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Interplay of Elementary Graphematic Calculi

Graphematic Fourfoldness of semiotics, Indication, Differentiation and Kenogrammatics

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Abstract

Extended *Notes* on the interplay of graphematic calculi: George Spencer Brown's *Laws of Form*, Mersenne calculi and Gunther's trito- and deutero-grammatic systems. Moshe Klein's second-level partitions, numeric represention of calculi, "serial", "parallel" and intermediary number concepts. Non-contradiction of Mersenne self-referentiality. Further elaboration of the *"Stirling Turn"*. Complementarity of Brownian and Mersenne calculi.

1. Complementarity of Mersenne and Brown calculi

1.1. Discussion

1.1.1. General situation

This paper is proposing some loosely connected notes about the connections between different graphematic systems, like Brownian, the newly postulated Mersenne calculus and calculi in the context of Stirling numbers of the second kind. Aspects of combinatorics are as far elaborated as necessary to understand the concepts and possible calculi, formal languages and cellular automata

For the special case of complexity and complication of value two, i.e. m=n=2, some interesting conceptual distinctions between Brownian (George Spencer Brown 1923 -)indication (multisets, bags) and Mersenne numbers (Marin Mersenne 1588 - 1648) might be to observed.

Types	**Example**	**Combinatorics**
Leibniz space	= {aa, bb, ab, ba},	$Sem_{(m, n)} = m^n$
Brownian space	= {aa, ab, bb},	$Ind_{(n, m)} = \binom{n+m-1}{n}$
Mersenne space	= {aa, ab, ba},	$M_n = 2^n - 1$
Stirling space	= {aa, ab}.	$\sum_{k=1}^{M} S(n, k)$

This study *"Interplay of Elementary Graphematic Calculi"* is a direct continuation of the previous paper *"Graphematic System of Cellular Automata"* which is studying 9 levels of graphematical inscription.
http://memristors.memristics.com/Graphematics/Graphematics%20of%20Cellular%20Automata.html

Stirling vs. Mersenne

Recursive Stirling numbers of the second kind :

$$\left\{{n \atop k}\right\} = \left\{{n-1 \atop k-1}\right\} + k\left\{{n-1 \atop k}\right\} \text{ with } \left\{{n \atop 0}\right\} = 0, \text{ for } n > 0 \text{ and } \left\{{n \atop n}\right\} = 1$$

Mersenne numbers

$$M_n = 2^{n-1} - 1$$

$$\left\{{n \atop 2}\right\} = Sn(n, 2) = Mers(n)$$

Hence, Mersenne numbers appear as a subset of Stirling numbers of the second kind. This purely combinatorial fact shows the combinatorial dependency between the Stirling and the Mersenne space. Again, combinatorial studies are not telling much about the conceptual characteristics of the specific types of calculi and their computational space.

Example

n **:** 2 3 4 5 6

$\left\{{n \atop 2}\right\}$: 1, 3, 7, 15, 31

$2^{n-1} - 1$: 1, 3, 7, 15, 31

Stirling vs. Mersenne via primes

Another interesting way to compare Stirling and Mersenne numbers is possible by an indirect comparison mediated by the concept of prime numbers of both types of numbers.
http://mathworld.wolfram.com/MersennePrime.html

Joe DeMaio , Stirling Numbers of the Second Kind and Primality
http://science.kennesaw.edu/~jdemaio/stirling%20second%20primes.pdf
http://cs.fit.edu/~wds/classes/adm/Slides/StirlingSecond.pdf

Mersenne vs. Brown numbers
In fact, there are still no specific Brown numbers.
Numbers in Brown systems like the calculus of indication are modeling classical natural numbers in the framework of the Calculus of Indication (CI).

The crucial fact, that the CI is defined on the base of a 2-dimensional semiotics with serial and parallel constructions is not yet understood for a genuine Brownian number concept and arithmetic. The case that the Brownian don’t wont to accept their own invention might also be considered. In this case the big claim seems to vaporize to a rather harmless endeavour.

Moshe Klein started some ideas to introduce genuine Brownian numbers but seems to have abandoned his trial.

