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Abstract

Morphogrammatics is a calculus of morphic patterns, called morphograms.
In a morphogrammatic calculus, morphograms are transformed by morphogrammatic operations.
Between morphograms relations of morphogrammatic equivalence is defined.
Morphograms might be composed together to new morphograms.
There are several types of compositions, concatenation, chaining and fusion.
Morphograms are combined in an additional sense, with concatenation.
Morphograms are combined to chains, with chaining operation.
Morphograms are combined in an overlapping sense, with fusion operation.
A morphic pattern, i.e. a morphogram, is build of monomorphies.
Monomorphies are taking place at a locus in a pattern.
Monomorphies are finite patterns of kenograms.
Kenograms are the marks we need to perceive morphograms. They are not signs in a technical sense.
Signs are marks with identity, they even may mean something. Kenograms mean nothing, there job is to mark the differences between the monomorphies for a morphogram.

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Morphogrammatics for Dummies: The Domino Approach

A gentle introduction to some elements of morphogrammatics

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Abstract

Dominoes, morphograms, cellular automata, memristics. Topics: possible continuation, coalitions, cooperations, substitution, morphic bisimilarity.

1. Introducing morphograms

Morphogrammatics is a calculus of morphic patterns, called *morphograms*.
In a morphogrammatic calculus, morphograms are transformed by morphogrammatic operations.
Between morphograms relations of morphogrammatic equivalence is defined.

Morphograms might be composed together to new morphograms.
There are several types of compositions, concatenation, chaining and fusion.

Morphograms are combined in an additional sense, whith *concatenation*.
Morphograms are combined to chains, whith *chaining* operation.
Morphograms are combined in an overlapping sense, whith *fusion* operation.

A morphic pattern, i.e. a morphogram, is build of *monomorphies*.

Monomorphies are taking place at a *locus* in a pattern.

Monomorphies are finite patterns of *kenograms*.

Kenograms are the marks we need to perceive morphograms. They are not signs in a technical sense. Signs are marks with identity, they even may mean something. Kenograms mean nothing, there job is to mark the differences between the monomorphies for a morphogram.

Another "gentle" introduction with different intention, in German, is written by Claus Baldus, Morgen und Morgen.
http://www.vordenker.de/cb/cb_morgen.pdf (copy blocked!)

Conventions
We start with two main conventions. This will help to simplify things.

First, we encounter texts, events, behaviours at a *position* in a contexture of texts, events or behaviors, which we perceive and accept as structured patterns. They shall be called *morphograms*.

Second, the encountered patterns of the morphograms are conceived as consisting of parts, organized over different loci. They shall be called *monomorphies*.

Hence morphograms are characterized by the triple :

<Position, Locality, Place>.
Position of the morphogram in a morphogrammatic system defined by *emanation* and *evolution*.
Locality of the monomorphies in a morphogram; loci are offering place for different monomorphies of the same or different structure.
Monomorphies might be reduced to homogeneous patterns, [aabccc] to [aa], [b], [ccc] or they might keep

some structuration, like [aab], [cc], [c].
Place of a kenom in a monomorphy depending on the length of the monomorphy.

Tectonics

Morphogrammatics : $\mathbf{MG}^{(m,n)} = [MG^{(m,n)}, Ops]$

Morphograms : $MG^{(m,n)}$

Monomorphies : mg_i^j

Kenomic interpretations of monomorphies : [kseq]

Semiotic interpretations of kenoms : (sign - seq).

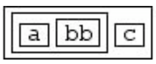
Position $MG^{(m,n)}$	
$MG^{(m)}$	locus
$Dec(MG^{(m)})$	monomorphy
$Ken(MG^{(m)})$	kenom

Notation

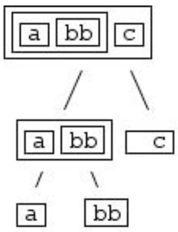
A morphogram might be depicted as a *diagrammatic* pattern of monomorphies.

```
[ □     [a]          ]      [  □     [mg1]           ]
        / \                          / \
[ [bb]   -    [c]    ]      [ [mg2]   -     [mg3]    ]
```

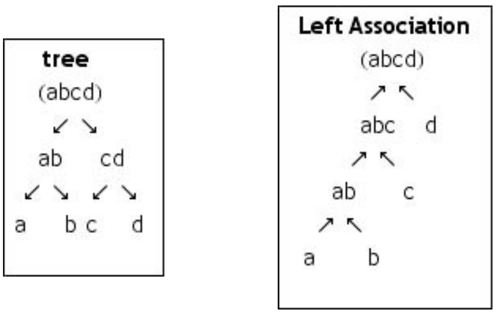
The same morphogram gets a *block* notation as:



The block notation gets a Left-Associative(LA) presentation which defines the morphogram:



An LA presentation is different to the usual binary tree presentation:



But morphograms consist of monomorphies and not of atomic signs.

How to start?

We might encounter a pattern somewhere in this world with the structure [aaaabbcccaaaabbbb]. Such a pattern shall be conceived as a *morphogram MG*. Hence, what counts is the pattern, not the signs, {a,b,c}, inscribing the pattern for *MG*. Furthermore we recognize the structure of the morphogram as build by parts: [aaaa], [bbb], [ccc], [aaaa] and [bbbb]. These parts are localized in the pattern, where they get their *locus* of inscription. Therefore, the parts [bbb] an [ccc] are different, not by structure but by the locus the occupy. The parts of the morphogram are the *monomorphies* of the morphogram.

The morphogram MG might be placed somewhere in the system of morphogrammatics, and will have properties of *emanative* and *evolutive* realizations. At this point of introduction, I will omit the aspects of emanation and evulation as such.

I precis the analysis of the morphogram MG in the following table:

MG = [aaaabbcccaaaabbbb]

MG	loc_1	loc_2	loc_3	loc_4	loc_5
Dec	mg_1	mg_2	mg_3	mg_1	mg_2
Ken	*a*	*b*	*c*	*a*	*b*
	a	*b*	*c*	*a*	*b*
	a	ø	*c*	*a*	*b*
	a	ø	ø	*a*	*b*

Dec(MG): the decomposition of the morphogram MG into its monomorphies *mg* placed at the loci *loc*.
Ken(MG): The marks of the monomorphies, called kenograms *kg* or kenoms.

Now we have an idea of a morphogram.
Morphograms might be encountered everywhere, not only as marks on a screen or on paper.

What can we do with it?

Like with domonos we might *reverse* a morphogram.

We proceed our little intro we the idea of *prolongation*. A morphogram might prolongate itself into a more complex morphogram by adding its own parts, i.e. monomorphies to itself.
We still have only one morphogram and only its monomorphies at our disposal.

Obviously, the morphogram is able to produce different prolongations.

Now, having some morphograms at our disposal, we are ready to ask: *How are morphograms interacting with each other?*

In analogy we might consider *coalition* building by “addition” and *cooperation* building by “multiplication”.

1.1. Domino-like continuations

An early attempt to formalize morpho- and kenogrammatics was modeled along the paradigm of recursive word arithmetics, i.e. regular formal languages as they are well known today in formal linguistics, the theory of computation and pogramming languages.

A very different approach was given by Roland Hausser’s *Left-Association Grammars* (1987). Until now, this approach wasn’t studied and applied explicitly for morpho- and kenogrammatics. One reason might be that for kenogrammatics the formal grammar approach has at first some clear conceptual advantages. But for monomorphy-based morphogrammatics priorities are changing to the reverse. Abstract grammars are much too clumsy to model morphogrammatics. Hausser’s Domino Approach on the other hand seems to be quite natural.
Hence, an application, which always is a transformation of the concept too, to morphogrammatics is reasonable and technically helpful.

The algorithm of Left-Associative Grammar (LA-grammar, TCS’92):
"LA-grammar is based on the principle of possible continuations. *This is in contrast to the algorithms commonly used in today’s linguistics, namely Phrase Structure Grammar (PSG) and Categorial Grammar (CG), which are based on the principle of possible substitutions.*
"Computing possible continuations models the time-linear *structure of natural language and permits us to handle turn-taking as the interaction of three kinds of LA-grammar, namely LA-hear, LA-think, and LA-speak.”* (Hausser)

Differences
The first difference, obviously, is that in contrast to LAG, morpgogrammatic continuations are not linear but tabular and retro-grade recursive with multiple simultaneous resultants.

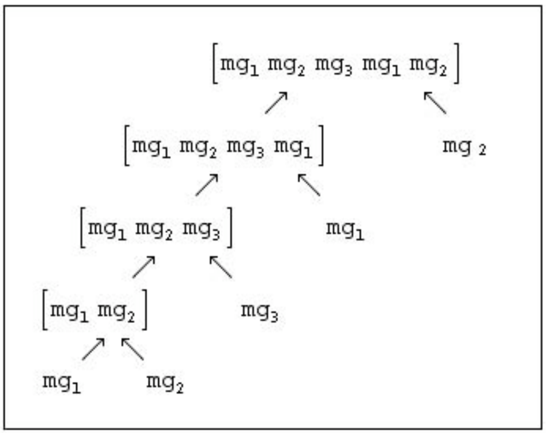
LAGs are designed for a modeling, formalization and computation of natural language with its time-linear structure.
Morphogrammatics is designed to make accessible not spoken language but writing. And writing is not understood in the tradition of subordination of writing under the general laws of spoken language with its ideality of atomicity, linearity and potential realizability in time, space and matter.
Writing for morphogrammatics is not modeled by natural language but by the possibilities of a general economy of writing, i.e. designing and using marks, signs in a pre-semiotic manner. Therefore the abstraction of morphogrammatic continuations is not linear but tabular retro-grade recursive, while linguistic possible continuations are linearly recursive with one or more successors.

