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Palmer's Pentalectics

A note about modeling pentalectics in a 3-contextural monoidal category

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Abstract

Bob Coecke has recently given an introduction to the mysteries of quantum mechanics with the help of his magically simplified diagrammatic approach to monoidal categories. Kent Palmer just published an extraordinary complex philosophy of *Emergent Design* culminating in his approach of *pentalectics* as a co-design of system and metasystem. My notes try to bring both approaches together on the playground of an emerging polycontextural diamond category theory. This happens with some non-authorized re-designs of bifunctoriality as well as with a wild application of pentalectics. Both approaches cooked together might win the palm to bridge the gaps between the separated worlds of qualitative conceptual and operative mathematical achievements to design in general.

1. Kent Palmer’s meta-systems

1.1. The concept of monoidal categories

"What strikes me about Abramsky and Coecke’s papers is the suggestive and amazingly simple new formalism. You can really imagine specifying systems in it (well, at least after the nice clear examples they give), which is as far as I know a first for this area.” (Allen on March 9, 2007 4:03 AM)

http://golem.ph.utexas.edu/category/2007/03/computer_science_and_physics.html

How is a ‘monoidal category’ defined?

"Bifunctoriality has a very clear conceptual interpretation:
If we apply an operation f *to one system and an operation* g *to another system, then the order in which we apply them doesn’t matter. Hence bifunctoriality expresses some notion of locality but still allows for the quantum type of non-locality.”* (Coecke, p. 7)

http://www.comlab.ox.ac.uk/people/bob.coecke/Cats.pdf

What are the pretensions?

"The $(\circ,\otimes)$*-logic is a* logic of interaction*. It applies to cooking processes,*

physical processes, biological processes, logical processes (i.e. proofs), or computer processes (i.e. programs). The theory of monoidal categories*, the subject of this chapter, is the mathematical framework that accounts for the common structure of each of these* theories of processes*. The framework of monoidal categories moreover enables* modeling *and* axiomatising *(or 'classify') the extra structure which certain families of processes may have."* (Coecke, Paquette, Categories for the practising physicist, p. 3)
web.comlab.ox.ac.uk/people/Bob.Coecke/ctfwp1_final.pdf

1.2. The concept of bifunctoriality

The concept of bifunctoriality, especially in monoidal categories, has recently become crucial for the thematization, formalization and computation in different fields of *interactivity*: quantum mechaics (Abramsky, Coecke), computational interaction (Milner), logic (Girard), and category theory itself (Selinger). Bifunctoriality comes as a generalization of distributivity and was also called "Vertauschungsgesetz in Doppelkategorien" (Hasse, 1966). Bifunctoriality was crucial for the development of the theorem prover LOLA for polycontextural logics (1992).

The pretensions of categorists are brightly going on top of the hill by claiming its ultimate applicability:

"a category is the exact mathematical structure of practicing physics!"
(Coecke)

This note, part of an extensive study, is playing with some category theoretic concepts, well aware that physics isn' t the game of my choice.

Explicit Operationalism (Coecke)

Primitive data :

processes / **operations** : f, g, h, ...

which are **typed** as A B, B C, A A, ...

where A, B, C, ... are kinds / **names** of systems.

Primitive connectives

Sequential composition is a primitive connective on processes / operations cf.

$$f \circ g : A \longrightarrow C \text{ for } f : A \longrightarrow \underline{B} \ \& \ g : \underline{B} \longrightarrow C$$

Parallel composition is a primitive connective both on systems and processes / operations cf.

$$f \otimes g : A \otimes C \longrightarrow B \otimes D \text{ for } f : A \longrightarrow B \ \& \ g : C \longrightarrow D$$

A different notation is emphasizing the sequential and parallel characteristics of the key concepts of monoidal categories.

