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NON-ARISTOTELIAN LOGICS

One of the deep-seated cravings of the human mind seems to be a desire for something permanent, for something eternally the same, changeless and absolute. Of such an object of reverence it can then be exclaimed, "Oh Thou Who Changest Not !" This worship of the changeless is by no means confined to religion, for, in the field of philosophy, as is known, no less a person than Plato argued that change is a mark of imperfection. In science this idea has also manifested itself. In Newtonian physics, for example, space, time, and matter are conceived as absolutes. Perhaps the physicist's demand for an absolute even appears in the relativity physics which has replaced Newtonian mechanics, for the principle of the constancy (absoluteness) of the velocity of light is an integral part of Einstein's system.

If one were to seek for a psychological explanation of this tendency of the mind to exalt the permanent and invariant, it might be found that since earlier (pre-scientific) man had to adapt himself to a changing environment, he did what all men do who develop an inferiority complex in the face of overmastering external forces: he created in his imagination an ideal world incorruptible by moths and rust. It appears to be true in religion that the "eternal verities" embody those absolutes which compensate for the defects of the physical environment. And in philosophy, in the case of the Platonic preference for the changeless, it may be, as one student of mathematics suggested, that this was so because the Greeks did not have a mathematical technique, the calculus of Newton and Leibniz, to deal with a world characterized by various modes and rates of change. And so, the argument continues, some of the Greeks (e.g. Zeno, with his paradoxes of motion) came to despise nature philosophically because they could not master it intellectually. Such psychoanalytic explanations are rarely satisfactory, but whatever the correct account may be, it is true that the Platonic metaphysics has exercised an important influence an the subsequent philosophies of Western European culture. The latest ramification of this influence is evidenced in the Platonic realism of the theistic view of Jeans, whose supreme Architect of the Universe knows more mathematics than Plato ever thought of, or could have wished.

In mathematics and logic the demand for something absolute has been no less insistent. This faith in the eternal certainty of something permanent was voiced by Lewis Carroll, that otherwise subtle critic of conventional habits of thought, when he stated that the charm of pure mathematics "lies chiefly in the absolute certainty of its results for that is what, beyond all mental treasure, the human intellect craves for. Let us be sure of something!" More recently, in commenting on the fact of the disappearance of absolutes from natural science, a mathematician exclaimed: "Thank God, mathematicians still have the law of contradiction !"

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This unwillingness to surrender the belief in something permanent and unchanging, abiding amidst universal flux, may by some be regarded as an example of human conservatism. Very few thinkers (only the extreme skeptics, who profess to believe nothing – not even that they believe that they believe nothing !) have sought to escape from this habit, or nullify its desire. Logic, like mathematics, dealing with the supposedly permanent and necessary forms of thinking, has also, until recently, shown a strong inclination towards intellectual conservatism. This conservatism was illustrated by the demand that logic search out those universal and invariant validating forms of inference which the human mind must employ if it is to think correctly, or even think at all. These forms may then be given concrete interpretations in the separate disciplines (sciences) in which logic may be employed. According to traditional logic the most fundamental regulatory principles are the so-called "laws of thought," presupposed in all valid thinking, whether deductive or inductive. The specific recognition of these principles is generally credited to Aristotle, and the acceptance of them is a part of the Aristotelian tradition in logic. *It is for this reason that any abandonment of the three laws of thought would constitute a non-Aristotelian logic*.

And now let us state these three laws of thought, as follows: (a) the *law of identity*; (b) the *law of contradiction* (or *non-contradiction*) ; (c) the *law of excluded middle*. As Professor C. I. Lewis[1] states : "From Aristotle down, the laws of logic have been regarded as fixed and archetypal ; and as such they admit of no conceivable alternatives. Often they have been attributed to the structure of the universe or to the nature of human reason; and in general they have been regarded as providing an Archimedean fixed point in the realm of thought." So deeply rooted is this tradition that any challenge to the view is likely to be looked upon as sacrilegious, and if by any chance such an attack should prove successful, this would appear to some logicians and mathematicians to mark the downfall of science and of intellectual system. As one Person has said: to talk about non-Aristotelian logic is like talking about illogical logic – a contradiction in terms. Of course, the extent to which various systems of philosophy have made use of the laws of thought has varied, but the extreme instance of the attempt to base a metaphysics an a logic is seen in the case of Fichte, who attempted to deduce an entire philosophy from the law of identity.

