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Title

Diamond Calculus of Formation of Forms

A calculus of dynamic complexions of distinctions as an interplay of worlds and distinctions

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A calculus of dynamic complexions of distinctions as an interplay of worlds and distinctions

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A new abstraction is introduced which enables to unify the beginnings "order" and "number" of the Calculus of Indication (CI) in George Spencer Brown's Laws of Form (LoF). This unification is produced by the abstraction of *interchangeability* of the two primary beginnings of "order" and "number". This unification as interchangeability gets generalized in a complexion of distributed LoF, which are mapped onto the kenomic matrix. Basic functions are analyzed in respect of a distributed reentry constellations, retrograde recursivity and exemplified with enaction, succession, addition (coalition) and multiplication (cooperation) of LoF systems. As a consequence, a chiasm and diamond between the hidden difference of world and distinction is established. Memristive behaviors of the calculus are shortly mentioned.

Topics

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- DRAFT - best results with **Wolfram *Mathematica* Player**, free download.
Good results: read the PDF in a Safari browser.

http://www.thinkartlab.com/pkl/media/Diamond Calculus/Diamond Calculus.nb

1. Why space isn't the place

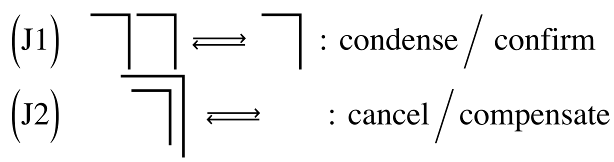
After well 40 years of training to deal with George Spencer Brown's *Laws of Form* (LoF)it might be time to start the game again with some more lively and less dogmatic conditions than celebrated especially by people sordid to hear any whisper of a new pries from abroad of the dictate of uniform formation and distinction.

> "*Although all forms, and thus all universes, are possible, and any particular form is mutable, it becomes evident that the laws relating to such forms are the same in any universe. It is this sameness, the idea that we can find a reality which is independent of how the universe actually appears, that lends such fascination to the study of mathemeatics."* (George Spencer Brown, Laws of Form, 1972 edition, p. v)

The conception of calculation

1. **Condensation**: Two instances of the form are equivalent to one instance of the form if they are placed in the same space.
2. **Cancellation**: Two instances of the form are equivalent to no instance of the form alias empty space if one of the forms is the argument of the other form.

Arithmetic of LoF



http://mathworld.wolfram.com/Spencer-BrownForm.html

Who wants to be placed in the same space? It's time to develop a calculus where things or persons, events or thoughts might be placed at different spaces, too. Not at one place alone but at an irreducible manifold of different spaces, places, fields, arenas and whatever. Not in isolation and separation but enabled to interaction, reflection and interventions.
Spaces that are enabled to become contents and contents that are able to change to spaces.

Academically, we might distinguish between G. Spencer-Brown's abstract and *mono-contextural* formal concept of unique distinction as "perfect continence" and a more 'qualitatively' disseminated complexion of distinctions as "co-creative togetherness" in a *polycontextural*, i.e. interactional and reflectional setting.

Recall, again

> "*The key idea in Spencer-Brown's representation of indication is that all distinctions in their formal sense are alike, and all domains in which distinctions are performed are also alike. This gives rise to the notion of* primary

> *distinction and* indicational *space. We erase every qualitatively difference of the criteria of distinctions, and simply reduce them to their essential quality: generating boundary in whatever domain."* (F. Varela, Principles of Biological Autonomy, 1979, p.110)

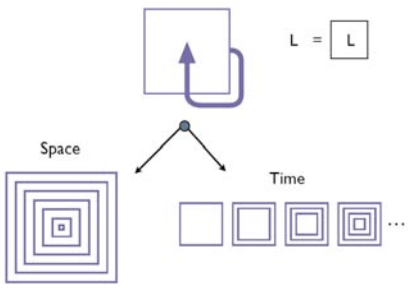
It might be the case that *"the laws relating such forms are the same in any universe"* (GSB). But sameness has not necessarily to lead to uniqueness, and to a unique universe with its unique Laws of Form. Sameness might be understood, not as equality but as equivalence, and equivalence is opening up the possibility of many kinds of Laws of Form, and therefore different forms for different universes, i.e. pluri- or poly-verses. Logically, the polyverse approach is playing with the difference of *equality* (Selbigkeit) and *equivalence* (Gleicheit) in contrast to *difference* (Verschiedenheit). Spencer Brown's Laws of Form are based on a simple unique distinction of a two-sided form.

If "*space is the place*" of distinctions (Schiltz) we have to disseminate this space, and place it at its different places.

Hence, a more radical understanding of the category of distinction is not involved in presuppositions of "time" and "space". Both are not pre-given but have to be created by distinctions.

Deparadoxing in time and space

Kauffman's classical "space" and "time" "solution" of the paradox reentry form. For Luhmann its is "Entparadoxierung in Raum und Zeit".



http://www.math.uic.edu/~kauffman/KauffSAND.pdf

Antidromic reentry forms

There is no reason either why the action of "*order*" and the action of "*number*" has to be taken in series, one after the other, and not in parallel, i.e. both at once. It might be argued that the initials J1 and J2 are holding both at once in the calculus but there is no formula which is expressing such a parallelism. In contrast, it is shown that the initials are independent, therefore separated and there is no third initial which rules their interplay.

Because of the "linear" order of the Calculus of Indication (CI), reentry form is reduced to "unidirectional" recursivity, i.e. 'one-way' circularity, and is therefore not able to formalize the antidromic complementary 'backwards' 'movement' of simultaneously - "*to eat and to be eaten"* - of non-egologically founded self-referentiality as it was asked by the 'late' Francesco Varela, in contrast to his early reentry form of the Extended Calculus of Indication.

Quadralectics

The quadralectic (tetralemmatic, diamond) notation is enabling operations on the parts of the diamond complexions consisting of *Inside*, *Outside* and *inside*, *outside,* i.e. [[*A* | *a*] | [*a* | *A*]], short: [*a* | *A* | *a*].
Those operations applied to the quadralectic complexion have to preserve the rules of retrograde recursivity.

[[*A* | *a*] | [*a* | *A*]]:
[Inside | Outside] **|** [outside| inside]:
[*Inside* of inside | *Outside* of inside] | [*outside* of Outside | *inside* of Outside].

The antidromicity of the Uroboros figure isn't such an obscure concept if one owns perception isn't restricted to hierarchy; a full reading of the Uroboros figures tells it all. Although Spencer Brown worked together with Ronald Laing, he didn't take the full advice.

> *One tries to get inside oneself*
> *that inside of the outside*
> *that one was once inside*
> *once one tries to get oneself inside what*
> *one is outside:*
>
> *to eat and to be eaten*
> *to have the outside inside and to be*
> *inside the outside.*
>
> R. D. Laing (1970, Knots. New York: Vintage Books., p. 83).

Hence, the distinction of '*order*' and '*number*', or serial and parallel processes, inscribed by the two first axioms of the *Calculus of Indication* might be applied on the calculus itself: *order* and *number* are 'ordered' and 'numbered' at once onto itself.

In a first step, *interchangeability* as a new abstraction of the interaction of *order* and *number,* shall be constructed.

The idea to start with 'circularity' and 'self-reference' instead of constructing it with reentry is not simply a change of the order of the architectonics of the calculus but to give it a new arena to play its new choreography. Hence, new abstractions that allow a dissemination of the original calculus are in demand.

The new kind of 'self-reference' is the *retrograde recursivity* of the re-configurations of complexions of LoFs.
GSB's LoF is monocontextural and therefore its concept of iterability and recursivity doesn't have the conceptuality for retrograde recursivity. LoF is limited to iteration and reentry of its forms inside a contexture (space) without any formal guarantee for the function to not to miss the re-entry of its form.

It might be said that the two initials J1 and J2 of LoF are formulating the parallel and serial aspect of the act of distinction. But both remain conceptually separated and there is also no theoretical explanation offered which could bridge this gap. Even if there are formulas formulating a kind of an interplay between J1 and J2, they are secondary in the architectonics of the LoF and not primarily constitutive for the formalism.

The proposed 'interchangeability' approach to serial and parallel features of the act of distinction is making clear from the very beginning of the calculus their inter-relatedness.
http://memristors.memristics.com/Polyverses/Polyverses.pdf

2. Complexions of LoF

Instead of postulating the so called obvious facts of distinction and their primary behavior as "*calling*" and as "*crossing*", formalized in the two beginnings of the arithmetics of LoF, I will set the act of distinction into a field of other distinctions. Therefore, every primary concept of distinction is embedded in a field of other distinctions. Hence, every distinction is surrounded with its neighbor distinctions.
The world supposed by G. Spencer-Brown for his LoF is without distinctions. It is not yet the place to determine the structure of a world within a field of different distinctions.

The simplest situation possible for a 'complex' LoF is introduced by the diamond of distinctions.
Before any initials J1 and J2 are introduced, GSB distinguishes the distinction as a mark $\rceil$.

Hence, a complex FoL starts not with a singular mark but with a chiasm, more precisely, with a diamond of marked distinctions: $\rceil_1$, $\rceil_2$, $\rceil_3$, $\rceil_4$ and their diamond inter-relations.
As a consequence, even the singular distinction is understood, not as an atomic element $\rceil$, but as a morphism onto itself: $\rceil \longrightarrow \rceil$.

Cancellation vs. memorization
If a distinction marks a state, then a distinction of a distinction marks the state of a state. Such a distinction of a distinction as $\rceil\!\rceil$ boils down in the LoF by cancellation to a no-distinction, "*alias empty space*", i.e. $\rceil\!\rceil \to \varnothing$. With that, all possibilities of an inscription of second-order states, i.e. of states of states that *memorize* their previous state, like it is typical for *memristive* systems, is lost from the very beginning of the calculus. Memory, then, and time, is introduced later by the LoF with the help of reentry flip-flops.

Unification of beginnings in a complexion

2.1.1. Distinction and kenogram

As a next step towards a complex LoF, c-LoF, the beginnings of LoF are put together on a second-order level of the distinction of the law of "*calling*" and the law of "*crossing*', that is on a level ruled by composition and mediation.

Why is it reasonable to map LoF onto morphogrammatic systems?

If a distinction marks a state, then a distinction of a distinctions marks the state of a state.

Such a distinction of a distinction as boils down in the LoF to no-distinction, i.e. → ∅. With that, all possibilities of an inscription of second-order states, i.e. of states of states that memorize the previous state, like it is typical for *memristive* systems, is lost from the very beginning of the calculus.

"Distinction is perfect continence." (LoF)

Morphograms, consisting of kenograms and monomorphies of kenograms, are *"perfect continence"*.
In logico-semantical terms we might state that *"perfect continence"* means or says that a kenogram contains both truth-values at once, the value for true and the value for false. There is nothing more a sentence, sign or mark might contain. Therefore, it is in the mode of "perfect continence".

In other words, *"perfect continence"* for a kenogram means that the distinction of "true" and "false" gets rejected as a whole. A kenogram is neither in the state of "true" nor in the state of "false"; both are rejected as such.

LoF disseminated onto morphogrammatic systems offers adequate mechanism of extending and transforming fields of distributed and mediated LoFs.

A logical interpretation of the distinction mark in LoF as 'containing' both truth-values "at once" was given by GSB himself. Also the citation suggest a succession of interpretations, i.e. "*we have the choice*", the form , 'as such', is incorporating both possibilities at once.

> *"We see, in logic, that 'not true' means the same as 'false', and that 'not false' also means 'true'. So we have the choice of whether to associate the unmarked state with truth and the marked state with untruth, to associate marked state with truth and the unmarked state with untruth."* (LoF, p. 113)

mapping as " *perfect continence* " : $\begin{pmatrix} \text{true} \\ \text{false} \end{pmatrix} \longrightarrow \left(\overline{|} \right)$.

The same holds for kenograms :

mapping as " *perfect continence* " : $\begin{pmatrix} \text{true} \\ \text{false} \end{pmatrix} \longrightarrow (\blacklozenge)$.

Kenograms are " *perfect continence* " of valuation.

2.1.2. Semiotic relationship of LoF and morphogrammatics

> *"... we can see that there is some symmetry in the relationship between G. S. Brown's commutative strings and G. Günther's kenograms:*
> *The former are invariant w.r.t. permutations of the index set {1,...,n}, while the latter are invariant w.r.t. permutations of the alphabet A."*
> *"As we noticed in the introduction, in the context of "Laws of Form" for any two terms "a" and "b" the concatenation results "ab" and "ba" are semiotically identical."*
> *"A third deviation from classical semiotics is less obvious: the commutativity of the concatenation operation. For any two terms "a" and "b" the terms "ab" and "ba" are identical."* (Matzka, 1993)

http://www.rudolf-matzka.de/dharma/semabs.pdf

Mediation of LoF and MG

Morphogrammatically based complexions of LoFs might therefore be understood as a mediation of two fundamentally different semiotic systems. The commutative *heap*-semiotics of LoFs and the identity-abstraction semiotics of MG.

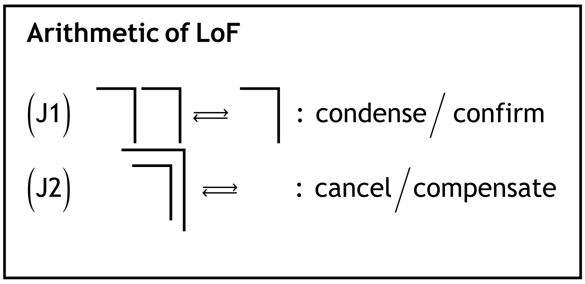
It is not possible to map morphogrammatics (MG) onto LoF but it is possible to map LoF onto MG.

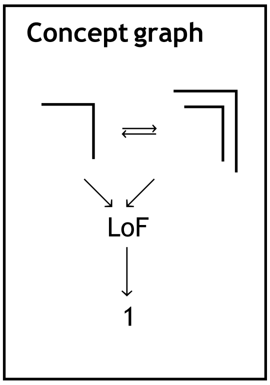
Also morphogrammatics is not localized on a level of conceptual and operational identity, morphogrammatics is much more complex than the architectonic design of LoF. Therefore, a dissemination of LoF over morphogrammatics opens up new insight into a possible calculus of complex forms.

2.1.3. Distinguishing the distinction of worlds and distinctions

"The theme of this book is that a universe comes into being when a space is severed or taken apart." (George Spencer Brown, Laws of Form, 1972 edition, p. v)

A) Inside a single world of distinction, the universal approach

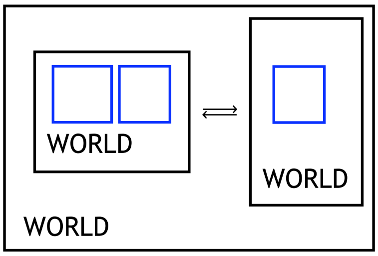


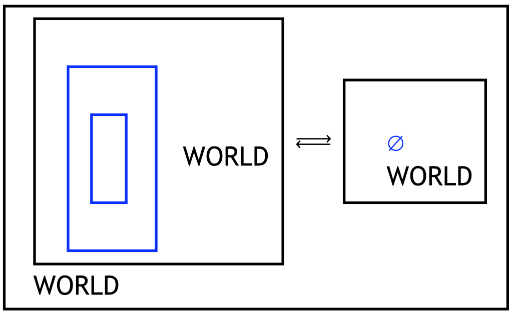


The arithmetics of LoF is based on two initials: J1 and J2. Introduced in LoF as "number" (J1) and as "order" (J2).
The concept graph is inscribing the conceptual relationship between the conceptual constituents of LoF. The calculus is defined in the realm of identity. According to the principle of *relevance*, the indication "1" of the concept graph, indicating the uniqueness of the calculus, might be omitted in LoF.

Any distinction and its marks, combined with its initials as operators are defined inside a world. Hence, neither distinctions nor operations on distinctions are leaving its world of distinctions.

This might be depicted by the following diagrams.





The main property of the universal approach to distinction is its 'topological' *closure*, or in other terms, its completeness.
Metatheoretically, LoF is complete (theorem 17), consistent and finite, and its axioms are independent. Hence, LoF is a sound calculus.

Theorem 1. Form
"*The form of any finite cardinal number of crosses can be taken as the form of an expression.*" (GSB, FoL, p. 12)

Taken the initials J1 and J2 as the operator J and x as a sequence of J, then we get the closure properties:

(A0) $\emptyset = J(\emptyset)$	[Einbettung]
(A1) $x \subseteq J(x)$	[Monotonie]
(A2) $J(J(x)) \subseteq J(x)$	[Abgeschlossenheit]

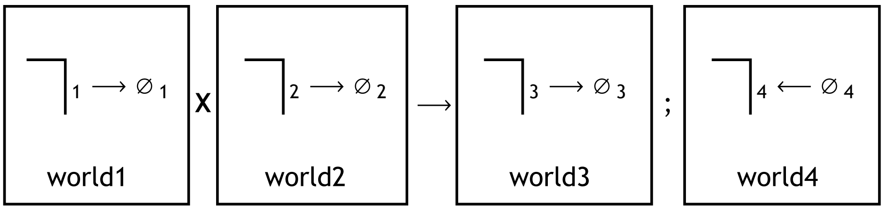
Topologically, the formalization of the universe of distinction of LoF is characterized by its *closure*.
Hence, for any distinction, there exists a universe U (world) such that the distinction is element (part) of it.

Principle of relevance and polycontextural place-designator
I might paraphrase, that the universality, in contrast to the polyversality, of the LoF doesn't need to be indicated because it is a property applicable to every indication of the LoF. (Seventh Canaon. Principle of relevance).

