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Rudolf Kaehr
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Some Refinements of Morphogrammatics

How to decompose complex polycontextural systems?

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Some very first attempts to apply Mathematica and Combinatorica to the topics in question are undertaken.

A concept of a double refinement of partitions is introduced additionally to the well known partitions and refinements of partitions.

The distinction of recursive and explicit definitions of morphograms are considered.

Decomposition strategies by partitions and refinements of partitions are crucial to deal with highly complex morphic and polycontextural systems

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Some Refinements of Morphogrammatics

How to decompose complex polycontextural systems?

Rudolf Kaehr, Dr. phil

ThinkArt Lab Glasgow

Abstract
Combinatorial methods to decompose morphogrammatic systems are introduced.
Some very first attempts to apply *Mathematica* and *Combinatorica* to the topics in question are undertaken. A concept of a double refinement of partitions is introduced additionally to the well known partitions and refinements of partitions. The distinction of recursive and explicit definitions of morphograms are considered.
Decomposition strategies by partitions and refinements of partitions are crucial to deal with highly complex morphic and polycontextural systems.

Keywords

Partitions, Stirling numbers, Bell numbers, refinements, morphograms

(work in progress, vers. 0.3, Jan 2014)

Download: http://www.wolfram.com/cdf-player/
http://memristors.memristics.com/Refinements/Refinements in Morphogrammatics.cdf

Motivations for a double refinement of morphogrammatic constellations

There are many disciplines where the concept of *polycontexturality* plays a crucial role. It is often proposed that systems that are involving different observational positions to be adequately observed, described, managed or produced, are necessarily involved in polycontextural considerations.

It is less known that the study of polycontexturality is not just given by a study of *polycontextural logic* as it was proposed by Gotthard Gunther (1900 - 1984) but also by *morphogrammatics* as Gunther himself introduced in the late 1960s.

This paper takes another turn. The question is: *How to decompose complex polycontextural systems (structurations)?*

Decomposition of systems is a kind of partition as it is well known in combinatorics. Henc, the question is concretized by the question: *How to partition complex polycontextural systems (structurations)?*

Partitions of natural numbers are well studied and taught at an early stage of education. The number 5 might be partitioned into parts like 5=4+1, 3+2, 2+2+1, 2+1+1+1 and 1+1+1+1+1.

Morphogrammatic interpretations of number systems as introduced by Gotthard Gunther in his paper *"Natural numbers in trans-Classical systems"* demands for new techniques of decomposition of complexions to deal with the partition of systems of morphograms.

More at: http://memristors.memristics.com/Fibonacci/Fibonacci%20Sequences%20in%20trans-Classical%20Systems.html

The concept and formalism of morphograms is based on the Stirling numbers of the second kind, StirlingS2.

Hence, StirlingS2 of the number 4 gives the partition: (1, 7, 6, 1).

A first refinement of this results is given by the *multinomial* partitions of the Stirling partitions.

Therefore, the refinement of Stirling (1, 7, 6, 1) is given by: (1, 4, 3, 6, 1).

All that is well known and elaborated in the literature of combinatorics. Albeit not necessarily in study groups of polycontexturality.

A less known possibility of refinement is proposed as a *second* refinement of the multinomial partitions that seems to be naturally possible. Because it is a refinement of the refinement it might be called a second-order refinement. Up to now, I haven’t found the concept and a formula for this refinement in the literature.

Such a second refinement of (1, 7, 6, 1) delivers: (1, 3+1, 3, 3+2+1, 1).

Thus, the complete chain of the numbers of partitions of the natural number 4 is given by:

[4]: (1, 7, 6, 1) $\Longrightarrow$ (1, 4, 3, 6, 1) $\Longrightarrow$ (1, 3+1, 3, 3+2+1, 1)

It seems that the second kind of refinement is depending or enabled by a morphogrammatic understanding of the results of the first refinement that is not anymore ruled by a strictly extensional or formalistic understanding of the distributions, numerical or differential, i.e. EN-structural.

Therefore two refinements, like [1,1,1,2] and [1,2,2,2] or [2,1,1,1] of the partition of the number 4: (3,1), which are formally representing the same refinement, are considered morphogrammatically as different in respect of a *second* refinement.

Polycontextural systems are based on morphogrammatic constellations. Fusion and decomposition of complex polycontextural systems are well based on the fusion/de-fusion principles of morphogrammatic systems.

A classical example in the literature of morphogrammatics for such de-fusion is the decomposition of morphograms into their *monomorphies*.

The strategy follows several steps of combinatorial decompositions.

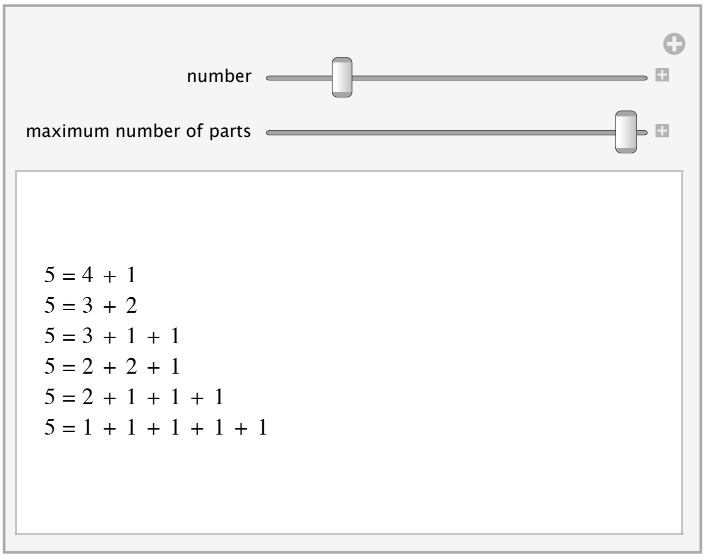
The steps are obvious:
Partitions, IntegerPartitions, StirlingS2, First Refinements T[m,n], Second refinements D(T[m,n])

This approach offers a 3 step classification and reduction of morphogrammatic complexions that are the basic patterns of polycontextural constellations.

A polycontextural thematization of a complexion of objects of any sorts applies simultaneously on all 3 basic levels of the classification.

Partitions P[n]

```
Manipulate[Pane[Text[Column[Row[{n, " = ", Row[#," + "]}]&/@Rest[IntegerPartitions[n,k]],Left]]
{{n,5,"number"},2,20,1},
{{k,20,"maximum number of parts"},2,20,1},
AutorunSequencing→{1}]
```



"Partitions of Integers" from the Wolfram Demonstrations Project
http://demonstrations.wolfram.com/PartitionsOfIntegers/
Contributed by: Stephen Wolfram

Stirling Partitions

The following considerations about a possible second-order refinement of partitions will take just three cases into account, i.e. for m=4,5,6.

It is easily possible that this concept of a second refinement has been treated *in extenso* in the literature for combinatorics. Unfortunately, I couldn't find any elaborations to this topic. Therefore, I present here my first, still descriptive and not yet fully operative, results.

Table[StirlingS2[4, m], {m, 4}] »

Input

```
Table[StirlingS2[4, m], {m, 4}]
```

Stirling numbers of the second kind

TableForm[Table[StirlingS2[n,k], {n, 0,7}, {k,0,n}]]

StirlingS2[n,k] ?

Values

n m	0	1	2	3	4	5	6	7
0	1	0	0	0	0	0	0	0
1	0	1	0	0	0	0	0	0
2	0	1	1	0	0	0	0	0
3	0	1	3	1	0	0	0	0
4	0	1	7	6	1	0	0	0
5	0	1	15	25	10	1	0	0
6	0	1	31	90	65	15	1	0
7	0	1	63	301	350	140	21	1

```
Column[Table[Binomial[i, j], {i, 0, 10}, {j, 0, i}], Center]
```

```
                    {1}
                  {1, 1}
                {1, 2, 1}
              {1, 3, 3, 1}
            {1, 4, 6, 4, 1}
         {1, 5, 10, 10, 5, 1}
       {1, 6, 15, 20, 15, 6, 1}
     {1, 7, 21, 35, 35, 21, 7, 1}
   {1, 8, 28, 56, 70, 56, 28, 8, 1}
{1, 9, 36, 84, 126, 126, 84, 36, 9, 1}
{1, 10, 45, 120, 210, 252, 210, 120, 45, 10, 1}
```

TableForm[Bell[n], {n, 0,7}, {,0,n}]]

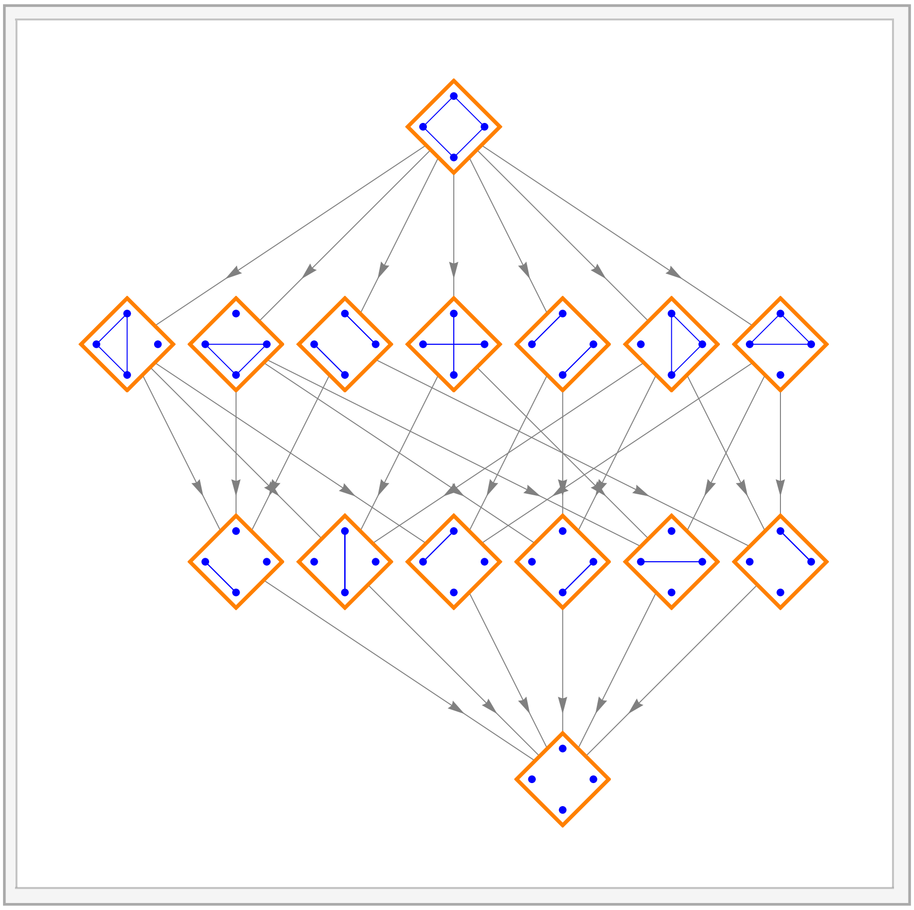
Bell[n]