The domino model of possible continuations is helpful to move away from the dominance of an alphabet and its rules to generate new constellations. In contrast to the abstract semiotic concatenation, possible continuations are more adjusted to the constellations they are prolongating.

Structure of a morphogram

MG = [aaaabbcccaaaabbbb] = $[mg_1\ mg_2\ mg_3\ mg_1\ mg_2]$

The Schema of Left Associative Concatenation Rules
http://www.linguistik.uni-erlangen.de/clue/fileadmin/docs/rrh/monographs/1989/cl-input.pdf



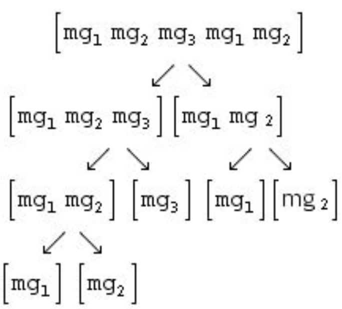
$$LA(MG) = [[[[mg_1] + mg_2] + mg_3] + mg_1] + mg_2]$$

$$MG = [a^4\ b^2\ c^3\ a^4\ b^4] \text{or} [1^4\ 2^2\ 3^3\ 1^4\ 2^4]$$

$$\text{lenght}(MG) = 4 + 2 + 3 + 4 + 4 = 17$$

What's the point to chose the LA composition instead of the binary tree?

Binary decomposition



$$BIN(MG) = [[[mg_1] + [mg_2]] + [mg_3] + [[mg_1] + [mg_2]]$$

Are BIN (MG) and LA (MG) morphogrammatically equal?

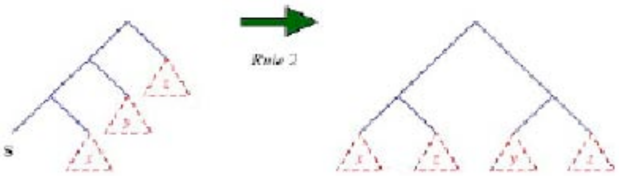
$$[[[[\,mg_1 + mg_2\,] + mg_3\,] + mg_1\,] + mg_2\,] ?? \; [[mg_1] + [mg_2]] + [mg_3] + [[mg_1] + [mg_2]]$$

Morphogrammatic opertions are not necessarily associative.

LA is botton up, BIN is top down. Are they morphogrammatically symmetric?

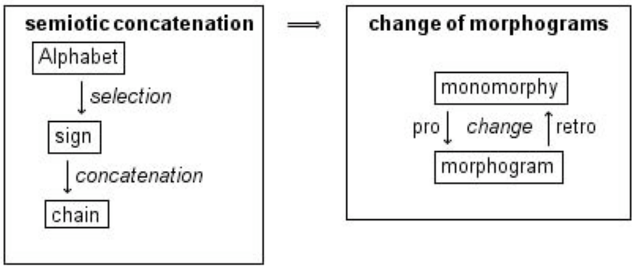
Combinatory logic
From a formal point of view of term calculi he answer is clear. The combinator **S** is transforming the formula "(x(y(z))" into the equivalent formula "xz(yz))".

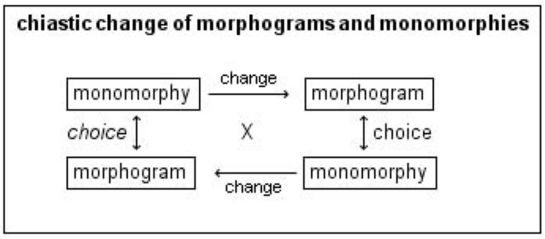
Combinator **S**: **S**(xyz) = xz(yz))



1.2. Structure of morphogrammatic continuations

In contrast to semiotic prolongations or continuations, morphogrammatic continuations are retro-grade recursive depending on the morphogram and its monomorphies to be prolongated.





2. The Metaphor of Dominoes

Trained as Westerners we are easily trapped into a way of thinking, which is determined as system of linearly structured signs, objects and operators. This is perfectly studied in disciplines like the theory of formal systems, logic, programming languages and more. Postmodern thinking, art, music, theater and dance tried to escape this terror to linearization and atomization. Modern physics and computer science are still dominated by Western logocentrism. Even if that is rejected as philosophical aestheticism.

This paper tries to give a gentle introduction to a difficult stuff that is in fact very simple presumed we are not being terrorized by linearity and are free to play a game of different form.
Therefore, I recommend to use the game of Dominoes as a metaphor to enter the game of morphograms.

"Each piece you add must be compatible with the previous one, and it in turn determines what can be added next. This is the idea of possible continuations. The continuation approach is based on linear

order of concrete pieces (word forms): The next piece con only be added at the end of the sequence."' (Hausser)

"Each domino is a rectangular tile with a line dividing its face into two square ends. Each end is marked with a number of spots *(also called pips) or is blank. The backs of the dominoes in a set are indistinguishable, either blank or having some common design."*

"The line of play is the configuration of played tiles on the table. Typically it starts *with a* single tile, *from which it grows in two opposite directions when the players add matching tiles."* (WiKi)

A start is arbitrary, it is defined by the start of the game not by its structure. Hence any, possible continuation is following this arbitrary start. We might say, a start is not pre-given to the player but the player are encountering the specific start.

A domino consists of two "monomorphies" with a range of 0 to 6 spots. A domino is linearly ordered in two directions with a 2-"end"(or head and tail) partition.
The back of a domino is not involved, syntactically, into the game. But it might be of significance for the players. Or it might be used to substitute a domino of the same spots.

There are also dominoes of multi-colored tiles in one set. This could lead to interesting speculative extensions of the original singular game towards a complex game of overlapping dominos.

The figure and number of the spots of a domino is stable. Hence, no prolongation of a domino by other dominoes will change the pattern of the starting domino. The same holds for the added domino. It will stay stable and not change by concatenation. The only variability possible is to change a domino of one back color with one of another back color. But this possibility is in fact not covered by the rules of the game.

2.1. Prolongations and reflections

What can be done? What is an easy *operator* on morphograms?
Like dominoes, you can reverse your morphogram, rev(aabbcc) = (ccbbaa). Obviously they are different and if the first is not fitting to the domino pattern, then the reverse might. Otherwise you have to use another domino.

But for morphograms things are different:
rev(aabbcc) = (ccbbaa), but (aabbcc) $=_{MG}$ (ccbbaa), hence

rev(rev(aabbcc)) = rev(aabbcc).

Obviously, the trick is a *symmetry* of the morphogram, therefore we state:

$$MG = (mg_1, \ mg_2, mg_3)$$
$$\forall \ mg_1, \ mg_3 \in MG:$$
$$mg_1 =_{mg} mg_3 \Longrightarrow refl(MG) =_{mg} MG$$

The first case is trivial:
$mg_1 =_{mg} mg_3$, ken(mg_1) $=_{ken}$ mg_3: ==> refl(mg_1) $=_{mg}$ mg_3,

$mg_1 =_{mg} mg_3$, $mg_1 =_{ken} mg_3$: ==> $mg_1 =_{mg} mg_3$.

Example: (aa, bbbb, aa) ==> ref(aa, bbbb, aa) = (aa, bbbb, aa).

The second hints to a morphogrammatic abstraction:
$mg_1 =_{mg} mg_3$, ken(mg_1) != $_{ken}$ ken(mg_3): ==> refl(mg_1) $=_{mg}$ mg_3,

$mg_1 =_{mg} mg_3$, mg_1 $!=_{ken}$ mg_3: ==> $mg_1 =_{mg} mg_3$.

Example: (aa, bbbb, cc) ==> ref(aa, bbbb, cc) = (cc, bbbb, aa) = (aa, bbbb, cc).

A morphogram consists of monomorphies.

In the example,

(aa, bbbb, cc) : [mg_1 mg_2 mg_3]
rev (aa, bbbb, cc) = (cc, bbbb, aa) : [mg_1 mg_2 mg_3]

And [mg_1 mg_2 mg_3] $=_{MG}$ [mg_1 mg_2 mg_3].

