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Catching Transjunctions

Steps towards an emulation of polycontextural transjunctions in memristic systems

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Abstract

How to understand Gunther's idea of transjunctional operators? And how to implement transjunctions in memristive systems?

1. Bifunctorial approach

1.1. Distribution of junctions

Example: MED $\left(\left[\text{Log}\right],\ \langle \text{et} \rangle\right)^{(3)}$:

$$\left(\mathcal{U}_1 \bigcap_{1.2} \mathcal{U}_2\right) \bigcap_{2.3} \mathcal{U}_3 = \emptyset$$

$$\mathcal{U}^{(3)} = \left(\mathcal{U}_1 \amalg_{1.2} \mathcal{U}_2\right) \amalg_{1.2.3} \mathcal{U}_3 :$$

$$\mathcal{U}_i = \left\{\left[\text{Log}\right]_i,\ \left[\text{et}\right]_i\right\},\ i = 1, 2, 3$$

$$\begin{aligned} \text{cod}(X1) &= \text{dom}(X2), \\ \text{dom}(X1) &= \text{dom}(X3), \\ \text{cod}(X3) &= \text{cod}(X3). \end{aligned}$$

$$\begin{bmatrix} [B]_1 & [B]_2 & [B]_3 \\ [A]_1 & [A]_2 & [A]_3 \end{bmatrix} :$$

$$\begin{pmatrix} \begin{pmatrix} [A]_1 \text{ et}^{1.0.0} [B]_1 \\ \amalg_{1.2.0} \\ [A]_2 \text{ et}^{0.2.0} [B]_2 \end{pmatrix} \\ \amalg_{1.2.3} \\ [A]_3 \text{ et}^{0.0.3} [B]_3 \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} [A]_1 \\ \amalg_{1.2.0} \\ [A]_2 \end{pmatrix} \\ \amalg_{1.2.3} \\ [A]_3 \end{pmatrix} \text{et}_{1.2.3} \begin{pmatrix} \begin{pmatrix} [B]_1 \\ \amalg_{1.2.0} \\ [B]_2 \end{pmatrix} \\ \amalg_{1.2.3} \\ [B]_3 \end{pmatrix}$$

Example : MED $\left(\left[\text{Log}\right],\ <\text{trans et et}>\right)^{(3)}$:

$$\left(\mathcal{U}_1 \bigcap_{1.2} \mathcal{U}_2\right) \bigcap_{2.3} \mathcal{U}_3 = \emptyset$$

$$\mathcal{U}^{(3)} = \left(\mathcal{U}_1 \amalg_{1.2} \mathcal{U}_2\right) \amalg_{1.2.3} \mathcal{U}_3 :$$

$$\mathcal{U}_i = \left\{\left[\text{Log}\right]_i, \left[\text{et}\right]_i\right\},\ i = 1, 2, 3$$

$$\begin{aligned} &\text{cod}(X1) = \text{dom}(X2), \\ &\text{dom}(X1) = \text{dom}(X3), \\ &\text{cod}(X3) = \text{cod}(X3). \end{aligned}$$

$$\begin{bmatrix} [B]_1 & [B]_2 & [B]_3 \\ [A]_1 & [A]_2 & [A]_3 \end{bmatrix} :$$

$$\left(\left(\begin{array}{c} [A]_1 \circ_{1.1} [B]_1 \\ \amalg_{1.2.0} \\ \left([A]_2 \circ_{2.2} [B]_2\right) \diamond_{2.1} \left([A]_1 \circ_{2.1} [B]_1\right) \end{array}\right) \begin{array}{c} \\ \amalg_{1.2.3} \\ \left([A]_3 \circ_{3.3} [B]_3\right) \diamond_{3.1} \left([A]_1 \circ_{3.1} [B]_1\right) \end{array}\right) = \left(\left(\begin{array}{c} [A]_1 \\ \amalg_{1.2.0} \\ [A]_2 \diamond_{2.1} [A]_1 \end{array}\right) \begin{array}{c} \\ \amalg_{1.2.3} \\ [A]_3 \diamond_{2.1} [A]_1 \end{array}\right) \left[\begin{array}{c} \circ_{1.1} - \\ \circ_{2.1} \circ_{2.} \\ \square \\ \circ_{3.1} - \circ \end{array}\right.$$

Distributivity of et and vel

$$\text{DISTR}\left(\left[\text{Log}\right],\ <\text{et},\ \text{or}>\right)^{(3)}:$$

$$\left(\mathcal{U}_1 \bigcap_{1.2} \mathcal{U}_2\right) \bigcap_{2.3} \mathcal{U}_3 = \emptyset$$

$$\mathcal{U}^{(3)} = \left(\mathcal{U}_1 \amalg_{1.2} \mathcal{U}_2\right) \amalg_{1.2.3} \mathcal{U}_3 :$$

$$\mathcal{U}_i = \left\{\left[\text{Log}\right]_i,\ \left[\text{et},\ \text{or}\right]_i\right\},\ i = 1, 2, 3$$

$$R_0 : \text{DISTR} : A \text{ et } \left(B \text{ or } C\right) = \left(A \text{ et } B\right) \text{ or } \left(A \text{ et } C\right)$$

$$\begin{bmatrix} [C]_1 & [C]_2 & [C]_3 \\ [B]_1 & [B]_2 & [B]_3 \\ [A]_1 & [A]_2 & [A]_3 \end{bmatrix} :$$

$$\left(\begin{array}{c} \left(\begin{array}{c} [A]_1 \text{ et}^{1.0.0} \left([B]_1 \text{ or}^{1.0.0} [C]_1\right) \\ \amalg_{1.2.0} \\ [A]_2 \text{ et}^{0.2.0} \left([B]_2 \text{ or}^{0.2.0} [C]_2\right) \end{array}\right) \\ \amalg_{1.2.3} \\ [A]_3 \text{ et}^{0.0.3} \left([B]_3 \text{ or}^{0.0.3} [C]_3\right) \end{array}\right) =$$

$$\left(\begin{array}{c} \left(\begin{array}{c} [A \text{ et } B]_1 \\ \amalg_{1.2.0} \\ [A \text{ et } B]_2 \end{array}\right) \\ \amalg_{1.2.3} \\ [A \text{ et } B]_3 \end{array}\right) \text{or}_{1.2.3} \left(\begin{array}{c} \left(\begin{array}{c} [A \text{ et } C]_1 \\ \amalg_{1.2.0} \\ [A \text{ et } C]_2 \end{array}\right) \\ \amalg_{1.2.3} \\ [A \text{ et } C]_3 \end{array}\right)$$

1.2. Transjunctions

$$\left(X <> \wedge \wedge Y\right) - \textbf{scheme}: \begin{bmatrix} <>_{1.1} & <>_{2.1} & <>_{3.1} \\ - & \wedge_{2.2} & - \\ - & - & \wedge_{3.3} \end{bmatrix}$$

$$\left(\begin{pmatrix} [A]_1 \circ_{1.1} [B]_1 \\ \sqcup_{1.2.0} \\ \left([A]_2 \circ_{2.2} [B]_2\right) \diamond_{2.1} \left([A]_1 \circ_{2.1} [B]_1\right) \end{pmatrix} \\ \sqcup_{1.2.3} \\ \left([A]_3 \circ_{3.3} [B]_3\right) \diamond_{3.1} \left([A]_1 \circ_{3.1} [B]_1\right)\right) =$$

$$\left(\begin{pmatrix} [A]_1 \\ \sqcup_{1.2.0} \\ [A]_2 \diamond_{2.1} [A]_1 \end{pmatrix} \\ \sqcup_{1.2.3} \\ [A]_3 \diamond_{2.1} [A]_1\right) \begin{bmatrix} \circ_{1.1} & - & - \\ & & \\ \circ_{2.1} & \circ_{2.2} & - \\ & \square & \\ \circ_{3.1} & - & \circ_{3.3} \end{bmatrix} \left(\begin{pmatrix} [B]_1 \\ \sqcup_{1.2.0} \\ [B]_2 \diamond_{2.1} [B]_1 \end{pmatrix} \\ \sqcup_{1.2.3} \\ [B]_3 \diamond_{2.1} [B]_1\right)$$

11 Tableaux rules for (X trans and and Y)

$$\frac{t_1\ X <> \wedge\wedge Y}{\begin{array}{c} t_1\ X \\ t_1\ Y \end{array}} \qquad \frac{f_1\ X <> \wedge\wedge Y}{\begin{array}{c} f_1\ X \\ f_1\ Y \end{array}}$$

$$\frac{t_2\ X <> \wedge\wedge Y}{\begin{array}{c|c} t_2\ X & f_1\ X \\ t_2\ Y & f_1\ Y \end{array}} \qquad \frac{f_2\ X <> \wedge\wedge Y}{f_2\ X \mid f_2\ Y \;\Big\|\; \begin{array}{c|c} f_1\ X & t_1\ X \\ t_1\ Y & f_1\ Y \end{array}}$$

$$\frac{t_3\ X <> \wedge\wedge Y}{\begin{array}{c||c} t_3\ X & t_1\ X \\ t_3\ Y & t_1\ Y \end{array}} \qquad \frac{f_3\ X <> \wedge\wedge Y}{f_3\ X \mid f_3\ Y \;\Big\|\; \begin{array}{c|c} f_1\ X & t_1\ X \\ t_1\ Y & f_1\ Y \end{array}}$$

$$\mathbf{val}\left(X <> \wedge \wedge Y\right): \begin{pmatrix} 1 & 3 & 3 \\ 3 & 2 & 3 \\ 3 & 3 & 3 \end{pmatrix} = \begin{bmatrix} T_{1.3} & F_{2.3} & F_3 \\ F_{2.3} & F_1 T_2 & F_2 \\ F_3 & F_2 & F_{2.3} \end{bmatrix}:$$

$$\left(\text{Junctors, transjunctors, transposition, mediation}\right): \; < \left(<> \wedge \wedge\right), \; \diamond, \; \amalg >$$