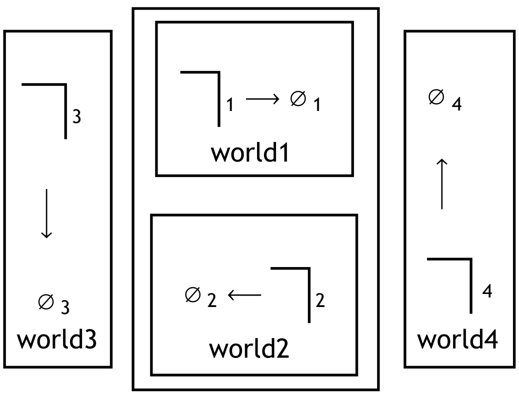
Mereotopology and Polycontexturality
http://www.thinkartlab.com/pkl/media/Mereo/Mereotopology.html (to come)

B) Interaction between worlds of distinctions, the polyversal approach

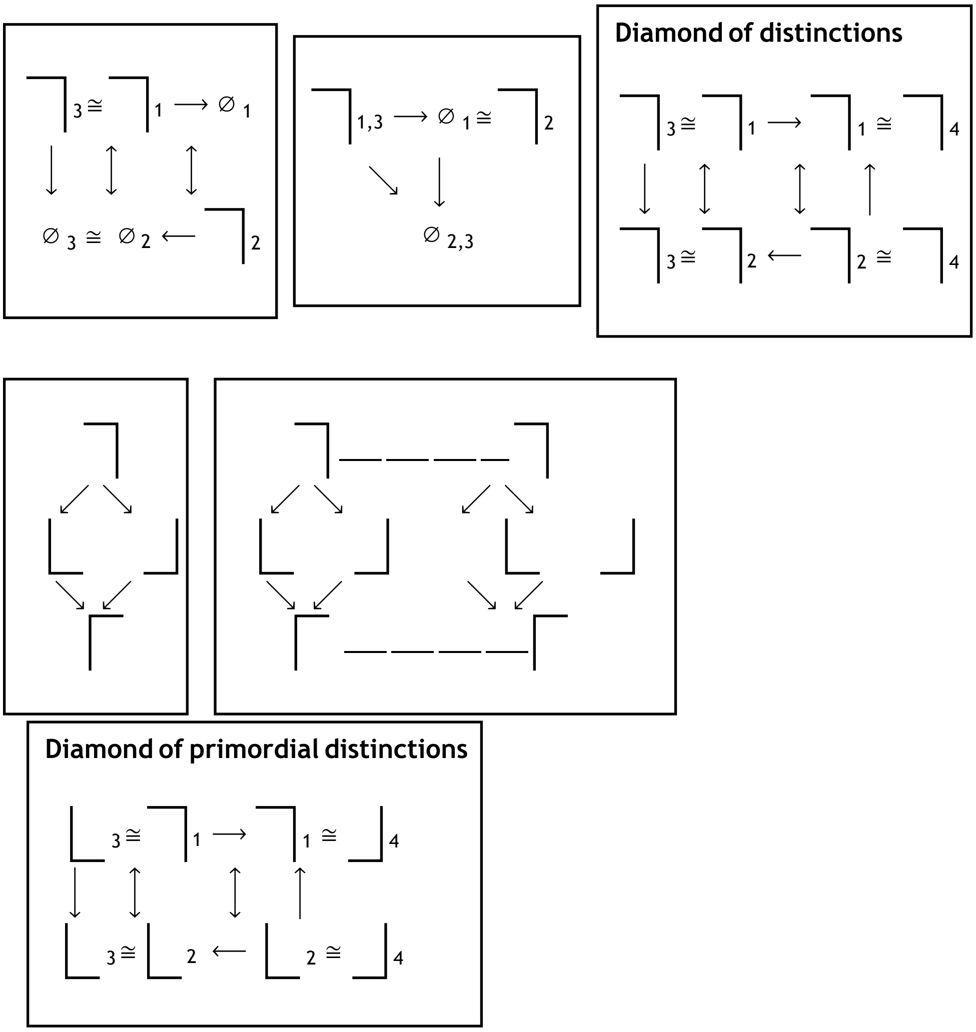
Because LoF is a sound calculus it is useless to try to extend it internally. On the other hand, it is a miserable life which is offered by this approach. What is not explicitly established inside the calculus of LoF is its presupposition of the necessity of a unique and universal world. LoF's one-world-assumption might be deconstructed towards a many-world-assumption in the sense of a polycontextural polyverse.



Chiastic form of composition



C) Concept graph of chiastic and diamond composition



First attempts for a dissemination of LoF goes back to 1980:
http://www.vordenker.de/ggphilosophy/rk_meta.pdf

From Universe to Polyverses
http://memristors.memristics.com/Polyverses/Polyverses.pdf

2.1.4. Diamond of distinction

Complementary and inverse forms

Because of the principle of "*perfect continence*", there are no dual forms in a logical sense in the calculi of forms. What is reflecting the formation of forms are parallax and complementary, i.e. diamond formations of form. This is mirrored first, by the systems of inverse forms. Hence, the basic, and not yet disseminated planar forms, are the forms of *complementarity* and *inversion*.

The aim is not to stay in a slavery obedience with the Boolean universe but to create diamond complementary and inverse universes, mediated in a polyverse interplay.

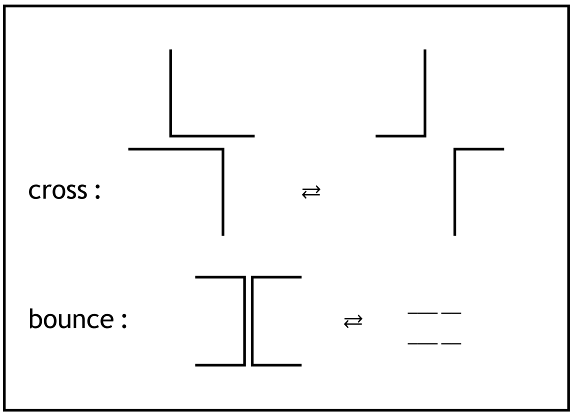
Distinctions between distinction systems

Beyond the systematics of planar distinctions, a polycontextural theory and calculus of distinctions, is demanding for distinctions between discontextural distinction systems. This might be realized by the introduction of topological and knot-theoretic constellations of distinction systems. A simple start could be a 3-dimensional distinction system with the set of planar distinctions and reentries at each contextural position and the transcontextural distinctions and reentry forms between distributed contextural distinction systems.

Map reformulation approach

A strictly different approach to "planar" distinctions is given by Spencer-Brown himself: *map reformulation,* (ed.) Lou Kauffman, princelet editions, 1986.

The new property of "The calculus of Idempotence" is: *"Common Boundaries Cancel"*.



"The Calculus of Idempositions is a Diagrammatic Language involving Closed Curves." (GSB)

Louis H. Kauffman, Reformulating the map color theorem.
http://www.math.uic.edu/~kauffman/MapReform.pdf

2.2. Quadralectic thematization

2.2.1. Quadralectic distinctions

A diamond calculus is not starting with the act of drawing a distinction to mark it by the complex activity of thematization. Thematization has to consider all formal aspects of the act of distinction. With the decision for a polyverse in contrast to a abstract universe of distinctions, the act of distinctions becomes itself distinguished by its “inside and outside” of distinction.
Therefore, polycontextural distinction becomes a 4-fold structure, and its dynamics are inscribed as a diamond of four different marks. This is in correspondence with Kent D Palmer’s quadralectics.
http://www.scribd.com/doc/30047691/Palmer-s-Pentalectics

According to Ronald Laing's writing, the structure of distinction gets a four-fold (quadralectic) of primary distinctions:

Quadralectic distinctions

Inside a contexture of distinction : ⌉

Outside a contexture of distinction : ⌈

The “inside of the outside” of distinction : ⌋

The “outside of the inside” of distinction : ⌊

This primary four-fold of distinction is placed in a polyverse by the proto-distinction of a place-designator for a distinctional system. In other words, primary distinctions are disseminated over the kenomic matrix, therefore placed and positioned and thus enabled for interactional and reflectional interplays.
Such proto-typical distinctions between contextures of distinctions, ruled by the place-designator, might establish a n-dimensional system of notation of marks.

Interestingly, the interpretation of the act of distinction by Matthias Varga von Kibéd and Rudolf Matzka is mentioning similar distinctions (of inside/outside of the distinctional/distinctive space):
"*d) Zur Form dieser Unterscheidung gehören nun der von der Unterscheidung gespaltene Raum zusammen mit dem gesamten Inhalt des Raums. Man könnte mit anderen Worten folgende Bestandteile der Form einer Unterscheidung auflisten:*
i) der Raum*, in dem die Unterscheidung stattfindet,*
ii) der Prozeß der Unterscheidung und zugleich sein Ergebnis, also die von der Unterscheidung erzeugte Grenze*,*
iii) das von dieser Grenze Umschlossene, das Innere*, Abgegrenzte, der markierte Zustand,*
iv) das außerhalb dieser Grenze Gelegene (das Äußere*, nicht Abgegrenzte, der nicht markierte Zustand)."*
Matthias Varga von Kibéd und Rudolf Matzka, Motive und Grundgedanken der »Gesetze der Form«, in: aus: Kalkül der Form, (Dirk Baecker, hrsg.), Suhrkamp, 1993, p.58-85.

Diamond Calculus of Formation of Forms

Construction

Draw a distinction, mark it : ┐

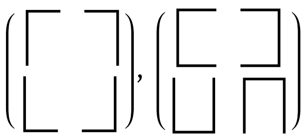
Distinguish the drawing, remark it : ┌

Reverse the distinction, sign it : ┘

Converse the drawing, resign it : └

Forms of Formation of planar Forms

Distinction set = {┐, ┌, └, ┘, ⊐, ⊏, ⊔, ⊓, |}

Distinction systems = 

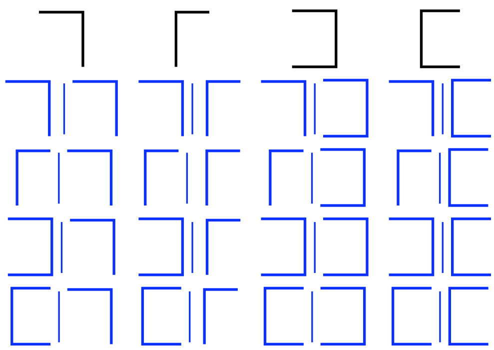
┐, ┌ : complementary distinctions

⊐, ⊏ : complementary reentries

┐, ┘; ┌, └ : inverse distinctions

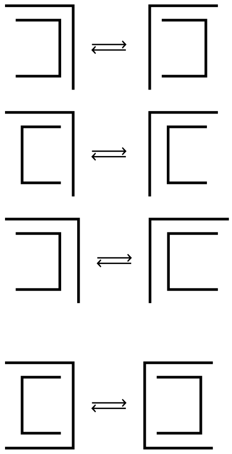
⊏, ⊔; ⊐, ⊓ : inverse reentries

Table1 of diamond formations



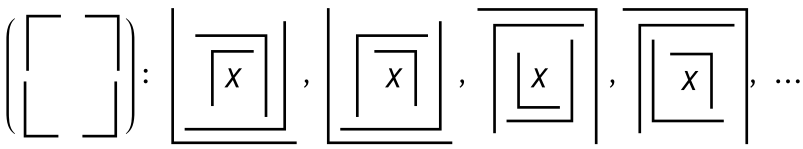
Distinctions are 'double faced',this becomes specially obvious with the double reading of the reentry form.

Complementarity of formations



Self-referential constellations

New form constellations of self-reference might be constructed as a composition of all types of distinction forms together.



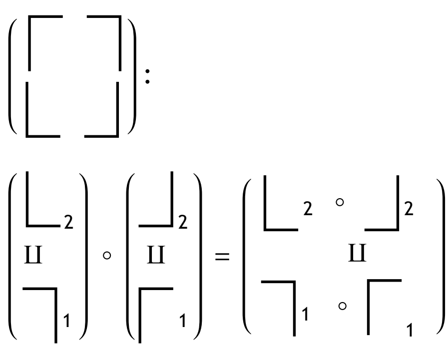
Interchangeability of Forms

$\mathcal{U}_2 = \{\text{inverse}_2,\ \text{antiinverse}_2\}$

$\mathcal{U}_1 = \{\text{form}_1, \text{antiform}_1\}$

$\mathcal{U}_1 \bigcap_{1.2} \mathcal{U}_2 = \emptyset$

$\mathcal{U}^{(2)} = \mathcal{U}_1 \amalg_{1.2} \mathcal{U}_2 :$



$\amalg$: mediation between contextures

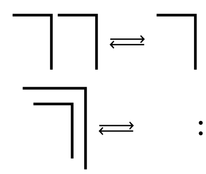
$\circ$: composition of morphisms

$=$: equivalence

2.2.3. Diamond structure of the calculus of indication

A closer look at the CI shows its hidden, i.e. silently presupposed, *diamond* structure. This becomes obvious if the matching conditions, necessary to repeat the marks in a linear concatenation and not somewhere else in the space, are properly considered too. As a further step, the results of diamondized distinction rules have to be applied to the whole theory and calculus of nonrestricted distinctions in polyverse constellations.

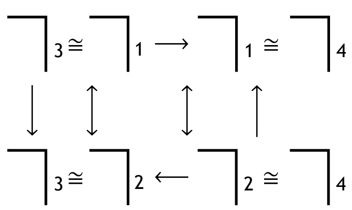
LoF initials



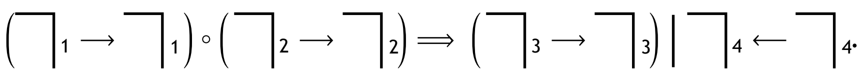
(J1) : condensation

(J2) : cancellation

Diamond distinction



Compositional notation for **Diamond distinction**



Adapting the the well known notation for categorical diamonds we get the *Diamond Distinction* as a diagram and the structure of diamond a the *Diamond Distinction*:

Diagram of Diamond Distinction

$$\begin{array}{c} \left(\urcorner_4 \longleftarrow \urcorner_4\right) \\ \shortmid \qquad \shortmid \\ \left(\urcorner_1 \longrightarrow \urcorner_1\right) \circ \left(\urcorner_2 \longrightarrow \urcorner_2\right) \\ \parallel \qquad\qquad\qquad\qquad \parallel \\ \left(\urcorner_3 \xrightarrow{\qquad\qquad\qquad\qquad\qquad} \urcorner_3\right) \end{array}$$

Diamond distinction

$$\begin{array}{ccccccc} \urcorner_3 & \cong & \urcorner_1 & \longrightarrow & \urcorner_1 & \cong & \urcorner_4 \\ \downarrow & & \updownarrow & & \updownarrow & & \uparrow \\ \urcorner_3 & \cong & \urcorner_2 & \longleftarrow & \urcorner_2 & \cong & \urcorner_4 \end{array}$$

Diamondization of J1 : $\urcorner\urcorner \rightleftarrows \urcorner$

$\urcorner \equiv \urcorner \longrightarrow \urcorner$: a single mark is understood as an identity morphism.

$\urcorner\urcorner \equiv (\urcorner \longrightarrow \urcorner) \circ (\urcorner \longrightarrow \urcorner)$:

With that,
the repetition of the marks is well defined as a composition of morphisms.

$\urcorner\urcorner \longrightarrow \urcorner \equiv (\urcorner \longrightarrow \urcorner) \circ (\urcorner \longrightarrow \urcorner) \longrightarrow (\urcorner \longrightarrow \urcorner)$:

the composition is resulting into a commutative diagram.

$(\urcorner \longrightarrow \urcorner)_1 \circ (\urcorner \longrightarrow \urcorner)_2 \longrightarrow (\urcorner \longrightarrow \urcorner)_3$.

Matching conditions for composition "$\circ$" :

cod $(\urcorner \longrightarrow \urcorner)_1$ = dom $(\urcorner \longrightarrow \urcorner)_2$. Trivially, $\urcorner_1 = \urcorner_2 = \urcorner$

Diamondizing of the matching conditions

$(\urcorner \longrightarrow \urcorner)_1 \circ (\urcorner \longrightarrow \urcorner)_2 \mid (\urcorner \longleftarrow \urcorner)_4$.

Hence,

$(\urcorner \longrightarrow \urcorner)_1 \circ (\urcorner \longrightarrow \urcorner)_2 \mid$

$(\urcorner \longleftarrow \urcorner)_4 \Longrightarrow (\urcorner \to \urcorner)_3 \mid (\urcorner \longleftarrow \urcorner)_4$

Diamondized initial J1 :

$(\urcorner \longrightarrow \urcorner)_1 \circ (\urcorner \longrightarrow \urcorner)_2 \Longrightarrow (\urcorner \longrightarrow \urcorner)_3 \mid (\urcorner \longleftarrow \urcorner)_4$.

Because of the commutative diagram for composition
the reverse reading is holding too, i.e. the morphisms

$(\urcorner \longrightarrow \urcorner)_1$ and $(\urcorner \longrightarrow \urcorner)_2$ are defined by the morphism $(\urcorner \longrightarrow \urcorner)_3$.

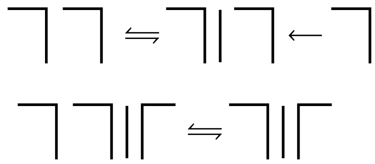
Hence, $(\urcorner \longrightarrow \urcorner)_3 \longrightarrow (\urcorner \longrightarrow \urcorner)_1 \circ (\urcorner \longrightarrow \urcorner)_2$.

Therefore,

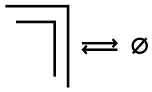
Diamond J1

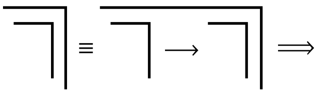
$$(\urcorner \longrightarrow \urcorner) \circ (\urcorner \longrightarrow \urcorner) \leftrightarrows (\urcorner \longrightarrow \urcorner) \mid (\urcorner \longleftarrow \urcorner)$$

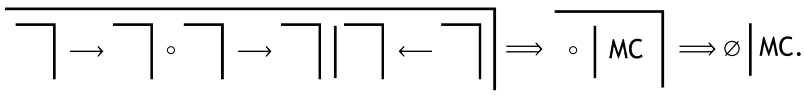
Diamond J1

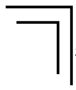


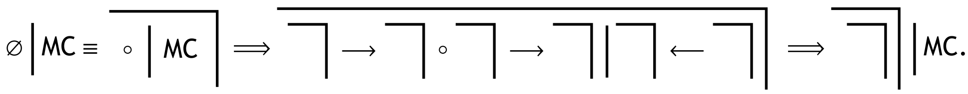
Wording
To recall is to call against a context.
A call of a recall in a context is a call against a context.