This study *“Interplay of Elementary Graphematic Calculi”* is a direct continuation of the previous paper *“Graphematic System of Cellular Automata”* which is studying 9 levels of graphematical inscription.
http://memristors.memristics.com/Graphematics/Graphematics%20of%20Cellular%20Automata.html

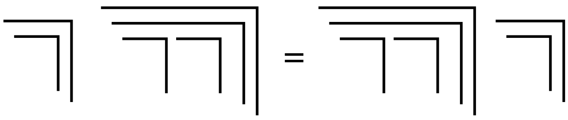
1.1.2. Indicational calculi

Indication (differentiation): heaps, multisets
The interest is in the differentiation of the elements on an identive level in contrast to a kenogrammatic level, hence (aa) ≠ (bb), while the differences of permutations of the elements are not of interest for the differentiation of the constellation, hence (ab)=(ba). This points to the fact that indicational terms are not sequences or sets of atomic signs but *heaps* (Matzka) of signs: {{a, b}} = {{b, a}}.

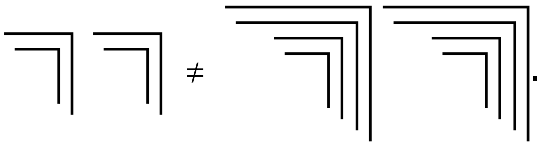
Multisets (heaps, bags) as models for indicational calculi
"There are for example 4 multisets of cardinality 3 with elements taken from the set {1,2} of cardinality 2 (n=2, k=3), namely :

{1,1,1}, {1,1,2}, {1,2,2}, {2,2,2}.
And there are also 4 subsets of cardinality 3 in the set {1,2,3,4} of cardinality 4 (n+k-1 = 4), namely :
{1, 2, 3}, {1, 2, 4}, {1, 3, 4}, {2, 3, 4}." WiKi
In multisets, as in sets and in contrast to tuples, the order of elements is irrelevant: The multisets {a, b} and {b, a} are equal." WiKi

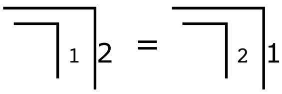
Example for commutativity of notation in CI:
(ab) = (ba):



(aa) ≠ (bb):



Certainly this holds for the *vertical* dimension too:



. Hence, there is no distinction involved, therefore the result is " ", i.e. no distinction: ∅.

1.1.3. Moshe Klein's second-level partitions and GSB's brackets

Gotthard Gunther always was glad to be able to irritate or even shock his contrahents when they argued with numbers. If someone insisted on the number 5, he simple asked: Which number 5 do you mean? I have at least 52 different exemplars of your number 5! Obviously he was referring to his trito-numbers of the kenogrammatic number system calculated by the Stirling numbers of the second kind. He could have been more generous and answering that he has at least 7 types of the number 5, referring to his deutero-numbers measured by the number of partitions.

Schadach has given an answer to the question: *How many numbers are represented by proto-, deutero- or trito-structures?*

Moshe Klein just introduced a new step in the analysis of partitions, the *second-level partitions* or the partitions of partitions. Equipped with the strategy of second-level partitions Gunther could have given a double answer: If you want a direct answer, I have 7 types of your single number 5. But if you wont a more reflected answer, I have 30 types of your single number 5.

Such argumentations have deep roots in Ancient Greek and Chinese thinking. It has taken Aristotle a lot of intriguing arguments to ridicule such a figurative or organic view of numbers (concepts, metaphors, images). Aristotle radical reductionism has won and opened up a mathematical foundation for science and financial politics. Today, this approach is exhausted and a new approach is needed. What the Ancient didn't have is a complex operative holistic number theory and calculus. Today, there are interesting beginnings for complex calculi to register. Gotthard Gunther's *"Natural Numbers in trans-Classic Systems"* (1971) is a promising step towards a liberation of numbers from the reductionist Aristotelian approaches.

Albeit Moshe Klein is not aware about the fundamental breakthrough of Gunther's elaborations it seems that Klein is working in a similar direction with his second-level partitions and a re-interpretation of Spencer Brown's Laws of Form.

This study of second-level partitions gets an interesting comparison with Spencer-Brown's calculus of indication.
How far those studies can be applied to Mersenne calculi has yet to be studied.