$$\mathbf{true}\left(X <> \wedge \wedge Y\right):$$

$$\left(\begin{pmatrix} t_1\left(\left[A_1\right] <>_{1.1} \left[B_1\right]\right) \\ \amalg_{1.2.0} \\ t_2\left(\left(\left[A_2\right] \wedge_{2.2} \left[B_2\right]\right) \diamond_{2.1} f_1\left(\left[A_1\right] \wedge_{2.1} \left[B_1\right]\right)\right) \\ \amalg_{1.2.3} \\ t_3\left(\left(\left[A_3\right] \bigwedge_{3.3} \left[B_3\right]\right) \diamond_{3.1} t_1\left(\left[A_1\right] \wedge_{3.1} \left[B_1\right]\right)\right) \end{pmatrix}\right) =$$

$$\left(\begin{pmatrix} t_1\left[A_1\right] \\ \amalg_{1.2.0} \\ t_2\left[A_2\right] \diamond_{2.1} f_1\left[A_1\right] \\ \amalg_{1.2.3} \\ t_3\left[A_3\right] \diamond_{3.1} t_1\left[A_1\right] \end{pmatrix}\right) \begin{bmatrix} \bigwedge_{1.1} & - & - \\ \bigwedge_{2.1} & \wedge_{2.2} & - \\ & \square & \\ \bigwedge_{3.1} & - & \bigwedge_{3.3} \end{bmatrix} \left(\begin{pmatrix} t_1\left[B_1\right] \\ \amalg_{1.2.0} \\ t_2\left[B_2\right] \diamond_{2.1} f_1\left[B_1\right] \\ \amalg_{1.2.3} \\ t_3\left[B_3\right] \diamond_{3.1} t_1\left[B_1\right] \end{pmatrix}\right)$$

$$\mathbf{false}\left(X <> \wedge \wedge Y\right):$$

$$\left(\begin{pmatrix} f_1\left(\left[A_1\right] <>_{1.1} \left[B_1\right]\right) \\ \amalg_{1.2.0} \\ f_2\left(\left[A_2\right] \wedge_{2.2} \left[B_2\right]\right) \diamond_{2.1} \left(\left(f_1\left[A_1\right] \wedge_{2.1} t_1\left[B_1\right]\right) \vee_{2.1} \left(t_1\left[A_1\right] \wedge_{2.1} f_1\left[B_1\right]\right)\right) \\ \amalg_{1.2.3} \\ f_3\left(\left[A_3\right] \bigwedge_{3.3} \left[B_3\right]\right) \diamond_{3.1} \left(\left(f_1\left[A_1\right] \wedge_{3.1} t_1\left[B_1\right]\right) \vee_{3.1} \left(t_1\left[A_1\right] \wedge_{2.1} f_1\left[B_1\right]\right)\right) \end{pmatrix}\right) =$$

$$\left(\begin{pmatrix} f_1\left[A_1\right] \\ \amalg_{1.2.0} \\ f_2\left[A_2\right] \diamond_{2.1} \left(f_1\left[A_1\right] \bigvee_{2.1} t_1\left[B_1\right]_1\right) \\ \amalg_{1.2.3} \\ f_3\left[A_3\right] \diamond_{3.1} \left(f_1\left[A_1\right] \bigvee_{3.1} t_1\left[B_1\right]_1\right) \end{pmatrix}\right) \begin{bmatrix} \bigwedge_{1.1} \\ \bigwedge_{2.1} \vee_{2.2} \\ \square \\ \bigwedge_{3.1} \vee_{3.3} \end{bmatrix} \left(\begin{pmatrix} f_1\left[B_1\right] \\ \amalg_{1.2.0} \\ f_2\left[B_2\right] \diamond_{2.1} \left(t_1\left[A_1\right] \bigvee_{2.1} f_1\left[B_1\right]\right) \\ \amalg_{1.2.3} \\ f_3\left[B_3\right] \diamond_{3.1} \left(t_1\left[A_1\right] \bigvee_{3.1} f_1\left[B_1\right]_1\right) \end{pmatrix}\right)$$

1.3. Discussion of functorial formalization of transjunctions

Distribution

$(A \text{ par } (A \text{ seq } B)) \text{ seq } (B \text{ par } (A \text{ seq } B)) = (A \text{ seq } B) \text{ par } ((A \text{ seq } B) \text{ seq } (A \text{ seq } B))$

R2: $(X \text{ par } Z) \text{ seq } (Y \text{ par } Z) = (X \text{ seq } Y) \text{ par } (Z)$: Distribution

R2 .1: $X \text{ seq } (Y \text{ par } Z) = (X \text{ seq } Y) \text{ par } Z$

R2 .2 : $(X \text{ par } Z) \text{ seq } Y = (Z \text{ seq } Y) \text{ par } Z$

Null

Bifunctoriality for f_1 $(<> \wedge \ \wedge)$:

R1: $(a \text{ par } c) \text{ seq } (b \text{ par } d) = (a \text{ seq } b) \text{ par } (c \text{ seq } d)$

$(a \text{ par nil}) \text{ seq } (b \text{ par nil}) = (a \text{ seq } b) \text{ par } (\text{nil seq nil})$

$a \text{ seq } b = a \text{ seq } b$

$f_1\, X \text{ et } f_1\, Y$

Null

Bifunctoriality for f_2 $(<> \wedge \ \wedge)$:

How to get the rules of interchangeability?

The rules of interchangeability (bifunctoriality) are abstracted from the full interpretation of the tableuax rul
while the tableaux rules are intepretations of the value matrix of logical functions.

Tableaux for **f_2** $(<> \wedge \ \wedge)$:

$(f_2\, X \text{ par } (f_1\, X \text{ et } t_1\, Y)) \text{ seq } (f_2\, Y \text{ par } (t_1\, X \text{ et } f_1\, Y)) = (f_2\, X \text{ seq } f\, Y_2) \text{ par } ((f_1\, X \text{ et } t_1\, Y) \text{ seq } (t_1\, X \text{ et } f_1$

$(f_2\, X \text{ par } (f_1\, X \text{ et } t_1\, Y)) \text{ vel } (f_2\, Y \text{ par } (t_1\, X \text{ et } f_1\, Y)) = (f_2\, X \text{ vel } f\, Y_2) \text{ par } ((f_1\, X \text{ et } t_1\, Y) \text{ vel } (t_1\, X \text{ et } f_1\, Y)$

$(A \text{ par } (A \text{ seq } B)) \text{ seq } (B \text{ par } (A \text{ seq } B)') = (A \text{ seq } B) \text{ par } ((A \text{ seq } B) \text{ seq } (A \text{ seq } B)')$, with

$a = A$

$c = (A \text{ seq } B)$,

$b = B$,

$c = (A \text{ seq } B)'$.

R1 (Bifunctoriality): $(a \text{ par } c) \text{ seq } (b \text{ par } d) = (a \text{ seq } b) \text{ par } (c \text{ seq } d)$.

$$\begin{pmatrix} A \\ \amalg \\ (A \text{ seq } B) \end{pmatrix} \text{seq} \begin{pmatrix} B \\ \amalg \\ (A \text{ seq } B)' \end{pmatrix} = \begin{pmatrix} (A \text{ seq } B) \\ \amalg \\ (A \text{ seq } B) \text{ seq } (A \text{ seq } B)' \end{pmatrix}$$

$$\begin{pmatrix} A \\ \amalg \\ C \end{pmatrix} \circ \begin{pmatrix} B \\ \amalg \\ D \end{pmatrix} = \begin{pmatrix} (A \circ B) \\ \amalg \\ (C \circ D) \end{pmatrix}.$$

Bifunctoriality for $\mathbf{f_3}\left(<> \wedge \wedge\right)$:

$(\text{a par c})\,\text{seq}\,(\text{b par d}) = (\text{a seq b})\,\text{par}\,(\text{c seq d}) \;:\; \text{R1 (Bifunctoriality)}$

$(f_3\, X\, \text{par}\, (f_1\, X\, \text{et}\, t_1\, Y))\ \text{seq}\ (f_3\, Y\, \text{par}\, (t_1\, X\, \text{et}\, f_1\, Y)) = (f_3\, X\, \text{seq}\, f\, Y_3)\, \text{par}\, ((f_1\, X\, \text{et}\, t_1\, Y)\, \text{seq}\, (t_1\, X\, \text{et}\, f_1\, Y$

$(f_3\, X\, \text{par}\, (f_1\, X\, \text{et}\, t_1\, Y))\ \text{vel}\ (f_3\, Y\, \text{par}\, (t_1\, X\, \text{et}\, f_1\, Y)) = (f_3\, X\, \text{vel}\, f\, Y_3)\, \text{par}\, ((f_1\, X\, \text{et}\, t_1\, Y)\, \text{vel}\, (t_1\, X\, \text{et}\, f_1\, Y)$

$\text{sig}_i\, X\, \text{seq}\, \text{sig}_j\, Y = \text{sig}(X\, \text{seq}\, Y),\ i = j$

Null

$$\mathbf{val}(X <> \wedge \wedge Y):\ \begin{pmatrix} 1 & 3 & 3 \\ 3 & 2 & 3 \\ 3 & 3 & 3 \end{pmatrix} = \begin{bmatrix} T_{1.3} & F_{2.3} & F_3 \\ F_{2.3} & F_1 T_2 & F_2 \\ F_3 & F_2 & F_{2.3} \end{bmatrix}:$$

$\mathbf{true}(X <> \wedge \wedge Y)$:

$$\left(\left(\begin{array}{c} t_1\left(\left[A_1\right] <>_{1.1} \left[B_1\right]\right) \\ \amalg_{1.2.0} \\ t_2\left(\left[A_2\right] \wedge_{2.2} \left[B_2\right]\right) \diamond_{2.1} f_1\left(\left[A_1\right] \wedge_{2.1} \left[B_1\right]\right) \\ \amalg_{1.2.3} \\ t_3\left(\left[A_3\right] \wedge_{3.3} \left[B_3\right]\right) \diamond_{3.1} t_1\left(\left[A_1\right] \wedge_{3.1} \left[B_1\right]\right) \end{array}\right)\right) =$$

$$\left(\left(\begin{array}{c} t_1\left[A_1\right] \\ \amalg_{1.2.0} \\ t_2\left[A_2\right] \diamond_{2.1} f_1\left[A_1\right] \\ \amalg_{1.2.3} \\ t_3\left[A_3\right] \diamond_{3.1} t_1\left[A_1\right] \end{array}\right)\right) \begin{bmatrix} \wedge_{1.1} & - & - \\ \wedge_{2.1} & \wedge_{2.2} & - \\ & \square & \\ \wedge_{3.1} & - & \wedge_{3.3} \end{bmatrix} \left(\left(\begin{array}{c} t_1\left[B_1\right] \\ \amalg_{1.2.0} \\ t_2\left[B_2\right] \diamond_{2.1} f_1\left[B_1\right] \\ \amalg_{1.2.3} \\ t_3\left[B_3\right] \diamond_{3.1} t_1\left[B_1\right] \end{array}\right)\right)$$

This approach doesn't imply that transjunctions are definable by junctions and negations only but that transjunctions are distributed and mediated over different logical loci and that at those loci junctional parts are placed.
Hence, under the condition of discontexturality and the use of the discontextural operator “transposition” transjunctions are ‘junctionally’ constructible.