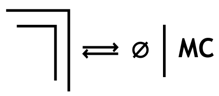
Diamondization of J2 : 





The distinction (reflection) of the composition rule, i.e. the conditions of distinctions, is not a distinction, therefore: ∅. But it is nevertheless the non-distinction of the conditions of distinction, thus the inverse action, from non-distinction ∅ (void of distinction) to the distinction of distinction, , is diamond-theoretically well defined. Obviously, the matching conditions, MC, don't get a notice in GSB's Calculus of Indication, CI, simply because they are kept in the mind of the designer and user of the calculus, and referred as obvious intuition and evidence.



Diamond J2 : 

http://www.thinkartlab.com/pkl/lola/Diamond-Theory-Collection.pdf

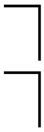
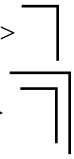
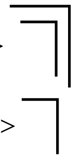
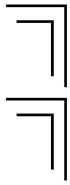
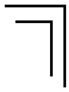
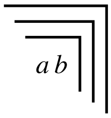
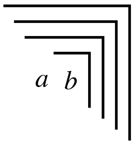
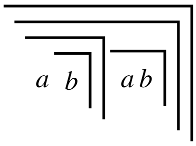
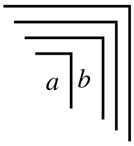
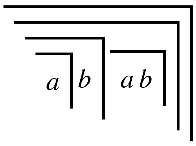
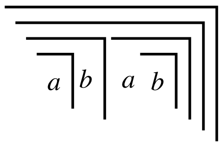
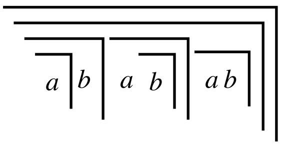
2.3. Boolean distinctions and morphograms

2.3.1. BOOLEANS and ANTI-BOOLEANS

$$\left(\text{TRUE} \quad \big|\!\overline{\,\big|\!\overline{\quad}}\quad 1\times 1\times 1\times 1\right)\ \textbf{complementary}\ \left(\text{FALSE}\quad \overline{\overline{\quad}\big|}\big|\quad 0\ \ 0\ \ 0\ \ 0\right)$$

$$\text{TRUE}\quad \overline{\overline{\overline{\overline{a|}\,\overline{b|}}\Big|\ \overline{\overline{a|}\,b}\Big|\ \overline{a\,\overline{b|}}\Big|\ \overline{a\,b}\Big|}\Big|}\Bigg|\ \ \textbf{complementary}\ \ \text{TRUE}\quad \big|\!\overline{\,\big|\!\overline{\quad}}$$

$$\left(\text{FALSE}\quad \overline{\overline{\quad}\big|}\big|\ \ \textbf{dual}\ \ \text{TRUE}\quad \overline{\overline{\overline{\overline{a|}\,\overline{b|}}\Big|\ \overline{\overline{a|}\,b}\Big|\ \overline{a\,\overline{b|}}\Big|\ \overline{a\,b}\Big|}\Big|}\Bigg|\right)$$

BOOLEAN Name	Form	*a* –> *b* –> 	*a* –> *b* –> 	*a* –> *b* –> 	*a* –> *b* –> 
FALSE		0	0	0	0
NOR		0	0	0	1
NOT *a* AND *b*		0	0	1	0
NOT *a*		0	0	1	1
a AND NOT *b*		0	1	0	0
NOT *b*		0	1	0	1
XOR		0	1	1	0
NAND		0	1	1	1

ANTI – BOOLEAN Name	Form	a –> b –>	a –> b –>	a –> b –>	a –> b –>
TRUE		1	1	1	1
OR	$a\,b$	1	1	1	0
a OR NOT b	$a\,b$	1	1	0	1
a	$a\,b \quad a\,b$	1	1	0	0
NOT a OR b	$a\,b$	1	0	1	1
b	$a\,b \quad a\,b$	1	0	1	0
XNOR	$a\,b \quad a\,b$	1	0	0	1
AND	$a\,b \quad a\,b \quad a\,b$	1	0	0	0

http://www.wolframscience.com/conference/2004/presentations/material/mschreiber-computational.nb

Morphogrammatics of Booleans

$$\left[\text{Morphogrammatics}^{(2,2)}\right] \Longrightarrow \begin{pmatrix} \text{BOOLEAN} - \\ \text{ANTI} - \text{BOOLEAN} - \end{pmatrix}$$

Morphogrammatics of Boolean Distinctions

$$\left[\text{Morphogrammatics}^{(4,2)}\right] \Longrightarrow \begin{pmatrix} \text{BOOLEAN} - \\ \text{ANTI} - \text{BOOLEAN} - \\ \text{INVERSE} - \text{BOOLEAN} - \\ \text{INVERSE ANTI} - \text{BOOLEAN} - \end{pmatrix}$$

2.3.2. Morphogrammatics and Boolean logic

Duality

ab **dual** *a* *b* *a* *b* *a* *b* *a b*

Inversion

ab inverse ab

Complementarity

ab complementary ab

Transversality

ab transversal ab

ab transversal ab

Orthogonality

ab orthogonal ab

ab orthogonal ab

Complementarity and morphograms

$\overline{\overline{}|}\,|$ complementary $|\,\overline{|\overline{}}$:

$$[\triangle\ \triangle\ \triangle\ \triangle] \Longrightarrow \begin{pmatrix} |\,\overline{|\overline{}}\ (\text{TRUE}) \\ \overline{\overline{}|}\,|\ (\text{FALSE} \end{pmatrix} =$$

$$\begin{pmatrix} \overline{\overline{\overline{\overline{a}|\ \overline{b}|}|\ \overline{a}|b|\ a\ \overline{b}|\,|\ \overline{a\,b}|\,|}\,|}\ (\text{TRUE}) \\ \overline{\overline{}|}\,|\ (\text{FALSE} \end{pmatrix}$$

$$|\,\overline{|\overline{}}\ (\text{TRUE}) \Longleftrightarrow$$

$$\overline{\overline{}|}\,|\ (\text{FALSE}) \Longrightarrow (1\ \ 1\ \ 1\ \ 1) =_{MG} (0\ \ 0\ \ 0\ \ 0) : [\triangle\ \triangle\ \triangle\ \triangle]$$

$$(\text{NOR})\ \overline{\overline{\overline{a\,b}|}\,|}\,| \Longleftrightarrow |\,\overline{|\,\overline{|\,\overline{a\,b}}}\ (\text{OR}) \Longrightarrow (1 \times 1 \times 1 \times 0) =$$

$$_{MG}(0\ \ 0\ \ 0\ \ 1) : [\triangle\ \triangle\ \triangle\ \bigcirc]$$

2.3.3. Junctional disseminations

Truth values

$0 \equiv \text{true}_{1.3}$

$1 \equiv \text{false}_{1,}\ \text{true}_{2}$

$2 \equiv \text{false}_{2.3}$

TRUE, TRUE, TRUE

$$\text{mediation}\left(\left\|\ulcorner_{1}\right\|\ulcorner_{1}\left\|\ulcorner_{3}\right)\right.:$$

$$\left[\text{True, True, True}\right]\left[(0000)_1\,(0000)_1\,(0000)_3\right] \Longrightarrow$$

$$\begin{pmatrix} [\text{True, True, True}] & \mathbf{0} & \mathbf{1} & \mathbf{2} \\ \mathbf{0} & 0 & 0 & 0 \\ \mathbf{1} & 0 & 0 & 0 \\ \mathbf{2} & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} \|\ulcorner_{1.1} & - & - \\ \|\ulcorner_{1.2} & - & - \\ - & - & \|\ulcorner_{3.3} \end{pmatrix}.$$

OR, AND, OR

$$\text{mediation}\left(\left|\overline{\left|\overline{\left|\overline{a\,b}\right.}\right.}\right._{1},\ \left|\overline{\left|\overline{\left|\overline{\left|\overline{a}\right.\ b\ \left|\overline{a\left|\overline{b}\right.}\right.\ \left|\overline{a\,b}\right.}\right.}\right.}\right._{2},\ \left|\overline{\left|\overline{\left|\overline{a\,b}\right.}\right.}\right._{3}\right):$$

$$[\text{OR, AND, OR}]\,[(0001)\,(1222)\,(0002)] \Longrightarrow$$

$$\begin{pmatrix} [\text{OR, AND, OR}] & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & 0 & 0 & 0 \\ \mathbf{1} & 0 & 1 & 2 \\ \mathbf{2} & 0 & 2 & 2 \end{pmatrix} =$$

$$\begin{pmatrix} \left|\overline{\left|\overline{\left|\overline{a\,b}\right.}\right.}\right._{1.1} & - & - \\ - & \left|\overline{\left|\overline{\left|\overline{\left|\overline{a}\right.\ b\ \left|\overline{a\left|\overline{b}\right.}\right.\ \left|\overline{a\,b}\right.}\right.}\right.}\right._{2.2} & - \\ - & - & \left|\overline{\left|\overline{\left|\overline{a\,b}\right.}\right.}\right._{3.3} \end{pmatrix}$$

$$[\text{OR, AND, OR}] = \left|\overline{\left|\overline{\left|\overline{a\,b}\right.}\right.}\right.\quad \left|\overline{\left|\overline{\left|\overline{\left|\overline{a}\right.\ b\ \left|\overline{a\left|\overline{b}\right.}\right.\ \left|\overline{a\,b}\right.}\right.}\right.}\right.\quad \left|\overline{\left|\overline{\left|\overline{a\,b}\right.}\right.}\right.$$

Matching Conditions

$\text{dom}(\text{OR}_1) = \text{dom}(\text{OR}_3)$

$\text{cod}(\text{OR}_1) = \text{dom}(\text{AND}_2)$,

$\text{cod}(\text{OR}_1) = \text{cod}(\text{OR}_3)$

IMP, IMP, IMP

$$\text{NOT } a \text{ OR } b \quad \overline{\overline{\overline{\overline{a}\; b}}} \quad 1\ \ 0\ \ 1\ \ 1 = \text{IMP}$$

$$[\text{IMP, IMP, IMP}]\left[(0100)_1\,(0100)_1\,(0200)_3\right] \Longrightarrow$$

$$\begin{pmatrix} [\text{IMP, IMP, IMP}] & \mathbf{0} & \mathbf{1} & \mathbf{2} \\ \mathbf{0} & 0 & 1 & 2 \\ \mathbf{1} & 0 & 0 & 1 \\ \mathbf{2} & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} \overline{\overline{\overline{\overline{a}\; b}}}_{\;1.1} & - & - \\ \overline{\overline{\overline{\overline{a}\; b}}}_{\;1.2} & - & - \\ - & - & \overline{\overline{\overline{\overline{a}\; b}}}_{\;3.3} \end{pmatrix}$$

2.3.4. Transjunctional disseminations

TRANS, OR, AND

$$[\text{TRANS, OR, AND}]\left[(0221)_1\,(1112)_2\,(0222)_3\right] \Longrightarrow$$

$$\begin{pmatrix} [\text{TRANS, OR, AND}] & \mathbf{0} & \mathbf{1} & \mathbf{2} \\ \mathbf{0} & 0 & 2 & 2 \\ \mathbf{1} & 2 & 1 & 1 \\ \mathbf{2} & 2 & 1 & 2 \end{pmatrix} =$$

$$\begin{pmatrix} \overline{\overline{\overline{\overline{a}\; b}}}_{\;1.1} & \overline{\overline{\overline{\overline{a}\; b}}}_{\;2.1} & \overline{\overline{\overline{\overline{a}\; b}}}_{\;3.1} \\ - & \overline{\overline{\overline{a\,b}}}_{\;2.2} & - \\ - & - & \overline{\overline{\overline{\overline{a}\; b}\;\; a\,\overline{b}\;\; \overline{a\,b}}}_{\;3.3} \end{pmatrix}$$

2.4. Interplay between worlds and distinctions

2.4.1. Chiasm of distinction and world

The LoF is ruled by a strict hierarchy of world and distinctions in this world. This hierarchy is transformed to a heterarchy between different worlds of different distinctions in a complex LoF.
But this is not simply a static organization of the systems. Between world and distinction a chiastic exchange relation rules. With that, worlds might become distinctions in another world, and vice versa, worlds might be turned to distinctions in another world.
Hence, world_1 becomes distinction_2 and distinction_1 becomes world_2 ruled the operator "$\diamond$". This happens on the base of the two compositions: $(\text{world}_1 \circ \text{distinction}_1)$ and $(\text{world}_2 \circ \text{distinction}_2)$.

Chiasm of wold and distinction

$$\begin{array}{lccc} LoF^1: & \text{world} & \longrightarrow & \text{distinction} \\ \square & \updownarrow & X & \updownarrow \\ LoF^2: & \text{distinction} & \longleftarrow & \text{world} \end{array}$$

Chiastic interchangeability of world and distinction

$$\begin{bmatrix} \text{distinction}_1 & \text{distinction}_2 \\ \text{world}_1 & \text{world}_2 \end{bmatrix}:$$

$$\begin{pmatrix} \text{world}_1 \\ \diamond \\ \text{world}_2 \end{pmatrix} \circ \begin{pmatrix} \text{distinction}_1 \\ \diamond \\ \text{distinction}_2 \end{pmatrix} = \begin{pmatrix} (\text{world}_1 \circ \text{distinction}_1) \\ \diamond \\ (\text{world}_2 \circ \text{distinction}_2) \end{pmatrix}$$

2.4.2. Metamorphosis of *world* and *distinction*

An important step in the project of dynamizing the laws of form in complex situations is achieved with the concept of metamorphosis.
Metamorphosis is ruling precisely the transformations of “world” into “distinction” and “distinction” into “world” by keeping the difference of “world” and “distinction” intact.
In contrast to the chiastic exchange between “world” and “distinction” on different levels of complexity, i.e. “world” becomes “distinction” and vice versa, metamorphosis is not based anymore on the is-abstraction but on the as-abstraction of its terms. Instead of “world” is “world”, it becomes “world_1 as $\text{distinction}'_2$ becomes world'_1 as distinction_2".

metamorphosis of *world* and distinction

$$\boxed{\text{world}_1 \approx \text{distinction}'_2} \amalg \boxed{\text{distinction}'_1 \approx \text{world}_2}$$

$$\diamond \setminus / \qquad \circ$$

$$\circ \qquad / \setminus \diamond$$

$$\boxed{\text{distinction}_1 \approx \text{world}'_2} \amalg \boxed{\text{world}'_1 \approx \text{distinction}_2}$$

Metamorphic chiasm of world and distinction

$$\left[\left(M, \sigma\right), \ \approx, \ \diamond, \ \circ, \ \amalg\right]:$$

$$\begin{pmatrix} \left(\left(\sigma_1 \approx M'_2\right) & \circ & \left(M'_1 \approx \sigma_2\right)\right) \\ \diamond & \amalg & \diamond \\ \left(\left(M_1 \approx \sigma'_2\right) & \circ & \left(\sigma'_1 \approx M_2\right)\right) \end{pmatrix}$$

$$\left[\begin{matrix} \left(M_2 \approx \right. & \left. M'_2\right) \\ \amalg & \diamond \\ \left(M_1 \approx \right. & \left. M'_1\right) \end{matrix}\right] \circ \left[\begin{matrix} \left(\sigma_2 \right. \\ \\ \left(\sigma_1 \right. \end{matrix}\right.$$

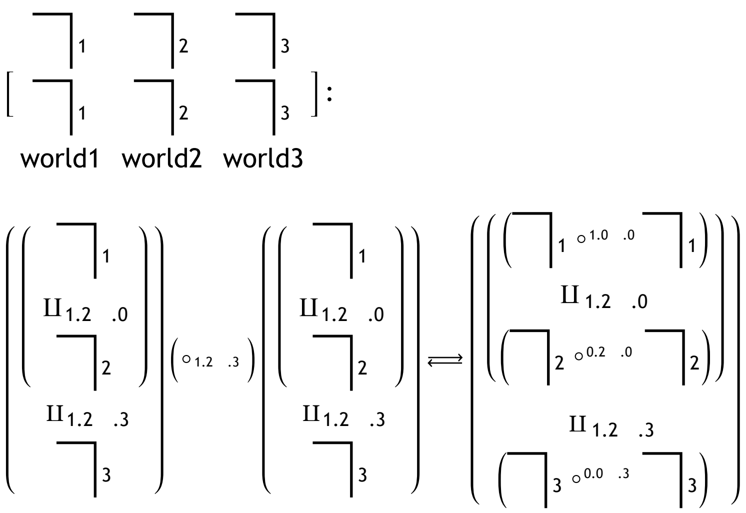
$$\left.\begin{matrix} \approx & \left. \sigma'_2\right) \\ \amalg & \diamond \\ \approx & \left. \sigma'_1\right) \end{matrix}\right] = \left[\begin{matrix} \left(M_2 \circ \sigma_2\right) \\ \amalg \\ \left(M_1 \circ \sigma_1\right) \end{matrix}\right] \approx \left[\begin{matrix} \left(M'_2 \circ \sigma'_2\right) \\ \diamond \\ \left(M'_1 \circ \sigma'_1\right) \end{matrix}\right]$$

$M \equiv$ world, $\sigma \equiv$ distinction

2.4.3. Interchangeability and unification

With the decision for a multitude of different worlds and therefore different systems of distinction the question of their interaction arises as the search for fundamentally new laws between distinctions. Interchangeability rules the basic structural inter-relationship between different worlds of distinction.