The newly introduced *Stirling turn* is based on the study of partitions, i.e. trito-structures, instead of elements and sets, and their functions, therefore, second-level partitions will be of direct interest for a further study of intrinsic properties of Stirling numbers. Schadach's analysis of the internal structure of partitions might hint into the same direction.

Catalan numbers and indicational calculi

> **"**Another problem with a similar answer is the counting of the number of parenthesis when there is *no significance to the order*. Let's look, for example, at parenthesis of order 2 ()();(()).
>
> When we look at parenthesis with order 3 there are 5 possibili-

ties

$()()(); (())(); (()()); ()(()); ((()))$.

The general number of possibilities is calculated with the Catalan numbers. But in the specific problem when the order is not important like in the problem of phylogenetic trees the two possibilities ()(())=(())() are identical.

"Now let us go one step further and distinguish between possibilities only by what they contain and by order.
For instance, from now on, ()(())=(())().

Inspired from *"Laws of Form"* written by Spencer Brown[10], we shall call the possibilities whose order is *insignificant*, "forms".
Now let us create a "sub-partition" definition that fits those forms. Note that these sub-partitions (or forms) are applicable and relevant, for instance in Biology or Computer Science, when counting the number of ways to arrange n membranes in space. The number of forms (not possibilities) of degree 3 is 4 and not 5 as before. The forms are:

$()()(); ()(()); (()()); ((()))$

We define (n) as the collection of all forms of parentheses that are wrapped with brackets and inside them there is a form of degree n.

For instance:

(2) = $\{(()()), ((()))\}$.

Note that

$(0)=\{()\}, (1)=\{(())\}$ and $(3)=\{(()()()), (()(())), ((()())), (((())))\}$."

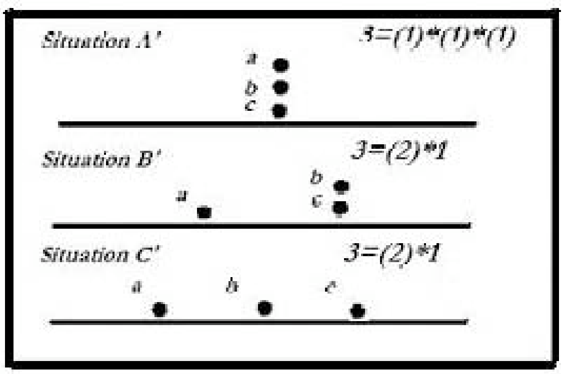
"Distinction is a very important part of our life. Similarly to Hilbert's analogy about the completeness of a mathematical theory, *Organic Mathematics* claims that any fundamental math-

ematical theory is incomplete if it does not deal with Distinction as first-order property of it. This presentation is focused on the structure of Whole Numbers." (Klein, 2009)
http://www.omath.org.il/image/users/112431/ftp/my_files/recurtion_over_partitions_118.pdf?id=8746401
http://www.youtube.com/watch?v=KoyiMz_-uew

Catalan numbers are a subset of Stirling numbers of the second kind and are coinciding with the values 1, 2, 3.

$$C_n = \frac{1}{n+1}\binom{2n}{n}$$

Table[CatalanNumber[n], {n, 10}]
{1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796}



Brown **Mersenne**
3 =(1)*(1)*(1) 3 = (1)*(1)*(1) Mersenne: situation A' equal situation C', and B' different B".
3 = (2)*1 3 = (2)*1 Brown: situation B' equal situation B", and A' different C'.
3 = (2)*1 3 = 1*(2)

It could be argued that *Brownian* partitions are not so much involved into differences as it is claimed. The reason of this argument is this: the internal structure of the Brownian number 3 with ()()(); ()(()); (()) is considering only one internal difference, namely ()(()) = (())(). The other two partitions, ()()() and (()) are internally homogenous, and are therefore not representing internal differences.

For the *Mersenne* calculus, the two cases ()(()) and (())() are representing internal differences albeit they are structurally symmetric. In other words, this reflection about different thematizations involves a kind of a meta-distinction over the distinctive ‘object’ represented by the chain of brackets. Hence, three in *series* and 3 in *parallel* are equal considering the complexity of their *internal* differences. One bracket and two superposed brackets, or two superposed bracket and one bracket are realizing an internal difference.