Input

```
BellB[n]
```

n	B_n
0	1
1	1
2	2
3	5
4	15
5	52
6	203
7	877
8	4140
9	21 147
10	115 975

Set Partition Refinement Lattice (Stirling)



Robert Dickau
"Set Partition Refinement Lattice"
http://demonstrations.wolfram.com/SetPartitionRefinementLattice/
Wolfram Demonstrations Project
Published: March 7, 2011

```
Manipulate[
 partitionLatticeDiagram[styleFn, n, show,
  Which[n < 3, 100, 3 ≤ n < 5, 50, 5 ≤ n, 15]],
 {{n, 3, "elements"}, 2, 6, 1, ControlType → SetterBar},
 {{show, True, "show insets"}, {True, False}},
 {{styleFn, LayeredGraphPlot, "graph type"},
  {LayeredGraphPlot → "layered graph plot", GraphPlot → "graph plot",
   GraphPlot3D → "3D graphics"}},
 SaveDefinitions → True]
```

The Stirling partition (1, 7, 6, 1) gets by a first refinement the values: (1, (4, 3), 6, 1)

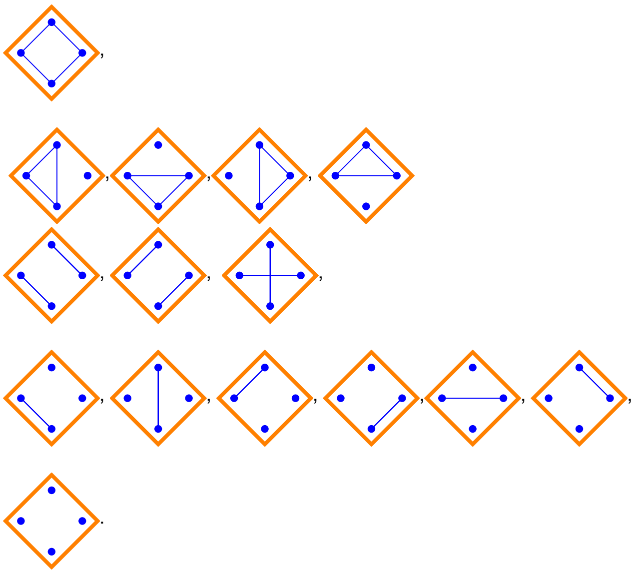
A next step of refinement produces the second-order refinement of [4] with (1, **4**, **3**, 6, 1) becoming: (1, (3+1, 3), 3+2+1, 1).

The refinement steps for a morphogrammatic system Morph[4] is therefore:

Morphogrammatic refinement for Morph[4]

StirlingS2		first refinement		second refinement
(1, 7, 6, 1)	$\Longrightarrow$	(1, 4 + 3, 6, 1)	$\Longrightarrow$	(1, 3 + 1, 3, 3 + 2 + 1, 1).

First refinement of the Set Partition



Second refinement by coloring or wighting

Instead of using different colors I will use integers as a tool to define the subclasses of the second refinement. With this numerical approach, different measures of ‘weight’ of a morphogram might be introduced to support further classifications.

This approach might not be strictly combinatorial but it seems to do the job for now.

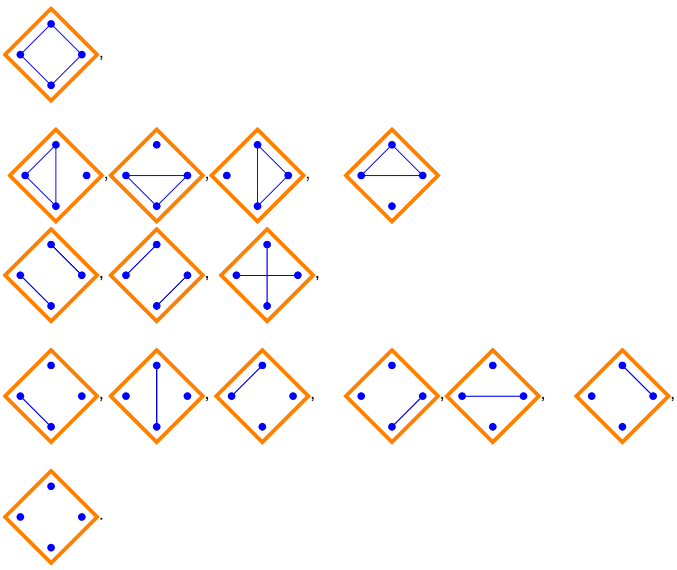
EXAMPLE

Enumeration scheme MG

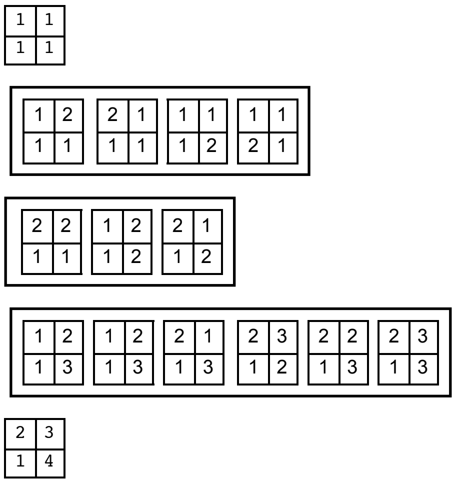
=

2	3
1	4

Weights are the sum of the components of the morphogram MG.



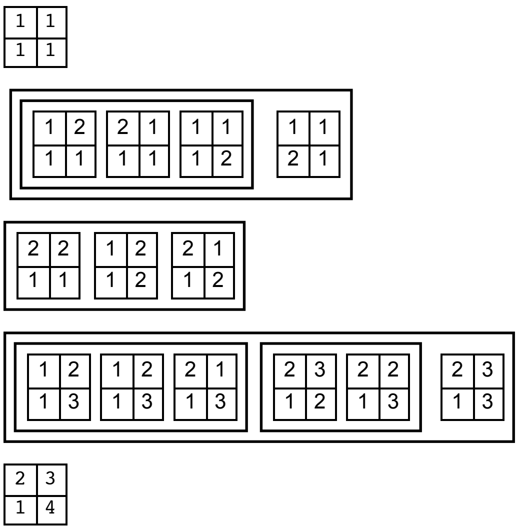
First refinement of MG[4]



W e i g h t s
4, (5, 7), 6, (6, 7, 8), 10.

Hence, the subclasses of morphograms corresponding to the classification by the weight of the morphograms are represented by the refinement: 1+(3+1)+3+(3+2+1)+1

Second refinement MG[4]



- **Tcontexture** 4; Bell 4 = 15 : {1,7,6,1}

No. Partition morphograms

S1 = [4]: [1,1,1,1] : (1)
S2 = [3,1]: [1,1,1,2], [1,1,2,1],[1,2,1,1]; [1,**2,2,2**] : (3+1)
S3 = [2,2]: [1,1,2,2], [1,2,1,2], [1,2,2,1] : (3)
S4 = [2,1,1]: [1,1,2,3], [1,2,1,3], [1,2,3,1]; [1,2,2,3], [1,2,3,2]; [1,2,3,3] : (3+2+1)
S5 = [1,1,1,1]: [1,2,3,4] : (1) .

Classification by dnf or permutation equivalence classes.