Until modern times the possibility of a non-Aristotelian logic was not taken seriously. But so deeply has the virus of criticism penetrated into the body of modern thought that the thing which our ancestors never considered as a possibility now has occurred. Now the last citadel of absolutism is being attacked. The three laws of thought mark the final battle line, and the fate of absolutism will be determined by the outcome. If the laws of thought should fall, then the most profound modification in human intellectual life will occur, compared to which the Copernican and Einsteinian revolutions are but sham battles. That famous river of Heraclitus, into which no man could step twice, then becomes a super-Protean flux into which one cannot even step once! A newer and more universal relativity

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[1]   "Alternative Systems of Logic," Monist, 1932, Vol. XLII, pp. 481-507.

is appearing which threatens to abolish the old landmarks, according to which we have hitherto set out intellectual compasses and gotten our spiritual bearings. But, you may be sure, before the fortress falls there will be another decisive battle of the world. For, as Professor F. A. Lindemann[2] says "the conventions and sanctions which bolstered up Euclidian space are as nothing to those which will be invoked to maintain inviolable the sanctity of logic."

The attacks upon the Aristotelian tradition have come from several different sectors along the battle line, and have not come simultaneously. Different motives are at work in different cases. The sources of non-Aristotelian logic may be classified into three groups, as follows

1. Evolutionary philosophy:
   A. Hegel's attack an the law of excluded middle.
   B. Dewey's dynamic logic.

2. Mathematics:
   A. Brouwer's criticisms from the point of view of the transfinite.
   B. The substitution of a "many-valued" logic for the Aristotelian two-valued logic, by Łukasiewicz and Tarski, and C. I. Lewis.

3. Physics:
   A. Count Korzybski's attack an the law of identity.

The common view of these laws of thought is that they are laws of *things* as well as laws of *thought*. To bring out this double reference (objective-subjective), I shall interpret the laws under two heads, ontological and epistemological, as laws of physical reality and as laws of mental operations. First, however, let us indicate how these laws are symbolized, respectively, in the Boole-Schroeder algebra, and in the logic of propositions:

		Calculus of Classes	*Calculus of Propositions*
1. *Law of Identity*	:	$a < a$	$\vdash . \; p \supset p$
2. *Law of Contradiction*	:	$a\,a' = 0$	$\vdash . \; p \supset \sim(\sim p)$
3. *Law of Excluded Middle*	:	$a + a' = 1$	$\vdash . \; p \supset \sim p \vee p$

Unlike the defenders of traditional logic, most modern symbolic logicians hold that these three laws are no more, and no less, important than the other logical primitives necessary to deductive system.

Now we consider the interpretations of these three "laws."

	As a Law of Reality	As a Law of Thought
Law of Identity, $A = A$	1. Whatever is, is. 2. A thind is what it is.	1. A word means what it means. 2. The meaning of a term must remain constant in any given discourse.
Law of Contradiction, $A </ A'$	1. A thing is not what it is not.	1. A word dous not mean what it does not mean.

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[2]   In his essay an "Physics" in the coöperative volume, *The Mind*.

	2. Whatever does not exist is non-existent.	2. Two negatives make an affirment.
Law of Excluded Middle A < B ∨ B'	1. A thing either exists or it does not exist. 2. An existent thing has a certain property or it does not. 3. A thing cannot have contradictory properties.	1. A proposition is either true or false. 2. Two contradictory propositions cannot both be true. 3. A class (or term) is either include in another class, or it is not.

It will be noticed that the law of excluded middle (L.E.M.) does not exclude the following possibilities

1. The same verbal proposition may be both true and false at the same time, but in different senses.

2. The same proposition may, in the same sense, be true (or false) at one moment, and false (or true) at another.

We may not know which of two contradictory propositions is true at any given instant of time, but one or the other necessarily is. Here is an illustration taken from J. S. Mill

(1) Matter is infinitely divisible.
(2) Matter is not infinitely divisible.

One or the other of these is true, though which it is we cannot say.