Interchangeability for *a* 3 – complex LoF



Topology-invariance

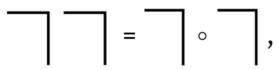
As Matzka and Varga pointed out, the semiotics of Spencer Brown's "*Laws of Form*" is based on a topology-invariant syntax based on *concatenation* and *enclosure*.

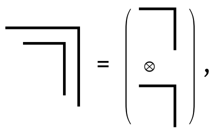
The main theorem, also not stated this way, might be called: *enclosure is reducible to concatenation.*

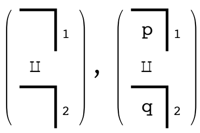
> *"In Laws of Form, there is a special semiotic atom, called the "*Cross*", which can be combined with other terms in two modes: by* concatenation *or by* enclosure*. This is an obvious deviation from standard semiotics, where concatenation is the only mode of combination. Combination by enclosure is also the basis for the "*reentrant *forms", another semiotic innovation. A third deviation from classical semiotics is less obvious: the* commutativity *of the concatenation operation. For any two terms "a" and "b" the terms "ab" and "ba" are identical. That this is indeed a semiotic identity (and not just a logical equality) has been stressed by Varga.*" (Matzka, 1993)
> http://www.rudolf-matzka.de/dharma/semabs.rtf

Interchangeability

Hence, **architectonics**:

composition: 

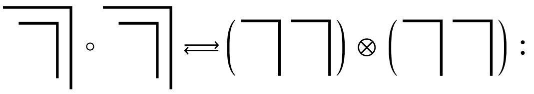
yuxtaposition: 

mediation: 

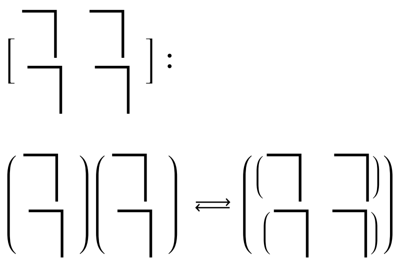
sop-reflection over the matrix.

Monoidal

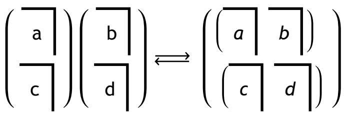
$$\left(f_1 \otimes f_2\right) \circ \left(g_1 \otimes g_2\right) = \left(f_1 \circ g_1\right) \otimes \left(f_2 \circ g_2\right)$$



Interchangeability arithmetics



Interchangeability algebra



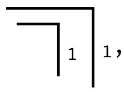
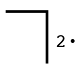
The law of functorial interchangeability has to be set, it can't be deduced from the original mono-contextural laws of Form. Interchangeability is introducing a new kind of *abstraction* beyond the forms for number and order.

How are number and order related in a bifunctoral approach?
There is a reason to apply bifunctoriality to the CI because it contains two operators which are similar to *yuxtaposition* and *composition*, i.e. to serial and parallel application.

It was said that awareness is a simultaneity of parallel and serial distinctions but there is still no calculus to deal with both features at once. Therefore, bifunctorial interchangeability for mono- as well as for polycontextural CI systems are appropriate demands.

In this case, the cross of the cross, is not reduced to cancellation but is establishing a connection between "*order*" and "*number*" of two different systems of distinction.

Memristics
Furthermore, it establishes, as a second-order term, the possibility of retrogradeness.

The state of the state of the cross of the cross, , is memorizing its previous state in form of the second-order state .

Obviously, *memristive* functions are not accessible in Spencer-Brown's calculus and its interpretation as a Boolean algebra. This observation about memristivity is inheriting features of morphogrammatic *retrograde* recursivity, it opens up an amazing new interpretation of the formation of the form of memristive systems and their distinctions. But there seems to be a new kind of memristive behavior genuine to the diamond calculus of distinction, introduced by the accretive understanding of the action of *double* crossing: the mechanism of *enaction*. Hence, retrograde recursivity and enaction are topics of distinctional memristivity. This alone might motivate further steps to study distinctional memristivity. Classical reentry has been connected with flip-flops, hence retrograde reentry forms might be appropriate for the study and construction of memristive flip-flop devices. Again, a feature unknown in the classical setting of LoF and its extensions.

> *"In other words, the history saved by the memristor are not the primary data but the data of the history. Historical data are data of data. Those second-order data might then be used to continue processing on the first-order level of the flip-flop.*
> *As a metaphor, the data of an observer of a data processing system are not the data of the observed system. But such observer-depending data of second-order might be given 'back' to the observed, i.e. first-order system to continue its game. Hence, the memristor is playing the game of an observer which is lending or giving away his data to the observed system. A memristive system is primarily storing the rules of the observed game and only secondarily the data involved."*
> http://memristors.memristics.com/Memory/Memory%20is%20more%20than%20Storage.pdf

2.5. Transitions between positions of LoFs

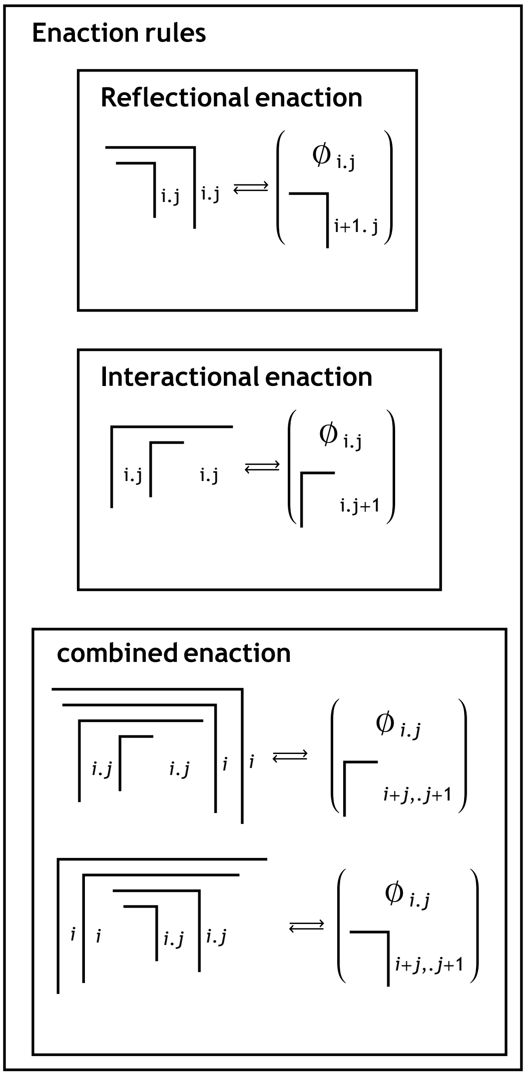
2.5.1. Enaction rules

A distinction of a distinction is conceived as both at once: as *annulation* and as *reflection* (enaction). Therefore, annulation is eliminating and destroying distinctions while reflection as enaction is not only creating new distinctions but also a new domain, i.e. world of distinctions, in which the new distinction and its further applications is realized.

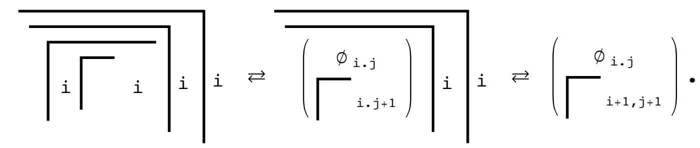
Enaction rule

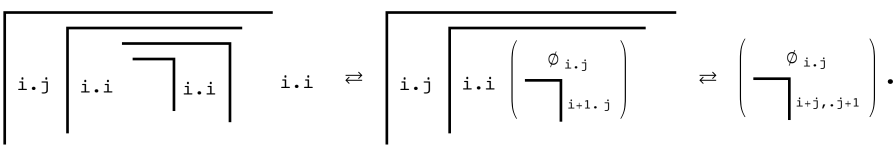
$$\overline{\overline{}\big|_{1}}\Big|_{1} \Longleftrightarrow \begin{pmatrix} \emptyset_1 \\ \overline{}\big|_{2} \end{pmatrix}:$$

$$\overline{}\big|_{2} \cong \emptyset_1 : \begin{pmatrix} \textbf{cancellation} \\ \text{anullment in Cl}^{1}: \ \overline{\overline{}\big|_{1}}\Big|_{1} \to \emptyset_1 \\ \textbf{enaction} \\ \text{distinction in Cl}^{2}: \ \overline{\overline{}\big|_{1}}\Big|_{1} \to \overline{}\big|_{2} \end{pmatrix}$$

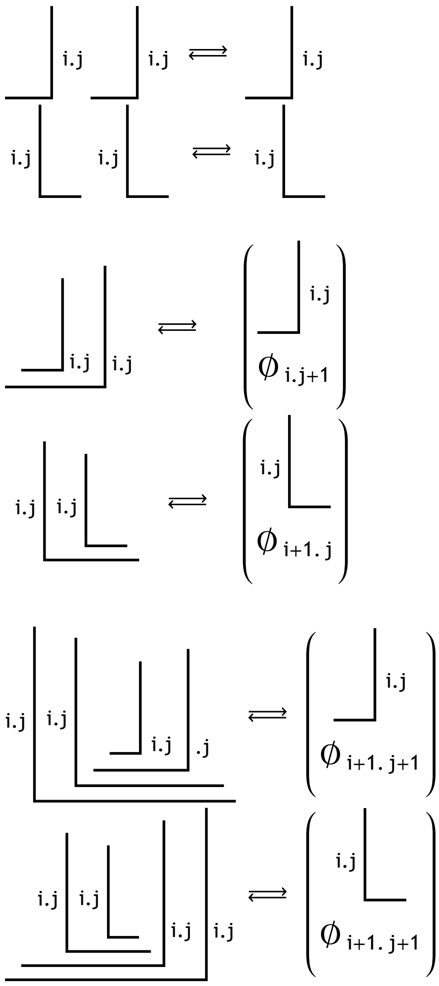

Enaction rules
Reflectional enaction
Interactional enaction
combined enaction

Example

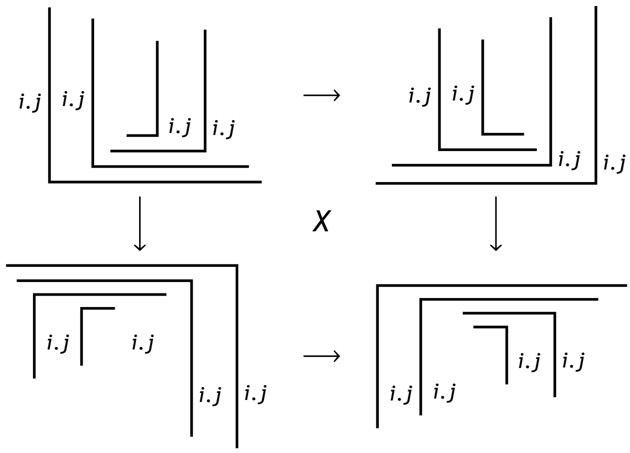




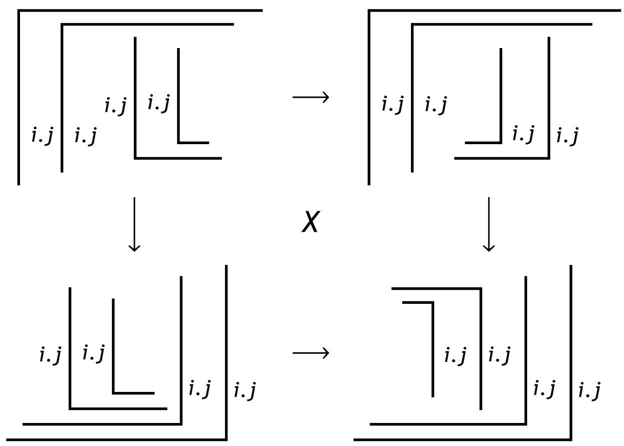
Reverse enaction rules



Basic constellation – 1 :

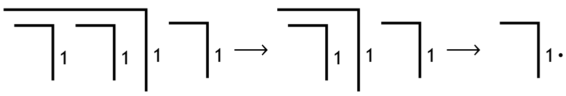


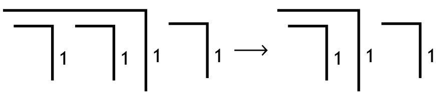
Basic constellation – 2 :

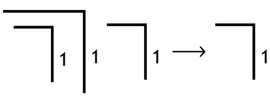


2.5.2. Examples for (reflectional) enactions

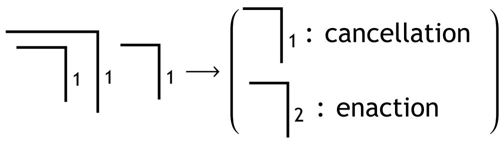
LoF (1) example : one world

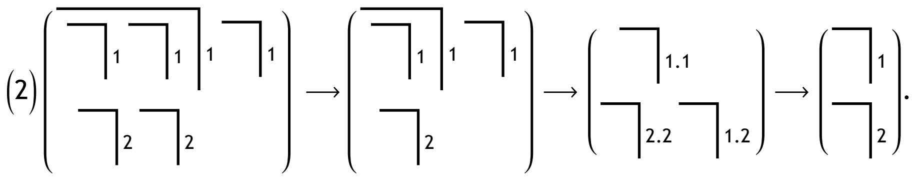
(1) 

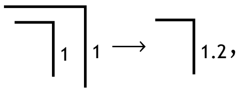
 : condensation

 : cancellation

LoF $^{(2)}$ example : two worlds

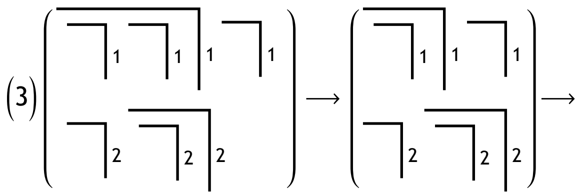


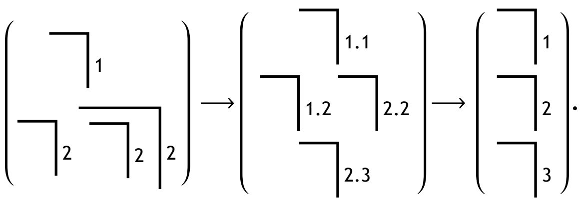


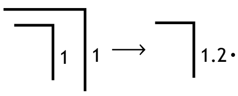
With condensation, cancellation and enaction on 

abstracting from position.

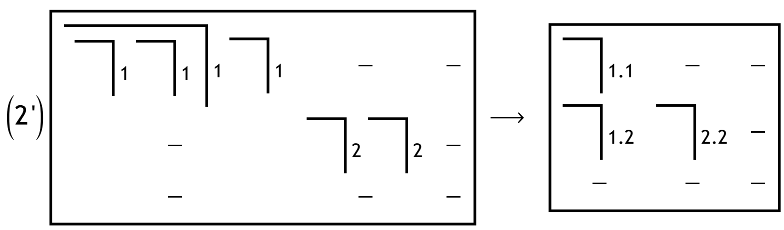
LoF $^{(3)}$ example : three worlds

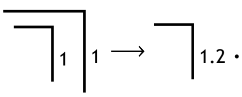




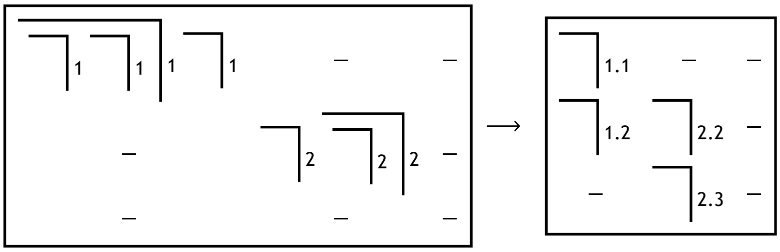
With condensation, cancellation and enaction on 

Positioning LoFs in the kenomic matrix



With condensation, cancellation , enactions and holding path on 

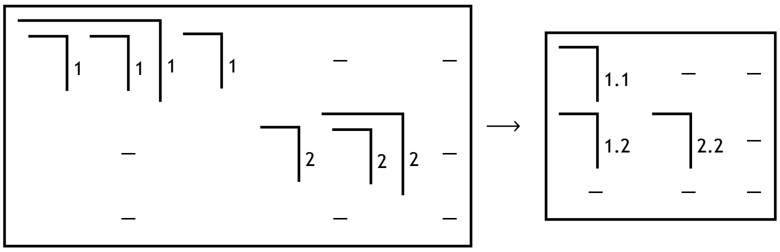
(3')



With condensation, cancellation,

enactions and holding path on $\overline{\overline{\ \ }|_1}|_1 \longrightarrow \overline{\ \ }|_{1.2}$ and $\overline{\overline{\ \ }|_2}|_2 \longrightarrow \overline{\ \ }|_{2.3}$.

(3'')

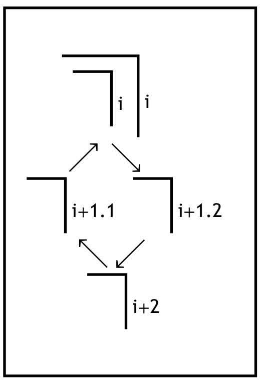


With condensation, cancellation , enaction and holding path on

$\overline{\overline{\ \ }|_1}|_1 \longrightarrow \overline{\ \ }|_{1.2}$ but omitting $\overline{\overline{\ \ }|_2}|_2 \longrightarrow \overline{\ \ }|_{2.3}$ for $\overline{\overline{\ \ }|_2}|_2 \longrightarrow \varnothing_2$.

With the precise *positioning* of the involved LoFs, not only a form development at each position is defined but the transition from one to another level of form by the use of the double character of the double cross is correctly marked. Each transition is also inscribing its path, showing (remembering) where it is from.