- Dcontexture 4:

val it = [[1,1,1,1],[1,1,2,2],[1,1,1,2],[1,1,2,3],[1,2,3,4]] : int list list
- dnf[1,1,2,2] = dnf[1,2,1,2];
val it = true : bool

- dnf[1,2,3,3] = dnf[1,1,2,3];
val it = false : bool

Diagram of the Second refinement

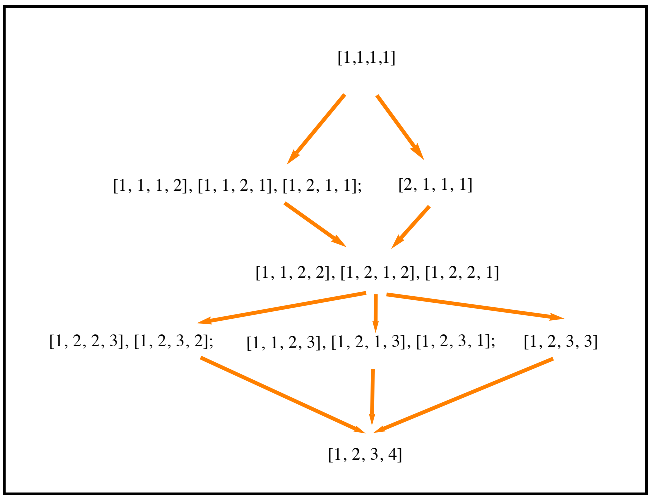
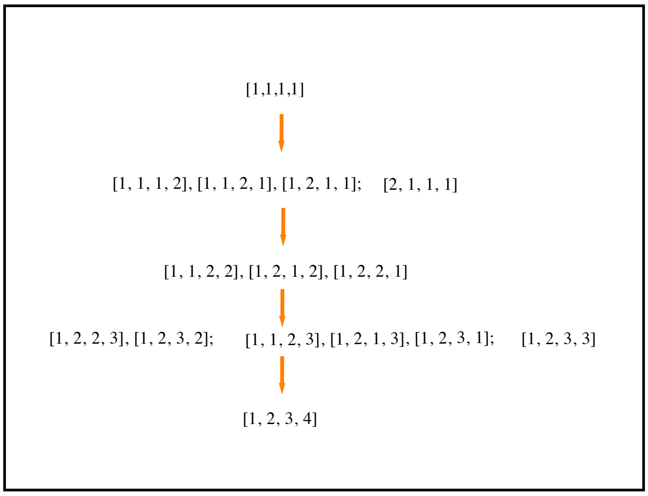
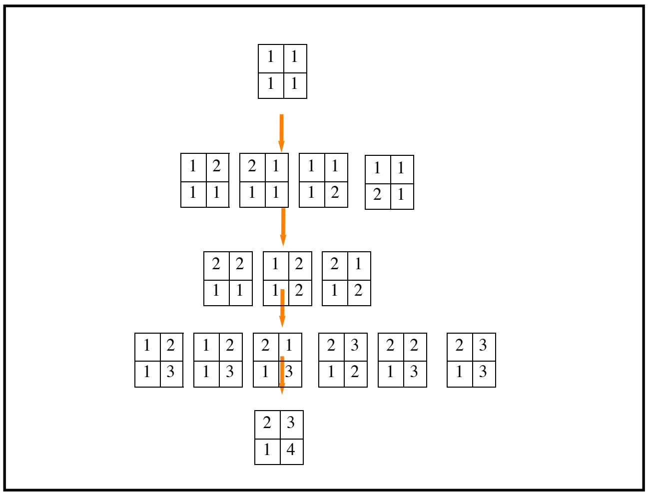
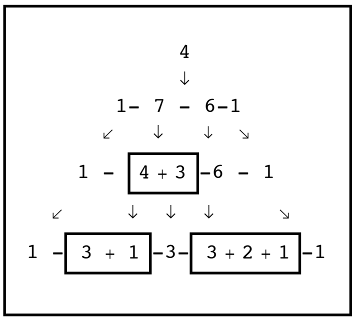


Diagram of the First refinement







First refinements of Partitions by T[m,n]

```
Column[runs[li : {__Integer}] := ((Length /@ Split[ # ])) &[Sort@ li];
 Table[Apply[Multinomial, Partitions[w], {1}] /
   Apply[Times, (runs /@ Partitions[w]) !, {1}], {w, 8}], Left]
```

```
{1}
{1, 1}
{1, 3, 1}
{1, 4, 3, 6, 1}
{1, 5, 10, 10, 15, 10, 1}
{1, 6, 15, 15, 10, 60, 20, 15, 45, 15, 1}
{1, 7, 21, 21, 35, 105, 35, 70, 105, 210, 35, 105, 105, 21, 1}
{1, 8, 28, 28, 56, 168, 56, 35, 280, 210,
 420, 70, 280, 280, 840, 560, 56, 105, 420, 210, 28, 1}
```

http : // oeis.org / A080575

A080575: Triangle of multinomial coefficients

4: (1,4,3,6,1)
5: (1,5,10,10,15,10,1)
6: (1, 6, 15, 15, 10, 60, 20, 15, 45, 15, 1)
7: (1, 7, 21, 21, 35, 105, 35, 70, 105, 210, 35, 105, 105, 21, 1)

"Row 4 represents 1*k(4)+4*k(3)*k(1)+3*k(2)^2+6*k(2)*k(1)^2+1*k(1)^4 and T(4,4)=6 since there are six ways of partitioning four labeled items into one part with two items and two parts each with one item." (Tilman Neumann)

http://oeis.org/A080575

The combinatorial meaning of the first refinement T[m,n] is expressed by the statement:
"T[n,m] = *count of set partitions of n with block lengths given by the m-th partition of n in the canonical ordering."*

http://www.tilman-neumann.de/index.html

For n=4 the 5 integer partitions in canonical ordering with corresponding set partitions and counts are:

[4] -> #{1234} = 1
[3,1] -> #{123/4, 124/3, 134/2, 1/234} = 4
[2,2] -> #{12/34, 13/24, 14/23} = 3
[2,1,1] -> #{12/3/4, 13/2/4, 1/23/4, 14/2/3, 1/24/3, 1/2/34} = 6
[1,1,1,1] -> #{1/2/3/4} = 1

Thus row 4 is [1, 4, 3, 6, 1].

*"Row 4 represents 1*k(4)+4*k(3)*k(1)+3*k(2)^2+6*k(2)*k(1)^2+1*k(1)^4 and T(4,4)=6 since there are six ways of partitioning four labeled items into one part with two items and two parts each with one item."*
http://oeis.org/A080575

This classical method for the determination of the first refinement also gives a direct hint to define the classifications for the *second* refinement.

▪ The idea of a second refinement

A further analysis would have to take the formula T[n,m] and its components into account find the solution for a mathematical definition of the second-order refinement of partitions out of the formula.

Here, I shall start with a first descriptive analysis.

Descriptive analysis

a.) [3,1]   -> #{123/4, 124/3, 134/2, **1/234**} = 4

This production has a refinement into
#{123/4, 124/3, 134/2 and
1/234}

delivering the morphograms:
[1,1,1,2],[1,1,2,1],[1,2,1,1], : 3
[1,2,2,2].   : 1

b.) [2,1,1]   -> #{12/3/4, 13/2/4, 1/**23**/4, 14/2/3, 1/**24**/3, 1/2/**34**} = 6

This production has a refinement into 3 groups:
#{12/3/4, 13/2/4, 14/2/3   : 3
1/**23**/4, 1/**24**/3   : 2
1/2/**34**   : 1

corresponding to the morphograms:
[1,1,2,3],[1,2,1,3],[1,2,3,1],
[1,2,2,3],[1,2,3,2],
[1,2,3,3].

In other words, the analysis of the 'fine' analysis gives a hint how to define the further step of a 'fine-analysis of the fine-analysis'.

The partition [2,1,1] has 6 canonical results:
1. {12/3/4, 13/2/4, 1/23/4} : is producing a repetition of "1",
2. {14/2/3, 1/24/3}   : is producing a repetition of "2",
3. {1/2/34}   : is producing a repetition of "3".

Therefore, the second fine-analysis of "[2,1,1]" produces the partition: (3,2,1) out of the integer partitions of number 6.

[2,1,1]   -> #{12/3/4, 13/2/4, 1/**23**/4, 14/2/3, 1/**24**/3, 1/2/**34**} = 6 = (3+2+1)

This procedure is represented by the T[m,n]-formula:
1*k(4)+4*k(3)*k(1)+3*k(2)^2+6*k(2)*k(1)^2+1*k(1)^4

Second-order Refinements

A further step in the analysis of Stirling numbers, additionally to the 'refined' analysis (based on the Bell coefficients) is achieved with a kind of a fine-analysis of T[m,n], i.e. a fine-analysis of the fine-analysis, that takes the *different* representations of the partitions into account.

D(T[m,n]) is classifying the results from T[m,n] into a *second* refinement.

Example

S2=[3,1]: [1,1,1,2],[1,1,2,1],[1,2,1,1],[1,2,2,2], :
(4) = 3+1: ([1,1,1,2],[1,1,2,1],[1,2,1,1]) + ([1,2,2,2]).

The representant [1,2,2,2] is differentiated from the other representants with the value 1 as repetition. Hence, [1,2,2,2] is different from the representants ([1,1,1,2],[1,1,2,1],[1,2,1,1]). Therefore, this difference supports a further analysis, the *fine-analysis of the fine-analysis*.
Especially, kref[1,1,1,2] $=_{MG}$ [1,2,2,2], and [2,1,1,1] $=_{MG}$ [1,2,2,2].

The canonical representation of the partitions is supposing a lexical ordering of its elements. Therefore, a morphogram [2,1,1,1] is not in lexical order and is therefore not accepted. It has to be replaced by the morphogrammatically equivalent pattern [1,2,2,2].

Comparison

First refinement of Bell coefficients	StirlingS2	Bell
1;		
1, 1;		
1, 3, 1;		
1, **4, 3**, 6, 1;	1, 7, 6, 1	: 15
1, **5, 10**, **10, 15**, 10, 1;	1, 15, 25, 10, 1	: 52
1, 6, 15, 15, 10, 60, 20, 15, 45, 15, 1;	1, 31, 90, 65, 15, 1	: 203
1, 7, 21, 21, 35, 105, 35, 70, 105, 210, 35, 105, 105, 21, 1;	1, 63, 301, 350, 140, 21,1	: 877

Small table of second-order fine-analysis of partitions

3: 1, 2+1, 1
4: 1, (3+1, 3), 3+2+1, 1
5: 1, (4+1, 6+4), (6+3+1, 12+2+1), 4+3+3, 1
6: 1, 6=5+1, 15=10+5, 10, 15=10+4+1, 60=30+15+15,
20=10+6+3+1, 15, 45=30+12+3, 15=5+4+3+2+1, 1.

Second-order refinement of [4] with 1, **4, 3**, 6, 1 becomes: 1, (3+1, 3), 3+2+1, 1.