Because memristive systems are able to emulate logical and arithmetical operations, a memristive transjunction or transjunctional memristive systems, i.e. memristive systems with transjunctional behaviors are constructible in the framework of presumed discontextural systems.

Hence, the challenge is to construct *discontextural* fields of memristivity and to establish its specific operators.

As for neurobiology, cognitive actions are not determined by isolated synapses only but by *assemblies* of neural activities. It is not a big departure form the tradition to postulate that assemblies are distinguishable what means that they are different. Hence, the whole system to work needs interactions between different neural assemblies. Interactions between neural assemblies are of a different kind, at least in their functionality, than the operators insid assemblies, which have to be “bridged” by interactions. Hence, interactions are different to the intra-systemic operations, i.e. logical operators, like conjunction or disjunction and negation. It might be reasonable to opt for polycontextural *transjunctions* to deal with the interactions between assemblies of neural activity.

Memristivity is a general property of nano-physical systems and occurs in different forms.
Even nano-electronic memristivity is definable under different conditions. All those differences between memristive systems might be used to define *discontextural* conditions for the realization of memristive transjunctions.

The technical equivalent for the set of logical signatures {True, False} are the couple {On, Off}.
In an earlier paper I postulated the concept of poly-layered crossbar systems as polycontextural opposites of mono-layered, i.e. multi-layered crossbar systems of mono-contextural memristic systems.

The double functionality, typical for memristors, is necessary and sufficient to construct mediation between discontextural domains.

This idea of a construction is not properly working without memristors. If transjunctional behavior gets modeled with CMOS devices and NOR or NAND logic, the crucial difference between domains is not realizable because all subsystems belong to the same systematic locus, i.e. all are defined by the very same kind of logical values and physically by the same electronic conditions.

The principle of *localization* of memristive operations is of crucial importance.

Where enters the crucial feature of memrsitors, its retrograde recursiveness, into the mechanism of mediation?
Retro-gradeness was analyzed as a chiastic structure which occurs in the process of continuation (concatenation, prolongation) of morphograms. Obviously, the mediation of logical functions, like in (AND, AND, AND), is a case of chaining (composing) functions and morphograms.

Mediation of AND_1 and AND_2 might category-theoretically be modeled by $cod(AND_1)= dom(AND_2)$, with the commuting morphism $(AND_3)$and the properties $dom(AND_1)= dom(AND_3)$and $cod(AND_2)= cod(AND_3)$ and the presumption of a single universe of objects and morphisms.

But this construction gets a chiastic interpretation and formalization in a polycontextural setting with diffferent, i.e. discontextural universes and morphisms.
Chiasms are retro-grade constructions working as an interplay of ‘memory'- and ‘computing'-functions of memristors in different roles.

Example of some combinations of mediation of junctors

$$\begin{pmatrix} \mathrm{dom} & \longrightarrow & \mathrm{cod} & \square & \square & : AND_1 \\ | & \square & \updownarrow & \square & \square & \amalg \\ | & \square & \mathrm{dom} & \longrightarrow & \mathrm{cod} & : AND_2 \\ | & \square & \square & \square & | & \amalg \\ \mathrm{dom} & \longrightarrow & \square & \longrightarrow & \mathrm{cod} & : AND_3 \end{pmatrix}$$

$$\begin{pmatrix} \mathrm{dom} & \longrightarrow & \mathrm{cod} & \square & \square & : AND_1 \\ | & \square & \updownarrow & \square & \square & \amalg \\ | & \square & \mathrm{dom} & \longrightarrow & \mathrm{cod} & : OR_2 \\ | & \square & \square & \square & | & \amalg \\ \mathrm{dom} & \longrightarrow & \square & \longrightarrow & \mathrm{cod} & : AND_3 \end{pmatrix}$$

$$\begin{pmatrix} \text{dom} & \longrightarrow & \text{cod} & \square & \square & :\text{OR}_1 \\ | & \square & \updownarrow & \square & \square & \amalg \\ | & \square & \text{dom} & \longrightarrow & \text{cod} & :\text{AND}_2 \\ | & \square & \square & \square & | & \amalg \\ \text{dom} & \longrightarrow & \square & \longrightarrow & \text{cod} & :\text{OR}_3 \end{pmatrix}$$

Each logical AND is emulated by memristive devices, memristors only or mixed with other mem-capacitators, mem-inductors, as Di Ventra et al defined and simulated it. Such memristive constructions are not yet focused on the time- and history dependence of the used memristors.

A new use of memristors enters the theater whith the *interactions* between memristors of different domains. This mechanism of connecting different domains happens in the mode of chiasms which are defined by their retro-grade properties.
Hence, memrisor-based logical junctions are mediated by memristive elements exploiting their memristive features of retro-gradeness. Hence, the focus on the roles of memristors are different, on a first-level interpretation, memristors are defining junctors without any involvement in retro-gradeness. On a second-order level, the memristive property of retro-gradeness o the behavior of memristors is in the focus and utilized for the mediation of memristive structures of different loci (domains).

Short remainder of the internal structure of chiastic mediation

$$\begin{array}{cccccc} \alpha_3 \equiv \alpha_1 = [M \mid r_1 r_2 r_2] & \longrightarrow & [M \mid r_1 r_2 r_2] = \omega_1 \\ \downarrow \quad \updownarrow & X & \updownarrow \\ \omega_3 \equiv \omega_2 = [M \mid r_1 r_2 r_2] & \longleftarrow & [M \mid r_1 r_2 r_2] = \alpha_2 \end{array}$$

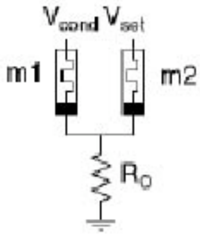
$$\begin{array}{ccc} \alpha_3 \equiv \alpha_1 & \longrightarrow & \omega_1 \\ \downarrow \quad \updownarrow & X & \updownarrow \\ \omega_3 \equiv \omega_2 & \longleftarrow & \alpha_2 \end{array}$$

With α as domain, and ω as codomain of the mediation of the levels (domains) 1 and 2.

2. Modeling and emulating logical operators

2.1. Modeling and emulating material implication

Lehtonen's example for the emulation of material implication with two memristors and 1 resistor only.

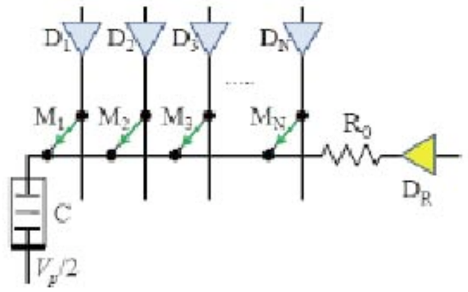


m_1	m_2	$m_2 = m_1 \rightarrow m_2$
0	0	1
0	1	1
1	0	0
1	1	1

Figure 2. Implication logic with memristors.

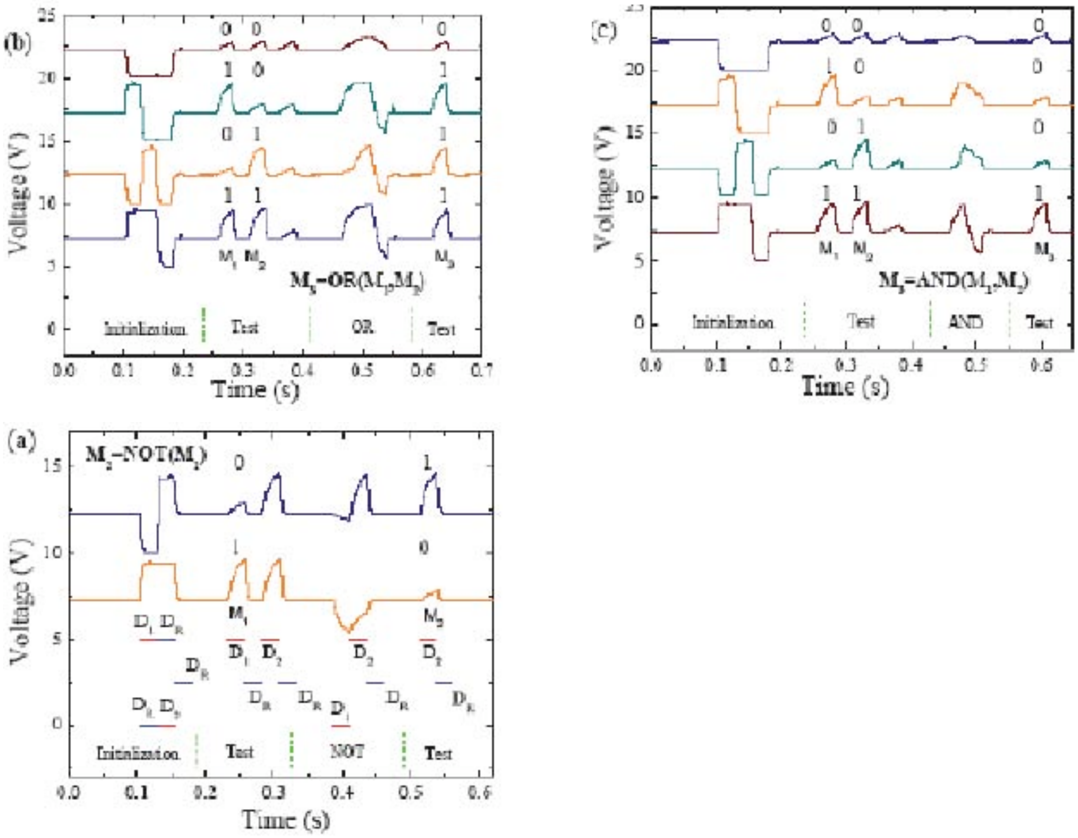
2.2. Memristive modeling and emulating of logical functions

