0, 3\} \rightarrow 0,$	$\{3, 0, 2\} \rightarrow 2,$ $\{3, 2, 0\} \rightarrow 2,$ $\{2, 0, 3\} \rightarrow 2,$  $\{2, 3, 0\} \rightarrow 2,$ $\{0, 2, 3\} \rightarrow 2,$ $\{0, 3, 2\} \rightarrow 2,$

### indCR<sup>(3,4)</sup> Rule set: ruleSetRCI

```
RuleSetRCI =
{
  rci[{{4440}}] := {0, 0, 0} → 0,
  rci[{{4441}}] := {0, 0, 0} → 1,
  rci[{{4442}}] := {0, 0, 0} → 2,
  rci[{{4443}}] := {0, 0, 0} → 3,

  rci[{{1110}}] := {1, 1, 1} → 0,
  rci[{{1111}}] := {1, 1, 1} → 1,
  rci[{{1112}}] := {1, 1, 1} → 2,
  rci[{{1113}}] := {1, 1, 1} → 3,

  rci[{{2220}}] := {2, 2, 2} → 0,
```

```

rcl[{2221}] := {2, 2, 2} → 1,
rcl[{2222}] := {2, 2, 2} → 2,
rcl[{2223}] := {2, 2, 2} → 3,

rcl[{3330}] := {3, 3, 3} → 0,
rcl[{3331}] := {3, 3, 3} → 1,
rcl[{3332}] := {3, 3, 3} → 2,
rcl[{3333}] := {3, 3, 3} → 3,

rcl[{1000}] :=
{
  {1, 0, 0} → 0,
  {0, 0, 1} → 0,
  {0, 1, 0} → 0
},
rcl[1001] :=
{
  {1, 0, 0} → 1,
  {0, 0, 1} → 1,
  {0, 1, 0} → 1
},
rcl[{1002}] :=
{
  {1, 0, 0} → 2,
  {0, 0, 1} → 2,
  {0, 1, 0} → 2
},
rcl[{1003}] :=
{
  {1, 0, 0} → 3,
  {0, 0, 1} → 3,
  {0, 1, 0} → 3
},
rcl[{2000}] :=
{
  {2, 0, 0} → 0,
  {0, 0, 2} → 0,
  {0, 2, 0} → 0
},
rcl[2001] :=
{
  {2, 0, 0} → 1,
  {0, 0, 2} → 1,
  {0, 2, 0} → 1
},
rcl[{2002}] :=
{
  {2, 0, 0} → 2,
  {0, 0, 2} → 2,
  {0, 2, 0} → 2
},
rcl[{2003}] :=
{
  {2, 0, 0} → 3,
  {0, 0, 2} → 3,
  {0, 2, 0} → 3
},
rcl[{3000}] :=
{

```

```

{3, 0, 0} → 0,
{0, 0, 3} → 0,
{0, 3, 0} → 0
},
rcl[3001] :=
{
{3, 0, 0} → 1,
{0, 0, 3} → 1,
{0, 3, 0} → 1
},
rcl[{3002}] :=
{
{3, 0, 0} → 2,
{0, 0, 3} → 2,
{0, 3, 0} → 2
},
rcl[{3003}] :=
{
{3, 0, 0} → 3,
{0, 0, 3} → 3,
{0, 3, 0} → 3
},

rcl[{1100}] :=
{
{0, 1, 1} → 0,
{1, 1, 0} → 0,
{1, 0, 1} → 0
},
rcl[{1101}] :=
{
{0, 1, 1} → 1,
{1, 1, 0} → 1,
{1, 0, 1} → 1
},
rcl[{1102}] :=
{
{0, 1, 1} → 2,
{1, 1, 0} → 2,
{1, 0, 1} → 2
},
rcl[{1103}] :=
{
{0, 1, 1} → 3,
{1, 1, 0} → 3,
{1, 0, 1} → 3
},
rcl[{1120}] :=
{
{1, 1, 2} → 0,
{2, 1, 1} → 0,
{1, 2, 1} → 0
},
rcl[{1121}] :=
{
{1, 1, 2} → 1,
{2, 1, 1} → 1,
{1, 2, 1} → 1
},

```

```

rcl[{1122}] :=
{
  {2, 1, 1} → 2,
  {1, 2, 1} → 2,
  {1, 1, 2} → 2
},
rcl[{1123}] :=
{
  {2, 1, 1} → 3,
  {1, 2, 1} → 3,
  {1, 1, 2} → 3
},
rcl[{1130}] :=
{
  {1, 1, 3} → 0,
  {1, 3, 1} → 0,
  {3, 1, 1} → 0
},
rcl[{1131}] :=
{
  {1, 1, 3} → 1,
  {1, 3, 1} → 1,
  {3, 1, 1} → 1
},
rcl[{1132}] :=
{
  {1, 1, 3} → 2,
  {1, 3, 1} → 2,
  {3, 1, 1} → 2
},
rcl[{1133}] :=
{
  {1, 1, 3} → 3,
  {1, 3, 1} → 0,
  {3, 1, 1} → 3
},
rcl[{1220}] :=
{
  {1, 2, 2} → 0,
  {2, 1, 2} → 0,
  {2, 2, 1} → 0
},
rcl[{1221}] :=
{
  {1, 2, 2} → 1,
  {2, 1, 2} → 1,
  {2, 2, 1} → 1
},
rcl[{1222}] :=
{
  {1, 2, 2} → 2,
  {2, 1, 2} → 2,
  {2, 2, 1} → 2
},
rcl[{1223}] :=
{
  {1, 2, 2} → 3,
  {2, 1, 2} → 3,
  {2, 2, 1} → 3
}

```

```

},
rcl[{1330}] :=
{
{1, 3, 3} → 0,
{3, 3, 1} → 0,
{3, 1, 3} → 0
},
rcl[{1331}] :=
{
{1, 3, 3} → 1,
{3, 3, 1} → 1,
{3, 1, 3} → 1
},
rcl[{1332}] :=
{
{1, 3, 3} → 2,
{3, 3, 1} → 2,
{3, 1, 3} → 2
},
rcl[{1333}] :=
{
{1, 3, 3} → 3,
{3, 3, 1} → 3,
{3, 1, 3} → 3
},
rcl[{2200}] :=
{
{0, 2, 2} → 0,
{2, 0, 2} → 0,
{2, 2, 0} → 0
},
rcl[{2201}] :=
{
{0, 2, 2} → 1,
{2, 0, 2} → 1,
{2, 2, 0} → 1
},
rcl[{2202}] :=
{
{0, 2, 2} → 2,
{2, 0, 2} → 2,
{2, 2, 0} → 2
},
rcl[{2203}] :=
{
{0, 2, 2} → 3,
{2, 0, 2} → 3,
{2, 2, 0} → 3
},
rcl[{3300}] :=
{
{0, 3, 3} → 0,
{3, 0, 3} → 0,
{3, 3, 0} → 0
},
rcl[{3301}] :=
{
{0, 3, 3} → 1,
{3, 0, 3} → 1,

```

```

{3, 3, 0} → 1
},
rcl[{3302}] :=
{
{0, 3, 3} → 2,
{3, 0, 3} → 2,
{3, 3, 0} → 2
},
rcl[{3303}] :=
{
{0, 3, 3} → 3,
{3, 0, 3} → 3,
{3, 3, 0} → 3
},
rcl[{2230}] :=
{
{2, 2, 3} → 0,
{2, 3, 2} → 0,
{3, 2, 2} → 0
},
rcl[{2231}] :=
{
{2, 2, 3} → 1,
{2, 3, 2} → 1,
{3, 2, 2} → 1
},
rcl[{2232}] :=
{
{2, 2, 3} → 2,
{2, 3, 2} → 2,
{3, 2, 2} → 2
},
rcl[{2233}] :=
{
{2, 2, 3} → 3,
{2, 3, 2} → 3,
{3, 2, 2} → 3
},
rcl[{2330}] :=
{
{2, 3, 3} → 0,
{3, 2, 3} → 0,
{3, 3, 2} → 0
},
rcl[{2331}] :=
{
{2, 3, 3} → 1,
{3, 2, 3} → 1,
{3, 3, 2} → 1
},
rcl[{2332}] :=
{
{2, 3, 3} → 2,
{3, 2, 3} → 2,
{3, 3, 2} → 2
},
rcl[{2333}] :=
{
{2, 3, 3} → 3,

```

```

{3, 2, 3} → 3,
{3, 3, 2} → 3
},
rcl[{1200}] :=
{
{0, 1, 2} → 0,
{2, 0, 1} → 0,
{2, 1, 0} → 0,
{1, 0, 2} → 0,
{1, 2, 0} → 0,
{0, 2, 1} → 0
},
rcl[{1201}] :=
{
{0, 1, 2} → 1,
{2, 0, 1} → 1,
{2, 1, 0} → 1,
{1, 0, 2} → 1,
{1, 2, 0} → 1,
{0, 2, 1} → 1
},
rcl[{1202}] :=
{
{0, 1, 2} → 2,
{2, 0, 1} → 2,
{2, 1, 0} → 2,
{1, 0, 2} → 2,
{1, 2, 0} → 2,
{0, 2, 1} → 2
},
rcl[{1203}] :=
{
{0, 1, 2} → 3,
{2, 0, 1} → 3,
{2, 1, 0} → 3,
{1, 0, 2} → 3,
{1, 2, 0} → 3,
{0, 2, 1} → 3
},
rcl[{1300}] :=
{
{0, 1, 3} → 0,
{3, 0, 1} → 0,
{3, 1, 0} → 0,
{1, 0, 3} → 0,
{1, 3, 0} → 0,
{0, 3, 1} → 0
},
rcl[{1301}] :=
{
{0, 1, 3} → 1,
{3, 0, 1} → 1,
{3, 1, 0} → 1,
{1, 0, 3} → 1,
{1, 3, 0} → 1,
{0, 3, 1} → 1
},
rcl[{1302}] :=