From a category-theoretic point of view, questions of bifunctoriality between “serial” and “parallel” constellations with *intermediary* cases are obvious but not yet considered in Brownian calculi.

Deeper than sets?

> *"Is it possible to provide an even deeper foundation for mathematics? A set is a particular type of distinction, namely, a distinction that creates a one from a many. But not every distinction is a set. For example, logical operators such as not are not sets. Thus, even more fundamental than the notion of a set is the notion of distinction, for every set is defined or created by making a distinction between what is contained in the set, and what is not (e.g., the set itself). So, the Pythagorean maxim then becomes:* everything is made of distinction*.”*
> Thomas J. McFarlane, Distinction and the Foundations of Arithmetic, 2011
> http://www.integralscience.org/lot.html

Comment

Despite the interesting combinatorial results, there is some criticism to mention.

Klein’s approach is **ad hoc** and is relating to George Spencer Brown’s calculus of indication which itself is also not giving a systematic introduction of the decisions fundamental for the indicational calculus.

There is no graphematic system which could give a systematic legitimation for the new intuition of “Organic Numbers".

Historically, there is also no mention of the work of Dieter Schadach et al at the BCL (1960s) towards combinatorial studies of different kinds of partitions for a theory of living systems and bio-computing.

The decision of “counting of the number of parenthesis when there is *no significance to the order”* is obviously of importance for the Brownian calculus, and to point to this "topological invariance" by Klein is supporting the approach of Varga as it is elaborated in my own constructions.

Klein: *"For instance, from now on,*

$$()(()) = (())()."$$

This is basic for the **Brownian** calculus.

The other equally reasonable decision is : *For instance, from now on,*

$$()() = (()).$$

This is basic for the complementary **Mersenne** calculus.

But for the same reasons we could state "counting of the number of parenthesis when there is *no significance to the number of the 'balanced' parenthesis",* the permutations are counting*.* That is, ()()=(()). Hence, the other decision is delivering

$()()(); (())(); ()()(()); ()(())(); (()()); (())()();$ This corresponds to:

$(())()(())$ instead of $()()(); ()(()); (()()); ((()))$.

aaa; bba; aab; aba abb baa bab instead of aaa; aab; abb; bbb (abc).

a	**b**	a	a	a b	b		
a	**b**	a	b	b a	a		(7)
a	a	b	a	b a	b		

()()(); (())(); ()()(()); ()(())(); (()()); (())()(); (())()(()) : The order has to be marked. Therefore 6 brackets are not enough.

The difference between Klein's and the proposed approach is, again, that the introduced new mathematical languages are well founded and located in a system of graphematics.

We might argue, if this decision makes sense for the Brownian calculus it equally will make sense for the Mersenne calculus too. And obviously, if we abandon both requisites, we get the *tritogrammatic* calculus, with $\{()()(); ()(())\}$.

With those steps we are back again in the game of (semiotics, indication, differentiation, tritogrammatics).

```
Semiotic convention

   semiotics
      ↙ ↘
Brwown   Mersenne
      ↘ ↙
 tritogrammatics
```

1.1.4. Deutero-numbers

Klein's second-level partitions can be interpreted as an analysis of the internal structure of the deutero-numbers of the general graphematic system of inscription. In contrast to trito-numbers and the importance of locality of monomorphies, deutero-numbers are non-local. The order of their kenograms (sub-numbers) is irrelevant, and therefore there are no monomorphies in the sense of tritograms involved. Deutero-numbers are the partitions of a number.

Deutero-numbers

P(4) = 5,
$sp_n(4) = 11$

P(4) : [4]-[3,1]-[2,2]-[2,1,1]-[1,1,1,1].

$sp_n(4)$: (3); ()(2); (1)(1); ()()(1); (()()()).