Elaborated examples for a double refinement of Tcontextures

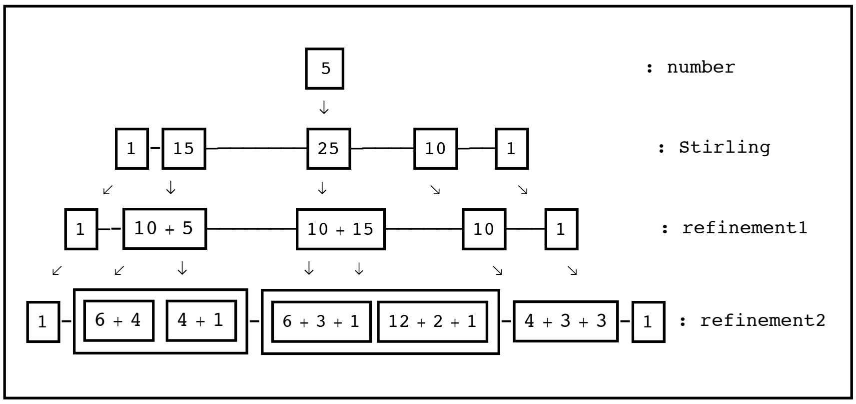
Tcontexture 5

No. Partition morphograms TM[5,5]: (1,5,10,10,15,10,1), (Bell 5 = 52)

- **(5):** (1)
  [1,1,1,1,1],
- **(4,1):** (4+1)
  [1,1,1,1,2], [1,1,1,2,1],[1,1,2,1,1],[1,2,1,1,1]; [1,2,2,2,2].
- **(3,2):** (6+4)
  [1,1,1,2,2],[1,1,2,1,2],[1,1,2,2,1],[1,2,1,1,2],[1,2,1,2,1],[1,2,2,1,1];
  [1,2,2,2,1],[1,2,2,1,2],[1,2,1,2,2],[1,1,2,2,2],
- **(3,1,1):** (6+3+1)
  [1,1,1,2,3],[1,1,2,1,3],[1,1,2,3,1],[1,2,1,1,3],[1,2,1,3,1],[1,2,3,1,1];
  [1,2,2,2,3],[1,2,2,3,2],[1,2,3,2,2]; [1,2,3,3,3].
- **(2,2,1):** (12+2+1)
  [1,1,2,2,3],[1,1,2,3,2], [1,1,2,3,3],[1,2,1,2,3],[1,2,1,3,2],[1,2,2,1,3],[1,2,2,3,1],[1,2,3,1,2];
  [1,2,3,2,1],[1,2,1,3,3],[1,2,3,1,3],[1,2,3,3,1];
  [1,2,2,3,3],[1,2,3,2,3],[1,2,3,3,2].
- **(2,1,1,1):** (4+3+3)
  [1,1,2,3,4],[1,2,1,3,4],[1,2,3,1,4],[1,2,3,4,1];
  [1,2,2,3,4],[1,2,3,2,4],[1,2,3,4,2];
  [1,2,3,3,4],[1,2,3,4,3],[1,2,3,4,4].
- **(1,1,1,1,1):** (1)
  [1,2,3,4,5] .

Second refinement: D(TM[5,5]): 1, (4+1, 6+4), (6+3+1, 12+2+1), 4+3+3, 1.

Refinement	1	5	10	10	15	10	1
SecondRef	1	4+1	6+4	6+3+1	12+2+1	4+3+3	1
Stirling	1	□	15	□	25	10	1



Tcontexture 6

Partition[6]

6" = "5 + 1
6" = "4 + 2
6" = "4 + 1 + 1
6" = "3 + 3
6" = "3 + 2 + 1
6" = "3 + 1 + 1 + 1
6" = "2 + 2 + 2
6" = "2 + 2 + 1 + 1
6" = "2 + 1 + 1 + 1 + 1
6" = "1 + 1 + 1 + 1 + 1 + 1

StirlingSn[6]

1, 31, 90, 65, 15, 1

First refinement TM[6,6]

1,6,15,15,10,60,20,15,45,15,1

Second refinement [6]

First Refinement	1	6	15	15	10	60	20	15	45	15	1
Second Refinement	1	5+1	10+5	10+4+1	10	30+15+15	10+6+3+1	15	30+12+3	5+4+3+2+1	1
Stirling	1	□	31	□	90	□	65	□	□	15	1

Elaboration of the second refinement for Tcontexture 6: D(TM[6,6])

No. Partition morphograms

• **6 : 1** [1,1,1,1,1,1]

• **(5,1)** : 6 = 5+1
[**1,1,1,1,1**,2],[1,1,1,1,2,1],[1,1,1,2,1,1],[1,1,2,1,1,1],[1,2,1,1,1,1];
[1,**2,2,2,2,2**].

• **(4,2)** : 15 = 10+5
[**1,1,1,1**,2,2],[1,1,1,2,1,2],[1,1,1,2,2,1],[1,1,2,1,1,2],[1,1,2,1,2,1],
[1,1,2,2,1,1],[1,2,1,1,1,2],[1,2,1,1,2,1],[1,2,1,2,1,1],[1,2,2,1,1,1];
[1,**2,2,2,2**,1],[1,2,2,2,1,2],[1,2,2,1,2,2],[1,2,1,2,2,2],[1,1,2,2,2,2].

• **(4,1,1)** : 15 = 10+4+1
[1,1,1,1,2,3],[1,1,1,2,1,3],[1,1,1,2,3,1],[1,1,2,1,1,3],[1,1,2,1,3,1],
[1,1,2,3,1,1],[1,2,1,1,1,3],[1,2,1,1,3,1],[1,2,1,3,1,1],[1,2,3,1,1,1];
[1,2,2,2,2,3],[1,2,2,2,3,2],[1,2,2,3,2,2],[1,2,3,2,2,2]; [1,2,3,3,3,3].

• **(3,3)** : 10
[1,1,1,2,2,2],[1,1,2,1,2,2],[1,1,2,2,1,2],[1,1,2,2,2,1],[1,2,1,1,2,2],
[1,2,1,2,1,2],[1,2,1,2,2,1],[1,2,2,1,1,2],[1,2,2,1,2,1],[1,2,2,2,1,1].

• **(3,2,1)** : 60 = 30+15+15
[**1,1,1**,2,2,3],[1,1,1,2,3,2],[1,1,1,2,3,3], (30)
[1,1,2,1,2,3],[1,1,2,1,3,2],[1,1,2,2,1,3],[1,1,2,2,3,1],[1,1,2,3,1,2],

[1,1,2,3,2,1],[1,1,2,1,3,3],[1,1,2,3,1,3],[1,1,2,3,3,1],[1,2,1,1,2,3],
[1,2,1,1,3,2],[1,2,1,2,1,3],[1,2,1,2,3,1],[1,2,1,3,1,2],[1,2,1,3,2,1],
[1,2,2,1,1,3],[1,2,2,1,3,1],[1,2,2,3,1,1],[1,2,3,1,1,2],[1,2,3,1,2,1],
[1,2,3,2,1,1],[1,2,1,1,3,3],[1,2,1,3,1,3],[1,2,1,3,3,1],[1,2,3,1,1,3],
[1,2,3,1,3,1],[1,2,3,3,1,1];
[1**,2,2,2**,1,3],[1,2,2,2,3,1],[1,2,2,1,2,3], (15)
[1,2,2,1,3,2],[1,2,2,3,2,1],[1,2,2,3,1,2],[1,2,1,2,2,3],[1,2,1,2,3,2],
[1,2,1,3,2,2],[1,2,3,2,2,1],[1,2,3,2,1,2],[1,2,3,1,2,2],[1,1,2,2,2,3],
[1,1,2,2,3,2],[1,1,2,3,2,2];
[1,1,2,**3,3,3**],[1,2,3,3,3,1],[1,2,3,3,1,3], (15)
[1,2,3,1,3,3],[1,2,1,3,3,3],[1,2,2,2,3,3],[1,2,2,3,2,3],[1,2,2,3,3,2],
[1,2,3,2,2,3],[1,2,3,2,3,2],[1,2,3,3,2,2],[1,2,3,3,3,2],[1,2,3,3,2,3],
[1,2,3,2,3,3],[1,2,2,3,3,3].

• **(3,1,1,1)** : 20 = 10+6+3+1
[**1,1,1**,2,3,4],[1,1,2,1,3,4],[1,1,2,3,1,4],[1,1,2,3,4,1],[1,2,1,1,3,4],
[1,2,1,3,1,4],[1,2,1,3,4,1],[1,2,3,1,1,4],[1,2,3,1,4,1],[1,2,3,4,1,1];
[1,**2,2,2**,3,4],[1,2,2,3,2,4],[1,2,2,3,4,2],[1,2,3,2,2,4],[1,2,3,2,4,2],
[1,2,3,4,2,2];
[1,2,**3,3,3**,4],[1,2,3,3,4,3],[1,2,3,4,3,3];
[1,2,3,**4,4,4**],

• **(2,2,2)** :15
[1,1,2,2,3,3],[1,1,2,3,2,3],[1,1,2,3,3,2],[1,2,1,2,3,3],[1,2,1,3,2,3],
[1,2,1,3,3,2],[1,2,2,1,3,3],[1,2,2,3,1,3],[1,2,2,3,3,1],[1,2,3,1,2,3],
[1,2,3,1,3,2],[1,2,3,2,1,3],[1,2,3,2,3,1],[1,2,3,3,1,2],[1,2,3,3,2,1],