```

```

{
  {0, 1, 3} → 2,
  {3, 0, 1} → 2,
  {3, 1, 0} → 2,
  {1, 0, 3} → 2,
  {1, 3, 0} → 2,
  {0, 3, 1} → 2
},
rcl[{1303}] :=
{
  {0, 1, 3} → 3,
  {3, 0, 1} → 3,
  {3, 1, 0} → 3,
  {1, 0, 3} → 3,
  {1, 3, 0} → 3,
  {0, 3, 1} → 3
},
rcl[{3200}] :=
{
  {3, 0, 2} → 0,
  {3, 2, 0} → 0,
  {2, 0, 3} → 0,
  {2, 3, 0} → 0,
  {0, 2, 3} → 0,
  {0, 3, 2} → 0
},
rcl[{3201}] :=
{
  {3, 0, 2} → 1,
  {3, 2, 0} → 1,
  {2, 0, 3} → 1,
  {2, 3, 0} → 1,
  {0, 2, 3} → 1,
  {0, 3, 2} → 1
},
rcl[{3202}] :=
{
  {3, 0, 2} → 2,
  {3, 2, 0} → 2,
  {2, 0, 3} → 2,
  {2, 3, 0} → 2,
  {0, 2, 3} → 2,
  {0, 3, 2} → 2
},
rcl[{3203}] :=
{
  {3, 0, 2} → 3,
  {3, 2, 0} → 3,
  {2, 0, 3} → 3,
  {2, 3, 0} → 3,
  {0, 2, 3} → 3,
  {0, 3, 2} → 3
},
rcl[{3210}] :=
{
  {3, 2, 1} → 0,
  {3, 1, 2} → 0,
  {2, 3, 1} → 0,
  {2, 1, 3} → 0,
}

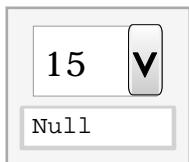
```

```

{1, 2, 3} → 0,
{1, 3, 2} → 0
},
rcl[{{3211}}] :=
{
{3, 2, 1} → 1,
{3, 1, 2} → 1,
{2, 3, 1} → 1,
{2, 1, 3} → 1,
{1, 2, 3} → 1,
{1, 3, 2} → 1
},
rcl[{{3212}}] :=
{
{3, 2, 1} → 2,
{3, 1, 2} → 2,
{2, 3, 1} → 2,
{2, 1, 3} → 2,
{1, 2, 3} → 2,
{1, 3, 2} → 2
},
rcl[{{3213}}] :=
{
{3, 2, 1} → 3,
{3, 1, 2} → 3,
{2, 3, 1} → 3,
{2, 1, 3} → 3,
{1, 2, 3} → 3,
{1, 3, 2} → 3
}
}

```

### indCR<sup>(3,4)</sup>ruleSetRCI: MenuView (todo)



### indCR<sup>(3,4)</sup> Rule scheme

```

ruleRCI[{a_, b_, c_, d_, e_, f_, g_, h_, i_, j_,
         k_, l_, m_, n_, o_, p_, q_, r_, s_}
      }]:=

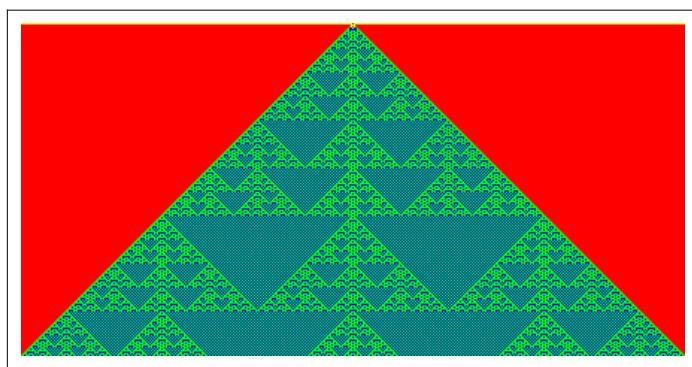
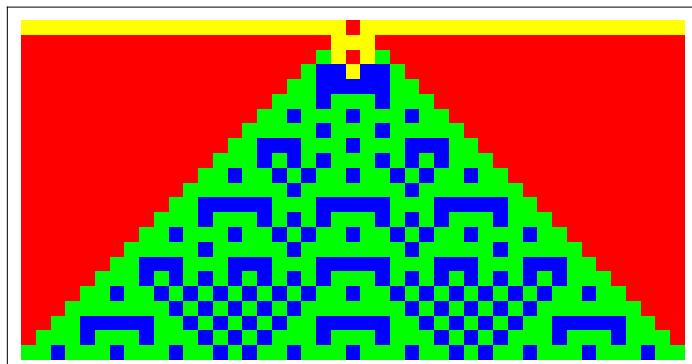
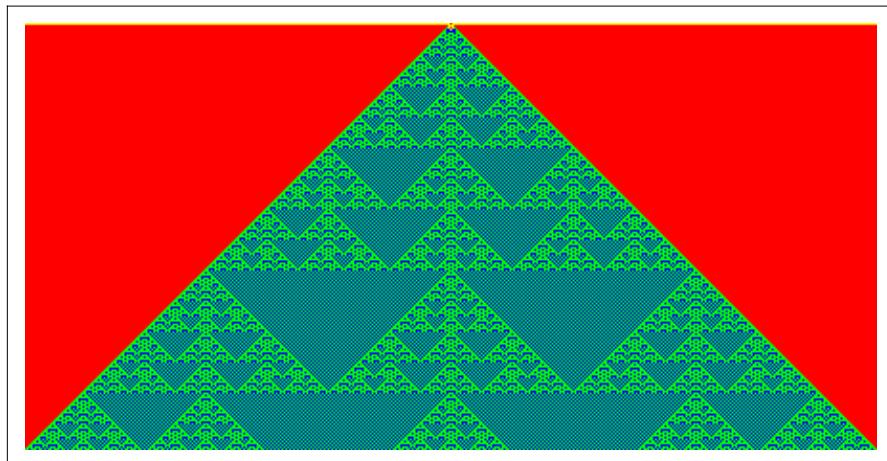
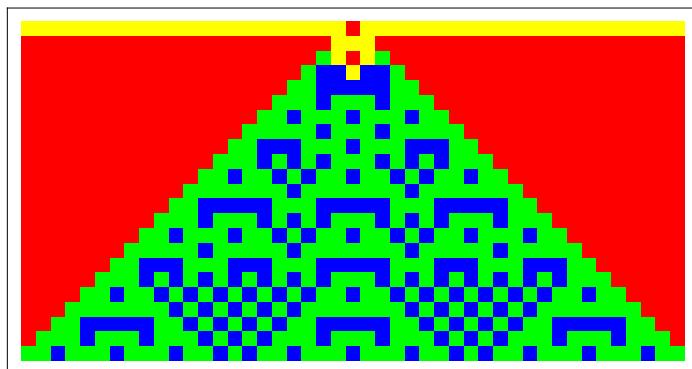
Flatten[{rcl[{a}], rcl[{b}], rcl[{c}], rcl[{d}],
         rcl[{e}], rcl[{f}], rcl[{g}], rcl[{h}],
         rcl[{i}], rcl[{j}], rcl[{k}], rcl[{h}],
         rcl[{l}], rcl[{m}], rcl[{n}], rcl[{o}],
         rcl[{p}], rcl[{q}], rcl[{r}], rcl[{s}]}
      }
]

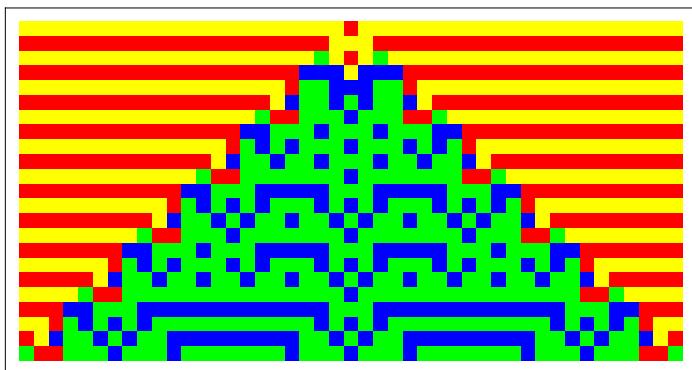
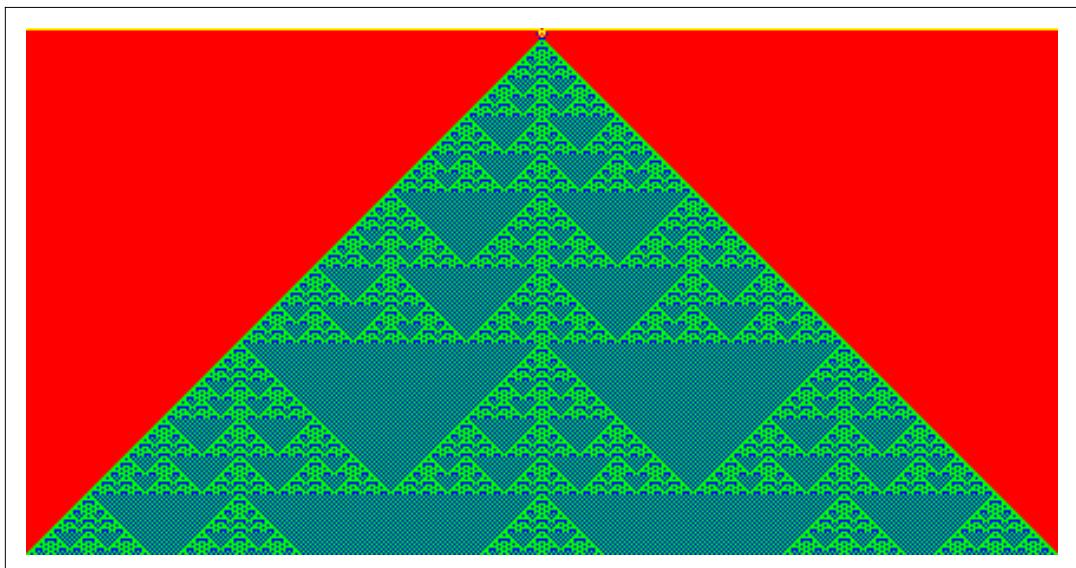
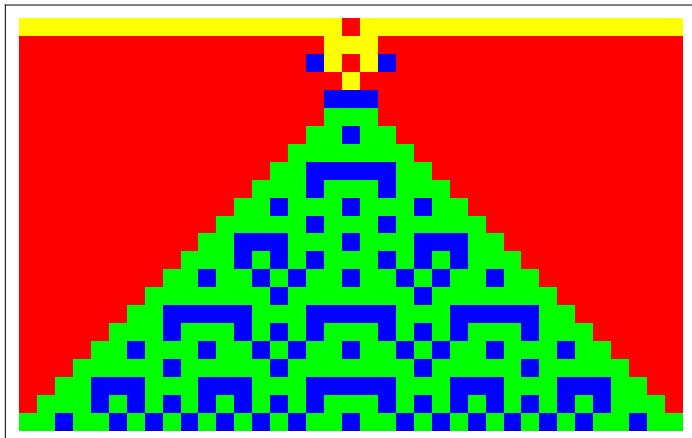
```