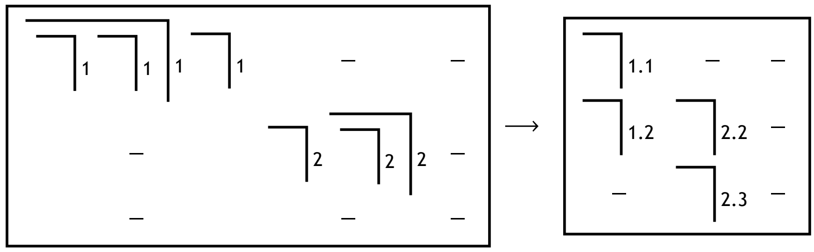
Because the transition formula (enaction) is defined in both directions, $\overline{\overline{\ \ }|_i}|_i \rightleftarrows \begin{pmatrix} \varnothing_i \\ \overline{\ \ }|_{i+1} \end{pmatrix}$, it is possible to move back to a lower level of distinction. Hence building a circular path between the levels of the complexions, which is strictly different to the self-referentiality of the reentry form.

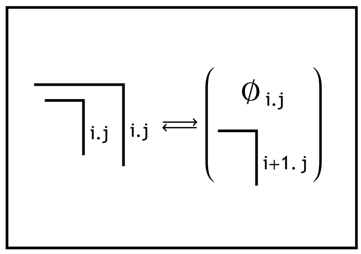


The (horizontal) arithmetic rules of LoF are holding on each level of the complexion. What has to be conceived and formalized are the new “vertical” arithmetical rules between the levels in the complexion. One such example might be the amazing possibility to draw circular distinction paths through a complexion of disseminated systems of distinction even before we have to deal with local reentry forms.

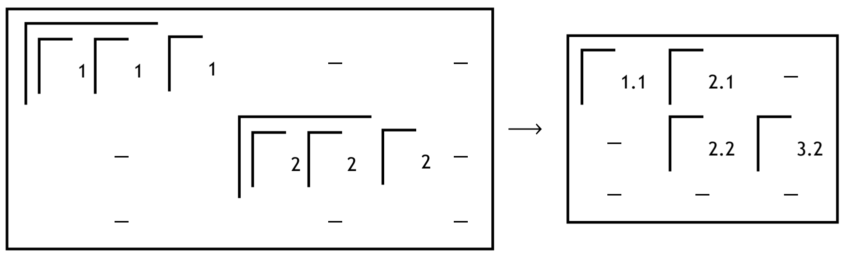
2.5.3. Reflectional and interactional enaction

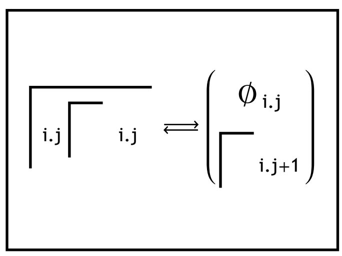
Reflectional enactions



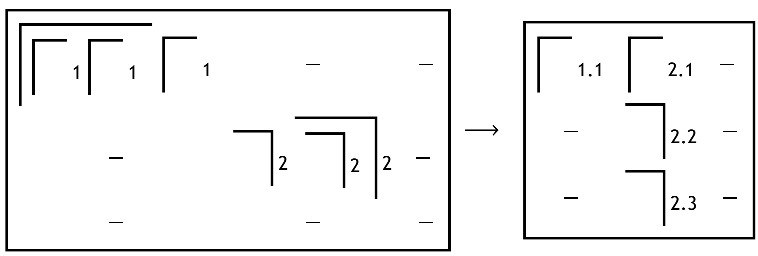


Interactional enactions

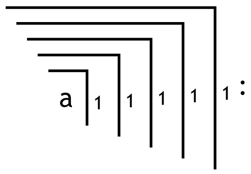


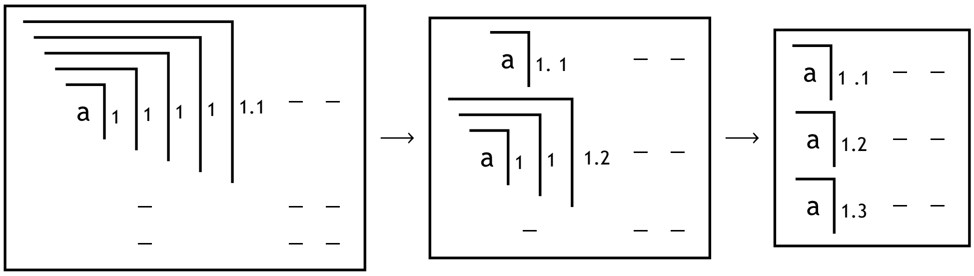


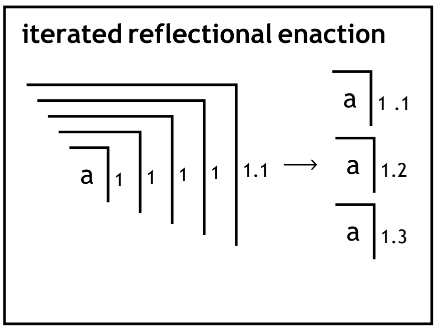
Interactional and reflectional enactions



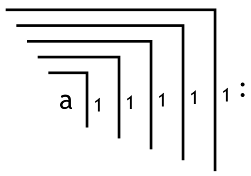
Iterated reflectional enactions

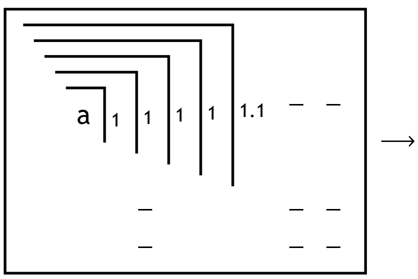


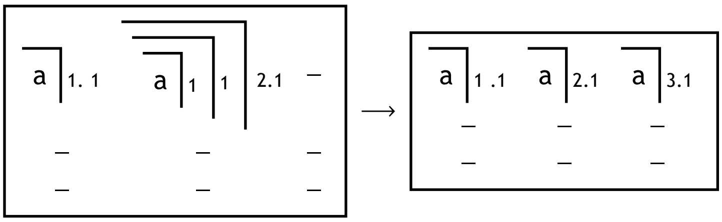




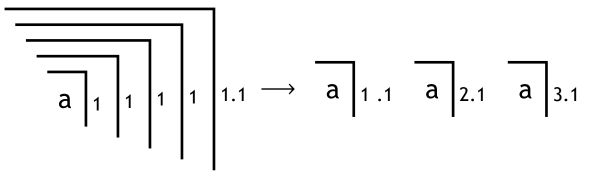
Iterated interactional enactions





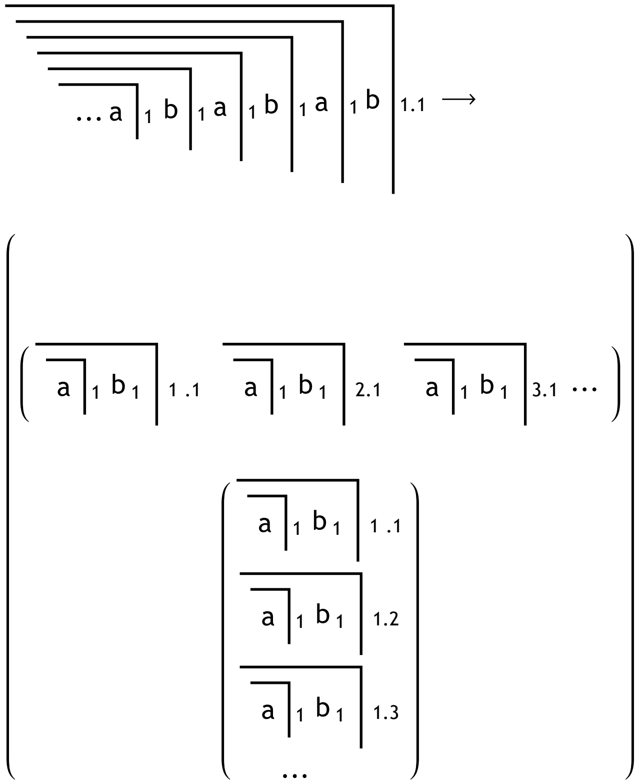


iterated interactional enaction

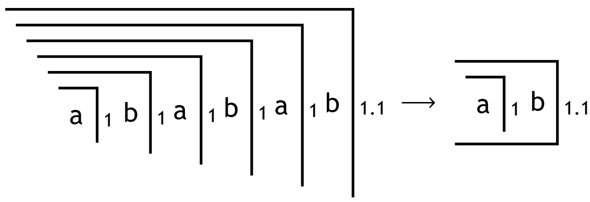


Reduction rules

E naction as reduction rules



cf. GSB, LoF, p. 55



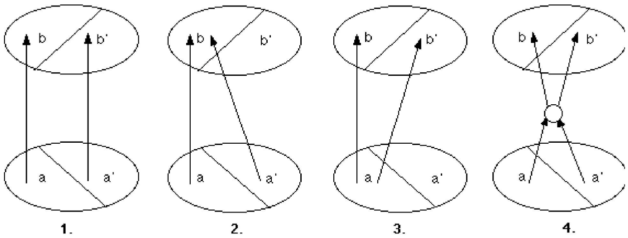
cf. GSB, LoF, p. 65

2.5.4. Distinction dynamics

In “DISTINCTION DYNAMICS: from mechanical to self-organizing evolution”, Francis Heylighen (1992) is sketching a different approach to “dynamic distinctions” which is enabled to create new distinctions. It seems that all 4 types of distinctions he is introducing are conceptually and operationally covered by the presented distinction-theoretic approach of “dynamic complexions of distinctions”.

"Complex dynamics is analysed as an example of a theory with a limited dynamics of

distinctions: distinctions can be destroyed but not created."



"**Figure 1**: four basic types of distinction processes: 1. *conservation*, 2. *destruction*, 3. *creation* and 4. *creation-and-destruction*." (Francis Heylighen) http://www.independent.co.uk/?CMP=ILC-refresh

A calculus of the *formation of forms* might include at least the 4 principls of distinction dynamics.

Laws of dynamics in complexions of LoFs

1. conservation : $\overline{\overline{\ \ }|}| \longrightarrow \overline{\ \ }|$: condensation

2. destruction : $\overline{\overline{\ \ }|}| \longrightarrow \varnothing$: cancelation

3. creation : $\overline{\overline{\ \ }|_i}|_i \longrightarrow \left(\overline{\ \ }|_{i+1} \right)$: enaction

4. creation & destruction : $\overline{\overline{\ \ }|_i}|_i \longrightarrow \left(\begin{matrix} \phi_i \\ \overline{\ \ }|_{i+1} \end{matrix} \right).$

3. Construction of complexity

3.1. Interchangeability with interaction and reflection

3.1.1. Interaction

Interchangeability of interaction as transposition

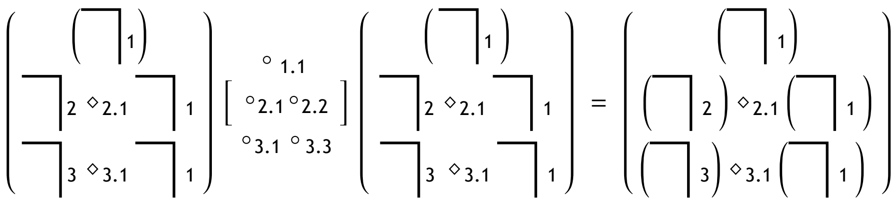
$$
\begin{pmatrix} \rceil_1 \\ \amalg_{1.2} \\ \rceil_2 \diamond_{2.1} \rceil_1 \\ \amalg_{2.3} \\ \rceil_3 \diamond_{3.1} \rceil_1 \end{pmatrix}
\begin{array}{c} \circ_{1.1} \\ \left[\circ_{2.1} \circ_{2.2} \right] \\ \square \\ \circ_{3.1} \circ_{3.3} \end{array}
\begin{pmatrix} \rceil_1 \\ \amalg_{1.2} \\ \rceil_2 \diamond_{2.1} \rceil_1 \\ \amalg_{2.3} \\ \rceil_3 \diamond_{3.1} \rceil_1 \end{pmatrix} =
$$

$$
\begin{pmatrix} \left(\rceil_1 \circ_{1.1} \rceil_1 \right) \\ \amalg_{1.2} \\ \left(\rceil_2 \circ_{2.2} \rceil_2 \right) \diamond_{2.1} \left(\rceil_1 \circ_{2.1} \rceil_1 \right) \\ \amalg_{2.3} \\ \left(\rceil_3 \circ_{3.3} \rceil_3 \right) \diamond_{3.1} \left(\rceil_1 \circ_{3.1} \rceil_1 \right) \end{pmatrix} =
$$

Notational abbreviations

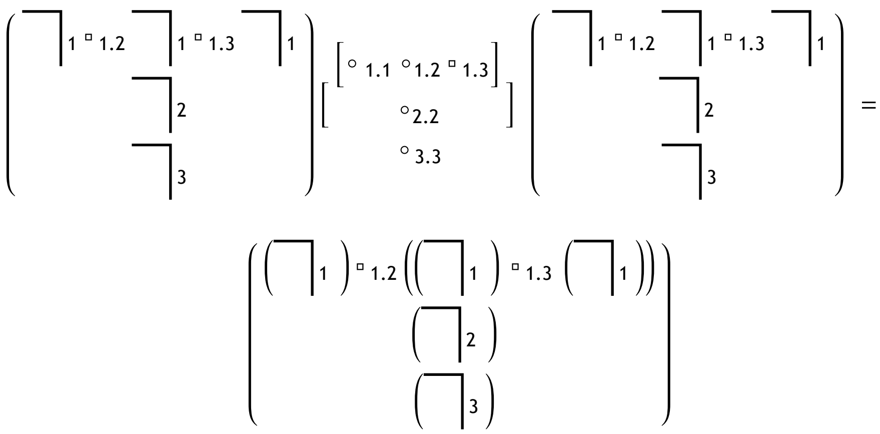
$$
\begin{pmatrix} \left(\rceil_1 \circ_{1.1} \rceil_1 \right) \\ \amalg_{1.2} \\ \left(\rceil_2 \circ_{2.2} \rceil_2 \right) \diamond_{2.1} \left(\rceil_1 \circ_{2.1} \rceil_1 \right) \\ \amalg_{1.2\ \ .3} \\ \left(\rceil_3 \circ_{3.3} \rceil_3 \right) \diamond_{3.1} \left(\rceil_1 \circ_{3.1} \rceil_1 \right) \end{pmatrix}
=>
\begin{pmatrix} \left(\rceil_1 \rceil_1 \right) \\ \left(\rceil_2 \rceil_2 \right) \diamond_{2.1} \left(\rceil_1 \rceil_1 \right) \\ \left(\rceil_3 \rceil_3 \right) \diamond_{3.1} \left(\rceil_1 \rceil_1 \right) \end{pmatrix}
$$

Interchangeability of interaction as transposition and condensation



3.1.2. Reflection

Interchangeability of reflection as replication, with condensation

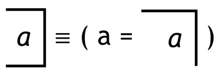


4. Retrograde recursivity

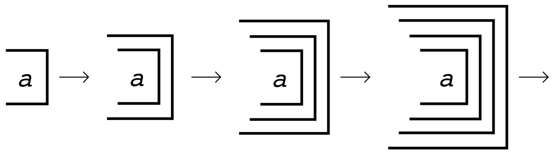
4.1. Iterative recursivity of LoF-reentry

Recursivity in LoF is ruled along the traditional lines of iterative reentry into the form. Therefore, there is nothing that could correspond diamond structures of repeatability.

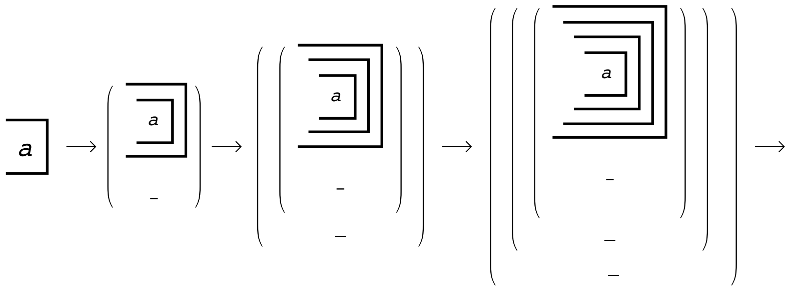
Iterability in LoF is not limited by any structural laws except of the abstract iteration of concatenation and crossing, i.e. its abstract expansion in "horizontal" and "vertical" directions.



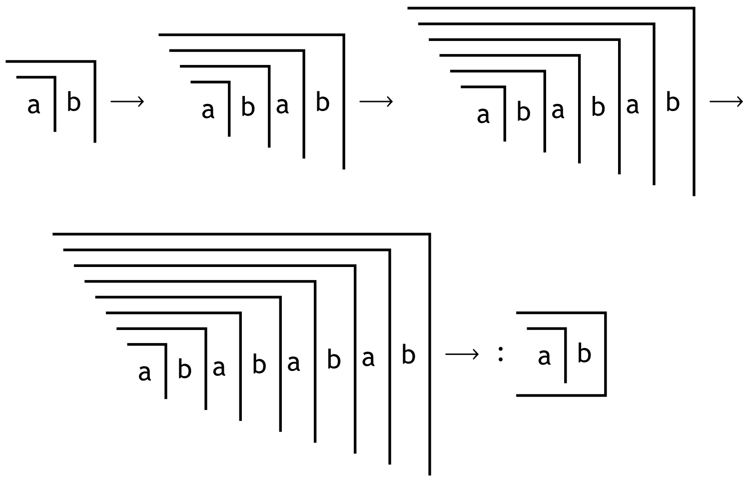
linear iterative reentry :



embedded linear iterative reentry :



LoF example

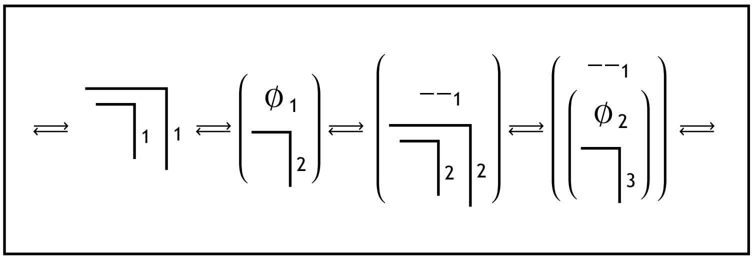


4.2. Retrograde extensions

Extensions of LoF-complexions are not arbitrary, they are determined retrograde recursively by the complexion to be extended. Any arbitrary extension would restore the abstract extensional characteristics of formal systems and would abandon the primary retrograde and holistic features of LoF-complexions based on morphogrammatics.