These laws have been regarded as so fundamental that they have been used as tests of the very existence of propositions. Even Bertrand Russell has declared that a proposition is a statement that is either true or false, and if it is neither, it is nonsense. To illustrate: applying the law of excluded middle, "A is either B or not-B," one might ponder whether it would be correct to say, "virtue is either square or not-square." Traditional logic (i.e., logic which accepts L .E.M.) would dispose of this statement by declaring that since it is neither true nor false, it is nonsense. So much by way of explanation.

Aristotle apparently did not formulate the law of identity. It might, however, be regarded as implicitly contained in the following statement: "Everything that is true must in every respect agree with itself."[3] But Aristotle does definitely formulate the principles underlying the law of contradiction and the law of excluded middle. A statement of the L.E.M. is found in the *Metaphysics* (Bk. III. Ch IV) . Here Aristotle argues that the most certain principle of all is that regarding which it is impossible to be mistaken. Such a principle is found in this, that "It is impossible that the same predicate can both belong and not belong to the same object at the same time, and in the same sense." In another place[4] he states: "If it is true to say that a thing is white, it must necessarily be white: if the reverse proposition is true, it will of necessity not be white. Again, if it is white, the

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[3]   *Anal. Priora*, 47a, 9.
[4]   *De Interpretatione*, 18b, 1-5.

proposition stating that it was white was true; if it was not white, the proposition to the opposite effect was true .... It may therefore be argued that it is necessary that affirmations or denials must be either true or false."

And now let us consider the criticisms of these laws which have appeared in the history of logic.

It would be difficult to state who was the first thinker to challenge any of the laws of thought, but it is possible that this doubtful honor belongs to Heraclitus, the evolutionist. In discussing the law of contradiction Aristotle says that "it is impossible for anyone to believe the same thing to be and not to be, *as some think Heraclitus says*" (Metaphysics, 1006a, 7. Italics mine.) But the first clearcut case of the denial of a law of thought comes again in connection with an evolutionary viewpoint. I refer here to Hegel, whose denial of the L.E.M. is part of his idealistic doctrine of change. For Hegel reality is process, a dynamic movement in which things grow, change, and pass away. Each thing passes beyond itself and becomes what it was not. A thing is a synthesis of opposites, an organic unity of differences. In this dialectical process, things (and thoughts too) start from (a) a *thesis*, and by an act of self-negation pass to the opposite pole of (b) the *antithesis*, from which the movement then passes into the union of the opposites, (c) the *synthesis*. And so it follows for Hegel[5] that "contradiction is the moving principle of the world: and it is ridiculous to say that contradiction is unthinkable." Instead of speaking of the maxim of excluded middle (which is the maxim of abstract understanding) we should rather say: "everything is opposite." In attacking this law, Hegel tries to show that A is both A and non-A, but critics point out that Hegel sometimes confused *contradictories* with *contraries*.

Since for Hegel the logic of reality and the logic of thought are the same, the notion of the concrete universal is fundamental. H. Wildon Carr maintained[6] that "it would be difficult to name a more perfect illustration of the concrete universal of Hegel than is offered to us by the modern electrical theory of matter." Carr here has in mind the concept of opposites kept apart and held together in a state of equilibrium in a field of force.

It is a curious fact that Hegel's system is at once one of the most obscure and most influential philosophies in all history. One needs to recall here only the influence of the Hegelian dialectic upon the development of the "dialectical materialism" of Karl Marx to see how apparently innocent philosophical ideas may assume momentous practical importance. But whatever the metaphysical and social effects of Hegelianism may have been, Hegel's criticisms of the laws of thought failed to bear fruit. Bosanquet's[7] Interpretation of Hegel's notion of the concrete universal in terms of the principle of *identity-in-difference* is perhaps the outstanding example of the historical influence of Hegel's logic, at least until the present

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[5]   *The Logic of Hegel*, translated by W. Wallace, 1892, p. 223.

[6]   *A Theory of Monads*, p. 284.

[7]   B. Bosanquet, *Logic*, 1911, Vol. I, p. 26 and *passim*.

time.[8] Modern opponents of the law of identity, Korzybski *et al.*, will need to consider carefully this principle before they abandon the law of identity in favor of a relativistic-organic-evolutionary view, for the law of identity, interpreted in terms of the principle of identity-in-diference, does not exclude change. As Hegel would say, the concrete universal is not a self-identical thing, but a form realizing itself in a historically changing system of things.