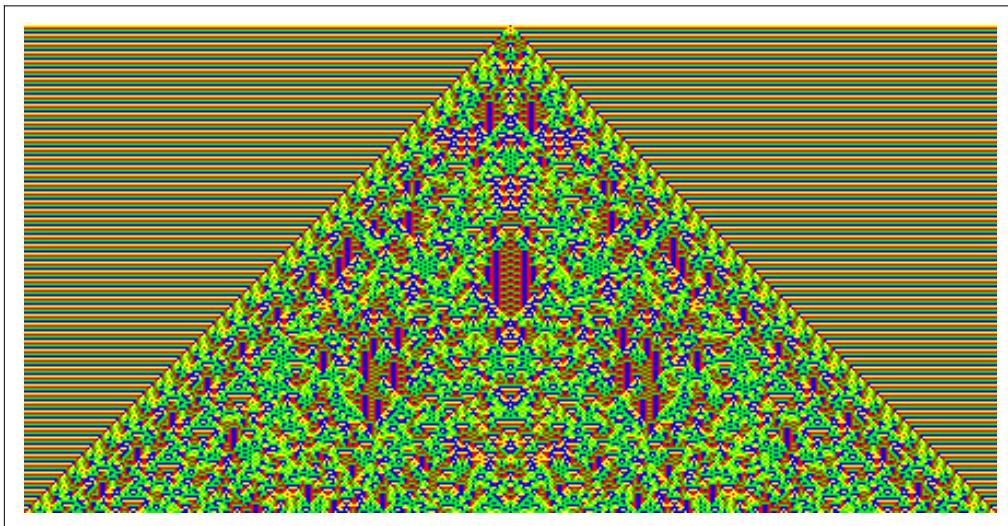
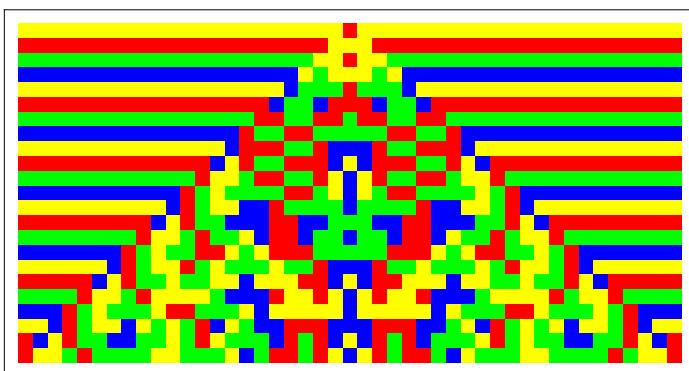
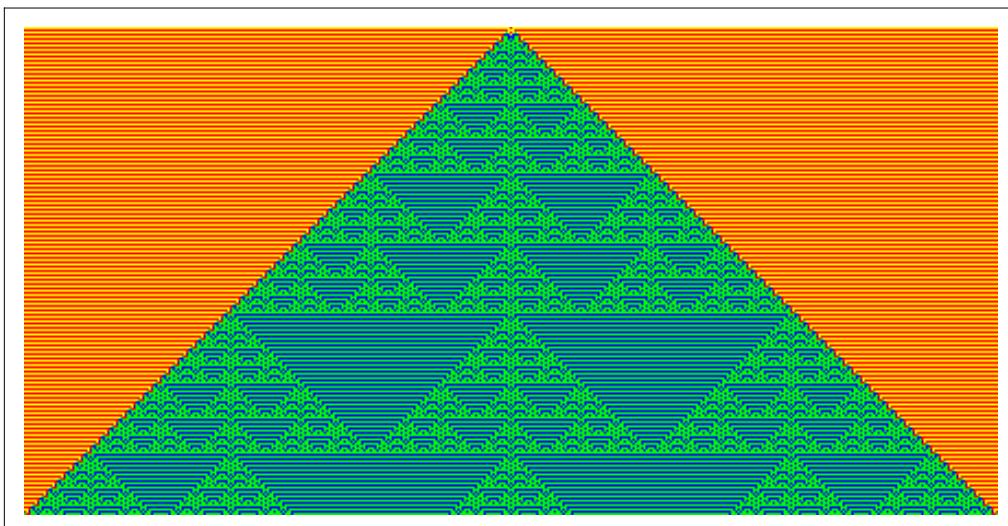
### Dynamic view of indCR<sup>(3,4)</sup> forms (todo)

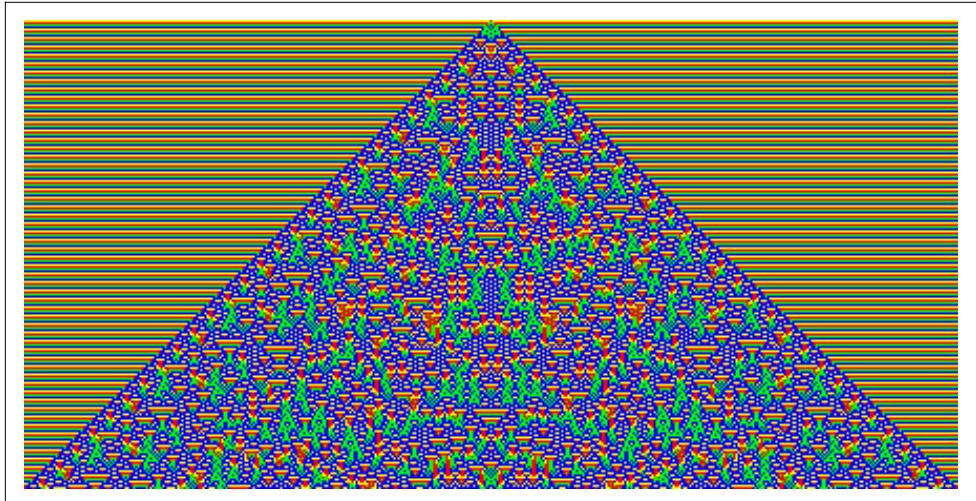
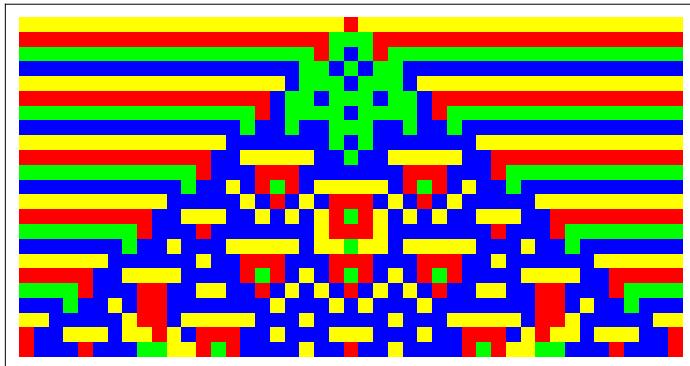
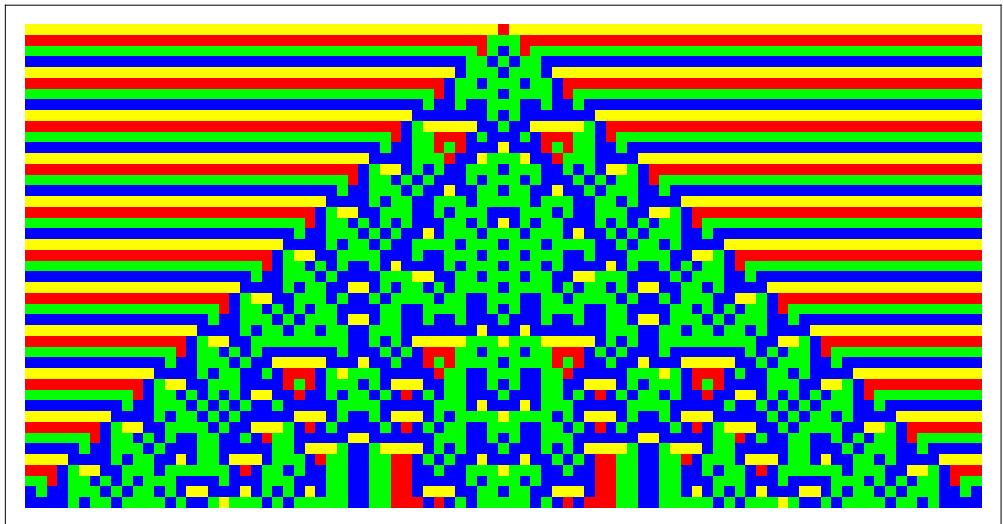
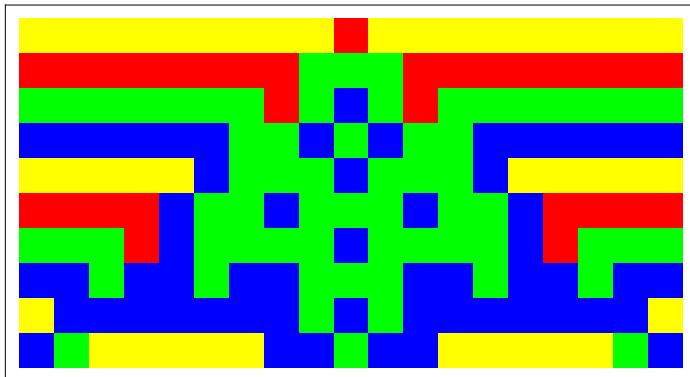
Examples of complete forms:

First: Symmetric indCA<sup>(4,3)</sup>

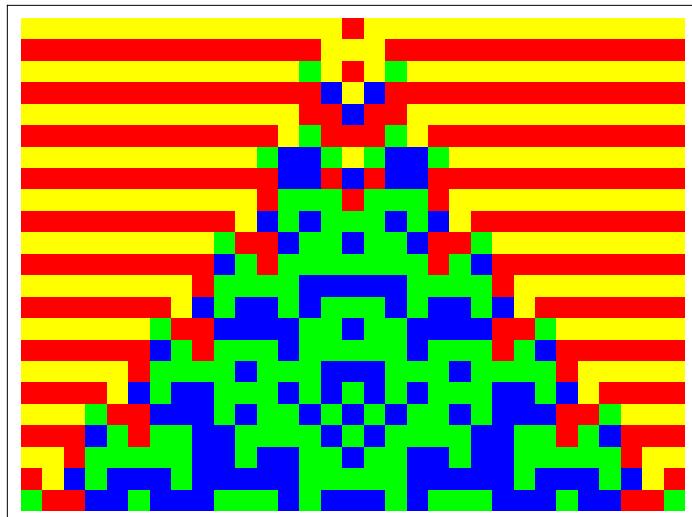
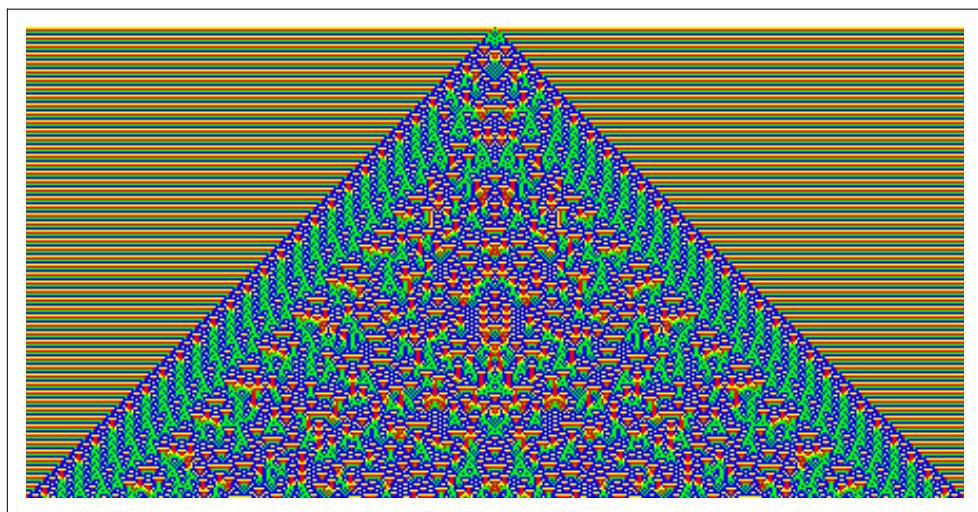
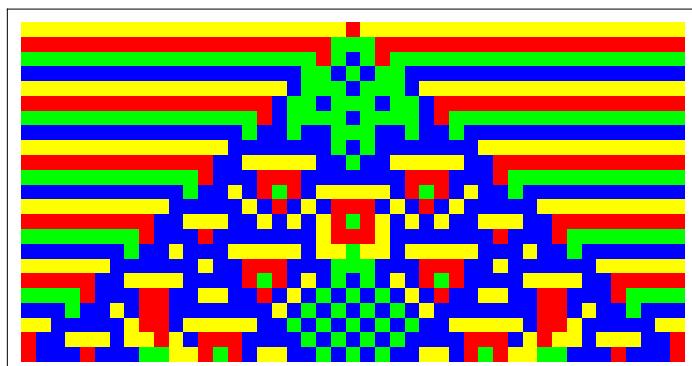