Objects

$A, B, C, D \in \mathbf{C}$: *objects* of category **C**

Sequentality

$(A, B) \longmapsto (A \longrightarrow B)$: *morphism*

$f \circ g : A \longrightarrow C$ for
$f : A \longrightarrow \underline{B}$ & $g : \underline{B} \longrightarrow C$: *composition*

Parallelity

$$\begin{pmatrix} A \\ \sharp \\ B \end{pmatrix} \longmapsto \begin{pmatrix} A \\ \otimes \\ B \end{pmatrix}$$: *tensor*

$$\begin{pmatrix} f \\ \otimes \\ g \end{pmatrix} : \begin{pmatrix} A & \xrightarrow{f} & B \\ \otimes & \otimes & \otimes \\ C & \xrightarrow{g} & D \end{pmatrix}$$: *juxtaposition*

for $$\begin{pmatrix} A \xrightarrow{f} B \\ \sharp \\ C \xrightarrow{g} D \end{pmatrix}$$ and $f : A \longrightarrow B$ & $g : C \longrightarrow D$

Bifunctoriality of composition and yuxtaposition

$$\frac{(f_1 \circ g_1) \otimes (f_2 \circ g_2)}{(f_1 \otimes f_2) \circ (g_1 \otimes g_2)}$$

$$\begin{bmatrix} g_1 & g_2 \\ f_1 & f_2 \end{bmatrix} : \begin{pmatrix} (f_1 \circ g_1) \\ \otimes \\ (f_2 \circ g_2) \end{pmatrix} = \begin{pmatrix} f_1 \\ \otimes \\ f_2 \end{pmatrix} \circ \begin{pmatrix} g_1 \\ \otimes \\ g_2 \end{pmatrix}$$

A formal definition of monoidal categories

Category
$(C1)\ g\circ f$ is defined iff $\mathrm{cod}(f)=\mathrm{dom}(g)$
$(C2)\ h\circ(g\circ f)=(h\circ g)\circ f$ when either is defined
$(C3)\ id\circ f=f$ and $f=f\circ id$

Identity : $id_I : I \longrightarrow I$

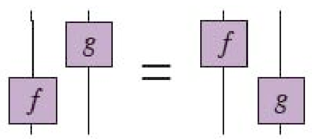
Monoidal category
$(M1)\ f\bigotimes(g\otimes h)=(f\bigoplus g)\otimes h$
$(M2)\ id_{\in}\bigotimes f=f\bigotimes id_{\in}$
$(M3)\ (f_1\bigotimes g_1)\circ(f_0\bigotimes g_0)=(f_1\circ f_0)\otimes(g_1\circ g_o)$.

Tensor product of $f : I_0 \longrightarrow I_1$ and $g : J_0 \longrightarrow J_1$ is defined iff $I_0\bigotimes J_0$ and $I_1\bigotimes J_1$ are both defined.

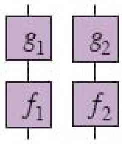
Explicitly,
$I_0\bigotimes J_0$ is defined iff $\mathrm{cod}(I_0)=\mathrm{dom}(J_0)$ and
$I_1\bigotimes J_1$ is defined iff $\mathrm{cod}(I_1)=\mathrm{dom}(J_1)$.
(Milner, Tutorial, p .14 / 15)]

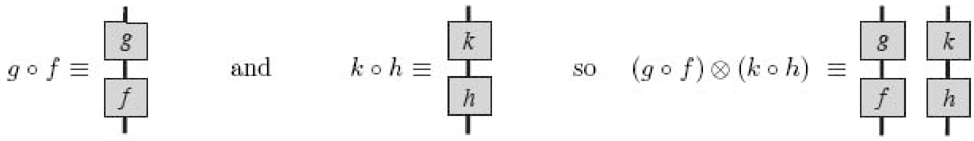
1.4. Graphic notation for bifunctoriality

Bifunctoriality. In the graphic language *bifunctoriality* stand for:

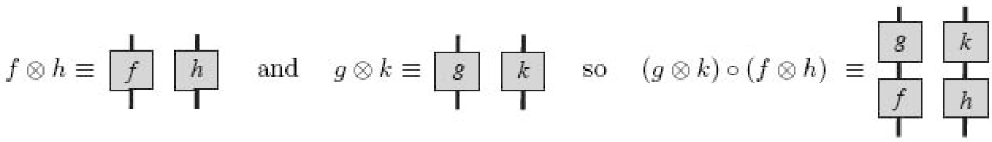


More explicitly:



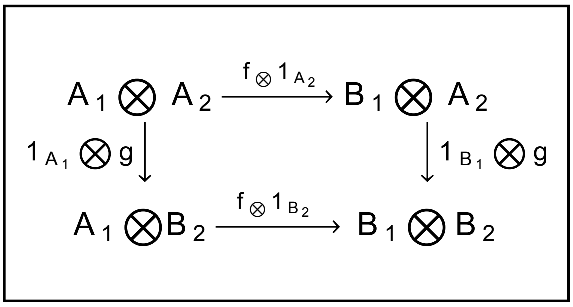


On the other hand we have:



"We read this as: since f_1 is causally before g_1 and f_2 is causally before g_2, the pair (f_1, f_2) is causally before (g_1, g_2) and vice versa [...]."

"The above pictorial equation can also be written down in term of a *commutative* diagram:



which expresses that both paths yield the same result." (Coecke)

Abstract mono – contextural bifunctoriality matrix :

$$\begin{bmatrix} \text{operating}_1 & \text{operating}_2 \\ \text{designing}_1 & \text{designing}_2 \end{bmatrix} \cong \begin{bmatrix} g_1 & g_2 \\ f_1 & f_2 \end{bmatrix} \cong \begin{bmatrix} g_1 & \otimes & g_2 & par \\ \circ & \square & \circ & \square \\ f_1 & \otimes & f_2 & par \\ seq & \square & seq & \square \end{bmatrix}$$

This (non-contextualized) matrix-scheme is easily to extend to more complex-/complicated situations.

(3, 2) – matrix in Cat^1

$$\begin{bmatrix} h_1 & h_2 \\ g_1 & g_2 \\ f_1 & f_2 \end{bmatrix} : \begin{pmatrix} (f_1 \circ g_1 \circ h_1) \\ \otimes \\ (f_2 \circ g_2 \circ h_2) \end{pmatrix} \longrightarrow \begin{pmatrix} f_1 \\ \otimes \\ f_2 \end{pmatrix} \circ \begin{pmatrix} g_1 \\ \otimes \\ g_2 \end{pmatrix} \circ \begin{pmatrix} h_1 \\ \otimes \\ h_2 \end{pmatrix}$$

(2, 3) – matrix in Cat^1

$$\begin{bmatrix} g_1 & g_2 & g_3 \\ f_1 & f_2 & f_3 \end{bmatrix} : \begin{pmatrix} (f_1 \circ g_1) \\ \otimes \\ (f_2 \circ g_2) \\ \otimes \\ (f_3 \circ g_3) \end{pmatrix} \longrightarrow \begin{pmatrix} f_1 \\ \otimes \\ f_2 \\ \otimes \\ f_3 \end{pmatrix} \circ \begin{pmatrix} g_1 \\ \otimes \\ g_2 \\ \otimes \\ g_3 \end{pmatrix}$$

Obviously, any non "linear" interpretations of the matrix, like *orthogonal* and *lateral*, are excluded from the game.

1.5. A simple application to physical and mental processes

1.5.1. Monoidal categories and their polycontextural distribution

The classic mono-contextural wording for physical processes is given by Coecke's cooking example:
"That is, 'boiling the potato and then *salting it,* while*, frying the carrot* and then *peppering it', is* equal *to 'boiling the potato* while *frying the carrot,* and then*, salting the potato* while *peppering the carrot'."* (Coecke)

A. Cooking while cooking

"That is,

*'boiling the potato (*and then *)*^{1} *salting the potato,*

*(*while)^1*,*

*frying the carrot (*and then)^1 *peppering the carrot',*

*is (*equal)^1 *to*

*'boiling the potato (*while)^1 *frying the carrot,*

*(*and then)^1*,*

*salting the potato (*while)^1 *peppering the carrot'."*

Category Cat1:

Objects: *potato*, *carrot*

Processes: *boiling*, *salting*, *peppering*, *frying*
Operations*: (and then), (while)*

$$\left(\begin{array}{c}\left(\text{boiling} \circ \text{salting}\right) \\ \otimes \\ \left(\text{frying} \circ \text{pepering}\right)\end{array}\right) = \left(\begin{array}{c}\text{boiling} \\ \otimes \\ \text{frying}\end{array}\right) \circ \left(\begin{array}{c}\text{salting} \\ \otimes \\ \text{peppering}\end{array}\right)$$

B. Cooking while thinking
Because terms and actions in the polycontextural model of the following example are distributed over different loci, the meanings have to belong to different contextures. One contexture might contain the *physical data* of the cooking example. Another contexture might contain the *mental data* accompanying the cooking processes.
Hence, “boiling” and “salting”, “carrot” and “potato”, belong to the physical contexture represented by the category Cat1.
While the accompanying mental processes of “evaluating” and “thinking” belong to the category Cat2.
The objects “carrot” and “potato” are appearing as physical objects in Cat1 and as representations, i.e. signs, in Cat2. Hence, the elements or objects of Cat1 and Cat2 are strictly separated: {Cat1} $\cap$ {Cat2) = $\emptyset$. Therefore, they are not elements of a common combined (product) category.

Both categories, Cat1 and Cat2, are mediated (reflected) and are part of a 3-contextural category $\text{Cat}^{(3)}$.
The interactivity of both thematizations, the physical and the mental, are represented by the bifunctoriality between the distributed operations of the different categories.

This is not the way category theory is defined and used. Hence, my ab/use consists in the mechanism of distributing categories over different loci. Each locus contains its own rationality: logic, semiotics, category theory, ontology, etc.

"That is, in the 2-contextural modelling*, the distribution is:*
*‘boiling the potato (*and then *)*$^{1.0}$ *salting the potato,*
*(*while)$^{1.2}$*,*
*thinking the carrot (*and then)$^{0.2}$ evaluating *the carrot’,*

*is (*equal)$^{1.2}$ *to*
*'boiling the potato (*while)$^{1.2}$ *thinking the carrot,*
*(*and then)$^{1.2}$*,*
*salting the potato (*while)$^{1.2}$ evaluating *the carrot'."*

Category Cat1:
Objects: *potato*, *carrot*
Processes: *boiling*, *salting*
Operations*: (and then), (while)*

Category Cat2:
Term-Objects: "*potato", "carrot"*
Processes: *thinking*, *evaluating*
Operations*: ("and then"), ("while")*

$$\begin{array}{l}\left(\text{boiling } \textit{and then} \text{ salting}\right)\\ \textit{while}\\ \left(\text{thinking } \textit{and then} \text{ evaluating}\right)\\ \textit{equal}\\ \left(\text{boiling } \textit{while} \text{ thinking}\right)\\ \textit{and then}\\ \left(\text{salting } \textit{while} \text{ evaluating}\right)\end{array}$$

$$\begin{pmatrix} \left(\text{boiling} \circ^{1.0} \text{salting}\right) \\ \otimes_{1.2} \\ \left(\text{thinking} \circ^{0.2} \text{evaluating}\right) \end{pmatrix} = \begin{pmatrix} \text{boiling} \\ \otimes_{1.2} \\ \text{thinking} \end{pmatrix} \circ_{1.2} \begin{pmatrix} \text{salting} \\ \otimes_{1.2} \\ \text{evaluating} \end{pmatrix}$$

Matrix notation

$$Cat^{1.0} \times Cat^{0.2} = \left[\begin{bmatrix} g_1 & - \\ f_1 & - \end{bmatrix}^1, \begin{bmatrix} - & g_2 \\ - & f_2 \end{bmatrix}^2 \right]:$$

$$\begin{pmatrix} \left(f_1 \circ^{1.0} g_1\right) \\ \otimes_{1.2} \\ \left(f_2 \circ^{0.2} g_2\right) \end{pmatrix} = \begin{pmatrix} f_1 \\ \otimes_{1.2} \\ f_2 \end{pmatrix} \circ_{1.2} \begin{pmatrix} g_1 \\ \otimes_{1.2} \\ g_2 \end{pmatrix}$$

1.5.2. Reflectional monoidal categories

What is missing in the above example of modeling mental and physical processes is an instance which is able to reflect on both categories. Hence, the 2-contextural categorical frame has to be augmented to 3-contextural categorical frame containing a new operator ”◆", able to reflect on the mental and physical processes of Cat1 and Cat2. This is again a kind of a “parallel” (simultaneity) operator not on processes but on the contextures containing cognitive and informatic processes.