• **(2,2,1,1)** : 45 = 30+12+3
[**1,1**,2,2,3,4],[1,1,2,3,2,4],[1,1,2,3,4,2],
[1,1,2,3,3,4],[1,1,2,3,4,3],[1,1,2,3,4,4],[1,2,1,2,3,4],[1,2,1,3,2,4],
[1,2,1,3,4,2],[1,2,2,1,3,4],[1,2,2,3,1,4],[1,2,2,3,4,1],[1,2,3,1,2,4],
[1,2,3,1,4,2],[1,2,3,2,1,4],[1,2,3,2,4,1],[1,2,3,4,1,2],[1,2,3,4,2,1],
[1,2,1,3,3,4],[1,2,1,3,4,3],[1,2,1,3,4,4],[1,2,3,1,3,4],[1,2,3,1,4,3],
[1,2,3,3,1,4],[1,2,3,3,4,1],[1,2,3,4,1,3],[1,2,3,4,3,1],[1,2,3,1,4,4],
[1,2,3,4,1,4],[1,2,3,4,4,1]
[1,**2,2**,3,3,4],[1,2,2,3,4,3],[1,2,2,3,4,4],[1,2,3,2,3,4],[1,2,3,2,4,3],
[1,2,3,3,2,4],[1,2,3,3,4,2],[1,2,3,4,2,3],[1,2,3,4,3,2],[1,2,3,2,4,4],
[1,2,3,4,2,4],[1,2,3,4,4,2]
[1,2,**3,3**,4,4],[1,2,3,4,3,4],[1,2,3,4,4,3]

• **(2,1,1,1,1)** : 15 = 5+4+3+2+1
[**1,1**,2,3,4,5],[1,2,1,3,4,5],[1,2,3,1,4,5],[1,2,3,4,1,5],[1,2,3,4,5,1];
[1,**2,2**,3,4,5],[1,2,3,2,4,5],[1,2,3,4,2,5],[1,2,3,4,5,2];
[1,2,**3,3**,4,5],[1,2,3,4,3,5],[1,2,3,4,5,3];
[1,2,3,**4,4**,5],[1,2,3,4,5,4];
[1,2,3,4,**5,5**].

• **(1,1,1,1,1,1)** : 1
[1,2,3,4,5,6] .

Test
Tcontexture 6 = 203
StirlingS2: 1, 31, 90, 65, 15, 1
Refinement2: 1+6+15+15+10+60+20+15+45+15+1;

All morphograms that are dnf-equal $mg_i =_{dnf} mg_j$, with i,j∈Dcontexture(n), belong to the same class of a double refinement.

How to define and detect second-order refinements mathematically?

One method is to apply the concept of 'weighted' *deutero*-normal form.
The other concept is to use *permutations* for the definition of classes of double-refinements.
Both are formally equivalent. But both are still just *descriptive* and not operative methods and not yet delivering directly combinatorial and numerical results.

An additional approach is using the ϵ/ν-structure of the morphograms. As a result, the *first* refinement is easily established by the number of the distinctions E and N. But the second refinement demands a further interpretation of those results.

Example m = 6

• **(2,1,1,1,1)** : 15 = 5+4+3+2+1
a.) [**1,1**,2,3,4,5],[1,2,1,3,4,5],[1,2,3,1,4,5],[1,2,3,4,1,5],[1,2,3,4,5,1],
b.) [1,**2,2**,3,4,5],[1,2,3,2,4,5],[1,2,3,4,2,5],[1,2,3,4,5,2],
c.) [1,2,**3,3**,4,5],[1,2,3,4,3,5],[1,2,3,4,5,3],
d.) [1,2,3,**4,4**,5],[1,2,3,4,5,4],
e.) [1,2,3,4,**5,5**],

From the set of morphograms of the class of the fist refinement (2,1,1,11), subclasses are defined by the permutation equivalence of morphograms.

Hence, the application of the permutation equivalence class building separates the class (2,1,1,1,1) into 5 subclasses a.) to e.).

Canonical forms for (3,1)
[1,1,1,2],[1,1,2,1],[1,2,1,1] : 3
[1,2,2,2]. : 1

The permutation of the canonical for [1,1,1,2] delivers the 2 reperesentations [1,1,2,1],[1,2,1,1].
But the morphogram [1,2,2,2] is not a permutation of [1,1,1,2] nor is the pattern [2,1,1,1] an accepted canonical form. If we accept the form as a canoncal permutation then the distinction between the 2 groups colapses.

If we want to save the unristringed permuation mode, we have to accept that the pattern [2,1,1,1] is allowed and is nevertheless morphogrammatically equal to [1,2,2,2] but it is not anymore separating the group into two.

Hence, the general result woud be:
(3,1): [1,1,1,2],[1,1,2,1],[1,2,1,1], [2,1,1,1].

Hence it seems that the permutation argument is not yet properly elaborated as a mechanism of refinement.

Canonical forms for (3,1,1,1)
For the case (3,1,1,1) of the partition p(6), there are 4 canonical representations to recognize;

I. [**1,1**,1,2,3,4]
II. [1,**2,2,2**,3,4]
III. [1,2,**3,3,3**,4]
IV. [1,2,3,**4,4,4**].

Permutations of canonical forms

The case IV.

[1,2,3,4,4,4] is singular, because there is only one canonical representation possible.

The case III.

[1,2,3,3,3,4] has just 2 different permutations that are canonically correct., i.e.

[1,2,3,3,4,3],[1,2,3,4,3,3].

The case II.

[1,2,2,2,3,4] has just 5 different permutations that are canonically correct. They are represented by:

[1,2,2,3,2,4],[1,2,2,3,4,2],[1,2,3,2,2,4],[1,2,3,2,4,2],[1,2,3,4,2,2].

The case I.

[1,1,1,2,3,4] has just 9 different permutations that are canonically correct. That is:

[1,1,2,1,3,4],[1,1,2,3,1,4],[1,1,2,3,4,1],[1,2,1,1,3,4],[1,2,1,3,1,4],
[1,2,1,3,4,1], [1,2,3,1,1,4],[1,2,3,1,4,1],[1,2,3,4,1,1].

■ First refinement of the ϵ/ν-structure of MG(4)

The EN-analysis of MG[4] delivers directly a *first refinement* indicated by the number of Es and Ns. Groups with the same number of Es and Ns are building a class of a classification by refinement.

What we can conclude is the fact that the EN-structures are mathematically directly corresponding to the combinatorics of the first refinement of Stirling partitions.

First refinement and the ϵ/ν-distribution are combinatorically equal.

This seems to be a new insight into the combinatorial structure of the descriptively developed morphogrammatics.

There is not yet a direct hint presented by this analysis for a *second*-order refinement.

A second-order refinement has to consider the structure of the ϵ/ν-patterns to get enough information for a further refinement.

How are those properties of the ϵ/ν-structures defined?

EN-analysis for MG(4)

- ENstructureEN [1,1,1,1];
val it = [[],[E],[E,E],[E,E,E]]

- map ENstructureEN [[1,1,1,2],[1,1,2,1],[1,2,1,1], [1,2,2,2]];
val it =
[[[],[E],[E,E],[N,N,N]],
[[],[E],[N,N],[E,E,N]],
[[],[N],[E,N],[E,N,E]],

[[],[N],[N,E],[N,E,E]]]

- map ENstructureEN [[1,1,2,2],[1,2,1,2],[1,2,2,1]];
val it =
[[[],[E],[N,N],[N,N,E]],
[[],[N],[E,N],[N,E,N]],
[[],[N],[N,E],[E,N,N]]]

- map ENstructureEN [[1,1,2,3],[1,2,1,3],[1,2,3,1], [1,2,2,3],[1,2,3,2], [1,2,3,3]];
val it =
[[[],[E],[N,N],[N,N,N]],
[[],[N],[E,N],[N,N,N]],
[[],[N],[N,N],[E,N,N]],

[[],[N],[N,E],[N,N,N]],
[[],[N],[N,N],[N,E,N]],

[[],[N],[N,N],[N,N,E]]]

- ENstructureEN [1,2,3,4];
val it = [[],[N],[N,N],[N,N,N]]

refinement TM[4,4]: 1,4,3,6,1.
How to find the second refinement out of the ϵ/ν-struture: 1, (3+1), 3, (3+2+1), 1?

Analysis

[[],[N],[N,N],[E,N,N]] :

N	□	□
N	N	□
E	N	N

: [1,2,3,1]

[[],[N],[N,N],[N,N,E]] :

N	□	□
N	N	□
N	N	**E**

: [1,2,3,3]

Trivially, the results for the first refinements are isomorphic to the ϵ/ν-structure. This is trivial, because there is a bijection between the morphograms and their ϵ/ν-structures. But the ϵ/ν-structure of the classification is independent of its numeric or alphabetic representation. Therefore, a case like [2,1,1,1] and [1,2,2,2] is treated as the same ϵ/ν-structure from the very beginning. They share the same ϵ/ν-structure, therefore they are the same.

What counts directly for the first refinement are just the number of Es and Ns.
But there is not yet any direct information left for the definition of a *second* refinement.