And now we consider the next heretic who has questioned the logical necessity of the traditional laws of thought, namely, the Dutch mathematician, L.E.J. Brouwer.[9]

Brouwer does not deny that the L.E.M. applies to all processes of thought concerned with finite classes of objects. But just as we recognize that the ordinary rules of arithmetic do not hold when we are dealing with transfinite aggregates (e.g., the rule that the whole is greater than any of its parts breaks down when we treat a line as composed of an infinite number of points) so, Brouwer holds, the law of *tertium non datur* (L.E.M.) must be abandoned. But first let us state the general thesis underlying this revolutionary view.

Brouwer is known as an *intuitionist* in mathematics only those things are accepted which can be recognized in consciousness as true. According to this view, in order that a proposition can be accepted it must be known either a priori to be true, or its truth must be capable of demonstration. But in the case of mathematical demonstrations involving an infinite sequence of operations the truth of any such proposition which is asserted is neither a priori evident nor capable of demonstration. Thus the proof that a certain number exists, or the question of whether the square root of a certain number is rational or irrational, may depend upon the use of an infinite construction, or unending mathematical induction. In such cases whatever assertion may be made must be justified by proof, but such proof cannot be exhibited, since the consciousness of an infinite sequence is impossible. If such an intuitionistic view were applied in all cases this would mean that Brouwer would not agree, e.g., that it is either raining or not raining, until he had looked to see ! Since, in the realm of the transfinite, one cannot intuit, Brouwer refuses to believe that propositions, the truth or falsity of which are in question, are subject to the I..E.M. Mathematical intuitionism, holding that a thing exists only after it has been exhibited, is thus unable to justify much of ordinary mathematics.

The next step in the development of a non-Aristotelian logic, as E. R. Hedrickl[10] has pointed out, is to show that the abandonment of the L. E. M. is not necessarily

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8 An interesting reinterpretation of the Hegelian dialectic in terms of mathematical logic is given by J.B. Burke in his book, *The Emergence of Life*. A follower of John Dewey, Dr. B. Bogoslovsky, seeks to revise the laws of thought to take care of the dynamic aspect of nature, and in doing so pays his compliments to Hegelian logic. For a criticism of Bogoslovsky's book, *The Technique of Controversy*, see A. E. Avey's paper, "The Law of Contradiction: Its Logical Status," *J. of Philos.*, 1929, Vol. 26, pp. 519-525.

9 See A. Dresden's paper, "Brouwer's Contribution to the Foundations of Mathematies," *Bull. Amer. Math. Soc.*, 1924, Vol. 30, pp. 31-40.

10 Cf, "Tendencies in the Logic of Mathematics," *Science*, 1933, Vol. 77, pp. 335-343.

tied up with Brouwer's intuitionism. In other words, one may admit that the L.E.M. is not a necessary part of logic or mathematics, whether one be an intuitionist or not.

This step was taken when some logicians abandoned the thesis that propositions can have only *two values* – truth and falsity. All the traditional systems of logic are two-valued logics. The Boole-Schroeder algebra of logic is a two-valued logic; even the Russell-Whitehead system of mathematical logic, which claimed to free itself from the limitations of the Aristotelian system, is two-valued, in the sense that propositions are considered to be true or false. The actual business of developing a logic in which this law is explicitly disregarded was carried through by two Polish investigators, Łukasiewicz[11]] and Tarski, who developed a three-valued logic, with a trichotomy of implications, in terms of truth, falsity, and uncertainty. From this it is clear that if we define an Aristotelian logic as a two-valued logic, then any logic with more than two truth values – three, four, or n-values-may be termed a non-Aristotelian logic. In a recent book,[12]] C. I. Lewis has developed a logic in terms of the conception of "strict implication," as against the "material implication" of the Whitehead-Russell *Principia Mathematica*. Here Lewis expounds the notion of multiple truth-value systems. Lewis declares[13]] that there are no "laws" of logic, in the sense that there are laws of physics. This view, similar to Hilbert's theory of mathematics, rests an the thesis that deductive system is the manipulaion of meaningless symbols according to arbitrarily selected rules of operation. As Lewis says, the source of necessary truth is in definitions, arbitrarily assigned. "Thus the tautology of any law of logic is merely a case of the general principle that what is true by definition cannot conceivably be false: it merely explicates, or follows from, a meaning which has been assigned, and requires nothing in particular about the Universe or the facts of nature. Thus any logical principle (and, in fact, any other truth which can be certified by logic alone) is tautological in the sense that it is an analytic proposition." If this view is correct, we must agree with Lewis when he states[14]] that the L.E.M. is not "writ in the heavens," but rather "reflects our stubborn adherence to the simplest of all possible modes of division, and our predominant interest in concrete objects as opposed to abstract concepts."