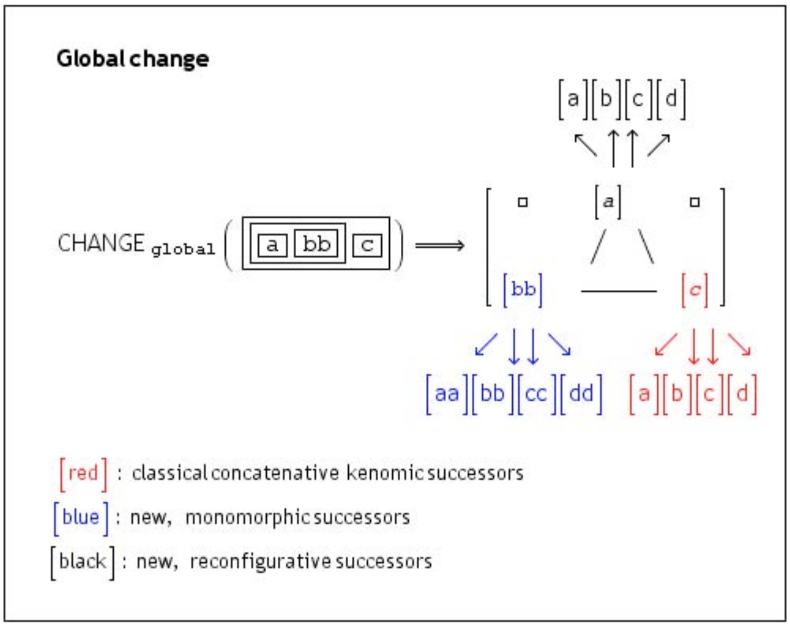
Thus we learn, the identity of the element of a monomorphy are not involved into the game. What counts is the pattern, i.e. the monomorphy, they are indicating.

This experience, that [mg_1] $=_{MG}$ [mg_3], or more concrete, that [aa] $=_{MG}$ [cc] is leading for the whole of morphogrammatics.

A single domino, or a single morphogram, is not yet enabling an interesting game.
Like dominoes have prolongation, similar holds for morphograms.

We might like to prolongate a morphogram.
Take the former example: MG_1= [mg_1 mg_2 mg_3].

A prolongation of MG_1 is possible at all places of monomorphies. Hence, the morphogram MG_1 is not simply getting a linear successor, like for numbers or words of a grammar, but each monomorphy offers the chance of prolongation. This is also in strict contrast to multi-successor systems and many-valued set theories. Multi-successor systems are not referring to their applicant but are adding arbitrary successors out of a given alphabet. Another difference to mark is that the applications for multi-successor system don't hold simultaneous but are build a set of results.



The order of the collection of prolongations is arbitrary.

We start with the monomorphy [a] of the morphogram [abbc]:
[a] gets a prolongation with [a], i.e. with itself.

[a] gets a prolongation with [b], i.e. because [a] is morphogrammatically equal [b],
[a] gets a prolongation with [c], i.e. because [a] is morphogrammatically equal [c],
[a] gets a prolongation with [d], i.e. because [a] is morphogrammatically equal [d],

All three cases are accepting the structure of the monomorphy [mg_1], i.e. its monadic structure indicated by the monad [a]. Furthermore, they are reflecting the structure of the whole morphogram, which is represented by the marks a, b, c. These marks are called *kenograms* to mark the difference to signs, which are ruled by identity.

But this not yet the whole story:

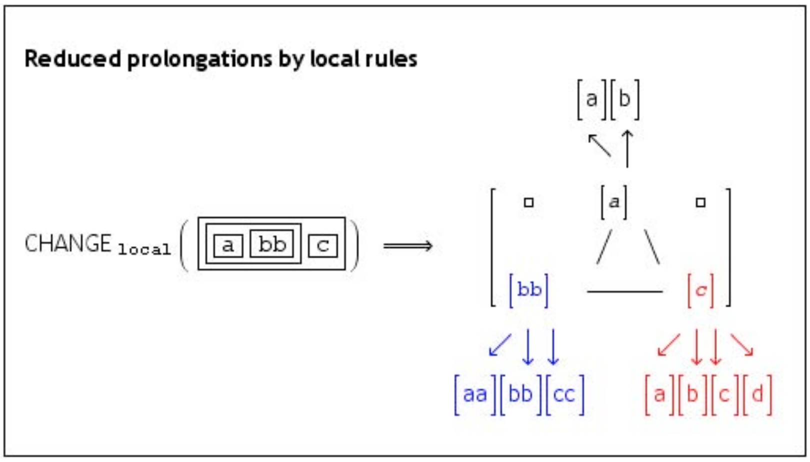
[a] gets a prolongation with [d], i.e. because [a] is morphogrammatically equal [d].
Now, it would be natural to continue with [e], [f], ...,[x], and whatever marks. But this is natural only for signs of an alphabet and the rule of identity.
For morphograms, [d] is morphogramamatically identical to [a], this would be true for other atomic marks, but [d] is indicating a new realization of the monomorphy [a]. And all other candidates would simply do the same. Therefore, it is enough that one monad, not included in the morphogram [abbc] is doing the job. Only by convention of a lexical order, [d] is chosen.

The exercise goes on with the monomorphies mg_2 = [bb] and mg_3= [c].

This, the prolongation of the morphogram MG_1, gets some more prolongations with the monomorphies [mg_2} and [mg_3], all together are producing:

Prolongations of MG_1= [abbc]:

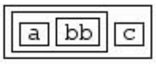
[a] => [a], [b], [c] and [d]
[bb] => [aa], [bb], [cc] and [dd], and
[c] => [a], [b], [c] and [d].



This kind of mophogrammatic continuations are defining an important departure from the prolongation rules of our introductory metaphor, the domino game. The domino game allows only a very restricted set of possible continuations. Morphogrammatic continuation is not restricted to head-tail concatenations but is also allowing continuations from every monomorphy of the morphogram.

2.1.1. Notations for morphograms

Start with a morphogram



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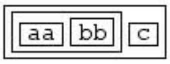
MG	loc_1	loc_2	loc_3
Dec	mg_1	mg_2	mg_3
Ken	*a*	*b*	*c*
	∅	*b*	∅

Prolongations



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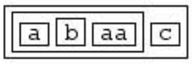
MG	loc_1	loc_2	loc_3
Dec	mg_1	mg_2	mg_3
Ken	*a*	*b*	*c*
	a	*b*	∅



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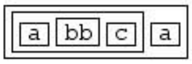
MG	loc_1	loc_2	loc_3	loc_4
Dec	mg_1	mg_2	mg_1	mg_3
Ken	*a*	*b*	*a*	*c*
	∅	∅	*a*	∅



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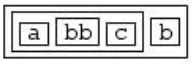
MG	loc_1	loc_2	loc_3	loc_4
Dec	mg_1	mg_2	mg_3	mg_3
Ken	*a*	*b*	*c*	*a*
	∅	*b*	∅	∅



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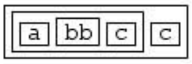
MG	loc_1	loc_2	loc_3	loc_4
Dec	mg_1	mg_2	mg_3	mg_2
Ken	*a*	*b*	*c*	*b*
	∅	*b*	∅	∅



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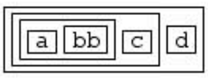
MG	loc_1	loc_2	loc_3
Dec	mg_1	mg_2	mg_3
Ken	*a*	*b*	*c*
	∅	*b*	*c*



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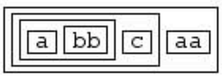
MG	loc_1	loc_2	loc_3	loc_4
Dec	mg_1	mg_2	mg_3	mg_4
Ken	*a*	*b*	*c*	*d*
	∅	*b*	∅	∅



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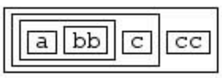
MG	loc_1	loc_2	loc_3	loc_4
Dec	mg_1	mg_2	mg_3	mg_1
Ken	*a*	*b*	*c*	*a*
	∅	*b*	∅	*a*



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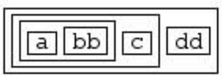
MG	loc_1	loc_2	loc_3
Dec	mg_1	mg_2	mg_3
Ken	*a*	*b*	*c*
	∅	*b*	*c*
	∅	∅	*c*



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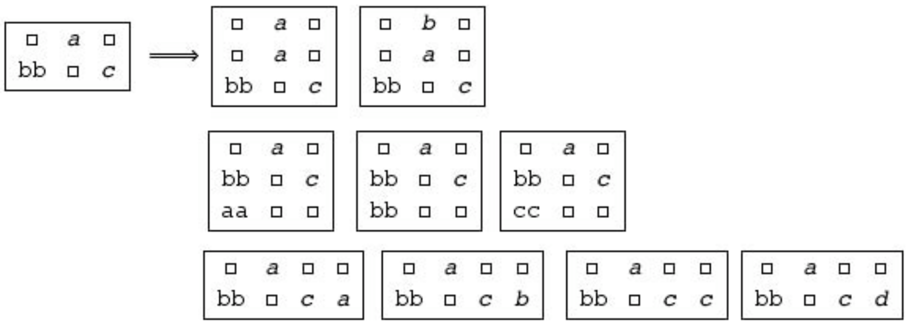
MG	loc_1	loc_2	loc_3	loc_4
Dec	mg_1	mg_2	mg_3	mg_4
Ken	*a*	*b*	*c*	*d*
	∅	*b*	∅	*d*

Table for cont(MG):

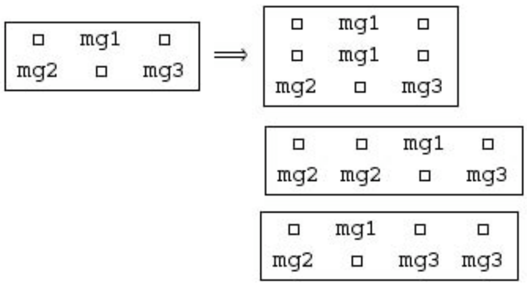
MG = (a b c / □ b □)	loc_1	loc_2	loc_3	loc_4
Dec	mg_1	mg_2	mg_3	–
con_{mg1}	a ∅	b b	c ∅	mg_1
con_{mg2}	MG			mg_2
con_{mg3}	MG			mg_3

2.1.2. Cellular notation

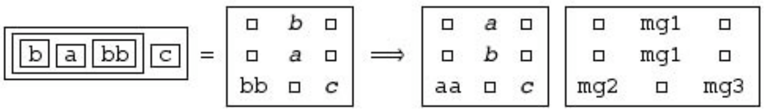
The cellular notation might prepare for a further formalization and programming of morphogrammatics in the framework of cellular automata.