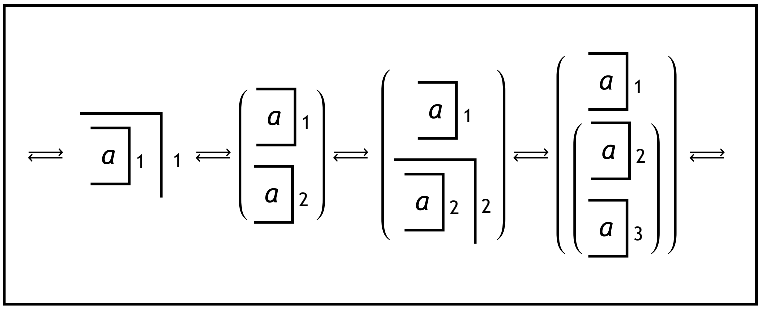
A kind of an extension chain based on the double function of the double cross in complex distinction systems might give a first idea how to extend retrograde recursively such complexions.

4.2.1. Enactional retrograde recursivity



4.2.2. Enactional recursivity for reentries

For FoL, a distinction of a reentry is a reentry $\overline{a\,}|$: $\overline{\overline{a\,}|\;}| = \overline{a\,}|$.



For a diamond calculus of forms, the distinction of a reentry form,

$\overline{\overline{a\,}|_{i}\;}|_{i}$ is a reentry form $\overline{a\,}|_{i}$ and *at once* an

enaction of the reentry form at another place $\overline{a\,}|_{i+1}$.

Hence, $\overline{\overline{a\,}|_{i}\;}|_{i} \Longleftrightarrow \left(\begin{array}{c} \overline{a\,}|_{i} \\ \overline{\overline{a\,}|_{i+1}} \end{array} \right)$.

Varela's approach

Initial 12.7 : $\overline{\overline{\;\;}|\;}| = \overline{\;\;}|$ (Constancy).

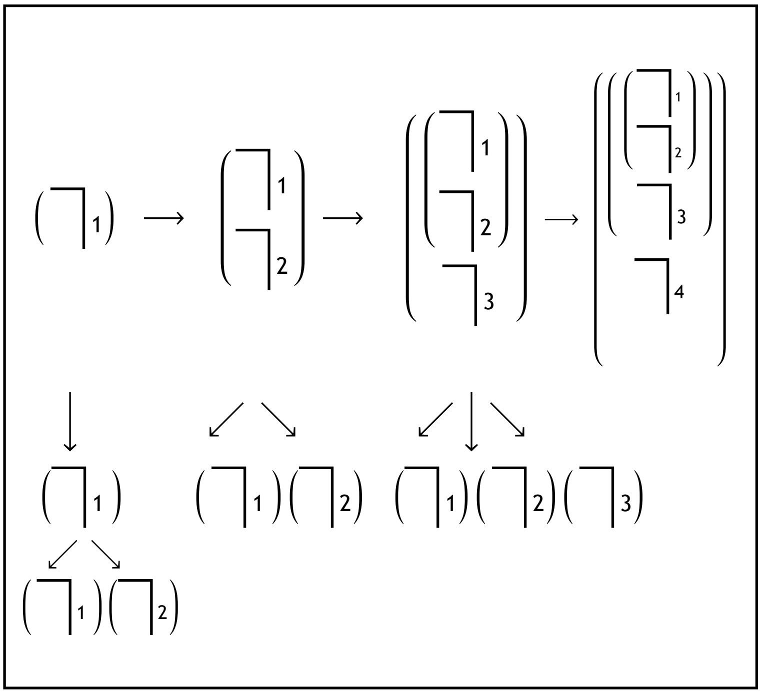
Initial 12.18 : $p\,\overline{\overline{\;\;}|\;}|\,p = p\,\overline{\;\;}|$ (Autonomy).

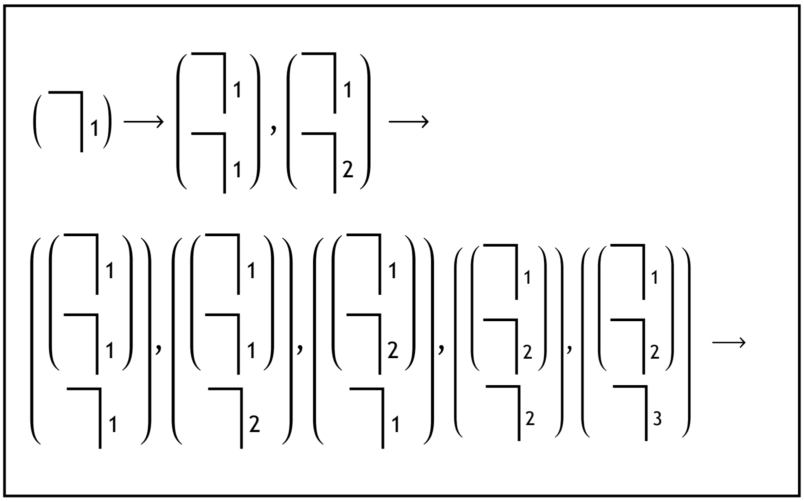
"Günther has been alone in pointing out that other possible interpretations of many-valued is as a basis for a 'cybernetic ontology', that is, for systems capable of self-reference, and precisely one additional value, he claims, must be taken as time. I follow here Günther's suggestion that a third value might be taken as time. But I have shown that

> *this third value can be seen at a level deeper than logic, in the calculus of indication, where the form of self-reference is taken as a third value in itself, and in fact confused with time as a necessary component for its contemplation. In the extended calculus, self-reference, time, and reentry are seen as aspects of the same third value arising autonomously in the form of distinction.”* F. Varela, Principles of Biological Autonomy, 1979, p.139

But Günther’s kenogrammatics is just the working approach, subversively positioned “*at a level deeper than logic*”. Varela’s “constancy” initial might be inspired by a traditional 3-valued logic, with neg(3) = 3, which is in no way approaching the declared demanding.

4.2.3. Concatenational retrograde recursivity

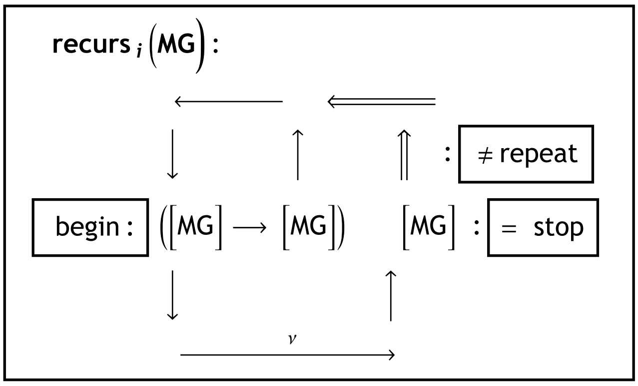




4.3. Accretive retrograde recursivity

On of the most fundamental features of kenogrammatics and morphogrammatics is its retrograde recursivity which is independent from a pre-given sign repertoire.
How does this fundamental feature enter into a polycontextural calculus of forms?

Changes between iterative and accretive reflections (reentry) are not arbitrarily set but are ruled by the morphogrammatic laws of retrograde recursivity. Therefore, polycontextural LoFs are produced by mapping the LoF calculus onto morphogrammatics.



http://memristors.memristics.com/MorphoReflection/ Morphogrammatics%20of%20Reflection-.pdf

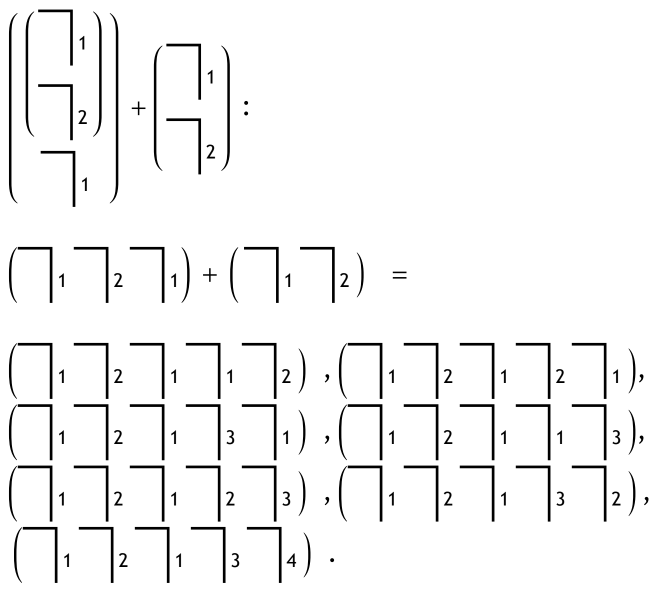
4.4. Coalitions and cooperations of distinction systems

4.4.1. Coalitions

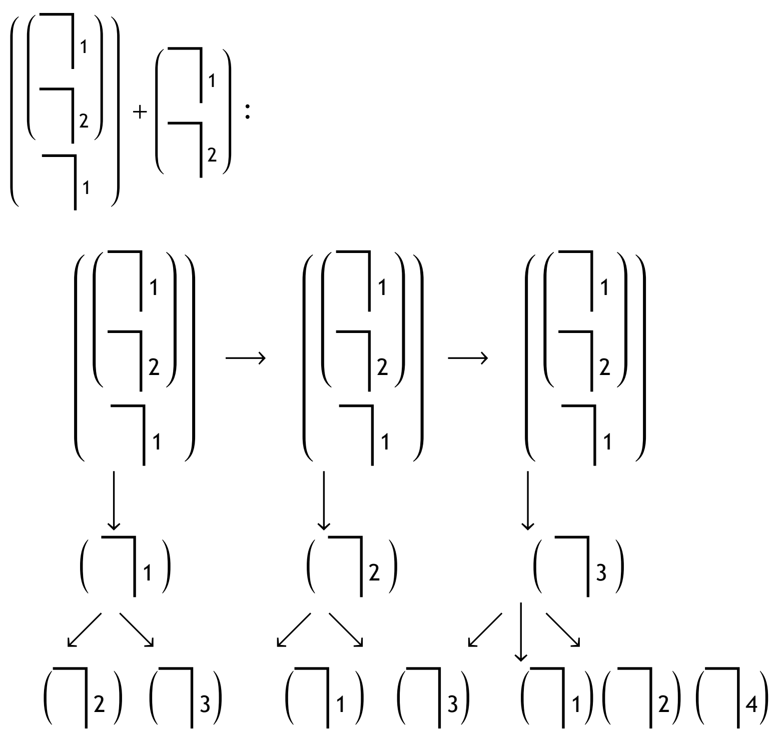
The field of possible coalitions is defined by the list of coalitions produced retrograde recursive by the operation of coalition.

$\mathrm{LoF}^{(3,2)}_{(1,2,1),(1,2)}$ is defining a field of possible coalitions with complication 5 and distribution 7, as shown in the example.

LoF –coalition $\mathbf{LoF}^{(3,\,2)}_{(1,2,1),\,(1,2)}$



Reursive decision graph for LoF $_{1.2\ \ .1}$ + LoF $_{1.2}$



4.4.2. Cooperations

Retrograde recursion scheme for cooperations

$(\text{succ}, \text{add}, \text{mult}) \in \text{CR}$:

$\text{mult}([\text{MG}], \emptyset) = \emptyset$

$\text{mult}(\emptyset, [\text{MG}]) = \emptyset$

$\text{mult}([\text{MG}_1], \text{succ}([\text{MG}_2])) = \text{add}(\text{mult}([\text{MG}_1], [\text{MG}_2]), [\text{MG}_1])$

Recursive generation of $(\urcorner_1 \urcorner_2) \odot (\urcorner_1 \urcorner_2)$

Recursive generation $(\urcorner_1 \urcorner_2) \odot (\urcorner_1 \urcorner_2)$

$\text{LoF}^{(2,2)}_{(\urcorner_{1.2}),(\urcorner_{1.2})}$:

$$\begin{pmatrix} \urcorner_1 \\ \urcorner_2 \end{pmatrix} \odot \begin{pmatrix} \urcorner_1 \\ \urcorner_2 \end{pmatrix} = (\urcorner_1 \urcorner_2) \odot (\urcorner_1 \urcorner_2)$$

$$(\urcorner_1 \urcorner_2)$$

$\downarrow$

$$\longrightarrow [\urcorner_1] \longrightarrow [\urcorner_1 \urcorner_2][\urcorner_1 \urcorner_2] \longrightarrow [\urcorner_1 \urcorner_2][\urcorner_1 \urcorner_3]$$

$\downarrow$

$$\longrightarrow [\urcorner_2] \longrightarrow [\urcorner_1 \urcorner_2][\urcorner_2 \urcorner_1] \longrightarrow [\urcorner_1 \urcorner_2][\urcorner_2 \urcorner_3]$$

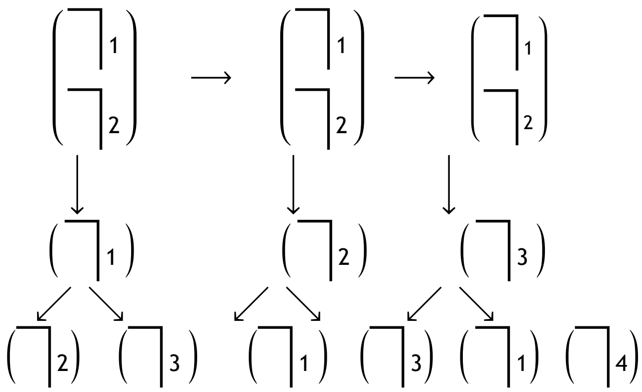
$\downarrow$

$$\longrightarrow [\urcorner_3] \longrightarrow [\urcorner_1 \urcorner_2][\urcorner_3 \urcorner_1] \longrightarrow [\urcorner_1 \urcorner_2][\urcorner_3 \urcorner_2] \notin \text{CR} \longrightarrow [\urcorner_1 \urcorner_2][\urcorner_3 \urcorner_4].$$

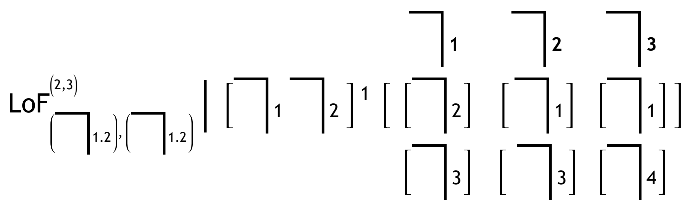
Collection table

$$[\urcorner_1 \urcorner_2 \urcorner_1 \urcorner_2],\ [\urcorner_1 \urcorner_2 \urcorner_1 \urcorner_3],$$
$$[\urcorner_1 \urcorner_2 \urcorner_2 \urcorner_1],\ [\urcorner_1 \urcorner_2 \urcorner_2 \urcorner_3],$$
$$[\urcorner_1 \urcorner_2 \urcorner_3 \urcorner_1],\ [\urcorner_1 \urcorner_2 \urcorner_3 \urcorner_4].$$

Reursive decision graph for LoF $_{1.2}$ $\odot$ LoF $_{1.2}$



Cooperation field for $\mathrm{LoF}^{(2,2)}_{(\urcorner_{1.2}),(\urcorner_{1.2})}$ **:**



Recursive generation of $(\urcorner_1 \urcorner_2 \urcorner_2) \odot (\urcorner_1 \urcorner_2 \urcorner_1)$

$$\mathrm{LoF}^{(3,3)}_{(\urcorner_{1.2\ \ .2}),(\urcorner_{1.2\ \ .1})} := (\urcorner_1 \urcorner_2 \urcorner_2) \odot (\urcorner_1 \urcorner_2 \urcorner_1)$$

Collection table

$$\left[\urcorner_1\ \urcorner_2\ \urcorner_2\right]\left[\urcorner_2\ \urcorner_1\ \urcorner_1\right]\left[\urcorner_1\ \urcorner_2\ \urcorner_2\right],$$
$$\left[\urcorner_1\ \urcorner_2\ \urcorner_2\right]\left[\urcorner_3\ \urcorner_4\ \urcorner_4\right]\left[\urcorner_1\ \urcorner_2\ \urcorner_2\right],$$
$$\left[\urcorner_1\ \urcorner_2\ \urcorner_2\right]\left[\urcorner_2\ \urcorner_3\ \urcorner_3\right]\left[\urcorner_1\ \urcorner_2\ \urcorner_2\right].$$

Multiplication table

$$\mathrm{kmul}\left(\left[\urcorner_1\ \urcorner_2\ \urcorner_2\right], \left[\urcorner_1\ \urcorner_2\ \urcorner_1\right]\right) =$$

kmul	$\urcorner_1$	$\urcorner_2$	$\urcorner_1$	$\urcorner_{2'}$	$\urcorner_{2''}$
$\urcorner_1$	$\urcorner_1$	$\urcorner_2$	$\urcorner_1$	$\urcorner_2$	$\urcorner_3$
$\urcorner_2$	$\urcorner_2$	$\urcorner_1$	$\urcorner_2$	$\urcorner_3$	$\urcorner_4$
$\urcorner_2$	$\urcorner_2$	$\urcorner_1$	$\urcorner_2$	$\urcorner_3$	$\urcorner_4$

Cooperation field for $\mathbf{LoF}^{(3,3)}_{(\urcorner_{1.2\ \ .2}),(\urcorner_{1.2\ \ .1})}$ **:**

$$\mathrm{LoF}^{(3,3)}_{(\urcorner_{1.2\ \ .2}),(\urcorner_{1.2\ \ .1})} \Big| \left[\urcorner_1\ \urcorner_2\ \urcorner_2\right]^{1.3} \left[\begin{array}{cc} \urcorner_{\mathbf{2}} & \urcorner_{\mathbf{3}} \\ \left[\urcorner_1\ \urcorner_1\right] & \left[\urcorner_4\ \urcorner_4\right] \\ \left[\urcorner_3\ \urcorner_3\right] & - \end{array} \right]$$

Domino Approach to Morphogrammatics
http://memristors.memristics.com/Dominos/Domino%20Approach%20to%20Morphogrammatics.html

5. Second-order diamond calculus

5.1. Second-order diamonds

Reflection on diamondization

A second-order diamondization is additionally to the first-order diamondization taking the diamond structure of the *environment* into account. This procedure becomes more plausible with the full notation of the automorphism as "○ ⟶ ○ ∘ ○ ⟶ ○", and therefore the environment as "□ ⟵ □". Hence there are two settings in the game: one is " [○ | □]" as the *distinction* with its environment and the other "[□ | ○]" as the *environment* with its distinction.