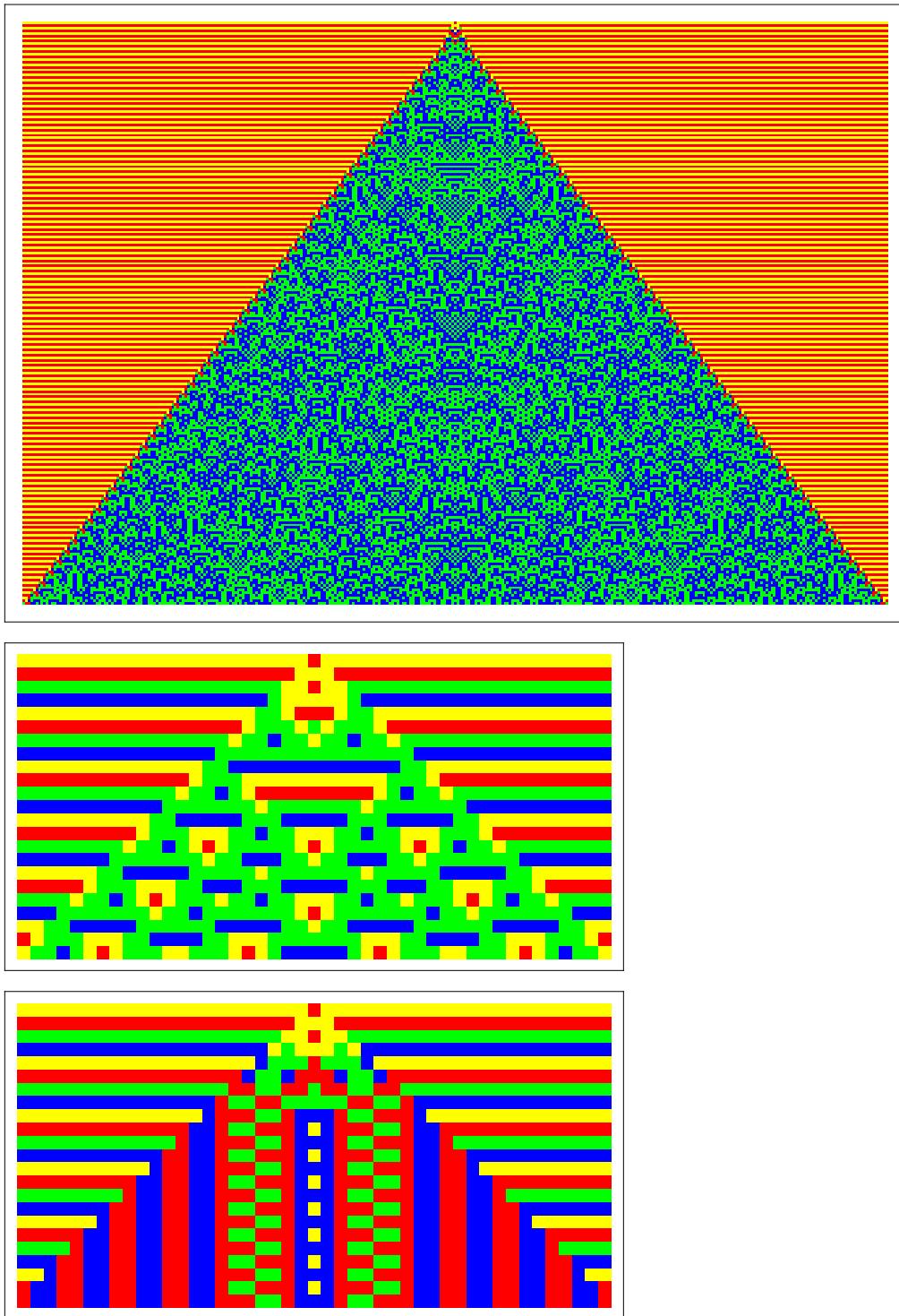


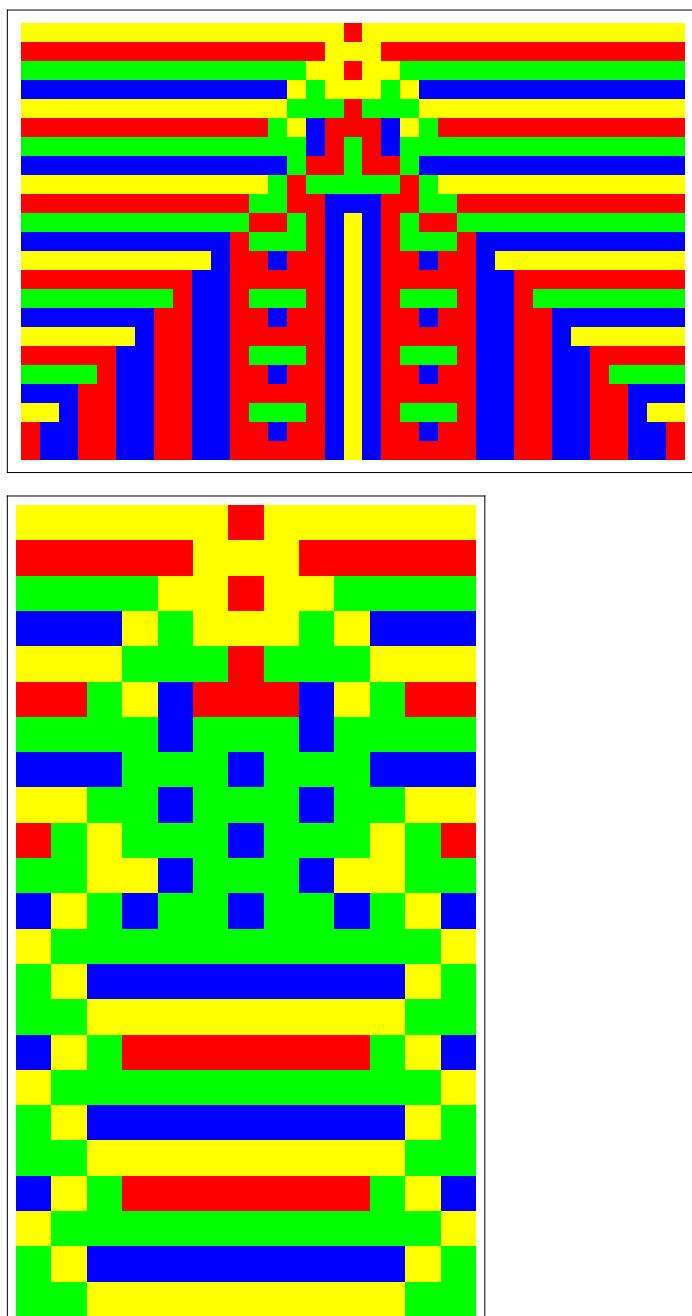


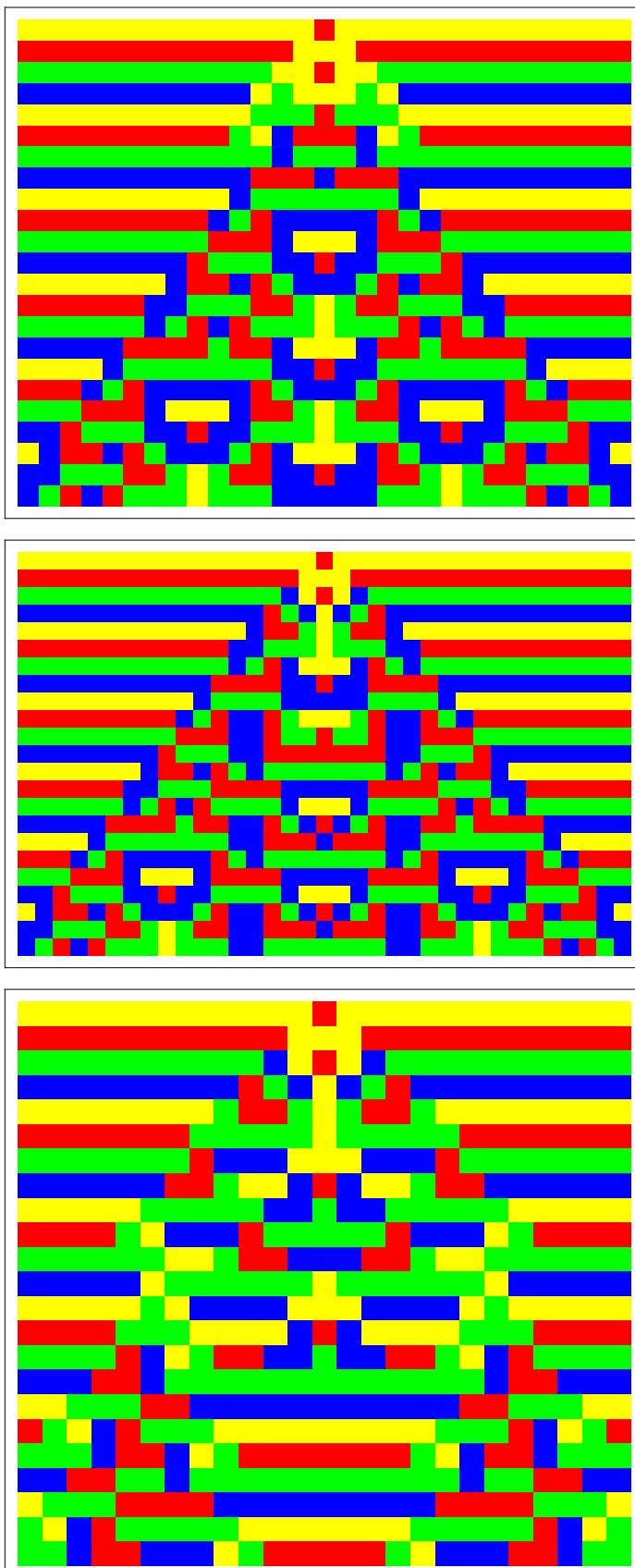


correction, two

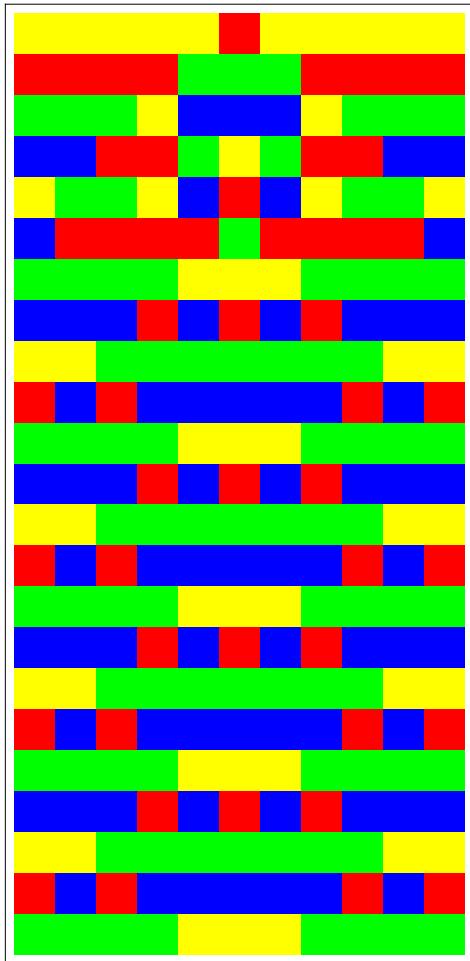


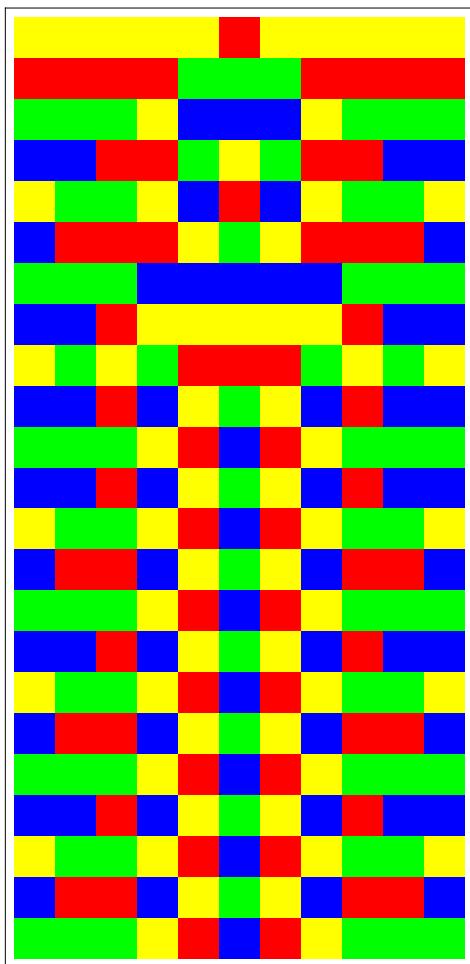




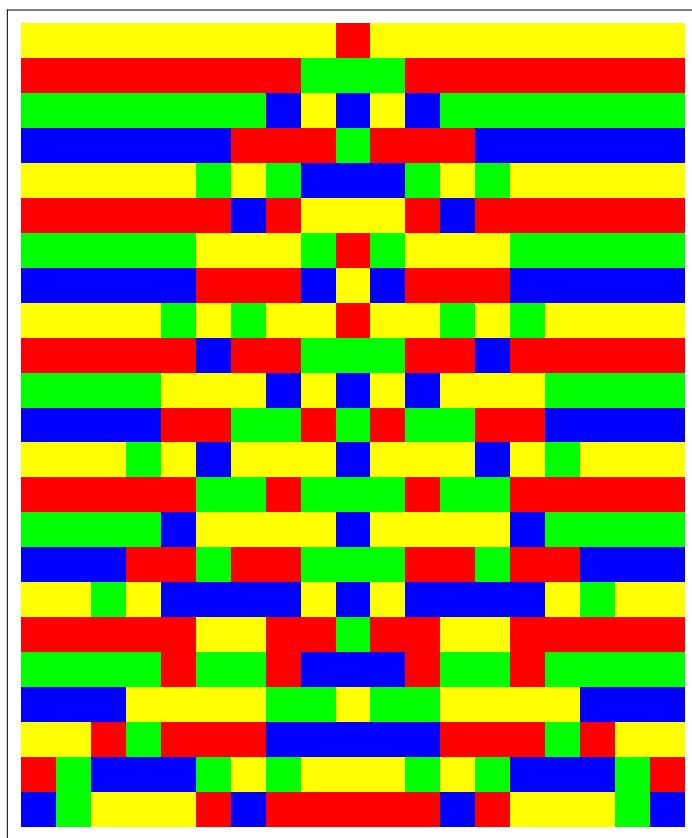


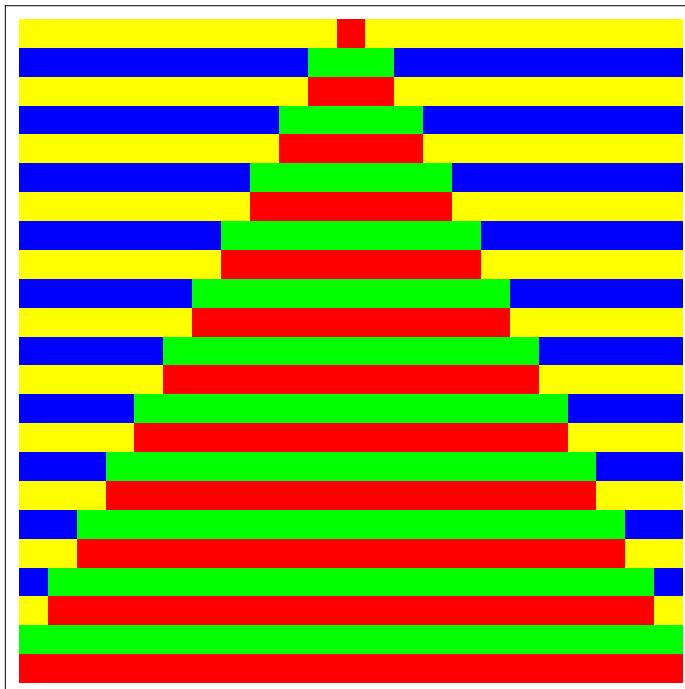
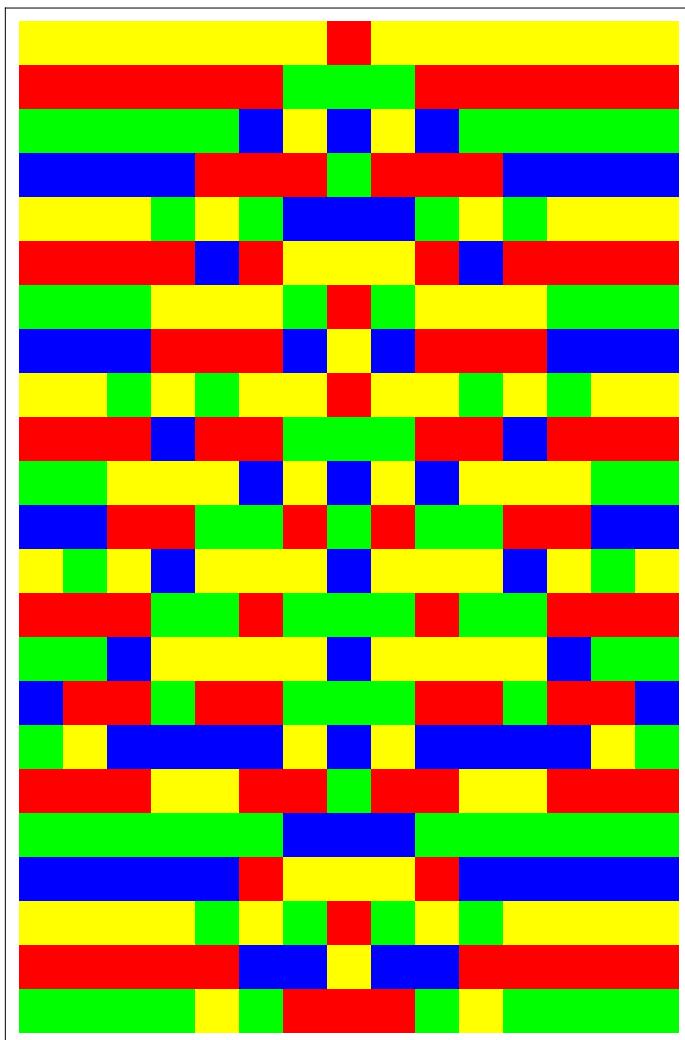
## Total trans-contextural patterns

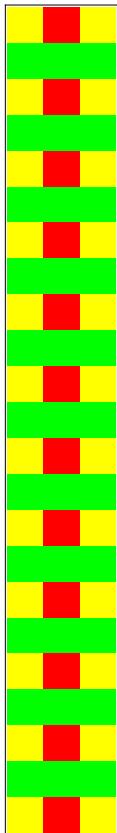




**alternative transcontextural**

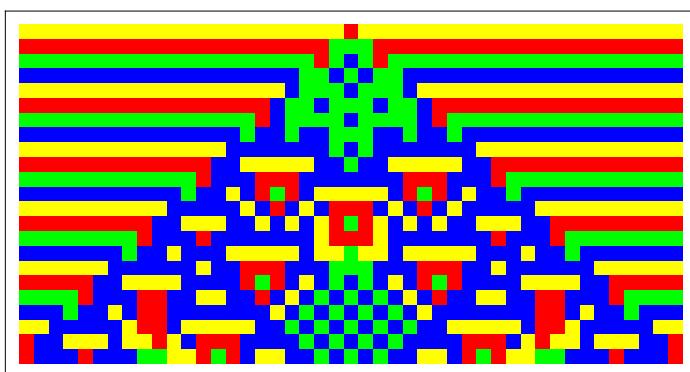




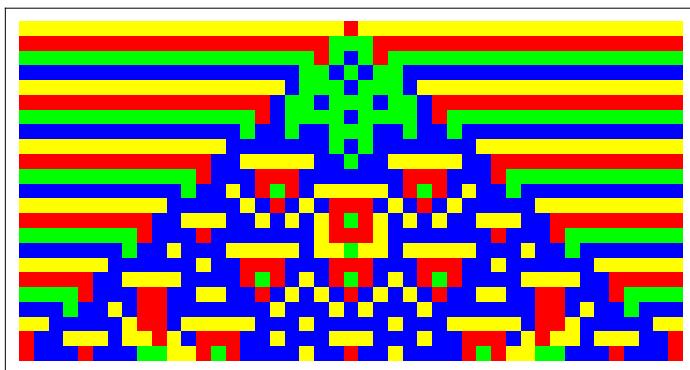


Deviant forms, overcomplete

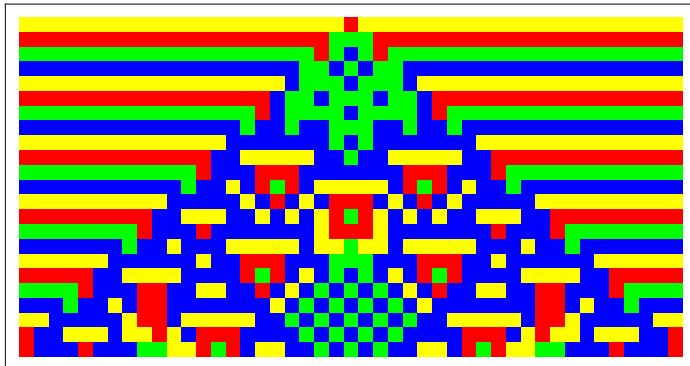
**deviant**



**correction, one**



correction, two = first



Examples of complete forms:

Second: Asymmetric indCA<sup>(3,4)</sup> ??

It seems that there are no asymmetric patterns for CA<sup>(n,m)</sup>, n = 3, m ≤ 4.

IndCA<sup>(3,2)</sup> is trivially symmetric. This corresponds to the Laws of Form.

IndCA<sup>(3,3)</sup> is in a not trivial sense symmetric (this has to be proven).

IndCA<sup>(3,4)</sup> is structurally symmetric too.

It contains interesting asymmetric patterns only as defects.

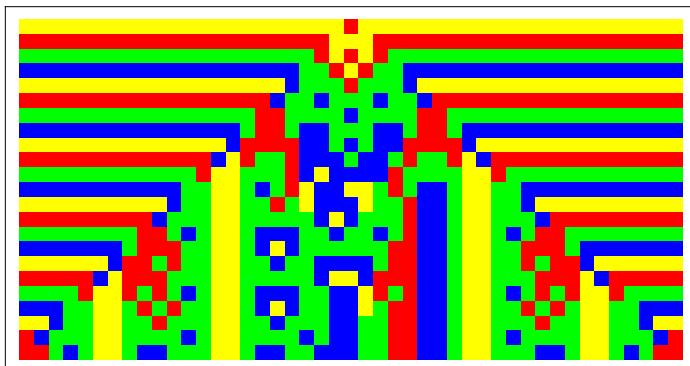
Are there genuine asymmetric patterns in a non – interactional and non – interventional calculus?

**Variations of the same asymmetric pattern: defect cases**

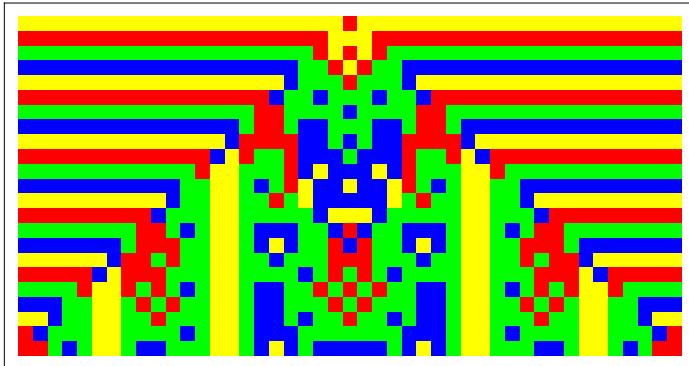
Same head

Asymmetry with defective function :