2.2.1. Di Ventra's physical model



"*Fig. 6. (Color online) Electronic circuit for Boolean logic and arithmetic operations.*
In this circuit, an array of N memristors M_i, memcapacitor *C and* resistor R_0 *are connected to a common (horizontal) line. The circuit operation involves charging the memcapacitor C through input memristors and discharging it through the output ones.*
Because of the circuit symmetry, each memristor can be used as input or output.
From the opposite (top) side, memristors are driven by 3-state (0V, V_p*, not connected) drivers.*
The right 4-state (0V, $V_p/2$*,* V_p*, not connected) driver DR connected through a* resistor *to the common line is used to initialize* memristors *and* memcapacitor. *This scheme employs threshold-type* bipolar *memristors assuming that the application of positive voltage to the top memristor electrode and 0V to the bottom electrode switches memristor from low resistance state (1 or ON) to the high resistance state (0 or OFF). It is also assumed that the memristors' threshold voltage* V_t *is between* $V_p/2$ *and* V_p*."* (Di Ventra)

Memristive simulation of the logical junctors AND, OR, NOT



"*Fig. 7. (Color online) Experimental realization of the basic logic gates with memristors based on the circuit shown in Fig. 6 with a capacitor C = 10μF instead of a memcapacitor.*
In our experimental circuit, the role of memristors is played by memristor emulators [...] governed by the threshold type model of Eqs. (3) and (4) with the following set of parameters: $V_t = 4V$*,* $a = 0$*,* $\beta = 62MOhms/(V\cdot s)$*.*
Each plot shows several measurements of the voltage between the top plate of the capacitor (the common line defined in the caption of Fig. 6) and ground taken for different possible

states of input memristors.
In (a) we also show an example of voltages applied to the drivers to generate the top voltage plot. The voltages on the drivers in (a) are in absolute values while the other curves were vertically displaced for clarity."
Yuriy V. Pershin and Massimiliano Di Ventra, Neuromorphic, Digital and Quantum Computation with Memory Circuit Elements

The simulation runs with 3 memristors, 1 memcapacitor and 1 resistor. If two memristors are enough to simulate logical function, a distribution of logical functions over 3 contextures needs therefore not more than 9 memristors. But with 9 memristors only *separated* and not mediated logical functions could be simulated in their specific domain. Additionally to the definition and realization of 3 different domains, which will be represented by different voltage-domains, at least 3x2 additional memristors are demanded to realize the mediation (connection) between the 3x3 distributed memristors.

2.2.2. Mediation of memristors

Types of combinations of memristors
Classical combinations: *serial* and *parallel*,
Transclassical combinations: *mediations*, i.e. interactional, reflectional and interventional mediations.

How are mediator defined?

In more abstract terms, mediators are realizing the matching conditions for the composition of morphisms (functions, operations).

$$\left(\mathrm{MEM}^1 \amalg \mathrm{MEM}^2\right) = \begin{pmatrix} \begin{pmatrix} \mathrm{on}_1 \\ \mathrm{off}_1 \end{pmatrix} \\ \mathrm{switch}_3 \\ \begin{pmatrix} \mathrm{on}_2 \\ \mathrm{off}_2 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} \mathrm{MEM}_1 & - \\ - & \mathrm{MEM}_3 \\ \mathrm{MEM}_2 & - \end{pmatrix}$$

MEM_1 and MEM_2 are memristors with an intra-contextural *selective* switch-function, basic for the emulation of logical and arithmetical operations. In contrast, the role of memristor MEM_3 is an *elective* trans-contextural function, realizing the functor 'mediation' ($\amalg$).

funct(MEM) = (election, selection)
selection = [ON/OFF] for digital and [ON,..., OFF] for analog operations in a contexture,
election = switch of contextures in the mode of transpositions, reflections and interventions.
Selection is realized by the first-order functionality of memristors in a crossbar system,
Election is realized by the second-order functionality of memristors between different crossbar systems.

First-order states of a memristive configuration are defining the type of primary action, i.e. logical junctions and basic arithmetic operations.

Second-order states of a memristive configuration are states of states, memorizing the first-order states of precessing states and as a new functionality, the context of those first-order states. Because states are always realized in a context, the memorization function of the state of the state delivers both, the former first-order state and its context (contexture).

"Because of the graphematical tabularity *of ConTextures there are not only two possibilities to perform a selection. For each contexture there are* intra-contexturally *only two possibilities to perform a selection. But between contextures,* transcontexturally, *there are as many new selectors as neighbor contextures. These new "selectors" should be called* electors. *Electors are*

electing the election for selectors to perform mutually each its selection.
In other words, such a general a selector *as any other successive or procedural the same contexture or trans-conA selector as a complexion can be realized at once in different contextures. It could therefore be called a* "poly-selector"*. Such a poly-selector can be defined as an overlapping of an intra-contextural selector "sel" and a trans-contextural selector, called* elector *'elect'."* (ConTeXtures, p. 20, 2005)

A prolongation of such a tupel of (state, context) is a double-function, realized by '*selection*' and '*election*'.
Hence, a further logical or arithmetical step always has first to select the context, i.e. to chose to stay in the actual context or to chose another context for prolongation. Both at once might happen too, to change and to stay in context. After this contextural decision (election), the well known logical and arithmetical operations might continue by selection.

2.3. Dissemination

2.3.1. Superoperators in action

Boiled down to the simplest structure avaible the whole game of dissemination (monoidal) categories and memristive crossbar systems too, is to introduce an additional operator to the known operators, i.e composition and yuxtaposition.
This operator, called disseminatior, consisting of the two compelemtary aspects "distribution" and "mediation", is abstracting the whole box of the theory of monoidal categories as a new unit and is distributing it with the operator of "election" over a kenomic matrix.
A specification of the distribution operator is given by the so called "super-operators":

id: identity
perm: permutation,
red: reduction,
repl: replication,
bif: bifurcation.

$$\mathrm{DISS}\left(\left(\circ\,,\otimes\right),3,3\right)=\boxed{\begin{array}{cccc}(\circ\otimes)^{(4,4)} & \mathrm{loc1} & \mathrm{loc2} & \mathrm{loc3}\\ \mathrm{pos1} & (\circ\otimes)_{1.1} & (\circ\otimes)_{1.2} & (\circ\otimes)_{1.3}\\ \mathrm{pos2} & (\circ\otimes)_{2.1} & (\circ\otimes)_{2.2} & (\circ\otimes)_{2.3}\\ \mathrm{pos3} & (\circ\otimes)_{3.1} & (\circ\otimes)_{3.2} & (\circ\otimes)_{3.3}\end{array}}$$

Some typical constellations for m = n, $\mathrm{DISS}((\circ, \otimes), 3, 3)$

Identity : $(\mathbf{id}, \mathbf{id}, \mathbf{id})$, **i = j**

$(\circ, \otimes)^{(3,3)}$	loc1	loc2	loc3
pos1	$(\circ\otimes)_{1.1}$	-	-
pos2	-	$(\circ\otimes)_{2.2}$	-
pos3	-	-	$(\circ\otimes)_{3.3}$

Iteration $(\mathit{id}, \mathit{iter}, \mathit{id})$

$(\circ, \otimes)^{(3,3)}$	loc1	loc2	loc3
pos1	$(\circ\otimes)_{1.1}$	-	-
pos	-	$(\circ\otimes)_{2.2.2.2}$	-
pos3	-	-	$(\circ\otimes)_{3.3}$

reduction : $(\mathbf{red}, \mathbf{red}, \mathbf{id})$

$(\circ, \otimes)^{(3,3)}$	loc1	loc2	loc3
pos1	$(\circ\otimes)_{1.1}$	-	-
pos2	-	$(\circ\otimes)_{1.1}$	-
pos3	-	-	$(\circ\otimes)_{3.3}$

permutation : $(\mathbf{perm}, \mathbf{perm}, \mathbf{id})$

$(\circ, \otimes)^{(3,3)}$	loc1	loc2	loc3
pos1	$(\circ\otimes)_{2.2}$	-	-
pos2	-	$(\circ\otimes)_{1.1}$	-
pos3	-	-	$(\circ\otimes)_{3.3}$

replication : $(\mathbf{repl}, \mathbf{id}, \mathbf{id})$

$(\circ, \otimes)^{(3,3)}$	loc1	loc2	loc3
pos1	$(\circ\otimes)_{1.1}$	-	-
pos2	$(\circ\otimes)_{1.2}$	$(\circ\otimes)_{2.2}$	-
pos3	$(\circ\otimes)_{1.3}$	-	$(\circ\otimes)_{3.3}$

Transjunction $(\mathbf{bif}, \mathbf{id}, \mathbf{id})$

2.4. Distribution of conjunction

$\mathbf{AND}^{(3)}_{\wedge 1 \wedge 2 \wedge 3}$ =

1. $\left(\text{AND}^1 \amalg_{1.2} \text{AND}^2\right) \amalg_{2.3} \text{AND}^3$:

conceptual mediation ($\amalg$) of the distributed implications AND.

2. $((\text{MEM}^1_1 \circ \text{MEM}^1_2) \amalg_{1.2} (\text{MEM}^1_1 \circ \text{MEM}^1_2)) \amalg_{2.3} (\text{MEM}^3_1 \circ \text{MEM}^3_2)$:

conceptual mediation of conjunction and realization of conjunction AND by two memristors per contexture.
R_0, respectively R^i_0, is omitted.

3. $((\text{MEM}^1_1 \circ \text{MEM}^1_2)\, \text{procm}_{1.2} (\text{MEM}^1_1 \circ \text{MEM}^1_2))\, \text{procm}_{2.3} (\text{MEM}^3_1 \circ \text{MEM}^3_2)$

realization of conjunction AND and mediation by two memristors per contexture and
one memristive processor between contextures (procm).

More concretely, the interaction of $\text{AND}^{1.0.0}$ and $\text{AND}^{0.2.0}$ is ruled by the operator of *mediation* "$\amalg$", which is the 'diagonal' mediation "$\amalg$" of the planar kenomic matrix.

This together constitutes a memristive system of distributed and mediated memristors in their double role as *memory* for the realization of the conjunction *AND* and in the role as *processors* for the mediation "$\amalg$" of the contextures of the distributed conjunctions.