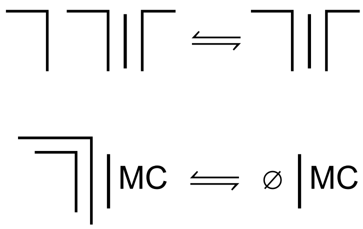
Metaphorics

Metaphorically, what is achieved is a formalization for the wording:" [*Inside* of the inside | *Outside* of inside] **|** [*outside* of the outside | *inside* of the outside]" as the metaphorical meaning of "[○ | □] **|** [□ | ○]".
Because the simultaneity of "*Inside* of the inside" and "*Outside* of inside" marked by "|" and the complementarity of the whole formula: "[○ | □] **|** [□ | ○]", a further formal explication is succeed by the mechanism of functorial *interchangeability*.

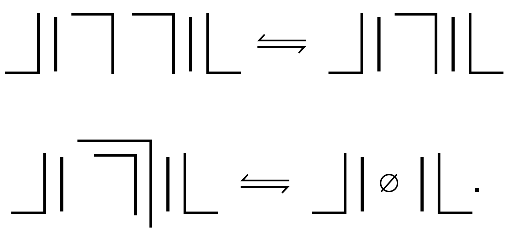
5.2. Second-order diamond arithmetics

Second-order diamond strategies shall be applied on the diamondization of the initials J1 and J2 of calculus of distincions.

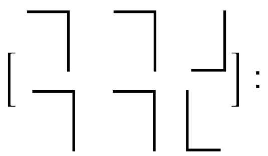
Diamond J1 and J2

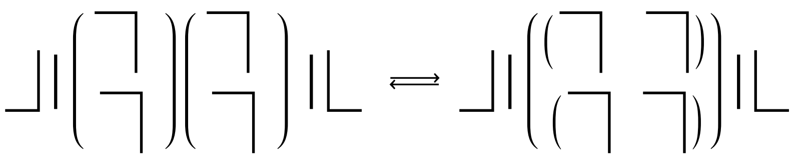


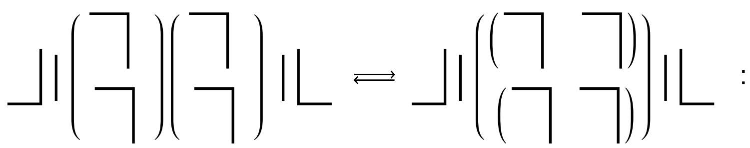
2. – order Diamond J1 + J2

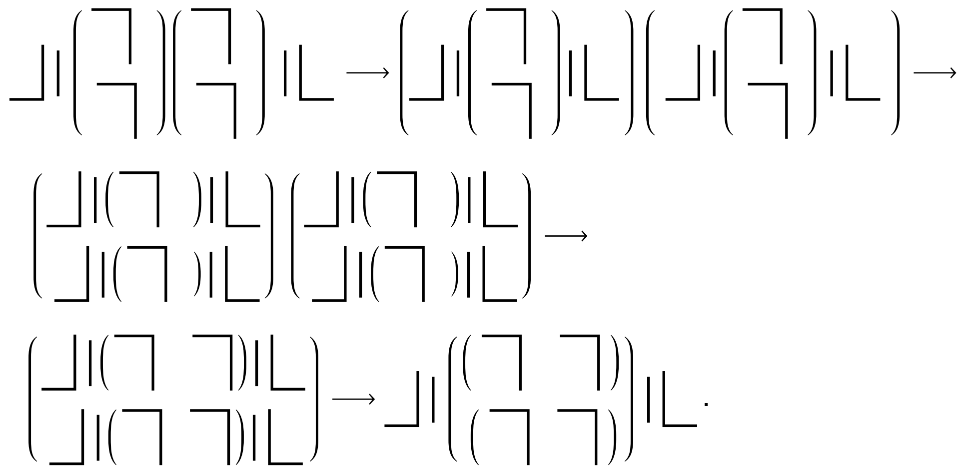


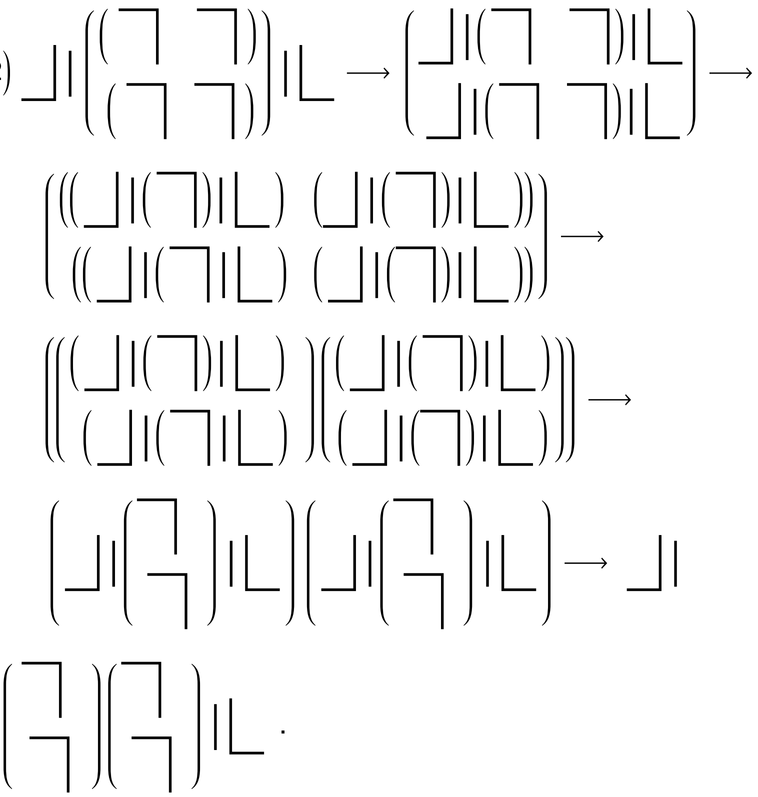
Interchangeability of J1 and J2 arithmetics





(0) 

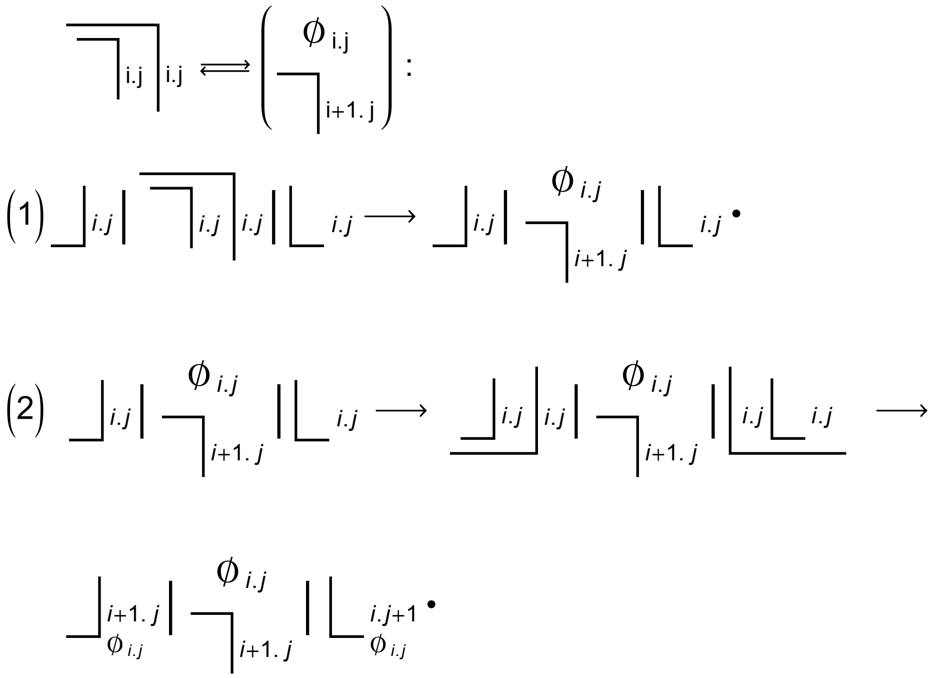
(1) 

(2) 

5.3. Diamond enaction

The new concept of enaction might be extended to a second-order concept of enaction in distinctional or distinctive diamond calculi. Enaction rules are interesting new features of memristive systems. The diamondization of the enaction rules is supporting memristivity of the contexts of the enacting distinctions. Context are themselves involved into enactions. Thus, a memristive behavior is not just placed at a place but involved into the enaction of the places too.

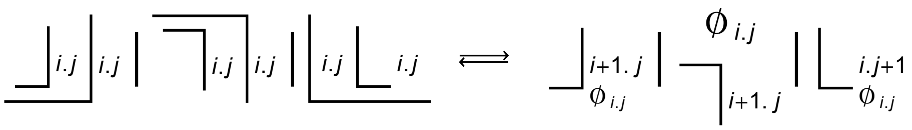
Some second – order enactional rules



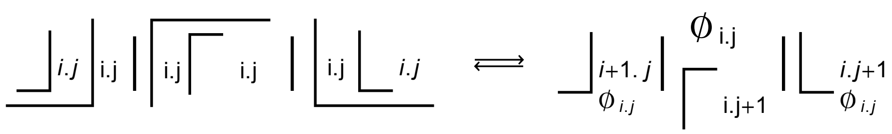
Null

Second – order enaction rules

Second – order reflectional enaction



Second – order interactional enaction



6. What's next?

6.1. What is computes.

Hence, what "is" it that is beyond or beneath or next to the world, space, domain of the universe of *"What is computes"*?

6.2. SPENCER-BROWN form 110 and WOLFRAM rule 110 are equivalent.

"A kind of form is all you need to compute. A system can emulate rule 110 if it can distinguish:
More than one is one but one inside one is none.
This is equivalent to DiscreteDelta in *Mathematica*.
A Form Principle of Computational Equivalence could thus be stated like:
Simple distinctions can be configured into forms which are able to perform universal computations."

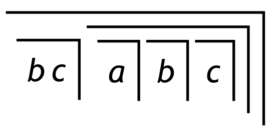
Michael Schreiber's Summary

"DiscreteDelta is a model for the SPENCER-BROWN form.

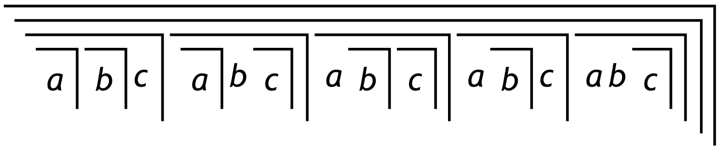
The SPENCER-BROWN form is functionally complete in the Boolean algebras of all degrees .

SPENCER-BROWN form {{b,c},{{a},{b},{c}}} emulates the universal elementary cellular automaton rule number 110 and might be a useful minimal example for interpretations of the new kind of scientific principle introduced by WOLFRAM:
'*There are various ways to state the Principle of Computational Equivalence, but probably the most general is just to say that almost all processes that are not obviously simple can be viewed as computations of equivalent sophistication.'* NKS 716-717." (Michael Schreiber, Computational Equivalence: Form 110, 2004)

Spencer Brown form 110



Wolfram rule 110

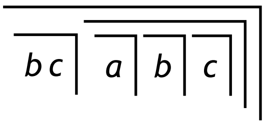


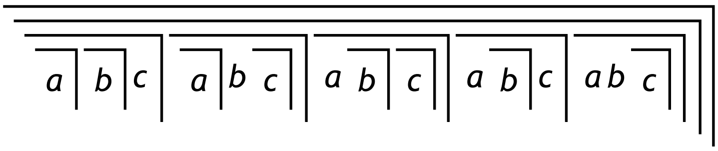
"Both of the above form terms are equivalent to rule number 110. The demonstration uses the simple representation and DiscreteDelta to evaluate each pair of brackets as a form." (M. Schreiber)
http://www.wolframscience.com/conference/2004/presentations/material/mschreiber-computational.nb

6.3. First steps beyond

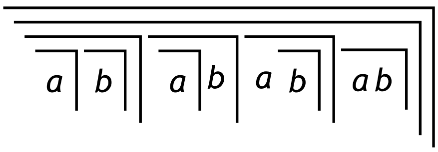
6.3.1. Complementary Wolfram rule?

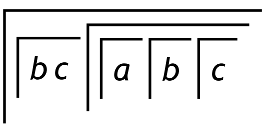
How are the distinctional and the ‘anti'-distinctional worlds related? Stephen Wolfram and George Spencer Brown are claiming to have found the elementary basic rule for universal computing (in a universal world of events).

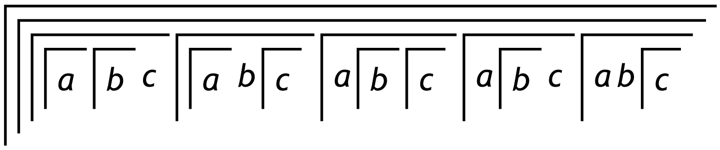
GSB: 

Wolfram: 

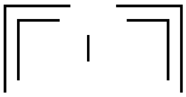
(TRUE $1 \times 1 \times 1 \times 1$) **complementary** (FALSE 0 0 0 0)

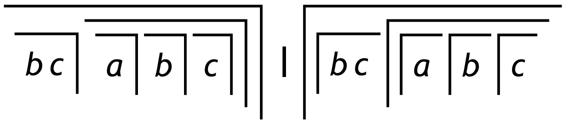
TRUE  **complementary** TRUE

Anti-GSB: 

Anti-Wolfram: 

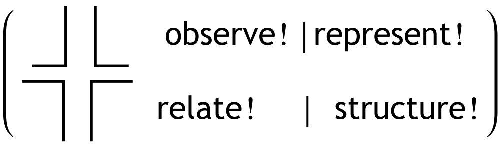
6.3.2. Complementary diamonds



GSB | Anti-GSB : 

6.4. Richard Herbert Howe's calculus of cognition (1970)

6.4.1. Calculus of cognition



Richard H. Howe is one of the very few thinker who succeeded to connect Gotthard Gunther's proemial relationship with George Spencer Browns *"Laws of Form"* under the inspirations given by Humberto Maturana and Heinz von Foerster at the BCL in the late 60s. Unfortunately, the announced monograph seems not to be accessible, and the BCL microfiches are not studied by many. I leave it to the reader to establish further connections.

Richard H. Howe writes:
"These five "key" words [of Maturana's approach to cognition, kae] are: description, observation, representation, relation and structure.
Within the text they are defined hierarchically but *recursively* in terms of each other.

Thus generally: a description or results from an observation of a representation of a relation of structures.
In the recursive domain of observation, however, such a description may be in turn encountered as a structure, or a representation, etc.

We may list:

*description*__________(d)____ n
*observation*__________(o)____n-1

*representation*_______(R)____n-2
*relation*____________ (r)_____n-2
*structure*____________(s)_____n-3

If we take as signs: | and – -, and let the directed crossings |-> and ↓- stand for the English particle "of"; and let the directed crossings ←| and ↑- stand for the English particulate phrase "with respect to", we may achieve a more compact notation for the relations given in the above list which will have other useful properties as well.

Thus in *canonical* form:

canonical form

$$d_j^n = \begin{array}{c|c} o_j^{n-1} & R_k^{n-2} \\ \hline r_l^{n-2} & s_m^{n-3} \end{array}$$

From the canonical form we may establish formulas giving definition to the other terms thus:

$$o_i^n = \begin{array}{c|c} d_j^{n+1} & R_k^{n-1} \\ \hline r_l^{n-1} & s_m^{n-2} \end{array} \;, \quad R_k^n = \begin{array}{c|c} o_j^{n+1} & d_k^{n+2} \\ \hline r_l^{n} & s_m^{n-1} \end{array} \;,$$

$$r_l^n = \begin{array}{c|c} o_j^{n+1} & R_k^{n} \\ \hline d_l^{n+2} & s_m^{n-1} \end{array} \;, \quad s_m^n = \begin{array}{c|c} o_j^{n+2} & R_k^{n+1} \\ \hline r_l^{n+1} & d_m^{n+3} \end{array}$$

We may easily optain operators corresponding to the imperative or injunctive *observe*!, *represent*!, *relate*!, and *structure*! from the form of the canonical form by taking that portion of the form in which the d_i^nterm appears as its sign.

Thus:

observe! __ __ __ __ _ ⌟

represent! __ __ __ __ ⌞

relate! __ __ __ __ __ ⌝

structure! __ __ __ __ ⌜

The injunction *describe*! will be given by the raising of the superscript or level on degree:

$$d_l^n \longrightarrow d_l^{n+1},$$

without reference to other expressions.