The weakest modeling might be realized by the mediating contexture between the first and the second contexture in a 3-contextural category. Nothing is lost: At each locus, i.e. for each distributed contexture, all the known category theoretical features are saved.

What is not yet in the game are the intrinsic rules of mediation, i.e. the matching conditions of mediation in contrast to the matching conditions for composition and juxtaposition, between the distributed contextures concerning concretely the interplay of the categorical notions.

Category Cat3 = (Cat1, Cat2):

3 – categorical modeling

(boiling *and then* salting)

$\left(f_1 \circ^{1.0\ \ .0} g_1\right)$

while $\otimes$

1.2 .0

(thinking *and then* evaluating)

$\left(f_2 \circ_{0.2\ \ .0} g_2\right)$

and reflecting on

◆ 1.2 .3

[(boiling *then* salting) *while* (thinking *then* evaluating)]

$\left(f_3 \otimes^{0.0\ \ .3} g_3\right)$

equal $=_1 =_2 =_3$

(boiling *while* thinking) *and reflecting on* (boiling *while* thinking))

$$\left(\left(\begin{array}{c} f_1 \\ \otimes_{1.2\ \ .3} \\ f_2 \end{array} \right) \begin{array}{c} \\ \blacklozenge\ 1.2\ \ .3 \\ f_3 \end{array} \right)$$

and then $_1$ *and* $_2$ *while* $_3$ $\circ_1\ \circ_2\ \otimes_3$

(salting *while* evaluating) *and reflecting*

on (salting *while* evaluating)

$$\left(\left(\begin{array}{c} g_1 \\ \otimes_{1.2\ \ .3} \\ g_2 \end{array} \right) \begin{array}{c} \\ \blacklozenge\ 1.2\ \ .3 \\ g_3 \end{array} \right)$$

Reflectional bifunctoriality in a 3 – contexture

$$\left[\left[\left[\begin{matrix} g_1 & -- \\ f_1 & -- \end{matrix}\right]^1, \left[\begin{matrix} -g_2 & - \\ -f_2 & - \end{matrix}\right]^2, \left[\begin{matrix} --g_3 \\ --f_3 \end{matrix}\right]^3\right]\right]:$$

$$\left(\begin{matrix} \left(\begin{matrix} \left(f_1 \circ^{1.0\ \ .0} g_1\right) \\ \otimes_{1.2\ \ .0} \\ \left(f_2 \circ^{0.2\ \ .0} g_2\right) \end{matrix}\right) \\ \blacklozenge_{1.2\ \ .3} \\ \left(f_3 \bigotimes_{0.0\ \ .3} g_3\right) \end{matrix}\right) = \left(\begin{matrix} \left(\begin{matrix} f_1 \\ \otimes_{1.2\ \ .0} \\ f_2 \end{matrix}\right) \\ \blacklozenge_{1.2\ \ .3} \\ f_3 \end{matrix}\right) \circ_1 \circ_2 \bigotimes_3 \left(\begin{matrix} \left(\begin{matrix} g_1 \\ \otimes_{1.2\ \ .0} \\ g_2 \end{matrix}\right) \\ \blacklozenge_{1.2\ \ .3} \\ g_3 \end{matrix}\right)$$

$$\left(f_3 \blacklozenge^{0.0\ \ .3} g_3\right) = \left(\begin{matrix} \left(f_1 \circ^{1.0\ \ .0} g_1\right) \\ \otimes_{1.2\ \ .0} \\ \left(f_2 \circ^{0.2\ \ .0} g_2\right) \end{matrix}\right)$$

$$f_3 = \left(\begin{matrix} f_1 \\ \otimes_{1.2\ \ .0} \\ f_2 \end{matrix}\right), \quad g_3 = \left(\begin{matrix} g_1 \\ \otimes_{1.2\ \ .0} \\ g_2 \end{matrix}\right)$$

1.6. Palmer's Pentalectics

Following Coecke's strategy of *simplifying* complex situations with the help of monoidal categories, a very first step towards a formalization of Palmer’s approach to the system/metasystem design process might be given by similarly simplified monoidal constructions.