Prolongation algebra

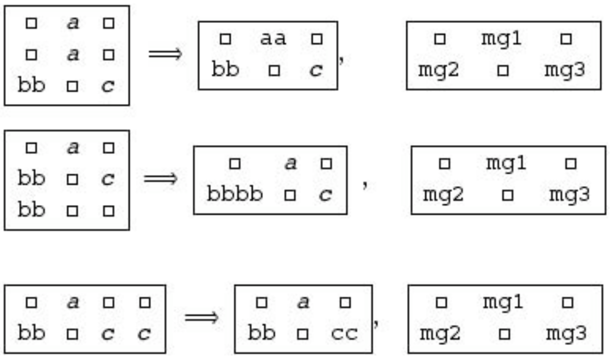


Normalizations



Domestication

There might be reasons to domesticate the neighbor pattern with the same kenograms, but equally, such a domestication might be refused with the insistence on the difference granted by the different locus the neighbor monomorphy occupies.



Further examples

MG = [abbc][*c*]

MG	loc_1	loc_2	loc_3	loc_4
Dec	mg_1	mg_2	mg_3	mg_3
Ken	*a*	*b*	*c*	*c*
	ø	*b*	ø	ø

⟹

MG	loc_1	loc_2	loc_3
Dec	mg_1	mg_2	mg_3
Ken	*a*	*b*	*c*
	ø	*b*	*c*

MG	loc_1	loc_2	loc_3
Dec	mg_1	mg_2	mg_3
Ken	*a*	*b*	*c*
	ø	*b*	ø

⟹

MG	loc_1	loc_2	loc_3
Dec	mg_1	mg_2	mg_3
Ken	*a*	*b*	*c*
	b	*b*	ø

=

MG	loc_1	loc_2	loc_3
Dec	mg_1	mg_2	mg_3
Ken	*a*	*b*	*c*
	ø	*b*	ø
	ø	*b*	ø

2.2. Coalition as addition

Example

MG_1 = [abba], MG_2 = [*a*]

[abba] + [*a*] **:**

MG	loc_1	loc_2	loc_3
Dec	mg_1	mg_2	mg_1
Ken	*a*	*b*	*a*
	ø	*b*	ø

coalition ⟹

MG	loc_1	loc_2	loc_3	loc_4
Dec	mg_1	mg_2	mg_1	mg_3
Ken	*a*	*b*	*a*	*a*
	ø	*b*	ø	ø

=

MG	loc_1	loc_2	loc_3
Dec	mg_1	mg_2	mg_1
Ken	*a*	*b*	*a*
	ø	*b*	*a*

MG	loc_1	loc_2	loc_3	loc_4
Dec	mg_1	mg_2	mg_1	mg_3
Ken	*a*	*b*	*a*	*b*
	ø	*b*	ø	ø

MG	loc_1	loc_2	loc_3	loc_4
Dec	mg_1	mg_2	mg_1	mg_3
Ken	*a*	*b*	*a*	*c*
	ø	*b*	ø	ø

As for the kenomic notation of prolongations, the monomorphic prolongations are restricted *retro-grade* to the morphogram. The monomorphic aspect is abstracting from the iterative/accretive distinction crucial for kenomic prolongations.

2.3. Simultaneity

Hence, the full and only possible prolongations are defined by the occurring monomorphies of a morphogram.

I have demonstrated the possible prolongation of the morphogram successively. But for conceptual reasons we have to accept the *simultaneity* of all the prolongations possible.
A successiveness would involve time. But there is no time necessary at the time now.
The prolongations are also not collected as a set or a heap, but as a complexion.

For advanced students I might mention the fact that a formalization of the simultaneity of prolongations is well done by category-theoretic *interchangeability*. There are many ways to model the described situation. To simplify the case I will chose a mix of iteration and mediation.

Interchangeability of the prolongations

$$\mathrm{prol}\left(\left[abbc\right]\right):$$

$$\left(\begin{array}{c}\left(\begin{array}{c}\left(\left[abbc\right]_1 \circ^{1.0.0} \left[a\right]_1 \parallel^{1.0.0} \left[b\right]_1 \parallel^{1.0.0} \left[c\right]_1 \parallel^{1.0.0} \left[d\right]_1\right) \\ \amalg_{1.2.0} \\ \left(\left[abbc\right]_2 \circ^{0.2.0} \left[aa\right]_2 \parallel^{0.2.0} \left[bb\right] \parallel^{0.2.0} \left[cc\right]_2 \parallel^{0.2.0} \left[dd\right]\right)\end{array}\right) \\ \amalg_{1.2.3} \\ \left(\left[abbbc\right]_3 \circ^{0.0.3} \left[a\right]_3 \parallel^{0.0.3} \left[b\right]_3 \parallel^{0.0.3} \left[c\right]_3 \parallel^{0.0.3} \left[d\right]_3\right)\end{array}\right) =$$

$$\left(\begin{array}{c}\left(\begin{array}{c}\left[abbc\right]_1 \\ \amalg_{1.2.0} \\ \left[abbc\right]_2\end{array}\right) \\ \amalg_{1.2.3} \\ \left[abbc\right]_3\end{array}\right) \circ_{1.2.3} \left(\begin{array}{c}\left(\begin{array}{c}\left[a\right]_1 \parallel^{1.0.0} \left[b\right]_1 \parallel^{1.0.0} \left[c\right]_1 \parallel^{1.0.0} \left[d\right]_1 \\ \amalg_{1.2.0} \\ \left[aa\right]_2 \parallel^{0.2.0} \left[bb\right]_2 \parallel^{0.2.0} \left[cc\right]_2 \parallel^{0.2.0} \left[dd\right]_2\end{array}\right) \\ \amalg_{1.2.3} \\ \left[a\right]_3 \parallel^{0.0.3} \left[b\right]_3 \parallel^{0.0.3} \left[c\right]_3 \parallel^{0.0.3} \left[d\right]_3\end{array}\right)$$

Interchangeability of the prolongations of the morphogram MG $_1$

$$MG_1 = \left[mg_1\ mg_2\ mg_3\right]$$

$$interch\left(prol\left(\left[mg_1\ mg_2\ mg_3\right]\right)\right):$$

$$\left(\begin{array}{c}\left(\begin{array}{c}\left(\left[MG\right]_1 \circ^{1.0.0} \left[mg\right]_1\right) \\ \amalg_{1.2.0} \\ \left(\left[MG\right]_2 \circ^{0.2.0} \left[mg\right]_2\right)\end{array}\right) \\ \amalg_{1.2.3} \\ \left(\left[M\right]_3 \circ^{0.0.3} \left[mg\right]_3\right)\end{array}\right) =$$

$$\left(\begin{array}{c}\left(\begin{array}{c}\left[MG\right]_1 \\ \amalg_{1.2.0} \\ \left[MG\right]_2\end{array}\right) \\ \amalg_{1.2.3} \\ \left[MG\right]_3\end{array}\right) \circ_{1.2.3} \left(\begin{array}{c}\left(\begin{array}{c}\left[mg\right]_1 \\ \amalg_{1.2.0} \\ \left[mg\right]_2\end{array}\right) \\ \amalg_{1.2.3} \\ \left[mg\right]_3\end{array}\right)$$

Properties of prolongation

In contrast to semiotic concatenation, i.e. the operation of connecting two sign sequences linearly together to produce a new sign sequence (word) out of both, the range morphogrammatic prolongation is depending directly on the morphogram (pattern) to be prolongated. All other arbitrary prolongations of the morphogram would be redundant, i.e. repeating a result already produced.

This property of morphogrammatic prolongations is called *retro-grade* recursivity.
Morphogrammatic prolongations are not only recursive, i.e. iterating abstractly the rules and objects, but are concretely determining retro-grade the range of the iteratively applied rules by the morphogram to be prolongated. Etymologically speaking, the term retro-grade recursivity sounds like a pleonasm: retro-re-current

2.4. Cooperations of morphograms

If we have constructed an idea of morphogrammatic prolongations, it seems to be straight forward to try it with morphogrammatic "addition", i.e. building of *coalitions*, and furthermore to introduce morphogrammatic "multiplication", i.e. cooperations.