The same wording holds for the distribution of *disjunction OR*.

Because of the lacalization of operators in memristic systems not all operators are placed on the "diagonal" of the kenomic matrix. For a distribution and mediation of *implication* further operators, like *replication* have to be applied.

Coincidence relation
For $ON_{1.3}$ a coincidence relation between $v(ON_1)$ and $v(ON_3)$ holds.

Also $V^1$and $V^3$are disjunct, $V^1 \cap V^3 = \varnothing$, they *coincede* at $ON_{1.3}$ with $v(ON_1) \equiv v(ON_3)$ and at $OFF_{2.3}$ with
$v(OFF_2) \equiv v(OFF_3)$.

Because $ON_1$and ON_3 are mediated by the operator $\amalg$, which is realized by a processing memristor, there is no logical or electronic contradiction or conflict involved in this mediating mechanism. The same holds for OFF_2 and OFF_3.

This is nothig mysterious. For the classical case, a logical interpretation of the physical values happens too. In many respects, electronics and computer hardware is a question of interpretation, i.e. hemeneutics, and not of naked physics.

"In this type of application, each memristor is used in the "digital" *mode of operation, namely only one bit of information is encoded in the memristor's state. We call "1"* (ON) *the state of lower resistance and "0"* (OFF) *that of higher resistance. The operation of the circuit sketched in Fig. 6 relies on charging a memcapacitor through input memristors and subsequent discharging through the output ones."* (Di Ventra)

What's definitively new and not yet tested is the mix of the voltage intepretation with the new domain interpretation.

Again, a value in a disconctxtural system is always a value of a contexture (domain), hence it has a double characteristics, one for the internal physical value (voltage) and one for the domain it belongs, i.e. its place-value marked by the place-designator.

Hence, “0” as “OFF” and “1” as “ON” has to be specified, i.e. indexed by its domain (contexture).

For distributed conjunction (AND AND AND)we get: ((AND $\amalg_{1.2}$ AND) $\amalg_{1.2.3}$ AND)

ON^1	$\amalg_{1.3}$	ON^3
OFF		--
--		OFF
OFF		--
$OFF^1 \amalg_{1.2}$	ON^2	--
	OFF	--
	--	OFF
	OFF	--
	$OFF^2 \amalg_{2.3}$	OFF^3

Again, there might be some obstacles. The interpretation $ON^1 \cong ON^3$ and $OFF^2 \cong OFF^3$ might have some plausibility. It seems to be more difficult to accept the interpretation: $OFF^1 \cong ON^2$.
But that’s not much more than the matching conditions between contexture1 and contexture2, with
cod(contexture1) ≅ dom(contexture2).

2.5. Formal modeling

A simplified categorical formulation for distributed memristors and implications based on memristors is shown with the following two formulas.

Interchangeability for $MEM^{(1.2.3)}$

$m = 3, \; n = 2,$

$$\begin{bmatrix} MEM_2^1 & - & MEM_2^3 \\ MEM_1^1 & MEM_2^2 & - \\ - & MEM_1^2 & MEM_1^3 \end{bmatrix}:$$

$$\begin{pmatrix} MEM_1^1 \\ \amalg_{1.2.0} \\ MEM_1^2 \\ \amalg_{1.2.3} \\ MEM_1^3 \end{pmatrix} \begin{bmatrix} \circ_{1.1} \\ \circ_{3.3} \\ \circ_{3.3} \end{bmatrix} \begin{pmatrix} MEM_2^1 \\ \amalg_{1.2.0} \\ MEM_2^2 \\ \amalg_{1.2.3} \\ MEM_2^3 \end{pmatrix} = \begin{pmatrix} \left(MEM_1^1 \circ_{1.0.0} MEM_2^1\right) \\ \amalg_{1.2.0} \\ \left(MEM_1^2 \circ_{0.2.0} MEM_2^2\right) \\ \amalg_{1.2.3} \\ \left(MEM_1^3 \circ_{0.0.3} MEM_2^3\right) \end{pmatrix}$$

If we accept this categorical construction for AND and OR for experimental reasons we might continue with the more intriguing situation of *interactions* between contextures ruled by *transjunctions*.

On the base of the proposed sketches, here and in the paper *“Poly-layered crossbars”*, an implementation of transjunctions on the base of memristive devices in the framework of discontexturality shouldn’t be an impossible task.

Hence the logical interpretation of transjunctions in cooperation with conjunctions, as studied before, are easily transposed into an electronic setting.

Trivially, True is ON, False is OFF

2.5.1. Interchangeability for ($<> \wedge \wedge$)

CASE for OFF

$\mathbf{OFF}_1 (<> \wedge \wedge)$:

$OFF_1 X$ et $OFF_1 Y$

$\mathbf{OFF}_2 (<> \wedge \wedge)$:

$(OFF_2\ X \text{ par } (OFF_1\ X \text{ et } ON_1\ Y)) \text{ vel } (OFF_2\ Y \text{ par } (ON_1\ X \text{ et } OFF_1\ Y))$

$(OFF_2\ X \text{ vel } OFF\ Y_2) \text{ par } ((OFF_1\ X \text{ et } ON_1\ Y) \text{ vel } (ON_1\ X \text{ et } OFF_1\ Y))$

$\mathbf{OFF}_3 (<> \wedge \wedge)$:

$(OFF_3\ X \text{ par } (OFF_1\ X \text{ et } ON_1\ Y)) \text{ vel } (OFF_3\ Y \text{ par } (ON_1\ X \text{ et } OFF_1\ Y)) =$

$(OFF_3\ X \text{ vel } OFF\ Y_3) \text{ par } ((OFF_1\ X \text{ et } ON_1\ Y) \text{ vel } (ON_1\ X \text{ et } OFF_1\ Y))$

CASE for ON

$\mathbf{ON}_1 (<> \wedge \wedge)$:

$ON_1\ X \text{ et } ON_1\ Y$

$\mathbf{ON}_2 (<> \wedge \wedge)$:

$(ON_2\ X \text{ par } OFF_1\ X) \text{ et } (ON_2\ Y \text{ par } OFF_1\ Y) = (ON_2\ X \text{ et } ON\ Y_2) \text{ par } (OFF_1\ X \text{ et } OFF_1\ Y)$

$\mathbf{ON}_3 (<> \wedge \wedge)$:

$(ON_3\ X \text{ par } ON_1\ X) \text{ et } (ON_3\ Y \text{ par } ON_1\ Y) = (ON_3\ X \text{ et } ON\ Y_3) \text{ par } (ON_1\ X \text{ et } ON_1\ Y)$.

2.5.2. Memristic emulation of transjunction and junctions

$$\mathbf{ON}\left(X <> \wedge \wedge Y\right):$$

$$\left(\begin{array}{c} ON_1\left(\left[A_1\right] <>_{1.1} \left[B_1\right]\right) \\ \amalg_{1.2.0} \\ ON_2\left(\left[A_2\right] \wedge_{2.2} \left[B_2\right]\right) \diamond_{2.1} OFF_1\left(\left[A_1\right] \wedge_{2.1} \left[B_1\right]\right) \\ \amalg_{1.2.3} \\ ON_3\left(\left[A_3\right] \wedge_{3.3} \left[B_3\right]\right) \diamond_{3.1} ON_1\left(\left[A_1\right] \wedge_{3.1} \left[B_1\right]\right) \end{array}\right) =$$

$$\left(\begin{array}{c} ON_1\left[A_1\right] \\ \amalg_{1.2.0} \\ ON_2\left[A_2\right] \diamond_{2.1} OFF_1\left[A_1\right] \\ \amalg_{1.2.3} \\ ON_3\left[A_3\right] \diamond_{3.1} ON_1\left[A_1\right] \end{array}\right) \left[\begin{array}{c} \wedge_{1.1} \; - \; - \\ \wedge_{2.1} \wedge_{2.2} \; - \\ \Box \\ \wedge_{3.1} \; - \; \wedge_{3.3} \end{array}\right] \left(\begin{array}{c} ON_1\left[B_1\right] \\ \amalg_{1.2.0} \\ ON_2\left[B_2\right] \diamond_{2.1} OFF_1\left[B_1\right] \\ \amalg_{1.2.3} \\ ON_3\left[B_3\right] \diamond_{3.1} ON_1\left[B_1\right] \end{array}\right)$$

$$\mathbf{OFF}\left(X <> \wedge \wedge Y\right):$$

$$\left(\begin{array}{c} OFF_1\left(\left[A_1\right] <>_{1.1} \left[B_1\right]\right) \\ \amalg_{1.2.0} \\ OFF_2\left(\left[A_2\right] \wedge_{2.2} \left[B_2\right]\right) \diamond_{2.1} \left(\left(OFF_1\left[A_1\right] \wedge_{2.1} ON_1\left[B_1\right]\right) \vee_{2.1} \left(ON_1\left[A_1\right] \wedge_{2.1} OFF_1\left[B_1\right]\right)\right) \\ \amalg_{1.2.3} \\ OFF_3\left(\left[A_3\right] \wedge_{3.3} \left[B_3\right]\right) \diamond_{3.1} \left(\left(OFF_1\left[A_1\right] \wedge_{3.1} ON_1\left[B_1\right]\right) \vee_{3.1} \left(ON_1\left[A_1\right] \wedge_{3.1} OFF_1\left[B_1\right]\right)\right) \end{array}\right) =$$

$$\left(\begin{array}{c} OFF_1\left[A_1\right] \\ \amalg_{1.2.0} \\ OFF_2\left[A_2\right] \diamond_{2.1} \left(OFF_1\left[A_1\right] \vee_{2.1} ON_1\left[B_1\right]\right) \\ \amalg_{1.2.3} \\ OFF_3\left[A_3\right] \diamond_{3.1} \left(OFF_1\left[A_1\right] \vee_{3.1} ON_1\left[B_1\right]\right) \end{array}\right) \left[\begin{array}{c} \wedge_{1.1} \\ \wedge_{2.1} \vee_{2.2} \\ \Box \\ \wedge_{3.1} \vee_{3.3} \end{array}\right] \left(\begin{array}{c} OFF_1\left[B_1\right] \\ \amalg_{1.2.0} \\ OFF_2\left[B_2\right] \diamond_{2.1} \left(ON_1\left[A_1\right] \vee_{2.1} OFF_1\left[\right.\right. \\ \amalg_{1.2.3} \\ OFF_3\left[B_3\right] \diamond_{3.1} \left(ON_1\left[A_1\right] \vee_{3.1} OFF_1\left[\right.\right. \end{array}\right.$$