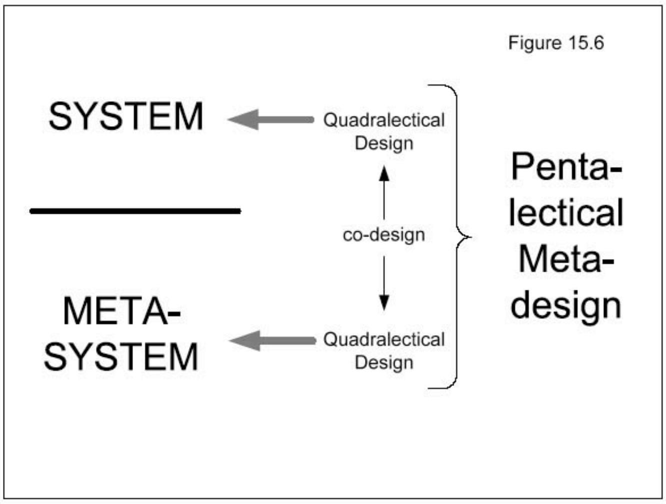
The advantage of using the apparatus of *monoidal categories* instead of the more profound constructions of *diamond (category) theory* is obvious: monoidal categories are well established and studied, diamond theory is still at its beginning and its significance is not yet generally recognized.

Therefore, additionally to Palmer's *“eidetic structuralism”*, monoidal categories might shed some light into this abstract realm of *“pentalectical meta-design”* (p. 508) by enabling some *operationality* between system and metasystem constructions.

> Kent Duane Palmer, EMERGENT DESIGN
> Explorations in Systems Phenomenology in Relation to Ontology, Hermeneu-

tics and the Meta-dialectics of Design, 2009
http://arrow.unisa.edu.au:8080/vital/access/manager/Repository/unisa:42392

Certainly, it should be more than an entertaining *paradox* to model highly complex *“eidetic structures”* of the post-Platonian realm with the help of a formalism adequate for a modeling of *“cooking potatoes and carrots”* (or quantum physics) in a sequential and parallel order.



"*If we could design the System and the Metasystem at the same time and in the same way by merely adding another moment to the Quadralectic, rather than by using two separate Quadralectics — see how efficient that would be?* The co-design of two Quadralectics, one aimed at the System and the other aimed at the Meta-system, is actually an expansion of the design process by the Pentalectic. As a result we can achieve efficiency by adding one more moment to the Quadralectic to achieve meta-design and this gives us an emergent benefit that is equal to two Quadralectics. *The Pentalectic is the core of the collision and collusion between the Quadralectic and anti-Quadralectic.*” (Palmer, p. 507)

General bifunctoriality of "system" and "meta-system" in a 2-contextural category as a distribution of *MASS* and *SET* terms might by sketched by the following formula.

It is crucial for an operative modeling to introduce some *terms* for SET and MASS as a kind of a multitude for the model of pentalectics. Otherwise, the differences between SET and MASS in their interplay (co-design) are collapsing and the scheme is loosing the possibility of a formal and operative modeling. In other words, the notational approach alone is emphasizing the complexity of the figure but not yet its complication. Only both together, *complexity* and *complication*, are enabling an operational treatment of the scheme in the proposed sense of categorical bifunctoriality.