2.4.1. Multiplication

Multiplication tables for kmul([*a*, *b*], [*a*, *b*])

kmul	*a*	*b*
1	*a*	x
2	*b*	y

kmul	*a*	*b*	*b*'	*b*"	*b*'"
a	*a*	*b*	*b*	*c*	*c*
b	*b*	*a*	*c*	*a*	*d*

$$\textbf{kmul}\left(\begin{bmatrix} a \\ b \end{bmatrix}, \begin{bmatrix} a \\ b \end{bmatrix}\right) = \left\{\begin{bmatrix} a & b \\ b & a \end{bmatrix}, \begin{bmatrix} a & b \\ b & c \end{bmatrix}, \begin{bmatrix} a & c \\ b & a \end{bmatrix}, \begin{bmatrix} a & c \\ b & d \end{bmatrix}\right\}$$

$\begin{bmatrix} a & b \\ b & a \end{bmatrix}$	loc_1	loc_2	loc_3	loc_4
Dec	mg_1	mg_2	mg_3	mg_4
	a	*b*	*a*	∅
	∅	*b*	∅	∅
Ken	x	y	z	u

$\begin{bmatrix} a & b \\ b & c \end{bmatrix}$	loc_1	loc_2	loc_3	loc_4
Dec	mg_1	mg_2	mg_3	mg_4
	a	*b*	*c*	∅
	∅	*b*	∅	∅
Ken	x	y	z	u

$\begin{bmatrix} a & c \\ b & a \end{bmatrix}$	loc_1	loc_2	loc_3	loc_4
Dec	mg_1	mg_2	mg_3	mg_4
	a	*b*	*c*	*a*
Ken	x	y	z	u

$\begin{bmatrix} a & c \\ b & d \end{bmatrix}$	loc_1	loc_2	loc_3	loc_4
Dec	mg_1	mg_2	mg_3	mg_4
	a	*b*	*c*	*d*
Ken	x	y	z	u

Non – commutativity

kmul([ab], [aaa)] ≠ kmul([aaa], [ab])
kmul([ab], [aaa]] = [ababab]
kmul([aaa], [ab]) = [aaabbb]

2.4.2. Substitution

Substitution, a retro-grade context depending operation
A next step which goes more into the mysteries of morphogrammatics, albeit well prepared by addition, is to define substitution of monomorphies in morphograms.
With the use of dominoes it is obvious that, e.g. a damaged domino might be replaced by another domino of the same form without changing the existing constellation.

Semiotic Substitution :
$\forall h,\ k_1 \in H_1,\ k_2 \in H_2,\ k_1 =_{sem} k_2$:
$\mathbf{H_1 =_{sem} H_2 \Leftrightarrow Subst_{h/k}(H_1) =_{sem} Subst_{h/k}(H_2)}$

$k_1 =_{sem} k_2$ iff $\text{length}(k_1) = \text{length}(k_2)\ \wedge$
$\forall i, j \in k_1, k_2$:
$\text{loc}_i(\text{atom}) = \text{loc}_j(\text{atom})$

Example
$H_1 = H_2 = (\text{aabbcde})$,
$h = (\text{bb}),\ k_1 = k_2 = (\overline{\text{lkmbc}})$:
$H_1 =_{sem} H_2 \Longleftrightarrow \text{Subst}_{\text{bb}/\overline{\text{klmbc}}}(\text{aabbcde}) = \text{Subst}_{\text{bb}/\overline{\text{klmbc}}}(\text{aabbcde})$
$(\text{aabbcde}) =_{sem} (\text{aabbcde}) \Longleftrightarrow (\text{aa}\ \overline{\text{lkmbc}}\ \text{bbcde}) = (\text{aa}\ \overline{\text{lkmbc}}\ \text{bbcde})$.

Monomorphic substitution : Type equal – length

Context rules for substitution CRS
$\forall h,\ m_1 \in H_1, m_2 \in H_2,\ m_1 =_{MG} m_2$,
$m_1 \neq_{sem} m_2,\ h \neq_{sem} m_1,\ m_2$,
$\text{length}(m_1) = \text{length}(m_2)$,
$\text{kenom}(m_1) \bigcap \text{kenom}(H_1) = \emptyset$,
$\text{kenom}(m_2) \bigcap \text{kenom}(H_2) = \emptyset$:
$\mathbf{H_1 =_{MG} H_2 \Longrightarrow}$
$\mathbf{Subst_{h/m_1}(H_1) =_{MG} Subst_{h/m_2}(H_2)}$;
modulo CRS

Example

$H_1 = [aabbacc],\ H_2 = [aaccabb],$

$H_1 =_{MG} H_2,\ H_1 \neq_{sem} H_2$

$Dec(H_1) = ([aa], [bb], [a], [cc]),$

$Dec(H_2) = ([aa], [cc], [a], [bb]),$

$h = [aa],\ m_1 = [ddd],\ m_2 = [eee],$

$length(m_1) = length(m_2),$

$m_1 \neq_{sem} m_2,\ h \neq_{sem} m_1,\ m_2,$

$sem(m_i) \bigcap sem(H_i) = \emptyset,\ i = 1, 2$

$Dec(H_1) = ([aa], [bb], [a], [cc])$

$Subst(H_1)_{[aa]/[ddd]}([aa], [bb], [a], [cc]) = ([ddd], [bb], [a], [cc])$

$Dec(H_2) = ([aa], [cc], [a], [bb])$

$Subst(H_2)_{[aa]/[eee]}([aa], [cc], [a], [bb]) = ([eee], [cc], [a], [bb])$

$H_1 =_{MG} H_2 \Longrightarrow Subst(H_1)_{[aa]/[ddd]} =_{MG} Subst(H_2)_{[aa]/[eee]}$

$$[aabbacc] =_{MG} [aaccabb] \Longrightarrow ([dddbbacc]) =_{MG} ([eeeccabb]).$$

Standard representation

$$[aabbacc] =_{MG} [aabbacc] \Longrightarrow ([aaabbcdd]) =_{MG} ([aaabbcdd]).$$

2.5. The magic of morphograms

2.5.1. More operators: chaining and fusion

There is a lot of magic with morphograms and morphogrammatics. But with the introduction of further operators, this magic gets some mind-boggling surprises.

Until now, our game lived from the fact that two morphograms are equal if they are at least of the same length.

But there is no need for such restricting conditions.

Take two automata. Both are doing the very same job, i.e. you cannot distinguish the results of the automata. They are the same. And as a user of the automata, that’s all you need.
Your question is: *What* are they doing?

You are not asking: *How* are they doing it?

Hence, it is easily possible, that both automata, albeit doing the same, are doing it very differently. The structure of the automata might be different but their behavior might be the same.

Because morphograms don’t have a representation in semiotics, they are in fact not given to any perception. Hence, what we can study is their behavior and interactions.

From the point of view of semiotics it is nonsense to state that two sign sequences might be equal if they are not at least of the same length.

That ecaxtly is the magic we experience with morphograms, they might be equal even if they are of different length. In other words, they might be the same even if their structure is of different complexity. What only counts, is their behavior.

The behavior of morphograms is mirrored by their *operators* and morphograms as "objects" are at the end inscribed as operators and not as sequences of signs.

Hence, two new operators, *chaining* and *fusion*, added to *concatenation*, will be involved to create the real magic of morphograms.

We even might speculate, that the new morphogrammatics starts with such magics, and everything else then is a derivation of this magic constellation, the theorem of morphic bisimilarity, i.e. of behavioral equivalence of morphograms. Such a turn is not surprising because semiotics starts with the concept of equality of signs, gathered by identification and the equality of sign sequences

I will not follow the obvious but tricky question: *Are then morphograms in fact machines?*

2.5.2. Morphogrammatic bisimularity

The trick to show the equality of dominos of different length doesn't work properly with physical dominos. A virtual representation of the dominos and some new rules would be required do it.

Hence, *abstractions* to define the length of morphograms happens over the group of possible *operators* and not, like for semiotics, on sets and equivalence relations over sets of signs. The operators are acting in the background as the hidden agents, the morphograms and the result of equivalence is a visible event.

A first striking result of such an application is the intriguing insight and construction of the possibility of the sameness, i.e. a bisimilarity, of morphograms of different kenomic length.

The prolongation of a domino chain is in fact not a simple concatenation or addition of the dominos. The tail of the chain and the head of the new domino have to be the same. This is defining a chain. But because of technical reasons, the overlapping of dominos, it is realized or played as a concatenation.

chain([aab] [abbbcc]) => [aabaaacc].

concatenation
$\text{length}(MG_1, MG_2) = \text{length}(MG_1) + \text{length}(MG_2)$

chain
$\text{length}(MG_1, MG_2) = \text{length}(MG_1) + \text{length}(MG_2) - 1$

fusion
$\text{length}(MG_1, MG_2) = \text{length}(MG_1) + \text{length}(MG_2) - n, \; 2 <= n <= \text{length}(MG_1)$

Therefore, monomorphy-based morphogrammatics starts with a bisimilarity theorem (Kaehr 1992) :

Theorem 1:*Two morphograms are the same iff they are decomposable into the same monomorphies.*

In other words: *Two morphograms are the same iff they behave the same.*

Example
A = [abba], B = [aba]
Morphograms A and B are morphogrammatically the same iff they are decomposable into the same monomorphies.