Recursivity for deutero-numbers (partitions)

According to *Morphogrammatik*, p. 74 we get :

Deutero-number : $D = [p_1, \ldots, p_{max}]$

Number of Deutero-successors :

$$n_{DTS}(D) = 2 + \sum_{i=1}^{max-1} \operatorname{sign}(p_i - p_{i+1}) \; .$$

Algorithm for Deutero-succession in ML :

```
fun DTS D =
[((pos 1 D)+1)::(tl D)] @
```

```
(remnils (map (fn i => if sign((pos i D) - (pos (i+1) D)) = 1
then replacepos (i+1) D ((pos (i+1) D)+1)
else [])
(fromto 1 ((length D)-1)))) @
[D@[1]]
```

Rule

$[1] \in \text{Deutero} \Rightarrow n_{\text{DTS}}(D) \in \text{Deutero}.$

Example for $n_{\text{DTS}}(D)$

D	$n_{\text{DTS}}(D)$	$\text{DTS}_1, \ldots, \text{DTS}_{\text{nDTS}(D)}$
$[3, 1]$	3	$[[4, 1], [3, 1], [3, 1, 1]]$
$[3, 2, 1]$	4	$[[4, 2, 1], [3, 3, 1], [3, 2, 2], [3, 2, 1, 1]]$

Addition of Deutero – numbers

Addition "$+_d$" of $D = [p_1, \ldots, p_{\text{maxD}}]$ and $E = [q_1, \ldots, q_{\text{maxE}}]$

is defined by :

Deutero – addition

$D = [p_1, \ldots, p_{\text{maxD}}], \; E = [q_1, \ldots, q_{\text{maxE}}]$

$$[p_1, \ldots, p_{\text{maxD}}] +_d [q_1, \ldots, q_{\text{maxE}}] =$$
$$\{[p_1, \ldots, p_{\text{maxD}}, q_1, \ldots, q_{\text{maxE}}],$$
$$[p_1, \ldots, p_i - 1, p_i - 1, p_{i+1}, \ldots, p_{\text{maxD}},$$
$$q_1, \ldots, q_j - 1, q_j - 1, q_{j+1}, \ldots, q_{\text{maxE}}, 1]\}.$$