Pentalectics between composition and juxtaposition

$$\left[\begin{bmatrix} MASS & - \\ SET & - \end{bmatrix}^1, \begin{bmatrix} - & MASS \\ - & SET \end{bmatrix}^2\right]:$$

$$\begin{pmatrix} \left(SET_1 \;\circ^{1.0}\; MASS_1\right) \\ \otimes_{1.2} \\ \left(SET_2 \;\circ^{0.2}\; MASS_2\right) \end{pmatrix} =^{1.2} \begin{pmatrix} SET_1 \\ \otimes_{1.2} \\ SET_2 \end{pmatrix} \circ_{1.2} \begin{pmatrix} MASS_1 \\ \otimes_{1.2} \\ MASS_2 \end{pmatrix}$$

objects : SET_1, SET_2 and $MASS_1$, $MASS_2$

operations :

$\bigotimes_{1.2}$: co – design,

$\circ_{1.2}$: design : $\begin{pmatrix} \circ^{1.0} : \text{system – design} \\ \circ^{0.2} : \text{meta – system – design} \end{pmatrix}$

"The Mass Foundational Mathematical Category is the dual of the Set. A Set is a projection of an ordering of difference but a Set (as a projection) does not need any elements in it. A Mass, on the other hand, does not exist if it has no instances. Thus, Sets are ideal and Masses are existential. When we engage in design, we design in Sets, but when we operate and execute Systems we operate in Masses. Both Masses and Sets have full logics, while all other Foundational Mathematical Categories have degenerate or surplus logics." (Palmer, p. 510)

"Both Masses and Sets have full logics" : the bifunctorial distribution scheme is emphasizing the difference and autonomy of both contextures, here interpreted as SET and MASS. At this level of modeling it has not yet to be specified what kind of logics might be involved. The modeling is not forcing any restrictions on both. Hence, SET might be modeled in the logic of categories and in contrast MASS in the logic of saltatories.
For simplicity it is supposed in this note that the operation of *composition* "∘" holds for SET and for MASS.

The advantage of bifunctoriality (as a generalization of distributivity) is its simple mechanism for the interaction (co-design) between SET and MASS.

Bifunctoriality in category theory simply rules the *interplay* between composition and yuxtaposition, i.e. between sequential and parallel processes.

SET: “*When we engage in design, we design in Sets”.*
MASS: “*when we operate and execute Systems we operate in Masses"*

After that, bifunctoriality is exposing the simplest rules of the interactivity between *design* and *execution (operation)*. It seems to be reasonable to conceive the SET-structure of design as a *hierarchical* sequentiality, hence in correspondence with *time*, while the executional and operative MASS-structure might be conceived as a *heterarchical* topology (parallelity), hence in correspondence with *space*. Both together as pentalectics might define the mechanism of Emergent Design.

In fact, what is ‘finally’ defining Emergent Design is not only the *interplay* between SET and MASS but the game of realizing the equality, .i.e. the *harmony* between both parts of the bifunctorial equation (proportion) of SET and MASS. This equality (equivalence) is part of the definition of classical bifunctoriality and reflects complication given by the number of terms. There is no space (tolerance) left for any interaction in the fulfilment of the categorical ”matching conditions”. In the examples of this note, the equality is presupposed too and is not yet constructed.

Bifunctoriality of *designing* and *operating* processes

$$\begin{pmatrix} \left(\text{designing}_1 \;\circ^{1.0}\; \text{operating}_1\right) \\ \otimes_{1.2} \\ \left(\text{designing}_2 \;\circ^{0.2}\; \text{operating}_2\right) \end{pmatrix} = \begin{pmatrix} \text{designing}_1 \\ \otimes_{1.2} \\ \text{designing}_2 \end{pmatrix} \circ_{1.2} \begin{pmatrix} \text{operating}_1 \\ \otimes_{1.2} \\ \text{operating}_2 \end{pmatrix}$$

Bifunctoriality vs. chiasms and diamonds
It is easy to see, that in non-formal conceptual terms, bifunctoriality is a *proportion*, i.e. A : B = C : D, and therefore has a hidden *chiastic* structure. The bifunctorial formula might be re-written as a proportional formula:

$(A \circ B) \otimes (C \circ D) = (A \otimes C) \circ (B \otimes D) \Longrightarrow (A : B) :: (C : D) = (A :: C) : (B :: D)$.

Hence, the whole modeling could be done within diamond theory and its chiasms. But this wouldn't be helpful for connecting to existing approaches. That is, monoidal categories are well known and are used here as an *intermediary* approach to further possible diamond theoretic modeling and formalizations.