The operators of *composition* (Vk, Vs) and of *decompositions* (EVk, EVs) are well defined operators of morphogrammatics. Hence, the *abstraction* over the interactions of the operators of composition and decomposition is well defined too.

Operators
Vk : concatenation (Verknüpfung)
Vs : melting, fusion (Verschmelzung)
EVk: concatenative decomposition (Entknüpfung)
EVs: melting decomposition (Entschmelzung)

A = [abba], B = [aba]

EVk(A) = {[ab], [ab]}
EVs(B) = {[ab], [ab]}
Vs([ab], [ab]) = [aba]
Vk([ab], [ab]) = [abba].

Existence

$$\exists A, B : A \neq_{MG} B \wedge A \simeq_{BIS} B$$

There exist morphograms A, B, such that if the morphograms A and B are not morphogrammatically identical, there exists a morphic bisimilarity between the morphograms A and B.

Recall, for identities: A = C, B = C => A = B

The morphogram A is defined as a concatenation (addition) Vk of [ab] and [ab], i.e. [abba].
Trivially, the reverse of the addition EVk of A is producing {[ab], [ab]} = C.

A = Vk([ab],[ab]) = [abba] iff EVk(A) = {[ab], [ab]}

Similar for the morphogram B.
The morphogram B is defined as a fusion (overlapping) Vs of [ab] and [ab], i.e. [abba]
Trivially, the reverse of the overlapping EVs of B is producing { [ab], [ab]} = C.

Hence for both morphograms A and B:

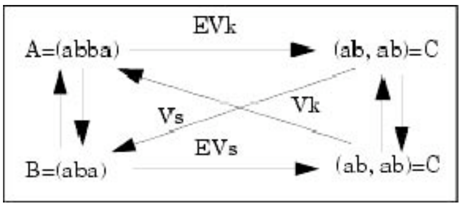
B = [aba] = Vs([ab], [ab]) iff EVs([aba]) = { [ab], [ab]}.

Therefore : There exists a morphic bisimilarity of A and B.

Again, that is : If [abba] $\neq_{MG}$ [aba], then there exists a bisimilarity for morphograms A and B,
[abba] $=_{BIS}$ [aba]. And if [abba] $=_{BIS}$ [aba], then there exists morphograms A and B with A $\neq_{MG}$ B.

Monomorphies in this example are considered with structure, e.g. [ab], while a decomposition into monomorphies by *Dec* would produce non-structured monomorphies:
Dec(A) = {[a], [bb], [a]} and Dec(B) = {[a], [b], [a]}.

All together is shown in the diagram:



XXX

Morphic bisimulation
"To a user, again, the state may remain hidden, it is irrelevant, as long as the automaton implements the desired regular expression. Again, two states may be identified, they behave the same way on the same input, which is to say, if they cannot be distinguished by any observation." (Peter Gumm)

Therefore, we have not to know the *internal* structure of the morphograms A and B. What we observe are the results of the operations EVk applied to A and EVs applied to B. Both results are equal. Hence, A and B are behavioral equivalent.

Additionally, I have to mention that bisimilarity which is a basic concept of co-algebra is based on the idea of a preference of final objects, while equivalence of algebra is based on initial objects. As shown elsewhere too, *morphogrammatic* "*bisimilarity*" is not based on initial nor final objects. It is in some way similar to autopoietic constructions (Maturana, Varela).

An application of "morphic bisimilarity" to quantum physics

Lee Smolin wrote in his interesting position paper, *The unique universe,* 2009:

"1. There is only one universe. There are no others, nor is there anything isomorphic to it.
This logically implies that there are no other universes, nor copies of our universe, whether within or without. The first is impossible as no subsystem can model precisely the larger system it is a part of, while the second is impossible because the one universe is by definition all there is. This principle also rules out the notion of a mathematical object isomorphic in every respect to the history of the entire universe, a notion that is more metaphysical than scientific."
http://physicsworld.com/cws/article/print/39306

I'm not doing God, neither cosmology. But there are always some texts, i.e. written statements and formulas, too, involved in such theories and postulations. Hence, I'm doing reading and writing.

"The first is impossible *as no* subsystem *can model precisely the larger system it is* a part of, *while the second is impossible because the one universe is by definition all there is."* (Smolin)

Probably, things are the other way round. Because Smolin believes into his *logical* impossibility theorem, that "*no subsystem can model precisely the larger system it is a part of"*, he concludes that the multi-verse assumption is impossible. Ironically, mathematical theories of whatever color are based on a *single universe* assumption (Grothendieck, Herrlich)). Hence, Smolin's approach is highly *circular*. Personally, I don't have a problem with circularity but for a single universe of mathematics, circularity is disastrous and is leading to antinomic situations even for a very big universe.

"This logically implies that there are no other universes, nor copies of our universe, whether within or without."
Therefore, it is the presumed *logic* which is denying the possibility of multi-verses and not *physics* or cosmology of experimental sciences.

Hence, what to do? First, physicists could learn that such questions of uni-/multi-/poly-versa are not genuinely parts of their branch "physics". Therefore, it is not their job to answer such questions. A *decision* for one of the possibilities still is a *legitimate decision* and should be marked as a decision or as a postulative *belief sentence*. With such a decision the whole debate, located in the framework of physics only, about uni/multiversa in physics becomes obsolete.

Nobody has to take into account the morphogrammatic construction just presented, there are other non-standard approaches in mathematics, which are offering interesting possibilities to overcome the logical impossibility theorem of parts and whole (A. Yessenin-Volpin, G. Gunther).

These arguments or hints are independent of the question of *time* in respect to a universe or to multi-/poly-versa.

2.5.3. Morphic bisimilarity and interchangeability

A poly-categorical thematization of monomorphy-based morphogrammatic equivalence shall be introduced.
How might this morphogrammatic interaction be caught by poly-categories and interchangeability?

Both type of interactions are structurally *discontextural* (disjunct) but mediated, i.e. $\mathcal{U}_1 \cap_{1.2} \mathcal{U}_2 = \emptyset$, with Vk, EVs$\in \mathcal{U}_1$ and Vs, Vk $\in \mathcal{U}_2$. Both, $\mathcal{U}_1$ and $\mathcal{U}_2$ are universes (contextures) of a polycontextural theory.
Both interactions, *concatenation* (Vk) and *melting* (Vs) as well as de-concatenation (EVk) and de-melting (EVs) are holding simultaneously, therefore they have to be mediated ($\amalg_{1.2}$) to stay in the game, i.e. $\mathcal{U}^{(2)} = \mathcal{U}_1 \amalg_{1.2} \mathcal{U}_2$.

Hence, a formalization as an *interchangeability* of composition and mediation in respect of the operations (Vk, Vs, EVk, EVs) applying to A and B seems natural.

EVk(A) ~= EVs (B) ==> Vs(EVk (A)) ~= Vk(EVs (B))

Example :

$$\left([MG], < op >\right)^{(2)} :$$

$$\mathcal{U}_1 \bigcap\nolimits_{1.2} \mathcal{U}_2 = \emptyset$$

$$\mathcal{U}^{(2)} = \mathcal{U}_1 \amalg_{1.2} \mathcal{U}_2 :$$

$$\mathcal{U}_i = \left\{[MG]_i, [op]_i\right\}, \; i = 1, 2$$

$$\begin{bmatrix} [EVs]_1 & [EVk]_2 \\ [Vk]_1 & [Vs]_2 \end{bmatrix} :$$

$$\begin{pmatrix} Vk_1 \\ \amalg_{1.2} \\ Vs_2 \end{pmatrix} \circ_{1.2} \begin{pmatrix} EVs_1 \\ \amalg_{1.2} \\ EVk_2 \end{pmatrix} = \begin{pmatrix} Vk_1 & \circ_1 & EVs_1 \\ & \amalg_{1.2} & \\ Vs_2 & \circ_2 & EVk_2 \end{pmatrix}$$

Because of the super-additivity of polycontextural mediations, the third system (contexture) has to be considered. It represents the contexture where the *result* of the interplay of the two contextures is explicitly stated, i.e. that, on the base of the interchangeability of the two contextures, the morphograms, A and B, are behaviorally equivalent. In other words, the interchangeability game of the two involved contextures gets a reflection, i.e. a own location where this interplay is thematized with the question of sameness of differentness of the involved kenomically different morphograms. In this sense, the third system (contexture) is reflecting and mediating the two mediated contextures.

3. Memristics

3.1. Memristics, the idea

It wouldn't be worth the efforts to study such tedious games, like morphogrammatics, if there wouldn't be a chance to apply it to some interesting real world requirements.
Luckily, there is an intriguing chance to speculate an application for memristive systems.
Hence, from dominos to morphogrammatics to memristics. That's the new scenario of exiting games.
As we learned with domino games and more explicitly with morphogrammatics, prolongations are fully determined by their history. Prolongations for morphograms are not products of an "abstraction of potential iterability" (Markov), which is abstracting from all limitations forced by matter, space and time.
Morphogrammatic prolongations are resource aware, bound by their own history of production.