$$\mathbf{OFF}\left(X <> \wedge \wedge Y\right) : \mathbf{index-free}$$

$$\left(\begin{pmatrix} \mathrm{OFF}_1\left(\left[A\right] <>_{1.1} \left[B\right]\right) \\ \amalg_{1.2.0} \\ \mathrm{OFF}_2\left(\left[A\right] \wedge_{2.2} \left[B\right]\right) \diamond_{2.1} \left(\left(\mathrm{OFF}_1\left[A\right] \wedge_{2.1} \mathrm{ON}_1\left[B\right]\right) \vee_{2.1} \left(\mathrm{ON}_1\left[A\right] \wedge_{2.1} \mathrm{OFF}_1\left[B\right]\right)\right) \\ \amalg_{1.2.3} \\ \mathrm{OFF}_3\left(\left[A\right] \bigwedge_{3.3} \left[B\right]\right) \diamond_{3.1} \left(\left(\mathrm{OFF}_1\left[A\right] \wedge_{3.1} \mathrm{ON}_1\left[B\right]\right) \vee_{3.1} \left(\mathrm{ON}_1\left[A\right] \wedge_{2.1} \mathrm{OFF}_1\left[B\right]\right)\right) \end{pmatrix}\right) =$$

$$\left(\begin{pmatrix} \mathrm{OFF}_1\left[A\right] \\ \amalg_{1.2.0} \\ \mathrm{OFF}_2\left[A\right] \diamond_{2.1} \left(\mathrm{OFF}_1\left[A\right] \bigvee_{2.1} \mathrm{ON}_1\left[B\right]\right) \\ \amalg_{1.2.3} \\ \mathrm{OFF}_3\left[A\right] \diamond_{3.1} \left(\mathrm{OFF}_1\left[A\right] \bigvee_{3.1} \mathrm{ON}_1\left[B\right]\right) \end{pmatrix}\right) \begin{bmatrix} \bigwedge_{1.1} \\ \bigwedge_{2.1} \vee_{2.2} \\ \square \\ \bigwedge_{3.1} \vee_{3.3} \end{bmatrix} \left(\begin{pmatrix} \mathrm{OFF}_1\left[B\right] \\ \amalg_{1.2.0} \\ \mathrm{OFF}_2\left[B\right] \diamond_{2.1} \left(\mathrm{ON}_1\left[A\right] \bigvee_{2.1} \mathrm{OFF}_1\left[B\right]\right) \\ \amalg_{1.2.3} \\ \mathrm{OFF}_3\left[B\right] \diamond_{3.1} \left(\mathrm{ON}_1\left[A\right] \bigvee_{3.1} \mathrm{OFF}_1\left[B\right]\right) \end{pmatrix}\right)$$

$$\mathbf{ON}\left(X \vee \wedge \wedge Y\right) :$$

$$\left(\begin{pmatrix} \mathrm{ON}_1\left(\left[A_1\right] \vee_{1.1} \left[B_1\right]\right) \\ \amalg_{1.2.0} \\ \mathrm{ON}_2\left(\left[A_2\right] \wedge_{2.2} \left[B_2\right]\right) \\ \amalg_{1.2.3} \\ \mathrm{ON}_3\left(\left[A_3\right] \bigwedge_{3.3} \left[B_3\right]\right) \end{pmatrix}\right) =$$

$$\left(\begin{pmatrix} \mathrm{ON}_1\left[A_1\right] \\ \amalg_{1.2.0} \\ \mathrm{ON}_2\left[A_2\right] \end{pmatrix} \\ \amalg_{1.2.3} \\ \mathrm{ON}_3\left[A_3\right]\right) \begin{bmatrix} \bigvee_{1.1} & - & - \\ \bigwedge_{2.1} & \wedge_{2.2} & - \\ & \square & \\ \bigwedge_{3.1} & - & \bigwedge_{3.3} \end{bmatrix} \left(\begin{pmatrix} \mathrm{ON}_1\left[B_1\right] \\ \amalg_{1.2.0} \\ \mathrm{ON}_2\left[B_2\right] \end{pmatrix} \\ \amalg_{1.2.3} \\ \mathrm{ON}_3\left[B_3\right]\right)$$

$$\mathbf{OFF}\left(X \vee \wedge \wedge Y\right) : \textbf{index - free}$$

$$\left(\begin{matrix} \left(\begin{matrix} OFF_1\left(\left[A\right] \vee_{1.1} \left[B\right]\right) \\ \amalg_{1.2.0} \\ OFF_2\left(\left[A\right] \wedge_{2.2} \left[B\right]\right) \end{matrix}\right) \\ \amalg_{1.2.3} \\ OFF_3\left(\left[A\right] \wedge_{3.3} \left[B\right]\right) \end{matrix}\right) =$$

$$\left(\begin{matrix} \left(\begin{matrix} OFF_1\left[A\right] \\ \amalg_{1.2.0} \\ OFF_2\left[A\right] \end{matrix}\right) \\ \amalg_{1.2.3} \\ OFF_3\left[A\right] \end{matrix}\right) \left[\begin{matrix} \vee_{1.1} \\ \wedge_{2.1} \vee_{2.2} \\ \square \\ \wedge_{3.1} \vee_{3.3} \end{matrix}\right] \left(\begin{matrix} \left(\begin{matrix} OFF_1\left[B\right] \\ \amalg_{1.2.0} \\ OFF_2\left[B\right] \end{matrix}\right) \\ \amalg_{1.2.3} \\ OFF_3\left[B\right] \end{matrix}\right)$$

2.6. Formal modeling negation

Negation is a fundamental operator for logic, and therefore for memristive implementations too.

From a formal point of view both aspects have to be considered, the *permutation* and the *mediation* of negators in a mediated complexion.

$$\begin{pmatrix} loc1 & loc2 & loc3 \\ \mathbf{ON} & - & \mathbf{ON} \\ \mathbf{OFF} & \mathbf{ON} & \square \\ - & \mathbf{OFF} & \mathbf{OFF} \\ MEM1 & MEM2 & MEM3 \end{pmatrix} \xrightarrow{neg1} \begin{pmatrix} loc1 & loc2 & loc3 \\ \mathbf{OFF}_1 & - & \mathbf{ON}_2 \\ \mathbf{ON}_1 & \mathbf{ON}_3 & \square \\ - & \mathbf{OFF}_3 & \mathbf{OFF}_2 \\ MEM1 & MEM3 & MEM2 \end{pmatrix} ; \xrightarrow{neg2} \begin{pmatrix} loc1 & loc2 & loc3 \\ \mathbf{ON}_3 & - & \mathbf{ON}_1 \\ \mathbf{OFF}_3 & \mathbf{OFF}_2 & \square \\ - & \mathbf{ON}_2 & \mathbf{OFF}_1 \\ MEM3 & MEM2 & MEM1 \end{pmatrix}$$

Monoidal permutation: $A_1 \otimes B_2 \longrightarrow B_2 \otimes A_1$.

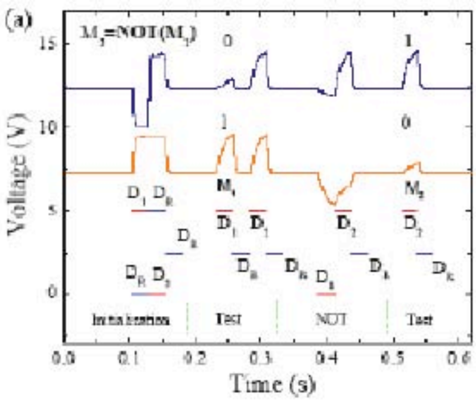
$$\left(\mathbf{neg}_1\right) :$$

$$\left(\begin{matrix} \left(\begin{matrix} neg_1\left[A\right] \\ \amalg_{1.2.0} \\ perm_2\left[A\right] \end{matrix}\right) \\ \amalg_{1.2.3} \\ perm_3\left[A\right] \end{matrix}\right) = \left(\begin{matrix} neg_1\left[A\right] \\ \amalg_{1.2.3} \\ \left(\begin{matrix} POS_3\left[A\right] \\ \amalg_{0.2.3} \\ POS_2\left[A\right] \end{matrix}\right) \end{matrix}\right)$$

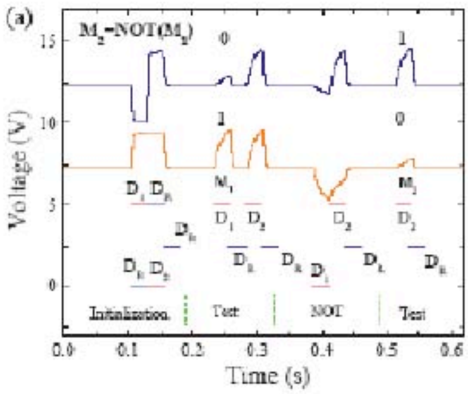
$$\left(\mathbf{neg}_2\right) :$$

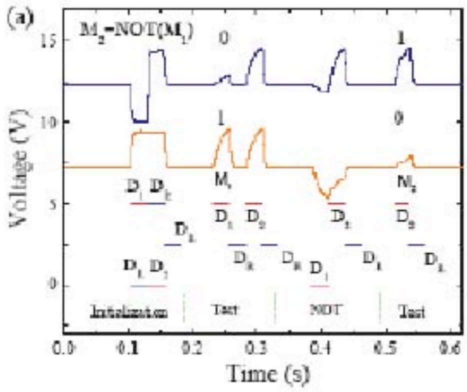
$$\left(\begin{matrix} \left(\begin{matrix} perm_1\left[A\right] \\ \amalg_{1.2.0} \\ neg_2\left[A\right] \end{matrix}\right) \\ \amalg_{1.2.3} \\ perm_3\left[A\right] \end{matrix}\right) = \left(\begin{matrix} POS_3\left[A\right] \\ \amalg_{1.2.3} \\ \left(\begin{matrix} neg_2\left[A\right] \\ \amalg_{0.2.3} \\ POS_2\left[A\right] \end{matrix}\right) \end{matrix}\right)$$