Pentalectical Meta-design

A further step in the modeling process is achieved by the modeling of a *reflection* on the interplay of system and meta-system, i.e. the thematization of the *co-design* of the two Quadralectical Designs. This reflection, called by Palmer *"pentalectical meta-design"*, shall be located at the place of a third contexture of a 3-contextural category, Cat3, which is "reflecting" the categories at place1 (SET) and place2 (MASS).

That is, the interplay between *design* and *execution* gets a structural definition as the *bifunctoriality* of SET and MASS, i.e. as the *co-design* of the two *quadralectics* involved in pentalectics.

Full Pentalectics between composition and juxtaposition

$$\begin{pmatrix} \begin{pmatrix} \left(f_1 \circ^{1.0\ \ .0} g_1\right) \\ \otimes_{1.2\ \ .0} \\ \left(f_2 \circ^{0.2\ \ .0} g_2\right) \end{pmatrix} \\ \blacklozenge_{1.2\ \ .3} \\ \left(f_3 \bigotimes_{0.0\ \ .3} g_3\right) \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} f_1 \\ \otimes_{1.2\ \ .0} \\ f_2 \end{pmatrix} \\ \blacklozenge_{1.2\ \ .3} \\ f_3 \end{pmatrix} \circ_1 \circ_2 \bigotimes_3 \begin{pmatrix} \begin{pmatrix} g_1 \\ \otimes_{1.2\ \ .0} \\ g_2 \end{pmatrix} \\ \blacklozenge_{1.2\ \ .3} \\ g_3 \end{pmatrix}$$

$$\left(f_3 \blacklozenge^{0.0\ \ .3} g_3\right) = \begin{pmatrix} \left(f_1 \circ^{1.0\ \ .0} g_1\right) \\ \otimes_{1.2\ \ .0} \\ \left(f_2 \circ^{0.2\ \ .0} g_2\right) \end{pmatrix}$$

$$f_3 = \begin{pmatrix} f_1 \\ \otimes_{1.2\ \ .0} \\ f_2 \end{pmatrix}, \quad g_3 = \begin{pmatrix} g_1 \\ \otimes_{1.2\ \ .0} \\ g_2 \end{pmatrix}$$

$f_{1,2,3}$, $g_{1,2,3}$: terms of (SET, MASS, PENTA)

$\bigotimes_{1.2}$: co – design of SET and MASS,

$\circ_{1.2}$: design : ($\circ^{1.0}$: *quadralectical* system – design ; $\circ^{0.2}$: *quadralectical* metasystem – design)

$\blacklozenge_{1.2\ \ .3}$: *pentalectical* meta – design of (SET, MASS)

Given this modeling of pentalectics, the whole apparatus of monoidal categories and polycontexturality might be applied for further studies.

This proposed modeling is still quite restricted. It is not yet considering the quadruples of terms necessary for quadralectic (chiastic) situations per level. Only two terms per level are involved in this modeling: f and g as: f_i, g_i, i=1,2,3. Neither are the logical and structural differences between MASS-quadralectics and SET-quadralectics characterized. (Except the hint to combinations as compositions and as saltisitions.)

Furthermore we might ask about the “numericity” of the pentalectic construction. Albeit Parker’s design of different realms of Being/Beyng, it seems that pentalectics comes as a *unique* general scheme. Therefore an important restriction appears by this fact that the bifunctorial formula for the interplay of design and execution is not involving any dissemination of “pentalectics” over different “societal” (or “cosmological”) instances and thus there is no structural space given (designed or accessible) for the interactionality, reflectionality and interventionality between different disseminated pentalectical metadesigns.

The same observation holds for Coecke’s quantum cooking. There is one and only one ultimate Maître de Cuisine employed for the design and execution of his unique Quantum World Kitchen. This is without doubt perfect for a Kindergarten performance (arXiv:quant-ph/0510032v1) to impress secretaries of the IUMS (International Unus Mundus Society) sponsoring committee, - nevertheless, it works for myself too :-) -, but it is probably not cooked enough to (let us) survive in the future.