A similar situation happens for memristive systems.

Hence, a modeling of "time- and history dependence" is naturally realized with morphogrammatic prolongations and their applications to coalitions and cooperations and more.

"Short for memory resistance, memristance is a property of an electronic component that lets it remember (or recall) the last resistance it had before being shut off."
http://www.webopedia.com/TERM/M/memristance.html

Check "*Memristics: Memristors, again?*" at: http://works.bepress.com/thinkartlab/37/

A compilation of the characteristics of the behavior of memristive systems is stated as follow:

"A finite state machine has a state but not a memory of a state.
A memristive machine has a state of a state, i.e. a *meta-state* as a memory, therefore a memristic machine is not a finite state machine.
A meta-state always can be taken as a simple state in the sense that a reduction from an as-abstraction to an is-abstraction is directly possible because the necessary informations are stored in the meta-state.
From "*x as y is z*" there is an easy way to *reduce* it to "*x is x*". Such a reduction of a second-order system to a first-order system is nevertheless losing the essential features of the reduced system." (Kaehr)

Towards Abstract Memristic Machines
http://memristors.memristics.com/Machines/Memristic%20Machines.pdf

A helpful "*orientation*" to the topic of memristics is avaiable at:
http://memristors.memristics.com/Machines/Orientation/orientation.html

3.2. Memristics, some formalizations

What is considered at this place are not questions about electronics, i.e. resistance, capacitance or inductance, but the structure of transitions within memristive systems. Therefore, the structure of the "time- and history-dependence" of memristive behaviors are in focus.
Following what we learnt about morphogrammatics, prolongations are retro-grade recursively connected with the morphogram to be prolongated.
As a possible interpretation of morphograms for memristive behaviors, I propose to interprete the morphogram [aa] by the memristic structure $[M|r_1r_1]$ or short $[M|r_{1.1}]$. The morphogram [ab] then corresponds to $[M|r_1r_2]$ or short $[M|r_{1.2}]$.
The same holds for [aba] ==> $[M|r_1r_2r_1]$ and [aab] ==> $[M|r_1r_1r_2]$.

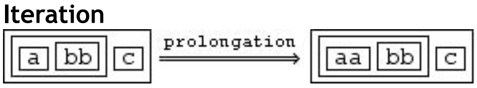
A more explicit notation for a memristic complexion $[M \mid r_1 r_2 r_2 r_1]$ shall be:

$$M^{(1,2,1)} = [M \mid r_1 r_2 r_2 r_1] : \begin{pmatrix} M^1 \mid r_1 \\ M^2 \mid r_{2.2} \\ M^1 \mid r_1 \end{pmatrix} = \begin{pmatrix} M^1 \mid r_1 \\ \amalg \\ M^2 \mid r_{2.2} \\ \amalg \\ M^1 \mid r_1 \end{pmatrix}$$

3.2.1. Prolongations

Morphogrammatic prolongation

Iteration

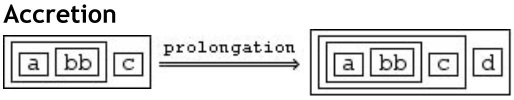


MG	loc_1	loc_2	loc_3
Dec	mg_1	mg_2	mg_3
Ken	*a*	*b*	*c*
	ø	*b*	ø

prolongation ⟹

MG	loc_1	loc_2	loc_3
Dec	mg_1	mg_2	mg_3
Ken	*a*	*b*	*c*
	a	*b*	ø

Accretion



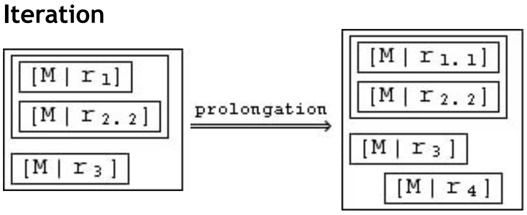
MG	loc_1	loc_2	loc_3
Dec	mg_1	mg_2	mg_3
Ken	*a*	*b*	*c*
	ø	*b*	ø

prolongation ⟹

MG	loc_1	loc_2	loc_3	loc_4
Dec	mg_1	mg_2	mg_3	mg_4
Ken	*a*	*b*	*c*	*d*
	ø	*b*	ø	ø

Memristic prolongation

Iteration



$$[M^{1.2.3}|r_{1.2.2.3}] \xrightarrow{\text{prolongation}} [M^{1.2.1}|r_{1.1.2.2.3.}]$$

$$\begin{pmatrix} M^1 & | & r_1 \\ M^2 & | & r_{2.2} \\ M^3 & | & r_3 \end{pmatrix} \xrightarrow{\text{prolongation}} \begin{pmatrix} M^1 & | & r_{1.1} \\ M^2 & | & r_{2.2} \\ M^3 & | & r_3 \end{pmatrix}$$

Accretion

MG	loc_1	loc_2	loc_3
Dec	M_1	M_2	M_3
Ken	r_1	r_2	r_3
	ø	r_2	ø

prolongation ⟹

MG	loc_1	loc_2	loc_3	loc_4
Dec	M_1	M_2	M_3	M_4
Ken	r_1	r_2	r_3	r_4
	ø	r_2	ø	ø

$$[M^{1.2.3} | r_{1.2.2.3}] \xrightarrow{\text{prolongation}} [M^{1.2.3.4} | r_{1.2.2.3.4}]$$

$$\begin{pmatrix} M^1 & | & r_1 \\ M^2 & | & r_{2.2} \\ M^3 & | & r_3 \end{pmatrix} \xrightarrow{\text{prolongation}} \begin{pmatrix} M^1 & | & r_1 \\ M^2 & | & r_{2.2} \\ M^3 & | & r_3 \\ M^4 & | & r4 \end{pmatrix}$$

Redundancy

A memristive system may have the complexity: [M $|r_{1223}$], then all possible memristive continuations of its behavior are given by the morphogrammatic continuation operations on [M $|r_{1223}$]. All other continuations would appear as artificial or redundant, e.g. [M $|r_{1223}$] => [M $|r_{122377}$], instead of [M $|r_{1223\mathbf{44}}$] $=_{mg}$ [M $|r_{1223\mathbf{77}}$]. Redundancy is an important feature in another consideration.

3.2.2. Cooperations

Multiplication tables for kmul([*a*, *b*], [*a*, *b*])

kmul	a	b
1	a	x
2	b	y

kmul	a	b	b'	b"	b'"
a	a	b	b	c	c
b	b	a	c	a	d

$$\text{kmul}\left(\begin{bmatrix} a \\ b \end{bmatrix}, \begin{bmatrix} a \\ b \end{bmatrix}\right) = \prod\left(\begin{bmatrix} a & b \\ b & a \end{bmatrix}, \begin{bmatrix} a & b \\ b & c \end{bmatrix}, \begin{bmatrix} a & c \\ b & a \end{bmatrix}, \begin{bmatrix} a & c \\ b & d \end{bmatrix}\right)$$

Memristic cooperation

$$\text{MEM}_1 = [M | r_1 r_2], \ \text{MEM}_2 = [M | r_1 r_2]$$

$$\text{kmul}(\text{MEM}_1, \text{MEM}_2) = \begin{pmatrix} \text{kmul}([M | r_{1.2}], [M | r_{2.1}]) = [M | r_1 r_2 r_2 r_1] \\ \amalg \\ \text{kmul}([M | r_{1.2}], [M | r_{2.3}]) = [M | r_1 r_2 r_2 r_3] \\ \amalg \\ \text{kmul}([M | r_{1.2}], [M | r_{3.1}]) = [M | r_1 r_2 r_3 r_1] \\ \amalg \\ \text{kmul}([M | r_{1.2}], [M | r_{3.4}]) = [M | r_1 r_2 r_3 r_4] \end{pmatrix}$$

The total memristance of the complexion kmul$(\text{MEM}_1\ \text{MEM}_2)$ is thus the field :

$$\text{MEM}^{(2,2)}_{(r_{1.2}),(r_{1.2})} \left| \prod ([r_1 r_2 r_2 r_1], [r_1 r_2 r_2 r_3], [r_1 r_2 r_3 r_1], [r_1 r_2 r_3 r_4]) \right.,$$

Table

$$\mathrm{MEM}^{(2,2)}_{(r_{1.2}),(r_{1.2})} \left| \; [r_1\, r_2]^1 \begin{bmatrix} \mathbf{r_2} & \mathbf{r_3} \\ [r_1] & [r_1] \\ [r_3] & [r_4] \end{bmatrix} \right.$$

$$\text{with} \begin{bmatrix} [r_2\, r_1] & [r_3\, r_1] \\ [r_2\, r_3] & [r_3\, r_4] \end{bmatrix} = \begin{bmatrix} \mathbf{r_2} & \mathbf{r_3} \\ [r_1] & [r_1] \\ [r_3] & [r_4] \end{bmatrix}$$