2.6.1. Speculative emulation for negations



neg





ON $(\mathbf{neg}_1)$ **:**

$$\begin{pmatrix} \begin{pmatrix} \mathrm{neg}_1\left(\mathrm{ON}\left(\left[A\right]\right)\right) \\ \amalg_{1.2.0} \\ \mathrm{perm}_2\left(\mathrm{ON}\left(\left[A\right]\right)\right) \end{pmatrix} \\ \amalg_{1.2.3} \\ \mathrm{perm}_3\left(\mathrm{ON}\left(\left[A\right]\right)\right) \end{pmatrix} = \begin{pmatrix} \mathrm{OFF}_1\left[A\right] \\ \amalg_{1.2.3} \\ \begin{pmatrix} \mathrm{ON}_3\left[A\right] \\ \amalg_{0.2.3} \\ \mathrm{ON}_2\left[A\right] \end{pmatrix} \end{pmatrix}$$

OFF $(\mathbf{neg}_1)$ **:**

$$\begin{pmatrix} \begin{pmatrix} \mathrm{neg}_1\left(\mathrm{OFF}\left(\left[A\right]\right)\right) \\ \amalg_{1.2.0} \\ \mathrm{perm}_2\left(\mathrm{OFF}\left(\left[A\right]\right)\right) \end{pmatrix} \\ \amalg_{1.2.3} \\ \mathrm{perm}_3\left(\mathrm{OFF}\left(\left[A\right]\right)\right) \end{pmatrix} = \begin{pmatrix} \mathrm{ON}_1\left[A\right. \\ \amalg_{1.2.} \\ \begin{pmatrix} \mathrm{OFF}_3\left[A\right. \\ \amalg_{0.2} \\ \mathrm{OFF}_2\left[\right. \end{pmatrix} \end{pmatrix}$$

ON $(\mathbf{neg}_2)$ **:**

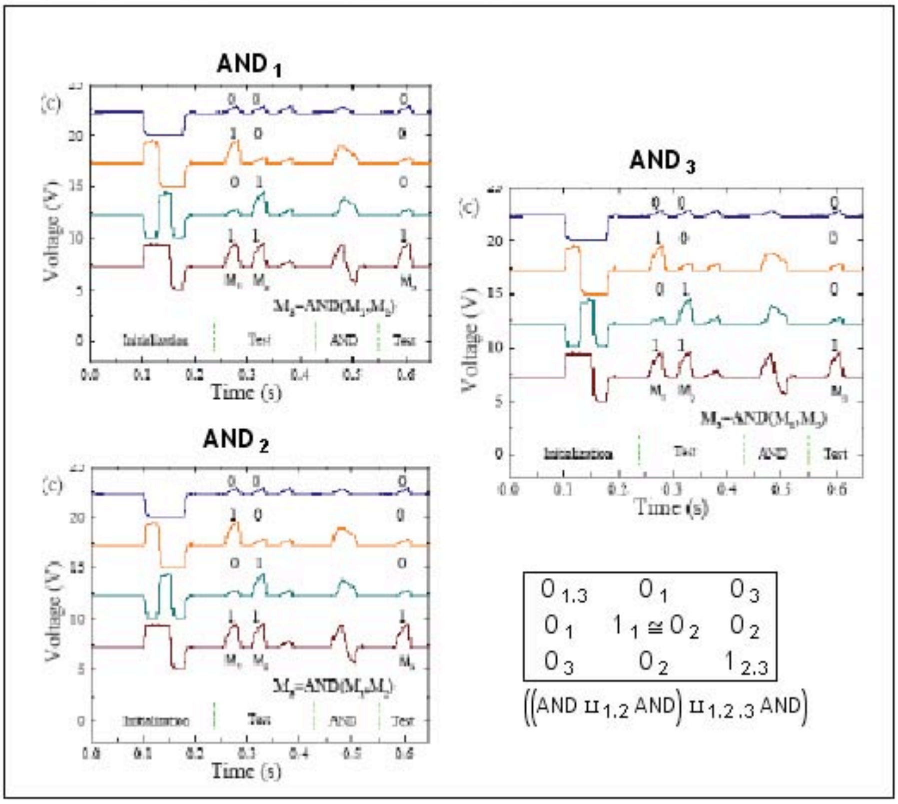
$$\begin{pmatrix} \begin{pmatrix} \mathrm{perm}_1\left(\mathrm{ON}\left(\left[A\right]\right)\right) \\ \amalg_{1.2.0} \\ \mathrm{neg}_2\left(\mathrm{ON}\left(\left[A\right]\right)\right) \end{pmatrix} \\ \amalg_{1.2.3} \\ \mathrm{perm}_3\left(\mathrm{ON}\left(\left[A\right]\right)\right) \end{pmatrix} = \begin{pmatrix} \mathrm{ON}_3\left[A\right] \\ \amalg_{1.2.3} \\ \begin{pmatrix} \mathrm{OFF}_2\left[A\right] \\ \amalg_{0.2.3} \\ \mathrm{ON}_1\left[A\right] \end{pmatrix} \end{pmatrix}$$

OFF $(\mathbf{neg}_2)$ **:**

$$\begin{pmatrix} \begin{pmatrix} \mathrm{perm}_1\left(\mathrm{OFF}\left(\left[A\right]\right)\right) \\ \amalg_{1.2.0} \\ \mathrm{neg}_2\left(\mathrm{OFF}\left(\left[A\right]\right)\right) \end{pmatrix} \\ \amalg_{1.2.3} \\ \mathrm{perm}_3\left(\mathrm{OFF}\left(\left[A\right]\right)\right) \end{pmatrix} = \begin{pmatrix} \mathrm{OFF}_3\left[\right. \\ \amalg_{1.2} \\ \begin{pmatrix} \mathrm{ON}_2\left[A\right. \\ \amalg_{0.2} \\ \mathrm{OFF}_1\left[\right. \end{pmatrix} \end{pmatrix}$$

2.7. Speculative emulation for (AND AND AND)

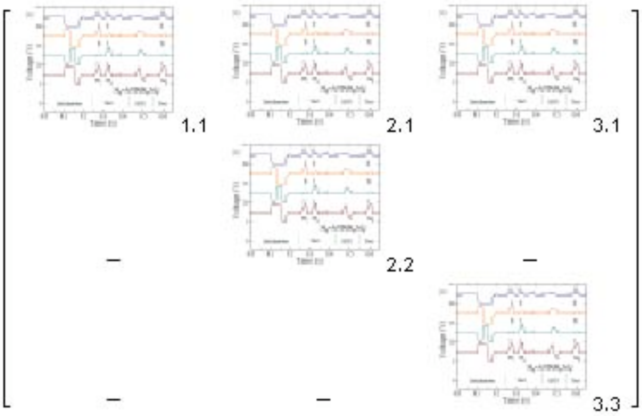
$vol(AND_1) = V_{t_1}$ is between $V_{p_1}/2$ and V_{p_1}.
$vol(AND_2) = V_{t_2}$ is between $V_{p_2}/2$ and V_{p_2}.
$vol(AND_1) = V_{t_3}$ is between $V_{p_3}/2$ and V_{p_3}.



2.7.1. Speculative modeling of transjunctions

$$(X <> \wedge \wedge Y) - \text{scheme}: \begin{bmatrix} <>_{1.1} & <>_{2.1} & <>_{3.1} \\ - & \wedge_{2.2} & - \\ - & - & \wedge_{3.3} \end{bmatrix}$$

$Mod_{elect}(X <> \wedge \wedge Y) =$



3. Morphogrammatics of transjunctions

3.1. Morphic abstraction of $(X <> \vee \wedge Y)$

$$\mathbf{val}\left(X <> \vee \wedge Y\right): \begin{pmatrix} 1 & 3 & 3 \\ 3 & 2 & 2 \\ 3 & 2 & 3 \end{pmatrix} = \begin{bmatrix} T_{1.3} & F_{2.3} & F_3 \\ F_{2.3} & F_1 T_2 & T_2 \\ F_3 & T_2 & F_{2.3} \end{bmatrix}:$$

$$\mathbf{Sys}_2\left(\vee\right): \mathrm{morph}\left(f_2\left(\left[A_2\right] \vee_{2.2} \left[B_2\right]\right),\ t_2\left(\left[A_2\right] \vee_{2.2} \left[B_2\right]\right)\right)$$
$$= \left[\vee_{2.2}\right]$$

$$\mathbf{Sys}_3\left(\wedge\right): \mathrm{morph}\left(f_3\left(\left[A_3\right] \wedge_{3.3} \left[B_3\right]\right),\ t_3\left(\left[A_3\right] \wedge_{3.3} \left[B_3\right]\right)\right)$$
$$= \left[\wedge_{3.3}\right]$$

$$\mathbf{Sys}_1\left(<>\right): \mathrm{morph}\left(\mathrm{TRANS}\right) = \left[\left[\mathrm{frame}_{1.1}\right], \left[\mathrm{core}_{2.1}\right], \left[\mathrm{core}_{3.1}\right]\right]:$$
$$\left(t_1\left(\left[A_1\right] <>_{1.1} \left[B_1\right]\right), f_1\left(\left[A_1\right] <>_{1.1} \left[B_1\right]\right) = \left[\mathrm{frame}_{1.1}\right]$$

$$f_1\left(\left[A_1\right] \wedge_{2.1} \left[B_1\right]\right),$$
$$\left(\left(f_1\left[A_1\right] \wedge_{2.1} t_1\left[B_1\right]\right) \vee_{2.1} \left(t_1\left[A_1\right] \wedge_{2.1} f_1\left[B_1\right]\right)\right) = \left[\mathrm{core}_{2.1}\right]$$
$$\left(\left(f_1\left[A_1\right] \vee_{2.1} t_1\left[A_1\right]\right) \wedge_{2.1} \left(t_1\left[B_1\right] \vee_{2.1} f_1\left[B_1\right]\right)\right)$$

$$t_1\left(\left[A_1\right] \wedge_{3.1} \left[B_1\right]\right),$$
$$\left(\left(f_1\left[A_1\right] \wedge_{3.1} t_1\left[B_1\right]\right) \vee_{3.1} \left(t_1\left[A_1\right] \wedge_{3.1} f_1\left[B_1\right]\right)\right) = \left[\mathrm{core}_{3.1}\right].$$
$$\left(\left(f_1\left[A_1\right] \vee_{3.1} t_1\left[A_1\right]\right) \wedge_{3.1} \left(t_1\left[B_1\right] \vee_{3.1} f_1\left[B_1\right]\right)\right)$$