ENstructureEN 5

- allENstructureEN 5;
val it =
[[[],[E],[E,E],[E,E,E],[E,E,E,E]], (1)
[[],[E],[E,E],[E,E,E],[N,N,N,N]],
[[],[E],[E,E],[N,N,N],[E,E,E,N]],
[[],[E],[N,N],[E,E,N],[E,E,N,E]], (5)

[[],[N],[E,N],[E,N,E],[E,N,E,E]],
[[],[N],[N,E],[N,E,E],[N,E,E,E]],

[[],[E],[E,E],[N,N,N],[N,N,N,E]], (10)
[[],[E],[N,N],[E,E,N],[N,N,E,N]],
[[],[E],[N,N],[N,N,E],[E,E,N,N]],
[[],[N],[E,N],[E,N,E],[N,E,N,N]],
[[],[N],[E,N],[N,E,N],[E,N,E,N]],
[[],[N],[N,E],[E,N,N],[E,N,N,E]],
[[],[N],[N,E],[N,E,E],[E,N,N,N]],
[[],[N],[N,E],[E,N,N],[N,E,E,N]],
[[],[N],[E,N],[N,E,N],[N,E,N,E]],
[[],[E],[N,N],[N,N,E],[N,N,E,E]],

[[],[E],[E,E],[N,N,N],[N,N,N,N]], (10)
[[],[E],[N,N],[E,E,N],[N,N,N,N]],[[],[E],[N,N],[N,N,N],[E,E,N,N]],
[[],[N],[E,N],[E,N,E],[N,N,N,N]],[[],[N],[E,N],[N,N,N],[E,N,E,N]],
[[],[N],[N,N],[E,N,N],[E,N,N,E]],[[],[N],[N,E],[N,E,E],[N,N,N,N]],
[[],[N],[N,E],[N,N,N],[N,E,E,N]],[[],[N],[N,N],[N,E,N],[N,E,N,E]],
[[],[N],[N,N],[N,N,E],[N,N,E,E]],

[[],[E],[N,N],[N,N,E],[N,N,N,N]],[[],[E],[N,N],[N,N,N],[N,N,E,N]], (15)
[[],[E],[N,N],[N,N,N],[N,N,N,E]],[[],[N],[E,N],[N,E,N],[N,N,N,N]],
[[],[N],[E,N],[N,N,N],[N,E,N,N]],[[],[N],[N,E],[E,N,N],[N,N,N,N]],
[[],[N],[N,E],[N,N,N],[E,N,N,N]],[[],[N],[N,N],[E,N,N],[N,E,N,N]],
[[],[N],[N,N],[N,E,N],[E,N,N,N]],[[],[N],[E,N],[N,N,N],[N,N,N,E]],
[[],[N],[N,N],[E,N,N],[N,N,E,N]],[[],[N],[N,N],[N,N,E],[E,N,N,N]],
[[],[N],[N,E],[N,N,N],[N,N,N,E]],[[],[N],[N,N],[N,E,N],[N,N,E,N]],
[[],[N],[N,N],[N,N,E],[N,E,N,N]],

[[],[E],[N,N],[N,N,N],[N,N,N,N]], (10)
[[],[N],[E,N],[N,N,N],[N,N,N,N]],[[],[N],[N,N],[E,N,N],[N,N,N,N]],
[[],[N],[N,N],[N,N,N],[E,N,N,N]],[[],[N],[N,E],[N,N,N],[N,N,N,N]],
[[],[N],[N,N],[N,E,N],[N,N,N,N]],[[],[N],[N,N],[N,N,N],[N,E,N,N]],
[[],[N],[N,N],[N,N,E],[N,N,N,N]],[[],[N],[N,N],[N,N,N],[N,N,E,N]],
[[],[N],[N,N],[N,N,N],[N,N,N,E]],

[[],[N],[N,N],[N,N,N],[N,N,N,N]]] (1)

First refinement (5): 1 5 10 10 15 10 1 .

Second Refinement (5): 1, (4+1, 6+4), (6+3+1, 12+2+1), 4+3+3, 1

Fusion and decomposition

The fusion or coalition of two morphic dyads, [1,1], [1,2], is producing in SML 15 morphograms:

```
- kconcat[1,1][1,1];
val it = [[1,1,1,1],[1,1,2,2]] : int list list  : S1+S3
- kconcat[1,1][1,2];
val it = [[1,1,1,2],[1,1,2,1],[1,1,2,3]] : int list list : S2+S4
- kconcat[1,2][1,1];
val it = [[1,2,1,1],[1,2,2,2],[1,2,3,3]] : int list list  : S2+S4
- kconcat[1,2][1,2];
val it = [[1,2,1,2],[1,2,2,1],[1,2,1,3],[1,2,3,1],[1,2,2,3],[1,2,3,2],[1,2,3,4]] : S3+S4+S5
```

The type of the fusion of the dyads is: (2,3,3,7).

In contrast, the system of the 15 morphograms is classified by the 2-refinement type: (1, (3+1), 3, (3+2+1), 1).

■ CASE TEST for the partition (3,1,1,1)

Also this case has just 3 Es, it is nevertheless divided into 4 subgroups.

Obviously, for the second refinement not only the number of Es and Ns are of importance but also their positions in the table.

The case of three Es:

- **(3,1,1,1)** : 20 = 10+6+3+1

[a.] [**1,1,1**,2,3,4],[1,1,2,1,3,4],[1,1,2,3,1,4],[1,1,2,3,4,1],[1,2,1,1,3,4],

[1,2,1,3,1,4],[1,2,1,3,4,1],[1,2,3,1,1,4],[1,2,3,1,4,1],[1,2,3,4,1,1],

b.) [1,**2,2,2**,3,4],[1,2,2,3,2,4],[1,2,2,3,4,2],[1,2,3,2,2,4],[1,2,3,2,4,2],[1,2,3,4,2,2],

c.) [1,2,**3,3,3**,4],[1,2,3,3,4,3],[1,2,3,4,3,3],

d.) [1,2,3,**4,4,4**] .

EN-analysis for m=6

a.) [**1,1,1**,2,3,4],[1,1,2,1,3,4],[1,1,2,3,1,4],[1,1,2,3,4,1],[1,2,1,1,3,4],
[1,2,1,3,1,4],[1,2,1,3,4,1],[1,2,3,1,1,4],[1,2,3,1,4,1],[1,2,3,4,1,1],

[[[],[E],[E,E],[N,N,N],[N,N,N,N],[N,N,N,N,N]],
[[],[E],[N,N],[E,E,N],[N,N,N,N],[N,N,N,N,N]],
[[],[E],[N,N],[N,N,N],[E,E,N,N],[N,N,N,N,N]],
[[],[E],[N,N],[N,N,N],[N,N,N,N],[E,E,N,N,N]],
[[],[N],[E,N],[E,N,E],[N,N,N,N],[N,N,N,N,N]],
[[],[N],[E,N],[N,N,N],[E,N,E,N],[N,N,N,N,N]],
[[],[N],[E,N],[N,N,N],[N,N,N,N],[E,N,E,N,N]],
[[],[N],[N,N],[E,N,N],[E,N,N,E],[N,N,N,N,N]],
[[],[N],[N,N],[E,N,N],[N,N,N,N],[E,N,N,E,N]],
[[],[N],[N,N],[N,N,N],[E,N,N,N],[E,N,N,N,E]],

b.) [1,**2,2,2**,3,4],[1,2,2,3,2,4],[1,2,2,3,4,2],[1,2,3,2,2,4],[1,2,3,2,4,2],[1,2,3,4,2,2],

[[],[N],[N,E],[N,E,E],[N,N,N,N],[N,N,N,N,N]],
[[],[N],[N,E],[N,N,N],[N,E,E,N],[N,N,N,N,N]],
[[],[N],[N,E],[N,N,N],[N,N,N,N],[N,E,E,N,N]],
[[],[N],[N,N],[N,E,N],[N,E,N,E],[N,N,N,N,N]],
[[],[N],[N,N],[N,E,N],[N,N,N,N],[N,E,N,E,N]],
[[],[N],[N,N],[N,N,N],[N,E,N,N],[N,E,N,N,E]],

c.) [1,2,**3,3,3**,4],[1,2,3,3,4,3],[1,2,3,4,3,3],

Mediation and interaction between classifications

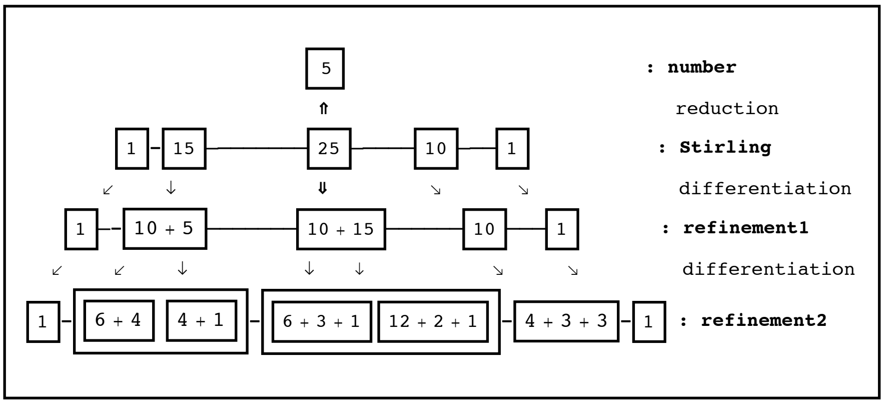
■ Hierachical versus heterarchical organizations

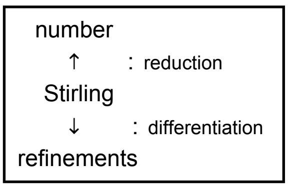
A hierachical order of the 3 different classifications of a partition system is naturally defined by the hierarchical steps of refinements.

The opposite movement of refinement is a kind of *specification* or generalization.

Hence, the start of the refinement/specification has not be identified as the start number n of the classification.

As natural as to start with the natural number n it is as natural to start with the Stirling numbers StirlingS2 (n).





This move prepare to distribute the classification over a contextural matrix where all levels are heterarchically mediated together by the polyfunctorial operations of Diamond Category Theory.