It must not be supposed from this that the defenders of the Aristotelian system are put to rout. Far from it! The reply may take several forms. In the first place the Aristotelians may argue that the view is self-refuting, in the sense that any one who denies the L.E.M. presupposes it; for if you say that the L.E.M. is not true, you are assuming that it is either true or not true (false?). Or the following[15]] ingenious argument, attributed to Aristotle, may be repeated: "If a proposition is neither true nor false, let us call it doubtful; but then if the Law of Excluded

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[11]   "Philosophische Bemerkungen zu mehrwertigen Systemen des Aussagenkalküls," by Łukasiewicz, J and Tarski, A., *Comptes Rendus Soc. de Varsovie*, 1930, Vol. 23, 111, pp. 51-77.

[12]   *Symbolic Logic*, by Lewis, C. I., and Langford, C. H.

[13]   Op. cit., p. 211.

[14]   *Monist*, Oct. 1932.

[15]   Quoted from F. P. Ramsey's book, *The Foundations of Mathematics*, p. 66.

Middle be false, it need not be either doubtful or not doubtful, so that we shall have not merely three possibilities but four, that it is true, that it is false, that it is doubtful, and that it is neither true, false, nor doubtful. And so an *ad infinitum*." If this is not sufficient to show the necessity for the L.E.M., it can be pointed out that in the Lewis system, which claims to establish a new logic bearing the same relation to the old logic which non-Euclidian geometry bears to Euclidian, there is an analogue to the old L.E.M. in the sense that no matter how many-valued your logic may be, a given proposition either possesses one of those values or it does not. This analogue is similar to what Paul Weiss[16] has termed the *excluded n+1*.

And now we come to the last possible attack upon the laws of thought.

If one were to ask which of the three laws was the most impregnable, the law of identity would undoubtedly be recommended for that position. And yet an attack upon that law has been made. In the main the doubt arises from unusual phenomena in physics. Thus when modern physics states that light and electrons are both undulatory and corpuscular, this seems to violate the law of identity, according to which a wave is a wave and a corpuscle is a corpuscle, and never the two can be one. Aside from such difficulties, which have suggested to some physicists that the fault of physics lies not so much in the stars and electrons as in ourselves, we have the proposal of Count Alfred Korzybski to develop a non-Aristotelian system of science in which the law of identity is conspicuous by its absence. Count Korzybski's book, *Science and Sanity*, is the first volume of the "International Non-Aristotelian Library," the aim of which is to solve all our problems by the introduction of a new method of thinking. Without attempting to state the views of this author, we will consider, quite independently, the possible criticisms of the law of identity, interpreted first as a law of thought and then as a law of reality.

The view here presented is that the law of identity, as a "law" of thinking, is both capable of being violated and incapable of beine violated – in different senses, however. The law of identity has two meanings: (a) it asserts that a word (or proposition) means what it means, and (b) that the meanings of our terms should remain constant in any given "universe of discourse." In the first sense the law of identity is a *descriptive* law, and in the second sense it is a *normative* law, or regulative principle. The law of identity is a descriptive law in the sense that at any given time one, and only one; meaning can be assigned to any specified term. This psychophysical impossibility of simultaneous duplicity of meaning, as the writer[17] has suggested, may rest upon some such physiological principle as the impossibility of the simultaneous innervation of reciprocal neuro-muscular patterns. If the bodily process underlying the thought of "A" is in process, the bodily process underlying the thought of "non-A" cannot also be in process at that same time. In this sense it is true that for "normal" persons the law of identity is incapable of being violated, though, to be sure, one might imagine that some sort of biological mutation (e.g., the addition of another supragranular layer to the cortex) might make it possible to think of both "A" and "non-A" simultaneously,