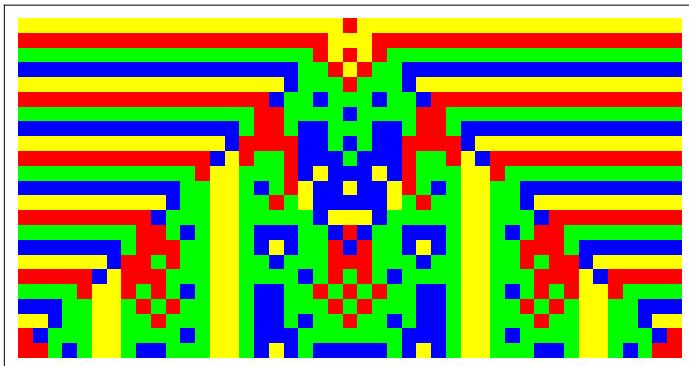
{3, 3, 1} → 2 instead of {3, 3, 1} → 3



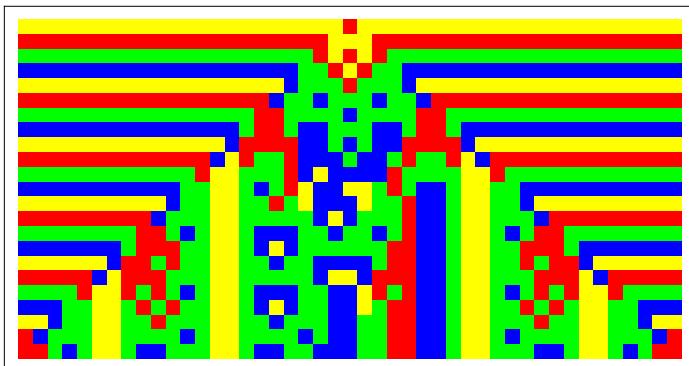
symmetry, corrected mutations



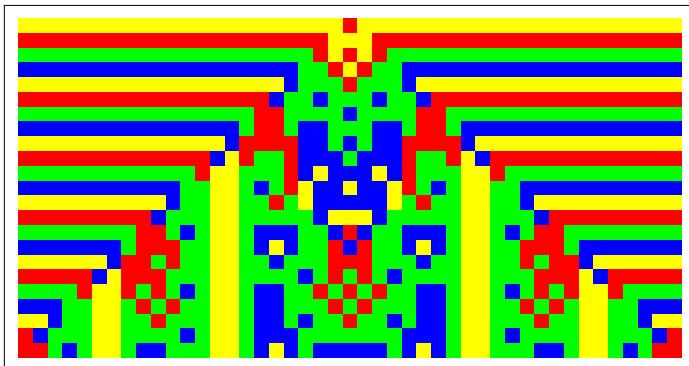
Corretion : {3, 3, 1} → 3, (\*{3,3,1}→2,\*)



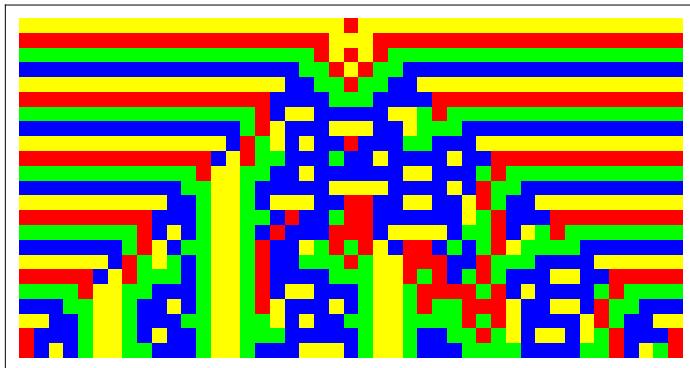
disturbed symmetry, same head



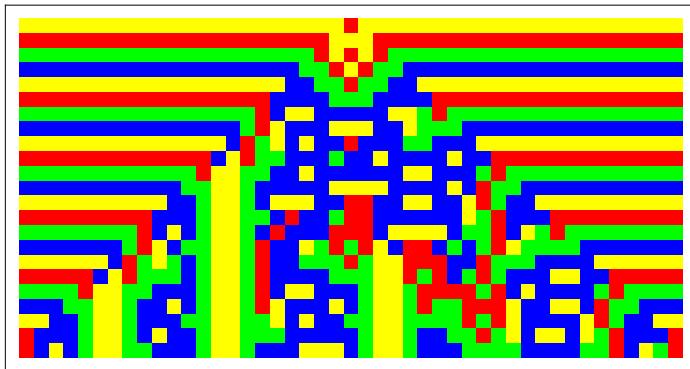
correction : {3, 3, 1} → 3, (\*{3,3,1}→2,\*)



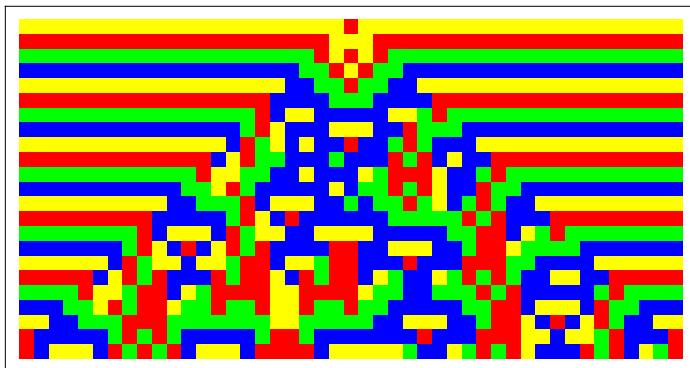
examples for same head asymmetric patterns



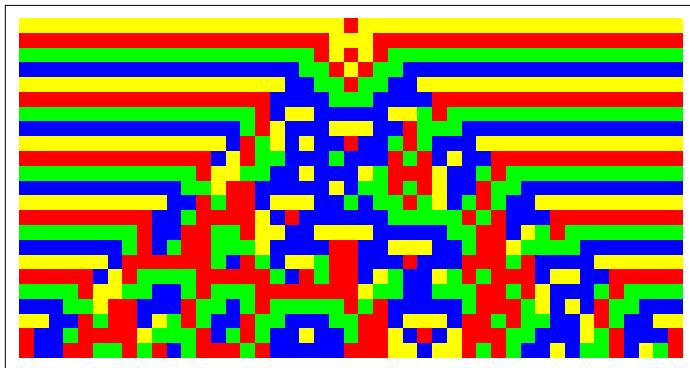
different code of example, same output

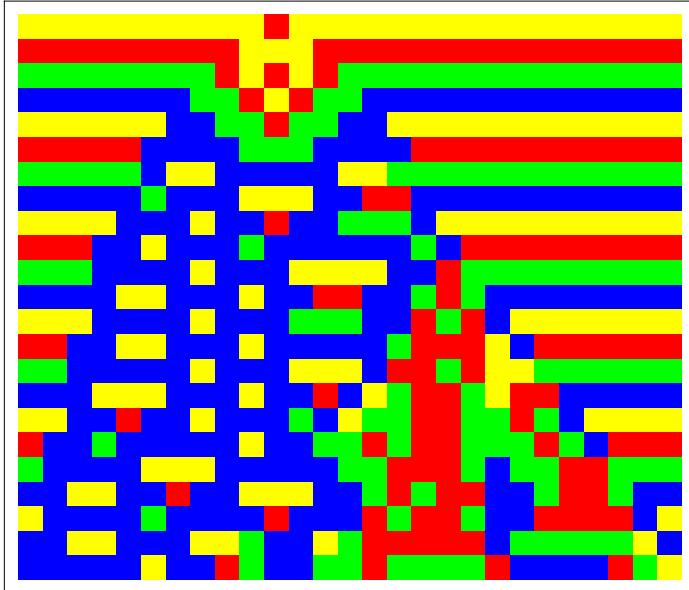
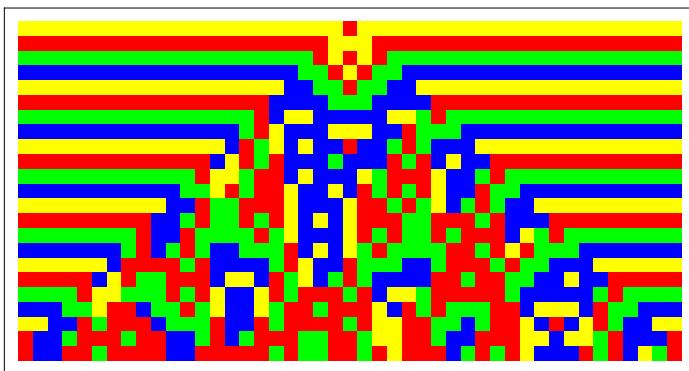