Decomposing polycontextural systems

■ How are polycontextural systems composed

The classical text that introduces polycontexturality in a general sense, is avaible at: http://www.vordenker.de/ggphilosophy/gg_life_as_polycontexturality.pdf

An introduction is here: http://www.vordenker.de/ggphilosophy/la_poly.htm

An analysis might be found here: http://memristors.memristics.com/Mereotopology/Mereotopology%20and%20Polycontexturality.pdf

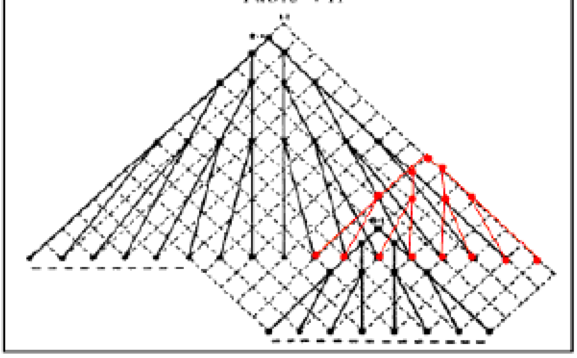
A neat characterization of the meaning of the term *polycontexturality* in the context of “dramatic texts” is sketched by the following paragraphs of Barbara Ventarola:

> *"1.In order to grasp the complexity of worldwide cultural networking, it is necessary to conceive of the cultural net as a universal structure consisting of several interacting, overlapping nets, as a world of worlds or - as Gotthard Gunther said - as a polycontextural structure of structures. The recourse to polycontextural theory allows the multi-directional circulation of conceptual and material forms to be taken into account without neglecting the "location of culture" (Bhabha) which should be borne in mind since it plays a particularly important role in the hierarchical colonial interactions of cultures.*
>
> *2. The same complexity governs dramatic texts, which can be re-conceived of as systems that model and evoke a (potentially) polycontextural world: Stemming from polycontextural subjects (their authors) and consisting themselves of a network of several sub-systems (the semiotic structures constituting the text), the dramas are able to refer to a plurality of cultural contexts at any time. One of the aspects of this textual multi-directionality, which until now has mostly been neglected, is its capacity to pursue several pragmatic aims at the same time by spreading them over diverse textual layers or ‘stages’. “*

http://www.geisteswissenschaften.fuberlin.de/we03/forschung/drittmittelprojekte/dramanet/Veranstaltungen/Ventarola_Abstract.pdf

A scheme of an interaction of 3 different decentralized hierarchical taxonomies is depicted by Table VII.

Table VII



Interchangeability of a 3 – contextural category with composition and mediation $(\amalg)$

$$\mathcal{U}^{(3)} = \left(\mathcal{U}_1 \amalg_{1.2} \mathcal{U}_2\right) \amalg_{1.2.3} \mathcal{U}_3$$

$$\left(\mathcal{U}_1 \bigcap_{1.2} \mathcal{U}_2\right) \bigcap_{1.2.3} \mathcal{U}_3 = \emptyset;$$

$$\mathcal{U}_i = \left\{f_i, g_i\right\}, \ i = 1, 2, 3$$

$$\begin{bmatrix} g_1 & \square & g_3 \\ f_1 & g_2 & \square \\ \square & f_2 & f_3 \end{bmatrix}$$

$$\left(\begin{pmatrix} \left(f_1 \circ_{1.0.0} g_1\right) \\ \amalg_{1.2.0} \\ \left(f_2 \circ_{0.2.0} g_2\right) \end{pmatrix} \begin{matrix} \\ \amalg_{1.2.3} \\ \left(f_3 \circ_{0.0.3} g_3\right) \end{matrix}\right) = \left(\begin{pmatrix} f_1 \\ \amalg_{1.2.0} \\ f_2 \end{pmatrix} \begin{matrix} \\ \amalg_{1.2.3} \\ f_3 \end{matrix}\right) \circ_1 \circ_2 \circ_3 \left(\begin{pmatrix} g_1 \\ \amalg_{1.2.0} \\ g_2 \end{pmatrix} \begin{matrix} \\ \amalg_{1.2.3} \\ g_3 \end{matrix}\right)$$

■ Interpretation

Morphograms are offering the structural localization and organization of the complexity and complication of different and same contextures.

What such an approach is intending is a societal analysis of the behavior of complex systems that is independent of *statistical* analysis as well as from *combinatorial* analysis that is based on the identity of its elements, events and behaviours.

Contextures of polycontextural compounds (Gunther) are located, they have their *'location of culture'* and their culture of location. Obviously, the term 'location' is a structural stratageme and is independent from any identificational localizations as it might be the case for mathematical group theory and for archaic cultures (human geography) too.

The term "contexture" is highly general. Without going into a philosophical exploration/explication the term is neither general nor particular. It is in a strict sense not even a term, notion or concept. It is characterized and differentiated only in the conceptual network of its neighbor terms: poly-, dis-, trans-, inter- and intra-conxtural, and in distinction to its morphogrammatics.

Polycontexturality is not just decentralized pluri-centrism.

It has a formal mathematical application, it appears in semiotic analysis, in text and drama theory. But it is also elaborated in *extenso* for structural questions of the system of international law (Teubner), and obviously, it is, or could be, of relevance, for countries that are in the process of separation (Basque country, Catalonia, Quebec, Scotland).

A contextural complexion, say of degree 4, is locating 6 different contextures together.

The complexity of the organized, i.e. mediated, contextures of MG[4] has a range of just 15 different constellations, reaching from full differentiation to zero differentiation.

This is formally represented by the 15 morphograms of complexity 4 and complication 1. And defines the field or range of polycontexturality for m=4.

$$\begin{array}{|c|c|}\hline 1&1\\\hline 1&1\\\hline\end{array} \begin{array}{|c|c|}\hline 1&2\\\hline 1&1\\\hline\end{array} \begin{array}{|c|c|}\hline 2&1\\\hline 1&1\\\hline\end{array} \begin{array}{|c|c|}\hline 1&1\\\hline 1&2\\\hline\end{array} \begin{array}{|c|c|}\hline 1&1\\\hline 2&1\\\hline\end{array} \begin{array}{|c|c|}\hline 2&2\\\hline 1&1\\\hline\end{array} \begin{array}{|c|c|}\hline 1&2\\\hline 1&2\\\hline\end{array} \begin{array}{|c|c|}\hline 2&1\\\hline 1&2\\\hline\end{array}$$

$$\begin{array}{|c|c|}\hline 1&2\\\hline 1&3\\\hline\end{array} \begin{array}{|c|c|}\hline 1&2\\\hline 1&3\\\hline\end{array} \begin{array}{|c|c|}\hline 2&1\\\hline 1&3\\\hline\end{array} \begin{array}{|c|c|}\hline 2&3\\\hline 1&2\\\hline\end{array} \begin{array}{|c|c|}\hline 2&2\\\hline 1&3\\\hline\end{array} \begin{array}{|c|c|}\hline 2&3\\\hline 1&3\\\hline\end{array} \begin{array}{|c|c|}\hline 2&3\\\hline 1&4\\\hline\end{array}$$

The morphogram $\begin{array}{|c|c|}\hline 1&1\\\hline 1&1\\\hline\end{array}$ of the places for 4 elementary contextures, i.e. contextures that are defined by a self-cycle, hence a kind of self-referentiality. Additionally, there are 2 contextures that are defined by the inter-relationship of the complexion MG[4].

But all 6 contextures are, despite their different localization, of the same type. They have the same structuration.

In contrast, the morphogram $\begin{array}{|c|c|}\hline 2 & 3 \\ \hline 1 & 3 \\ \hline\end{array}$ enables a contextural differentiation of its 4+2 contextures.

The question now is: *How to decompose the whole field of polycontexturality into contexturally similar sub-fields?*

Separtion in this context is not involved in any logical and set-theoretical disjunctions and divisions.

How it works was just demonstrated by the introduction of the 2 strategies of refinements.

Again, the scheme of the numbers of the refinements for Morph[4] is given by:

Morphogrammatic refinement for Morph[4]

StirlingS2 first refinement second refinement

$(1, 7, 6, 1) \Longrightarrow (1, 4+3, 6, 1) \Longrightarrow (1, 3+1, 3, 3+2+1, 1)$

There are other, less systematic classifications in use:
• junctional versus transjunctional morphograms
• palindromic versus non-palindromic morphograms.

■ Some possible interpretative concretizations

Supposed the national system is reasonably structured by just $4+2$ contextures.
What is the possible meaning of the morphograms ?

$MG(4, 1) = \begin{array}{|c|c|}\hline 1 & 1 \\ \hline 1 & 1 \\ \hline\end{array}$: this homogeneous pattern corresponds to the general ideology of one nation,

one family, one truth, as it is the program of *a* central government.

The morphograms MG(4, 2) are allowing a minimal differentiation that is distributed over the 4 loci. Say, there is one homogeneous ideology that allows just one difference. And that is the acceptance of one and only one reality in contrast to the rejected reality of a environment.

In fact, it is the case for the classical dichotomic (two-valued) logic, ontology and semiotics, and the politics based on it.

This difference occurs at different places.

It could be a difference in the educational system or even in the legal system as it appears in federalist countries.

Naturally, MG(4, 3) offers a further differentiation, distributed over the 4 places. This difference enable the complexion to draw a distinction in itself. It has therefore an internal and an external environment.

The difference might be with the legal, the educational and the language system.

But there is still some tolerance in the distribution of the contextures.

This changes with the organizational structure of MG(4,4). Each place is occupied by a different contexture.

This allows to differentiate between the self-differentiated system of MG[4,3] and its reflection by MG[4,4].

But morphograms are permutation invariant in respect of their valuations. Therefore, such an organization is not involved in any identifications with a systematic location of a value as it is necessary in hierarchical systems. Thus all permutations of MG(4,4) are morphogrammatically equivalent. Permutations of MG(4,3) and MG(4,2) are morphogrammatically equivalent in respect of their valuation, but not in respect of possible transpositions.

For example, `Permutations[{1, 2, 2, 3}]` delivers value permutations and but a change of locations too. Therefore, morphograms are equal under permutations only if their order is respected.

Hence, just the permutation {3,2,2,1} of {1,2,2,3} is morphogrammatically equivalent. All other permutations diver.