Pattern: [bif, id, id] for transjunction

$[\oplus \vee \wedge]$	S_1^1	S_2^1	S_3^1	S_1^2	S_2^2	S_3^2	S_1^3	S_2^3	S_3^3
1	○	–	–	–	–	–	○	–	○
2	–	–	–	□	–	–	□	–	–
3	–	–	–	–	–	–	–	–	○
4	–	–	–	□	–	–	□	–	–
5	△	–	–	△	△	–	–	–	–
6	–	–	–	–	△	–	–	–	–
7	–	–	–	–	–	–	–	–	○
8	–	–	–	–	△	–	–	–	–
9	–	–	–	–	□	–	–	–	□

$[\oplus \vee \wedge]$	$O1$	$O2$	$O3$
$M1$	$[trans]_1$	$[trans]_1$	$[trans]_1$
$M2$	$\varnothing$	$[or]_2$	$\varnothing$
$M3$	$\varnothing$	$\varnothing$	$[and]_3$

$[\oplus \vee \wedge]$	1	2	3
1	○	□	□
2	□	△	△
3	□	△	□

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3.2. General schemes for transjunctions and reflectors

3.2.1. Transjunction schemes

Transjunction scheme for $\oplus_i$:

$$\left[\bigoplus_{i,j}\right] = \begin{bmatrix} \circ & \square \\ \square & \triangle \end{bmatrix}_{i,j} :$$

$$\left[\left[S_i^i\right], \left[S_i^{i+1}\right], \left[S_i^{i+2}\right]\right] = \left[\left[\circ\ \triangle\right], \left[\square\ \square\ \triangle\right], \left[\circ\ \square\ \square\right]\right]$$

$$\left[\left[\bigoplus_{1.1}\right], \left[MG_{2.2}\right], \left[MG_{3.3}\right]\right] =$$

$$\left(\begin{array}{c}\left(\begin{array}{c}\left[\oplus_{1.1}\right]\\ \amalg_{1.2.0}\\ \left[MG_{2.2}\right]\end{array}\right)\\ \amalg_{1.2.3}\\ \left[MG_{3.3}\right]\end{array}\right) = \left(\begin{array}{c}\left(\begin{array}{c}\left[S_1^1\right]\\ \amalg_{1.2.0}\\ \left[MG\right]_2 \diamond_{2.1} \left[S_1^2\right]\end{array}\right)\\ \amalg_{1.2.3}\\ \left[MG\right]_3 \diamond_{3.1} \left[S_1^3\right]\end{array}\right)$$

$$\left[\left[\bigoplus_{1.1}\right], \left[MG_{2.2}\right], \left[MG_{3.3}\right]\right] =$$

$$\left(\begin{array}{c}\left(\begin{array}{c}\left[\oplus_{1.1}\right]\\ \amalg_{1.2.0}\\ \left[MG_{2.2}\right]\end{array}\right)\\ \amalg_{1.2.3}\\ \left[MG_{3.3}\right]\end{array}\right) = \left(\begin{array}{c}\left(\begin{array}{c}\left[S_1^1\right] \diamond_{2.1} \left[S_1^2\right] \diamond_{3.1} \left[S_1^3\right]\\ \amalg_{1.2.0}\\ \left[MG\right]_2\end{array}\right)\\ \amalg_{1.2.3}\\ \left[MG\right]_3\end{array}\right)$$

Transjunction as bifurcation

$\bigoplus$ = bifurcation "bif".

$$\mathrm{bif}_1\left(\left[\left[MG_{1.1}\right], \left[MG_{2.2}\right], \left[MG_{3.3}\right]\right]\right) = \left[\left[\bigoplus_{1.1}\right], \left[MG_{2.2}\right], \left[MG_{3.3}\right]\right]$$

$$\left[S_1^1\right] = \left[\circ - - \triangle\right], \left[S_1^2\right] = \left[- \square\ \square\ \triangle\right], \left[S_1^3\right] = \left[\circ\ \square\ \square\ -\right]$$
$$\mathrm{dom}\left(\left[S_1^1\right]\right) = \mathrm{dom}\left(\left[S_1^3\right]\right) = \mathrm{dom}\left(\left[S_3^3\right]\right)$$
$$\mathrm{cod}\left(\left[S_1^1\right]\right) = \mathrm{cod}\left(\left[S_1^2\right]\right) = \mathrm{dom}\left(\left[S_2^2\right]\right)$$
$$\mathrm{cod}\left(\left[S_2^2\right]\right) = \mathrm{cod}\left(\left[S_3^3\right]\right)$$

$$\left[\oplus_{1.1}\ MG_{2.2}\ \ MG_{3.3}\right] : [\text{bif, id, id}]$$

$$\begin{pmatrix} \begin{pmatrix} [\oplus_{1.1}] \\ \amalg_{1.2.0} \\ [MG_{2.2}] \end{pmatrix} \\ \amalg_{1.2.3} \\ [MG_{3.3}] \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} [S_1^1] \diamond_{2.1} [S_1^2] \diamond_{3.1} [S_1^3] \\ \amalg_{1.2.0} \\ [MG_{2.2}] \end{pmatrix} \\ \amalg_{1.2.3} \\ [MG_{3.3}] \end{pmatrix}$$

$$\left[\oplus\oplus\ MG_{3.3}\right] : [\text{bif, bif, id}]$$

$$\begin{pmatrix} \begin{pmatrix} [\oplus_{1.1}] \\ \amalg_{1.2.0} \\ [\oplus_{2.2}] \end{pmatrix} \\ \amalg_{1.2.3} \\ [MG_{3.3}] \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} [S_1^1] \diamond_{1.2} [S_1^2] \diamond_{1.3} [S_1^3] \\ \amalg_{1.2.0} \\ [S_2^1] \diamond_{2.1} [S_2^2] \diamond_{2.3} [S_2^3] \end{pmatrix} \\ \amalg_{1.2.3} \\ [MG_{3.3}] \end{pmatrix}$$

$$\left[\oplus\oplus\oplus\right] : [\text{bif, bif, bif}]$$

$$\begin{pmatrix} \begin{pmatrix} [\oplus_{1.1}] \\ \amalg_{1.2.0} \\ [\oplus_{2.2}] \end{pmatrix} \\ \amalg_{1.2.3} \\ [\oplus_{3.3}] \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} [S_1^1] \diamond_{1.2} [S_1^2] \diamond_{1.3} [S_1^3] \\ \amalg_{1.2.0} \\ [S_2^1] \diamond_{2.1} [S_2^2] \diamond_{2.3} [S_2^3] \end{pmatrix} \\ \amalg_{1.2.3} \\ [S_3^1] \diamond_{3.1} [S_3^2] \diamond_{3.2} [S_3^3] \end{pmatrix}$$

3.2.2. Reflector

$$\text{Refl}_1\left(\left[[\oplus_{1.1}], [MG_{2.2}], [MG_{3.3}]\right]\right) = \left(\left[[\oplus_{1.1}], [\oplus_{2.2}], [MG_{3.3}]\right]\right)$$

$$\begin{pmatrix} \begin{pmatrix} \text{refl}_1[\oplus_{1.1}] \\ \amalg_{1.2.0} \\ [MG_{2.2}] \end{pmatrix} \\ \amalg_{1.2.3} \\ [MG_{3.3}] \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} \text{refl}_1[S_1^1] \\ \amalg_{1.2.0} \\ [MG]_1 \diamond_{2.1} \text{refl}_1[S_1^2] \end{pmatrix} \\ \amalg_{1.2.3} \\ [MG]_3 \diamond_{3.1} \text{refl}_1[S_1^3] \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} \text{refl}_1[\oplus_{1.1}] \diamond_{2.1} [S_1^3] \diamond_{3.1} [S_1^3] \\ \amalg_{1.2.0} \\ [\oplus_{2.2}] \diamond_{1.2} [S_1^2] \diamond_{3.2} [S_1^2] \end{pmatrix} \\ \amalg_{1.2.3} \\ [MG_{3.3}] \end{pmatrix}$$

$$\left[\left[\circ\ \triangle\right]1,\ \left[\square\ \square\ \triangle\right]2,\ \left[\circ\ \square\ \square\right]3\right] \qquad \left[\left[\triangle\ \circ\right]1,\ \left[\triangle\ \square\ \square\right]3,\ \left[\square\ \square\ \circ\right]2\right]$$

$$\begin{bmatrix} \circ & \square \\ \square & \triangle \end{bmatrix}_{i,j} : \mathrm{refl}\left(\begin{bmatrix} \circ & \square \\ \square & \triangle \end{bmatrix}\right) = \begin{bmatrix} \triangle & \square \\ \square & \circ \end{bmatrix}$$

$$\left[\oplus \vee \wedge\right] = \boxed{\begin{matrix} \circ & \square & \square \\ \square & \triangle & \triangle \\ \square & \triangle & \square \end{matrix}} \qquad \boxed{\begin{matrix} \triangle & \square & \square \\ \square & \circ & \triangle \\ \square & \triangle & \square \end{matrix}}$$

$$\mathrm{Refl}_1\left(\left[\oplus \vee \wedge\right]\right) = \left[\oplus \oplus \wedge\right]$$

$$\left(\begin{matrix} \left(\begin{matrix} \mathrm{refl}_1\left[S_1^1\right] \\ \amalg_{1.2.0} \\ \left[\mathrm{MG}\right]_1 \diamond_{2.1} \mathrm{refl}_1\left[S_1^2\right] \end{matrix}\right) \\ \amalg_{1.2.3} \\ \left[\mathrm{MG}\right]_3 \diamond_{3.1} \mathrm{refl}_1\left[S_1^3\right] \end{matrix}\right) = \left(\begin{matrix} \left(\begin{matrix} \mathrm{refl}_1\left[\oplus_{1.1}\right] \diamond_{2.1} \left[S_1^3\right] \diamond_{3.1} \left[S_1^3\right] \\ \amalg_{1.2.0} \\ \left[\oplus_{2.2}\right] \diamond_{1.2} \left[S_1^2\right] \diamond_{3.2} \left[S_1^2\right] \end{matrix}\right) \\ \amalg_{1.2.3} \\ \left[\mathrm{MG}_{3.3}\right] \end{matrix}\right)$$