Morphogrammatic multiplication for

kmul([abb], [aba]) =

kmul	a	b	a	b"	b'"
a	a	b	a	b	c
b	b	a	b	c	d
b	b	a	b	c	d

Total memristance of $\mathbf{MEM}^{(3,3)}_{(r_{1.2.2}),(r_{1.2.1})}$ **:**

$$\mathrm{MEM}^{(3,3)}_{(r_{1.2.2}),(r_{1.2.1})} \left| \; [r_1\, r_2\, r_2]^{1.3} \begin{bmatrix} \mathbf{r_2} & \mathbf{r_3} \\ [r_1\, r_1] & [r_4\, r_4] \\ [r_3\, r_3] & \square \end{bmatrix} \right.$$

Field of memristivity for MEM :

$$\begin{array}{ll} [r_1\, r_2\, r_2][r_2\, r_1\, r_1][r_1\, r_2\, r_2] & [r_1\, r_2\, r_2][r_3\, r_4\, r_4][r_1\, r_2\, r_2] \\ [r_1\, r_2\, r_2][r_2\, r_3\, r_3][r_1\, r_2\, r_2] & \square \end{array}$$

Field of memristivity of $\mathbf{MEM}^{(2,3)}_{(r_{1.2.2}),(r_{1.2.3.1})}$ **:**

$$\mathrm{MEM}^{(3,4)}_{(r_{1.2.2}),(r_{1.2.3.1})} \left| \; [r_{1.2.2}]^{1.4} \begin{bmatrix} \mathbf{r_{2.1.1}} & \mathbf{r_{2.3.3}\, r} & \mathbf{r_{3.1.1}} & \mathbf{r_{3.4.4}} \\ r_{3.4.4} & r_{3.1.1} & r_{2.3.3} & r_{2.1.1} \\ \square & r_{3.4.4} & r_{2.4.4} & r_{2.3.3} \\ \square & r_{4.1.1} & r_{4.3.3} & r_{2.5.5} \\ \square & r_{4.5.5} & r_{4.5.5} & r_{4.1.1} \\ \square & \square & \square & r_{4.3.3} \\ \square & \square & \square & r_{4.5.5} \\ \square & \square & \square & r_{5.1.1} \\ \square & \square & \square & r_{5.3.3} \\ \square & \square & \square & r_{5.6.6} \end{bmatrix} \right.$$

Field of memristivity for $MEM^{(3,4)}_{(r_{1.2.2}),(r_{1.2.3.1})}$ □ □

(I, II)	(III)	(IV)
$[r_{1.2.2}][r_{2.1.1}][r_{3.4.4}][r_{1.2.2}]$		
$[r_{1.2.2}][r_{2.3.3}][r_{3.1.1}][r_{1.2.2}]$	$[r_{1.2.2}][r_{3.1.1}][r_{2.3.3}][r_{1.2.2}]$	$[r_{1.2.2}][r_{3.4.4}][r_{2.1.1}][r_{1.2.2}]$
$[r_{1.2.2}][r_{2.3.3}][r_{3.4.4}][r_{1.2.2}]$	$[r_{1.2.2}][r_{3.1.1}][r_{2.4.4}][r_{1.2.2}]$	$[r_{1.2.2}][r_{3.4.4}][r_{2.3.3}][r_{1.2.2}]$
$[r_{1.2.2}][r_{2.3.3}][r_{4.1.1}][r_{1.2.2}]$	$[r_{1.2.2}][r_{3.1.1}][r_{4.3.3}][r_{1.2.2}]$	$[r_{1.2.2}][r_{3.4.4}][r_{2.5.5}][r_{1.2.2}]$
$[r_{1.2.2}][r_{2.3.3}][r_{4.5.5}][r_{1.2.2}]$	$[r_{1.2.2}][r_{3.1.1}][r_{4.5.5}][r_{1.2.2}]$	$[r_{1.2.2}][r_{3.4.4}][r_{4.1.1}][r_{1.2.2}]$
□	□	$[r_{1.2.2}][r_{3.4.4}][r_{4.3.3}][r_{1.2.2}]$
□	□	$[r_{1.2.2}][r_{3.4.4}][r_{4.5.5}][r_{1.2.2}]$
□	□	$[r_{1.2.2}][r_{3.4.4}][r_{5.1.1}][r_{1.2.2}]$
□	□	$[r_{1.2.2}\, r_{3.4.4}][r_{5.3.3}][r_{1.2.2}]$
□	□	$[r_{1.2.2}][r_{3.4.4}][r_{5.6.6}][r_{1.2.2}]$

3.2.3. Memristic bisimilarity

Bisimilarity of A and B

$<$ Vs, Vk, EVk, EVs $>$:

$$\left([M|r_1 r_2 r_2 r_1],[M|r_1 r_2 r_1]\right) \longrightarrow \left([M|r_1 r_2 r_2 r_1],[M|r_1 r_2 r_1]\right):$$

$$\begin{pmatrix} [M|r_1 r_2 r_2 r_1] & \xrightarrow{EVk} & [M|r_1 r_2],[M|r_1 r_2] \\ \Updownarrow & Vs \times VK & \Updownarrow \\ [M|r_1 r_2 r_1] & \xrightarrow{EVs} & [M|r_1 r_2],[M|r_1 r_2] \end{pmatrix}$$

$$\Longrightarrow$$

$$[M|r_1 r_2 r_2 r_1 r_1] =_{BIS} [M|r_1 r_2 r_2].$$

$A = [M|r_1 r_2 r_2 r_1]$

$B = [M|r_1 r_2 r_1]$

$C = [M|r_1 r_2]$

3.3. Philosophical remarks on time

Why is time crucial?

It was shown that the behavior of memristors is time-dependent in the sense that each change of memristance is retro-grade mediated with the last state of the memristive system. Hence, temporality of memristive behavior is not conceived by the physical Newtonian time structure but in a "*historical*", i.e. self-referential and retro-grade time setting of the events. This structuration of events into retro-grade time dimensions is well modeled with the concept of morphogrammatic prolongation, which has in itself an organization of retro-grade tabular recursivity.

But time is much more complex. Time is not anymore conceived as a stream but as a field. Husserl has shown that time-consciousness demands for a two-dimensionality of a "*Längsintentionalität*" and "*Querintentionalität*" of the time-structure even for subjective awareness of time. Intersubjective time-structures are demanding for complex time-fields.

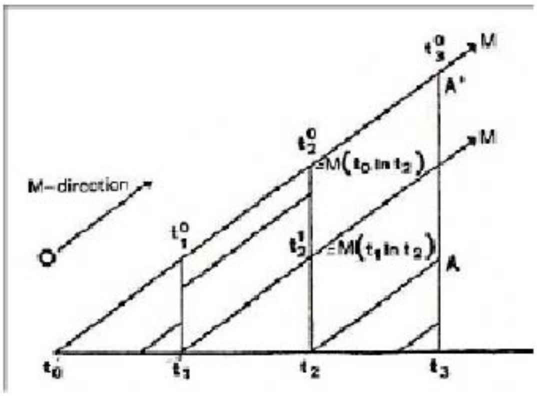
A very first approach to such a 2-dimensional organization of time for physical systems is proposed with the “multiplication”, i.e. *cooperation* of memristive behaviors. Such cooperation is designing a field of memristivity of a complexity depending on the actors of cooperation. But there is no need to restrict the temporality of the cooperativity of memristive systems to 2-dimensions only.

Husserl on time

"Bergson, James and Husserl realized that if our consciousness were structured in such a way that each moment occurred in strict separation from every other (like planks of a picket fence), then we never could apprehend or perceive the unity of our experiences or enduring objects in time otherwise than as a convoluted patchwork.
To avoid this quantitative view of time as a container, Husserl’s phenomenology attempts to articulate the conscious experience of lived-time as the prerequisite for the Newtonian, scientific notion of time’s reality as a march of discrete, atomistic moments measured by clocks and science.”

Husserl’s *Längsintentionalität* and *Querintentionalität*

"The living-present marks the essence of all manifestation, for in its automatic or passive self-givenness the living-present makes possible the apprehension of the elapsed phases of the life of consciousness and thereby the elapsed moments of the transcendent spatio-temporal object of which the conscious self is aware. This is possible, Husserl argues, because the “flow” of conscious life enjoys two modes of simultaneously operative intentionality.
"One mode of intentionality, which he terms Längsintentionalität, *or horizontal intentionality, runs along protention and retention in the flow of the living-present.*
The other mode of intentionality, which Husserl terms the Querintentionalität, *or transverse intentionality, runs from the living-present to the object of which consciousness is aware.”*
http://www.iep.utm.edu/phe-time/



"Rather than being a simple, undivided unity, self-manifestation is consequently characterized by an original complexity, by a historical heritage. The present can only appear to itself as present due to the retentional modification. Presence is differentiation; it is only in its intertwining with absence.” (Derrida 1990, 120)."

If there will be a new wave of Artificiality research of cognition & volition and robotics based on memristive technology, it will be confronted with new challenges of time, logic and computation. Contributions to morphogrammatics and polycontextural diamond category theory might offer some orientations.