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16   "The Nature of Systems," *Monist*, 1929, Vol. 39, p. 283.
17   "Biological Relativity," *J. of Philos*., 1931, Vol. 28, p. 714.

or instantaneously assign two contradictory meanings to the same term. Perhaps this simultaneous innervation of alternative (exclusive) patterns, that ordinarily function reciprocally, may actually occur in dreams. (At least a friend informs me that in dreams he is both himself and not himself simultaneously.)

According to the second meaning of the law of identity, as a law of thought, we must recognize that in the course of time it is possible to change the meanings of terms. We have new experiences; the nervous system is constantly in flux; and so words acquire new meanings. If it is true that the organism is never in exactly the same state twice, it is doubtful whether we can ever think exactly the same thought twice. Thus the law of identity, incapable of violation at any given moment, is capable of violation when time does its work. Logic recognizes that we are free to redefine our terms, but then we must also revise everything we have said while using such terms.

And now for the law of identity as a law of reality.

The law states that (a) *at any given time*, and (b) *from some selected point of view*, a thing remains identical with itself in all respects. This does not exclude the possibility that a thing may, in the course of time, cease to be what it was and become what it is (something else – e.g., the stockings of Sir John Suckling). The precise point of transition at which "A" ceases to be "A" and becomes "non-A" involves a nice problem of definition. It is also true, as Professor Whitehead states, that "nature at an instant" is an artifact, and that it requires time for a thing (even a hydrogen atom) to be itself. But this need not negative the law of identity. Again, as Korzybski has pointed out, difficulties arise when we say that a thing is identical with itself in *all* respects, especially if "all" means an infinite number of respects. Moreover, it is also true that a thing is what it is (to a limited extent) because of the environment it is in, and that "A" (e.g., an organism and an electron) may behave in one way in one environment and in another in some other environment. So that what a thing is "in itself" we may never know, unless we can specify *all* the properties a thing has, potentially and actually, in *all* possible environments. But then we again face the difficulty which Brouwer recognizes in the case of the L. E. M. as applied to infinite sets.

The final difficulty in applying the law of identity arises in connection with the circumstance that a thing (e.g., a star) viewed from one "frame of reference" may appear to possess certain properties (i.e., appear red) , and viewed from another may appear to possess other properties (i.e., appear blue, in the case of the star) . The world as we know it is subject to (a) a relativity due to physical motion, and (b) a relativity due to the biological constitution of the perceiving organism, and (c) a relativity due to the cultural status of the society of which the observer (scientist) is a member. Only in the case of ourselves are we permitted (through introspection) to view a thing from its "own" point of view. In a sense a brain can know itself, from its own point of view, but a star can never know what it is from its own point of view. A star is a star, of course, but what it is in itself we can never know. We can know an object, "A," only in terms of what it appears to us to be, in the given environment of a certain "cosmic epoch."

From all this it turns out that the law of identity as a law of physical reality occupies the same status as the equally famous principle of the uniformity of nature. As laws of reality both are incapable of being proved, and any attempt at empirical verification begs the question, or presupposes the point to be proved. But as a law of thought the law of identity is to be accepted, both as a descriptive and as a normative principle. We are here face to face with what H. M. Sheffer calls the "logo-centric predicament," or what Marvin Farber calls the "logic-centric predicament." We are reasoning about logic, and in doing so we are employing logic, and the logic we employ cannot itself be subject to criticism by itself. If we reason about identity, we assume that the meaning of the word identity remains identical with itself. In this sense the law of identity is a presupposition of all reasoning. But as already admitted, some future biological mutation may alter this, though what the superlogic of the coming superman may be we cannot now anticipate. Only a superman could duplicate the reasoning of a superlogician, and I, for one, do not claim to be a superman. (In this Sense we are all "humanists.") This, however, does not invalidate the claim of those who would substitute the many-valued logic for the two-valued logic based upon the Aristotelian law of excluded middle. The claims for this type of non-Aristotelian logic will have to be decided an their own merits, and no attempt is made in the present paper to prejudge this issue.