another different code

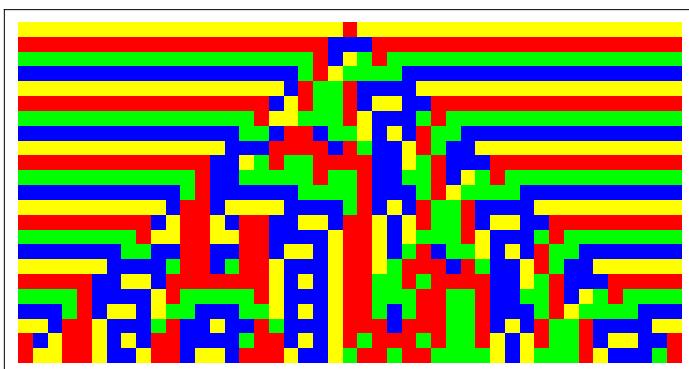


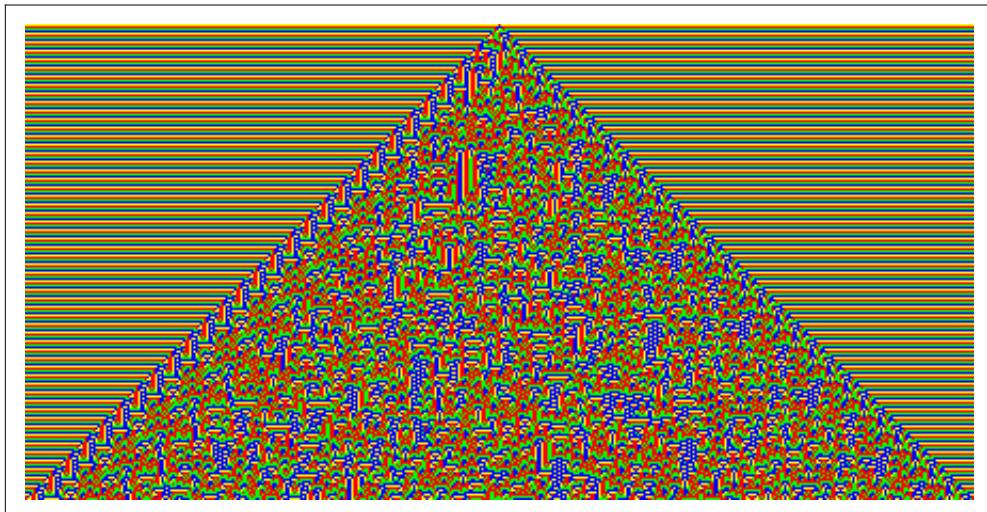
further different examples



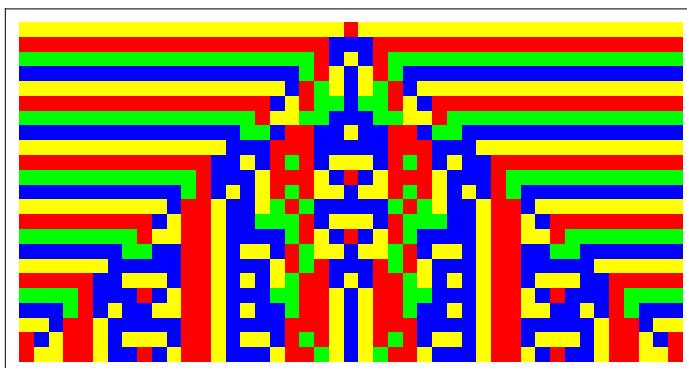


Different head  
and defectuos asymmetry

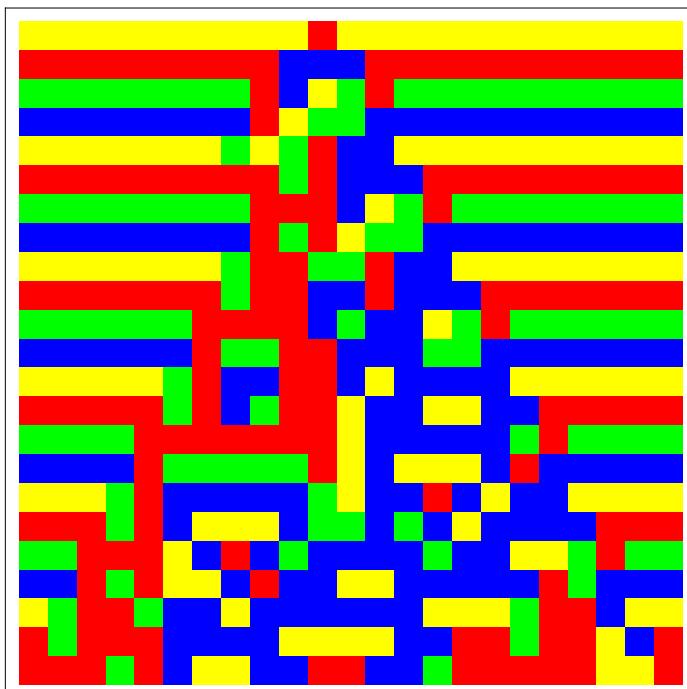




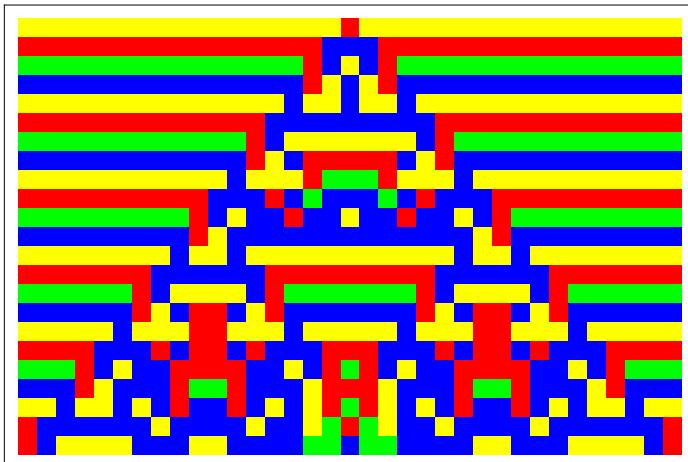
correction : {3, 3, 1} → 3, (\*{3,3,1}→2,\*); symmetry



defect, asymmetric pattern

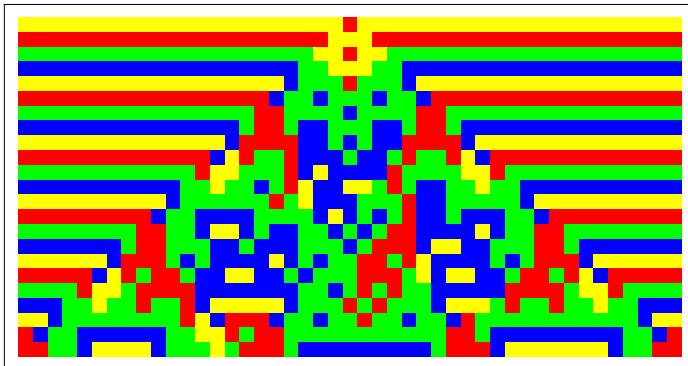


correction : {3, 3, 1} → 3, (\*{3,3,1}→2,\*)



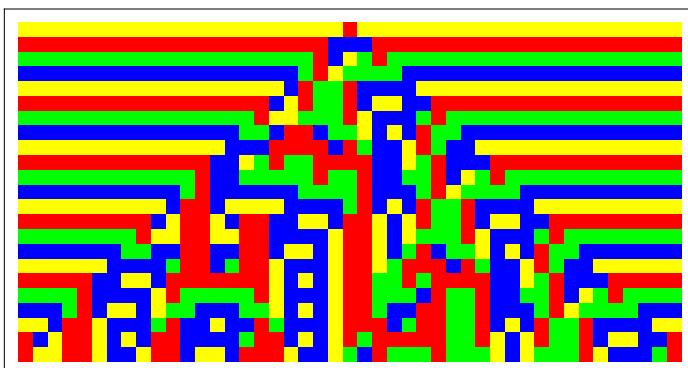
Non defect asymmetric pattern???  
Overdetermination as defect

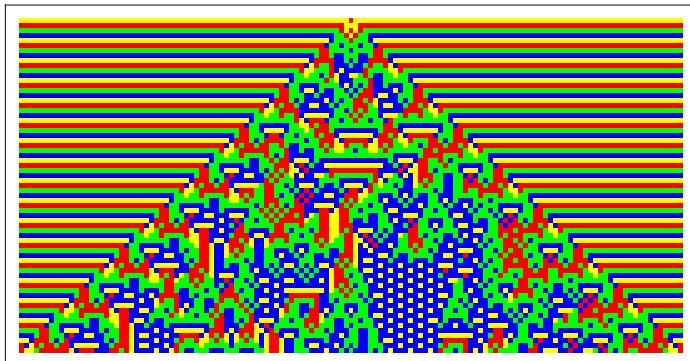
Asymmetry by redundancy



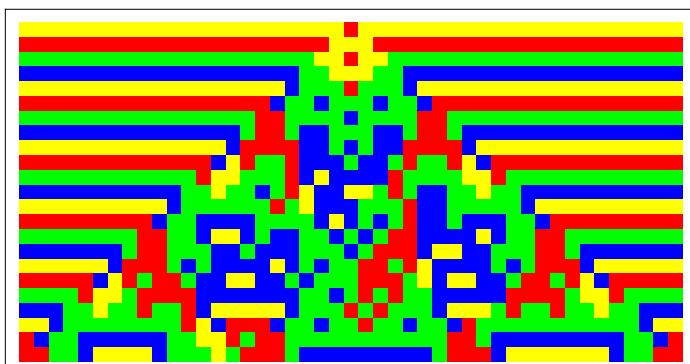
Asymmetric mutation by defect

one

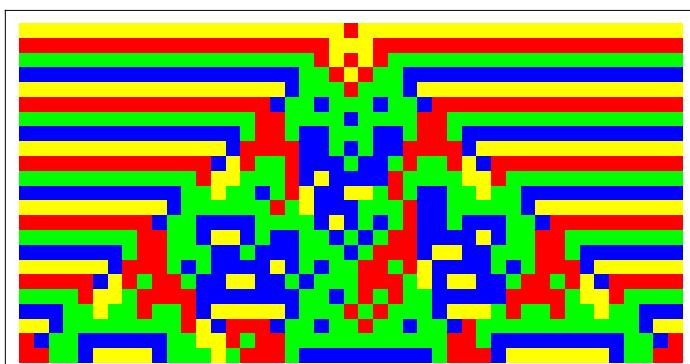




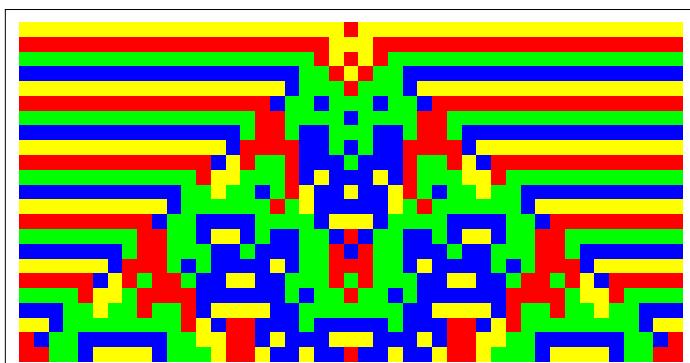
two

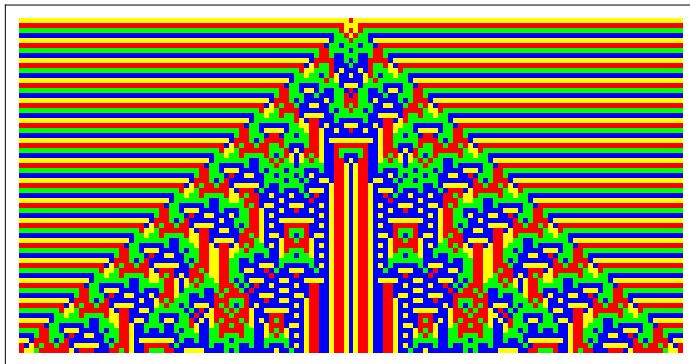


three, equal two



corrected, {3, 3, 1} → 3, (\*{3,3,1}→2,\* ) !!!, symmetric





Symmetric variations