Then for **example** if:

$$d_i^n = \begin{array}{c|c} o_j^{n-1} & R_k^{n-2} \\ \hline r_l^{n-2} & s_m^{n-3} \end{array}$$

then

$$\underline{\left| d_i^n \right.} = \underline{\left| \begin{array}{c|c} o_j^{n-1} & R_k^{n-2} \\ \hline r_l^{n-2} & s_m^{n-3} \end{array} \right.}$$

and by extension of the boundary sides of the operator:

$$\underline{\left| d_i^n \right.} = \begin{array}{c|c} o_j^{n-1} & \begin{array}{c|c} o_j^{n-1} & R_k^{n-2} \\ \hline r_l^{n-2} & s_m^{n-3} \end{array} \\ \hline r_l^{n-2} & s_m^{n-3} \end{array}$$

which condenses to:

$$\underline{\left| d_j^n \right.} = \begin{array}{c|c} o_j^{n+1} & R_k^n \\ \hline r_l^n & s_m^{n-1} \end{array} .$$

Each d, o, R, r and s in these expressions is a complex expression in its own right, then, if any of these be reducible under the operators given to a form which is equivalent to another position, we may contract our notation thus:

$$\begin{array}{c|c} o & R \\ \hline r & \begin{array}{c|c} [s.o] & R \\ \hline r & s \end{array} \end{array}$$

and accomodate any degree of ambiguity without loss.

□	Rv	so	Rv	so	□
□	so	Rv	so	Rv	□
□	Rv	so	Rv	so	□
□	so	Rv	so	Rv	□

[...]
From this matrix, applications as operators of the lines delineating the rows and columns may, in some circumstances allow the entire matrix to be reduced to a form in which first the s then the r, then the R, and finally the o elements are brought to the n-n=0 level, and drop out, leaving the expression:

$$\mathbf{d}_{\mathbf{i}}^{\mathbf{1}} \equiv \left.\underline{(o^{o})}\right| \equiv \textbf{\textit{observe}}\ !$$

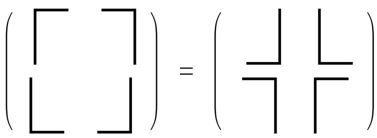
But this form is again equivalent to a distinction drawn between the space of distinction and that of no-distinction, and thus we are returned to our *beginning* [already discussed in section β and γ]."

R. H. Howe, Linguistic Composition of an Arithmetic of Cognition, pp. 54-70 (1970), BCL Report # 70.2, Fiche # 127/3

6.4.2. Diamond calculus

Following a purely abstract characteriszation of the *modi* of distinctions, Howe’s formula might be restated in the framework of Diamond Distinctions. Hence, any connection to a theory of cognitive systems with the properties of observation, description, representation and structure are at firstly omitted in favor of strict formal characteristics of the category of distinction. All modi of distinction together are defining a planar complexion of distinction. (The wording of the modi is still quite arbitrary.)

`Distinction system` =



Construction

Draw a distinction, mark it	:	$\rceil$	distinction
Distinguish the drawing, remark it	:	$\lceil$	complement
Reverse the distinction, design it	:	$\rfloor$	reverse
Converse the drawing, redesign it	:	$\lfloor$	converse

There are other useful interpretations of the quadralectics of primordial distinctions:

$\begin{pmatrix} \rfloor & \lfloor \\ \rceil & \lceil \end{pmatrix} \cong$ quadralectic distinctional systems

$\begin{pmatrix} \mathrm{sS} & \mathrm{oO} \\ \mathrm{sO} & \mathrm{oS} \end{pmatrix} \cong$ epistemological distinction system (Gunther, Toth)

http://mathematical-semiotics.com/pdf/Surreale%20Nacht.pdf

$\begin{pmatrix} \mathrm{I} & \mathrm{O} \\ \mathrm{M} & \mathrm{Q} \end{pmatrix} \cong$ semiotic distinctional system (Bense, Toth)

$\begin{pmatrix} \text{Thirdness} & \text{Secondness} \\ \text{Firstness} & \text{Zeroness} \end{pmatrix} \cong$ semiotic metaphysical system (Peirce, Bense, Toth)

$\begin{pmatrix} \text{Source} & \text{Origin} \\ \text{Boundary} & \text{Arena} \end{pmatrix} \cong$ system-theoretical quadralectic system (Kent Palmer)

http://www.scribd.com/doc/30047691/Palmer-s-Pentalectics

canonical hierarchical quadralectics of forms

$$\text{distinction}_j^n = \frac{\text{reverse} - o_j^{n-1} \;\big|\; \text{converse} - R_k^{n-2}}{\text{decision} - r_l^{n-2} \;\big|\; \text{complement} - s_m^{n-3}}$$

canonical hierarchical quadralectics of forms

$$\text{distinction}_j^n = \frac{\text{de sign!} - o_j^{n-1} \;\big|\; \text{redesign!} - R_k^{n-2}}{\text{mark!} - r_l^{n-2} \;\big|\; \text{demark!} - s_m^{n-3}}$$

canonical form for quadralectics

$$q_j^n = \frac{\left(\lrcorner\right)_j^{n-1} \;\big|\; \left(\lrcorner\right)_k^{n-2}}{\left(\urcorner\right)_l^{n-2} \;\big|\; \left(\ulcorner\right)_m^{n-3}}$$

Following Howe's approach to a quadralectics of distinction some direct applications and further exercises are demonstrated. Obviously, it's all a very first step which will be elaborated in a separate paper.

From the canonical form we may establish formulas giving definition to the other terms thus:

$$\left(\lrcorner\right)^{n}_{i} = \frac{\begin{array}{c|c} \left(\lrcorner\right)^{n+1}_{j} & \left(\lrcorner\right)^{n-1}_{k} \end{array}}{\begin{array}{c|c} \left(\urcorner\right)^{n-1}_{l} & \left(\ulcorner\right)^{n-2}_{m} \end{array}}, \qquad \left(\llcorner\right)^{n}_{k} = \frac{\begin{array}{c|c} \left(\lrcorner\right)^{n+1}_{j} & \left(\llcorner\right)^{n+2}_{k} \end{array}}{\begin{array}{c|c} \left(\urcorner\right)^{n}_{l} & \left(\ulcorner\right)^{n-1}_{m} \end{array}},$$

$$\left(\urcorner\right)^{n}_{l} = \frac{\begin{array}{c|c} o^{n+1}_{j} & \left(\llcorner\right)^{n}_{k} \end{array}}{\begin{array}{c|c} \left(\urcorner\right)^{n+2}_{l} & \left(\ulcorner\right)^{n-1}_{m} \end{array}}, \qquad \left(\ulcorner\right)^{n}_{m} = \frac{\begin{array}{c|c} \left(\lrcorner\right)^{n+2}_{j} & \left(\llcorner\right)^{n+1}_{k} \end{array}}{\begin{array}{c|c} \left(\urcorner\right)^{n+1}_{l} & \left(\ulcorner\right)^{n+3}_{m} \end{array}}$$

Examples : Recursive quadralectics

General

$$\text{quadralectics}^{n}_{\text{dist}} = \frac{\begin{array}{c|c} \left(\lrcorner\right)^{n-1}_{j} & \left(\llcorner\right)^{n-2}_{k} \end{array}}{\begin{array}{c|c} \left(\urcorner\right)^{n-2}_{l} & \left(\ulcorner\right)^{n-3}_{m} \end{array}}$$

$$\text{succ}_{\text{dist}} : \text{quadralectics}^{n}_{\text{dist}} \longrightarrow \text{quadralectics}^{n+1}_{\text{dist}} :$$

$$\text{succ}_{\left(\lrcorner\right)} : \frac{\begin{array}{c|c} \left(\lrcorner\right)^{n-1}_{j} & \left(\llcorner\right)^{n-2}_{k} \end{array}}{\begin{array}{c|c} \left(\urcorner\right)^{n-2}_{l} & \left(\ulcorner\right)^{n-3}_{m} \end{array}} \Longrightarrow \frac{\begin{array}{c|c} \left(\lrcorner\right)^{n+2}_{j} & \left(\llcorner\right)^{n+1}_{k} \end{array}}{\begin{array}{c|c} \left(\urcorner\right)^{n+1}_{l} & \left(\ulcorner\right)^{n}_{m} \end{array}}$$

$$\text{succ}_{\left(\llcorner\right)} : \frac{\begin{array}{c|c} \left(\lrcorner\right)^{n-1}_{j} & \left(\llcorner\right)^{n-2}_{k} \end{array}}{\begin{array}{c|c} \left(\urcorner\right)^{n-2}_{l} & \left(\ulcorner\right)^{n-3}_{m} \end{array}} \Longrightarrow \frac{\begin{array}{c|c} \left(\lrcorner\right)^{n+1}_{j} & \left(\llcorner\right)^{n}_{k} \end{array}}{\begin{array}{c|c} \left(\urcorner\right)^{n}_{l} & \left(\ulcorner\right)^{n-1}_{m} \end{array}}$$

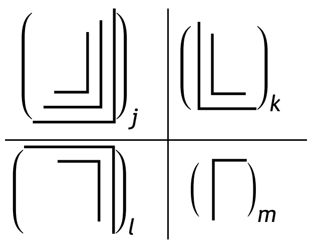
$$\mathrm{succ}_{(\urcorner)}:\ \frac{\left(\lrcorner\right)_j^{n-1} \;\Big|\; \left(\llcorner\right)_k^{n-2}}{\left(\urcorner\right)_l^{n-2} \;\Big|\; \left(\ulcorner\right)_m^{n-3}} \Longrightarrow \frac{\left(\lrcorner\right)_j^{n+1} \;\Big|\; \left(\llcorner\right)_k^{n}}{\left(\urcorner\right)_l^{n} \;\Big|\; \left(\ulcorner\right)_m^{n-1}}$$

$$\mathrm{succ}_{(\ulcorner)}:\ \frac{\left(\lrcorner\right)_j^{n-1} \;\Big|\; \left(\llcorner\right)_k^{n-2}}{\left(\urcorner\right)_l^{n-2} \;\Big|\; \left(\ulcorner\right)_m^{n-3}} \Longrightarrow \frac{\left(\lrcorner\right)_j^{n} \;\Big|\; \left(\llcorner\right)_k^{n-1}}{\left(\urcorner\right)_l^{n-1} \;\Big|\; \left(\ulcorner\right)_m^{n-2}}$$

Special

n = 4

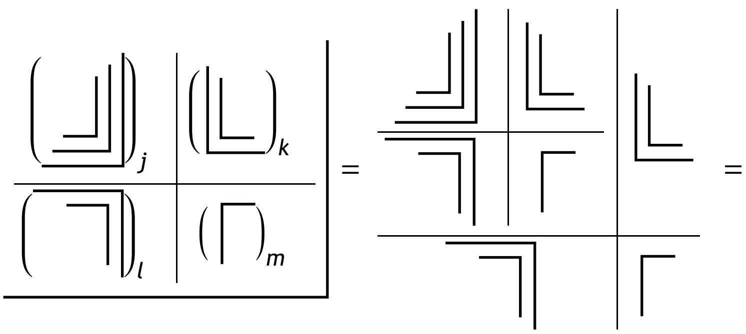
$$\mathrm{quadralectics}^4_{\mathrm{dist}} = \frac{\left(\lrcorner\right)_j^{3} \;\Big|\; \left(\llcorner\right)_k^{2}}{\left(\urcorner\right)_l^{2} \;\Big|\; \left(\ulcorner\right)_m^{1}} =$$

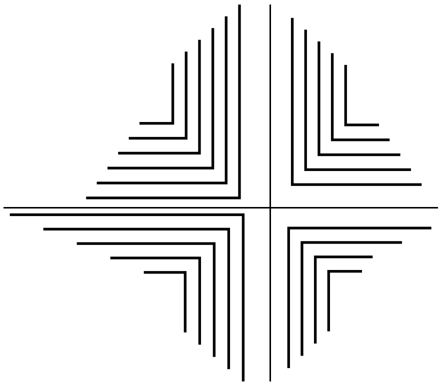


n = 5

$$\mathrm{succ}:\ \mathrm{quadralectics}^4_{\mathrm{dist}} \longrightarrow \mathrm{quadralectics}^5_{\mathrm{dist}}:$$

$$\underline{\mathrm{quadralectics}^4}\Big| = \mathrm{quadralectics}^5{}_{\lrcorner} =$$





$$= \begin{array}{c|c} (\lrcorner)_j^6 & (\llcorner)_k^5 \\ \hline (\urcorner)_l^5 & (\ulcorner)_m^4 \end{array}$$

$$\text{quadralectics}^4 = \text{quadralectics}^5_{\llcorner} =$$

$$\begin{array}{c|c} (\lrcorner)_j^3 & \begin{array}{c|c} (\lrcorner)_j^3 & (\llcorner)_k^2 \\ \hline (\urcorner)_l^2 & (\ulcorner)_m^1 \end{array} \\ \hline (\urcorner)_l^2 & (\ulcorner)_m^1 \end{array} = \begin{array}{c|c} (\lrcorner)_j^5 & (\llcorner)_k^4 \\ \hline (\urcorner)_l^4 & (\ulcorner)_m^3 \end{array}$$

$$\text{quadralectics}^4 = \text{quadralectics}^5_{\urcorner} =$$

$$\begin{array}{c|c} (\lrcorner)_j^3 & (\llcorner)_k^2 \\ \hline \begin{array}{c|c} (\lrcorner)_j^3 & (\llcorner)_k^2 \\ \hline (\urcorner)_l^2 & (\ulcorner)_m^1 \end{array} & (\ulcorner)_m^1 \end{array} = \begin{array}{c|c} (\lrcorner)_j^5 & (\llcorner)_k^4 \\ \hline (\urcorner)_l^4 & (\ulcorner)_m^3 \end{array}$$

$$\text{quadralectics}^4 = \text{quadralectics}^5_{\ulcorner} =$$

$$\begin{array}{c|c} (\lrcorner)_j^3 & (\llcorner)_k^2 \\ \hline (\urcorner)_l^2 & \begin{array}{c|c} (\lrcorner)_j^3 & (\llcorner)_k^2 \\ \hline (\urcorner)^2 & (\ulcorner)^1 \end{array} \end{array} = \begin{array}{c|c} (\lrcorner)_j^3 & (\llcorner)_k^2 \\ \hline (\urcorner)_l^2 & (\ulcorner)_m^1 \end{array} \bullet$$

Interchangeability

Given the quadruple structure of the formation of the general form, different possibilities of applying the interchangeability abstraction are opened up.
One first step might be achieved to understand the distinction forms “reverse” and “converse” as being in discontextural parallelism to the distinction forms “distinction” and “ complement”. While a kind of an order between “reverse” and “converse”, and “distinction” and “ complement” might be established. Therefore, the well-known functorial abstraction of interchangeability might apply.
Because of the super-additive structure of the combination of discontextural compositions, a third pair of distinctions, ruling the interaction between the two first pairs of distinctions has to be introduced. A candidate might be “*system*” and “*environment*” of the two primary distinction pairs.

With the introduction of “*internal*” and “*external*” environment, the usual distinctions for diamond category theory are introduced.

Interchangeability with super – additivity for distinctions, simplified notation

$$\left[\begin{array}{cccc|c} & \text{compl}_1 & - & \text{intern}-\text{env}_3 & \text{extern}-\text{env}_4 \\ \text{dist}_1 & \text{conv}_2 & - & & - \\ - & \text{rev}_2 & \text{intern}-\text{syst}_3 & & \text{extern}-\text{syst}_4 \end{array}\right]:$$

$$\left(\begin{array}{c} \left(\begin{array}{c} \left(\text{dist}_1 \circ^{1.0\ \ .0} \text{compl}_1\right) \\ \left(\text{rev}_2 \circ^{0.2\ \ .0} \text{conv}_2\right) \end{array}\right) \\ \left(\text{syst}_3 \circ^{0.0\ \ .3\ \ .0} \text{env}_3\right) \Big| \left(\text{syst}_4 \circ^{0.0\ \ .0\ \ .4} \text{env}_4\right) \end{array}\right) =$$

$$\left(\begin{array}{c} \left(\begin{array}{c} \text{dist}_1 \\ \text{rev}_2 \end{array}\right) \\ \left(\text{syst}_3 \,\Big|\, \text{syst}_4\right) \end{array}\right) \circ_{1.2\ \ .3\ \ .4} \left(\begin{array}{c} \left(\begin{array}{c} \text{compl}_1 \\ \text{conv}_2 \end{array}\right) \\ \left(\text{env}_3 \,\Big|\, \text{env}_4\right) \end{array}\right)$$

http://www.thinkartlab.com/pkl/lola/Diamond-Category-Theory.pdf
Rudolf Kaehr, Double Cross Playing Diamonds, Understanding interactivity in/between bigraphs and diamonds
in: Uwe Seifert, Jin Hyun Kim, Anthony Moore (eds.), Paradoxes of Interactivity:
Perspectives for Media Theory, Human-Computer Interaction, and Artistic Investigations, 2008
http://www.thinkartlab.com/pkl/lola/Interactivity.pdf

6.4.4. Metadistinction system of the Laws of Form

Although the calculus of *Laws of Form* starts with a single mark as the notation of a distinction, it needs several further kinds of distinctions, decisions and presuppositions to start the game and to stay in it. The Laws of Form are written as a simple calculus. But its introduction and interpretation is demanding for characterizations, i.e. distinctions which are not explicitly included in the calculus and don't get any *notational* realizations in the technicalities of the calculus as such.
Instead of denying primordial complexity and circularity in favor of a simple beginning, it might be reasonable, not to suspend such topics but to opt for a strategy to just begin with a complex and self-referential design of formalization.

Rudolf Kaehr: Disseminatorik: Zur Logik der „Second Order Cybernetics“. In: Kalkül der Form, (hg.) Dirk Baecker, 1993)
http://works.bepress.com/thinkartlab/6/