In other words, morphograms are not permutation invariant but invariant under valuation. Just for the cases MG(m,m) and MG(m,1) valuation and permutation of the values are coinciding.

```
Permutations[{1, 2, 2, 3}]
```

```
{{1, 2, 2, 3}, {1, 2, 3, 2}, {1, 3, 2, 2}, {2, 1, 2, 3}, {2, 1, 3, 2}, {2, 2, 1, 3},
 {2, 2, 3, 1}, {2, 3, 1, 2}, {2, 3, 2, 1}, {3, 1, 2, 2}, {3, 2, 1, 2}, {3, 2, 2, 1}}
```

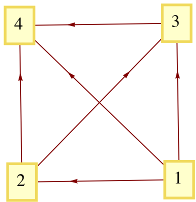
The morphogrammatic pattern MG[4,4] = $\begin{array}{|c|c|}\hline 2 & 3 \\ \hline 1 & 4 \\ \hline\end{array}$ has 24 non-redundant representations.

These are the permutations of the set {1,2,3,4}.

```
g = {1 → 2, 2 → 3, 3 → 4}

GraphPlot[g, VertexLabeling → True, DirectedEdges → True]
```



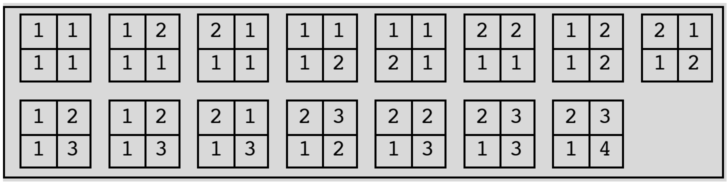
```
g = {1 → 2, 2 → 3, 3 → 4, 1 → 3, 2 → 4, 1 → 4 }

GraphPlot[g, VertexLabeling → True, DirectedEdges → True]
```



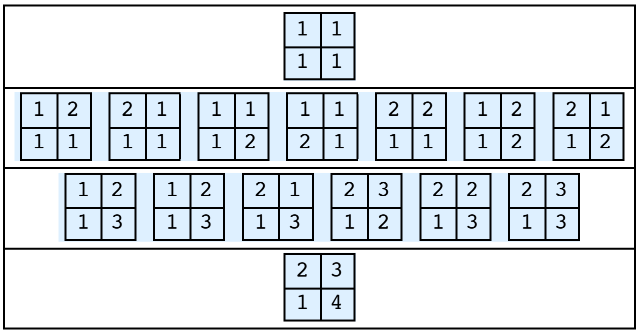

```
Permutations[{1, 2, 3, 4}]

{{1, 2, 3, 4}, {1, 2, 4, 3}, {1, 3, 2, 4}, {1, 3, 4, 2}, {1, 4, 2, 3}, {1, 4, 3, 2},
 {2, 1, 3, 4}, {2, 1, 4, 3}, {2, 3, 1, 4}, {2, 3, 4, 1}, {2, 4, 1, 3}, {2, 4, 3, 1},
 {3, 1, 2, 4}, {3, 1, 4, 2}, {3, 2, 1, 4}, {3, 2, 4, 1}, {3, 4, 1, 2}, {3, 4, 2, 1},
 {4, 1, 2, 3}, {4, 1, 3, 2}, {4, 2, 1, 3}, {4, 2, 3, 1}, {4, 3, 1, 2}, {4, 3, 2, 1}}
```

Patterns of possible decompositions and coalitions on the base of structural similarity

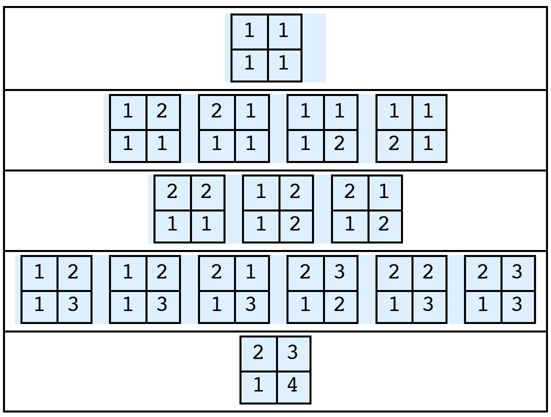
The undifferentiated field of MG[4] as a system



The field MG[4] with its Stirling differentiation



The field MG[4] with the first refinement of the Stirling differentiation



The field MG[4] with the second refinement of the Stirling refinement

$\begin{matrix}1&1\\1&1\end{matrix}$		
$\begin{matrix}1&2\\1&1\end{matrix}$	$\begin{matrix}2&1\\1&1\end{matrix}$	$\begin{matrix}1&1\\1&2\end{matrix}$
$\begin{matrix}1&1\\2&1\end{matrix}$		
$\begin{matrix}2&2\\1&1\end{matrix}$	$\begin{matrix}1&2\\1&2\end{matrix}$	$\begin{matrix}2&1\\1&2\end{matrix}$
$\begin{matrix}1&2\\1&3\end{matrix}$	$\begin{matrix}1&2\\1&3\end{matrix}$	$\begin{matrix}2&1\\1&3\end{matrix}$
$\begin{matrix}2&3\\1&2\end{matrix}$	$\begin{matrix}2&2\\1&3\end{matrix}$	
$\begin{matrix}2&3\\1&3\end{matrix}$		
$\begin{matrix}2&3\\1&4\end{matrix}$		

As it is obvious from the presented analysis, the decompositions, and in an inverse turn, the coalitions, are based on structural consideration only, and are thus not depending on semantics, traditions, believes or other identifiable cultural entities.

Morphograms are not just structural patterns but also morphic rules of morphogrammatic cellular automata.

Hence, the different classifications of morphogrammatic systems (structurations) are introducing a new classification of poly-morphic cellular automata too.

■ Towards an application

Philip J. Koopman, Jr , A Taxonomy of Decomposition Strategies Based on Structures, Behaviors, and Goals (1995) https://www.ece.cmu.edu/~koopman/decomp/decomp.html

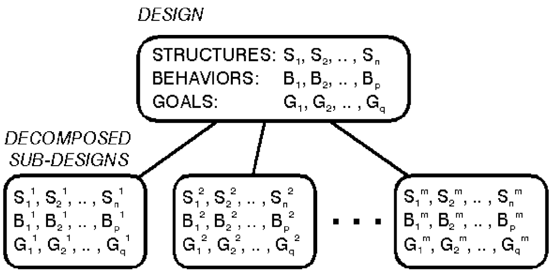
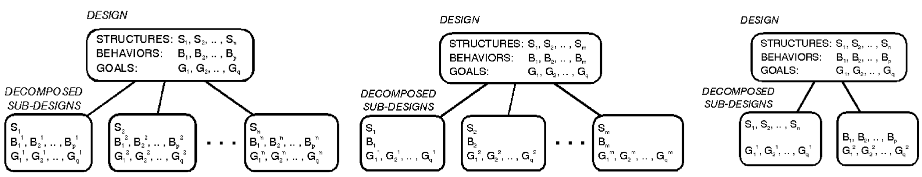


Figure 1 shows an ad-hoc decomposition in which a design has structures S_1, S_2, .. , S_n; behaviors B_1, B_2, .. , B_p; and goals G_1, G_2, .. , G_q. The multiple subdesigns *1* through *m* resulting from decomposition contain potentially modified versions of the original structure, behavior, and goal attributes. For example, if G_1 is a weight goal, $G_1{}^1$ through $G_1{}^m$ would be the weight goals for the subdesigns *1* through *m*. Similarly, S_1 might be a structure which is actually an assembly of components $S_1{}^1$ through $S_1{}^m$, and B_1 might be a behavior which emerges from an interaction of behaviors $B_1{}^1$ through $B_1{}^m$.

Different modi of modeling



□	categories	elements		
	structures	s1	s2	
	behaviors	b1	b2	
	goals	g1	g2	
	environments	e1	e2	
	observers	o1	o2	

Static modeling versus dynamic modeling

□	categories	elements		
	structures	s1	s2	
	behaviors	b1	b2	
	goals	g1	g2	
	environments	e1	e2	
	observers	o1	o2	

INTERCHANGE OF OBSERVERS
$\text{Reflection}_{o1-o2}[\text{Complexion}[(s_i, s_{i+1}), (b_i, b_{i+1}), (g_i, g_{i+1}), (e_i, e_{i+1}), (o_i, o_{i+1})]]$
$\Longrightarrow$
$\text{Complexion}[(s_{i+1}, s_i), (b_{i+1}, b_i), (g_{i+1}, g_i), (e_{i+1}, e_i), (o_{i+1}, o_i)]$

INTERCHANGE OF OBSERVER and environment
$\text{Reflection}_{o1-e2}[\text{Complexion}[(s_i, s_{i+1}), (b_i, b_{i+1}), (g_i, g_{i+1}), (e_i, e_{i+1}), (o_i, o_{i+1})]]$
$\Longrightarrow$
$\text{Complexion}[(s_{i+1}, s_i), (b_{i+1}, b_i), (g_{i+1}, g_i), (o_{i+1}, e_i), (e_{i+1}, o_i)]$

$\text{Refl}_{(o1-e2)}[x, (e_i, e_{i+1}), (o_i, o_{i+1})] \Longrightarrow [x, (e_i, o_{i+1}), (o_i, e_{i+1})]$

▪ Mediation of classifications

As much as the undifferentiated system of MG[4] represents the ontological reality of the taxonomy, all the further classifications, Stirling and its refinement, are just thematizations of a morphogrammatic design.

Therefore, it is reasonable to conceive them as holding simultaneously as thematizations, and are thus enabled to be mediated to a complexion of different, i.e. discontextural thematizations of a possible structuration, determined by the complexity of 4 and the complication of 1.