Examples for deutero – additions

$D = [1], \; E = [1]:$

$$[1] +_d [1] = \{[2], [1, 1]\}.$$

$D = [2],\ E = [1]:$

$[2] +_d [1] = \{[3],\ [2, 1]\}.$

$D = [2],\ E = [1, 1]:$

$[2] +_d [1, 1] = \{[3, 1], [2, 1, 1]\}.$

$D = [1, 1],\ E = [1, 1]:$

$[1, 1] +_d [1, 1] = \{[1, 1, 1, 1], [2, 1, 1]\}.$

$D = [2, 1],\ E = [1, 1]:$

$[2, 1] +_d [1, 1] = \{[3, 1], [2, 2],\ [2, 1, 1]\}.$

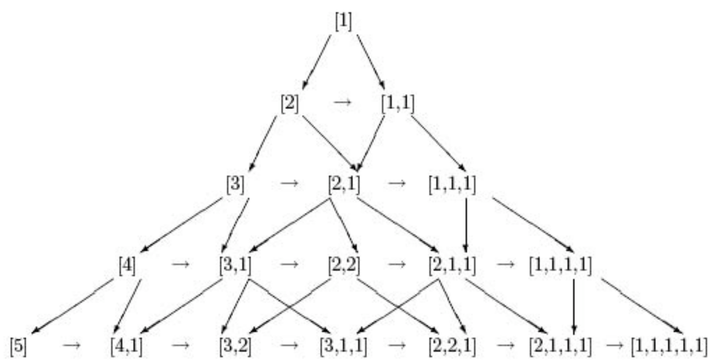
$D = [3, 1],\ E = [1]:$

$[3, 1] +_d [1] = \{[4, 1], [3, 2],\ [3, 1, 1]\}.$

$D = [3, 2, 1],\ E = [1]:$

$[3, 2, 1] +_d [1] = \{[4, 2, 1], [3, 3, 1],\ [3, 2, 2],\ [3, 2, 1, 1]\}.$

Deutero – graph



Numeric Deutero – number rules

$$R0: \Rightarrow [1]$$

$$R1.1: [n] \Rightarrow [n+1] \mid [n,1]$$

$$R1.2: [1,1] \Rightarrow [n+1,1] \mid [1,1,1]$$

$$R1.3: [n,1] \Rightarrow [n+1,1] \mid [n,2] \mid [n,1,1]$$

Second-level partitions introduced by Moshe Klein

> "Considering a more interesting example, since {3}, {2 + 1} and {1 + 1 + 1} are the partitions of 3, we believe that now it does in fact make sense to look at *second-level* partitions.
>
> As before, it is clear that it is meaningless to perform sub-partitions on the partition {1 + 1 + 1}, and on the partition {3}, as this would lead again to an infinite number of sub-partitions, via recursion. Thus, using the number 3 to help us finding a proper definition, we see that recursions may be used, yet must be applied carefully.
>
> For instance, in that case, it only makes sense to perform a sub-partition on the element {2 + 1} only. The number 2 has two different partitions: *"partition-a"* which is {2} and *"partition-b"* which is {1 + 1}. The process of performing a sub-partition on the number 3 by using a partition of the number 2 will thus lead to splitting the partition {2 + 1} into two sub-partitions: if we replace the summand 2 in the element {2 + 1} by its *"partition-a"* we get {{2}+ 1} and if we replace the summand 2 in the element {2 + 1} by its *"partition -b"* we get {{1 + 1} + 1}. As this results from a sub-partitioning of the original partition {2 + 1}, we consider the element {{1 + 1} + 1} to be different from the element {1 + 1 + 1}, for the reasoning explained above." (Klein)

Continuing Moshe Klein's approach to second-level partitions we get:

Theorem 3 The number of sub-partitions of n, sp_n, satisfies the following recurrence relation

$$sp_n = 1 + \sum \prod_{j=1}^{m} \binom{sp_{k_j} + s_j - 1}{s_j} \qquad \text{(Moshe Klein)}$$

where the sum is over all partitions

$\lambda = (k_1)^{s_1} (k_2)^{s_2} \ldots (k_m)^{s_m}$ of n such that

$n - 1 \geq k_1 > k_2 > \ldots > k_m \geq 1.$

n	$P(n)$	sp_n	diff
1	1	1	–
2	2	2	–
3	3	4	1
4	5	11	6
5	7	30	23
6	11	96	85

$$\text{diff} = |sp_n| - |P(n)|$$

Hence, for the number 3 with 3 partitions
we get 3 partions plus 1 sub – partition :

$\{3\}, \{\{2\}+1\}, \{\{\mathbf{1+1}\}+\mathbf{1}\}, \{1+1+1\}.$

1.1.5. Second-level partitions for trito-number

In general, the new second-level partitions has to realize the conditions of the type of partition in question. Hence, trito-second level partitions are following the trito-rules, and Mersenne 2-level distinctions are following their Mersenne rules.

So, what are the advantages of a *"second-level partition"* for the understanding of graphematic combinatorics?

As for deutero-numbers which are represented as partitions, second-level differentiation of trito-numbers are directly accessible. Trito-numbers, which are mathematically represented by the Stirling numbers of the second kind, are preserving the order of partitions. Therefore, a deutero-number 5 = [3,1,1] is represented by 3 trito-numbers: [3,1,1], [1,3,1], [1,1,3] or in a different notation, [aaabc], [abbba], [abccc], for n=5, m=3.

deutero (3) : $[1^3]$, $[1^1, 2^2]$, $[1^1,1^1,1^1]$

trito (3) : $[1^3]$, $[1^2, 2^1]$, $[1^1, 2^1, 1^1]$, $[1^1, 2^2]$, $[1^1, 2^1, 3^1]$.

2-trito(3) : $[1^2, 2^1] = [(1^1 1^1), 2^1]$,

$[1^1, 2^2] = [1^1, (2^1, 2^1)]$

trito (5) : $[1^1, 2^3, 3^1]$,

2-trito (5) : $[1^1, (2^2 2^1), 3^1]$, $[1^1, (2^1 2^2), 3^1]$,

$[1^1, (2^1 2^1 2^1), 3^1]$

Deutero(4): $[1^1,1^1,1^1,1^1]$, ...

Second-level Stirling numbers

A trito-2-partition is counted by the 2-partition of number *m* added by the permutation of detero(m, n):

m = 3, n = 2:

2-deutero(3): $pf_n(3) = 4$,

perm(3,1), perm(3,3) = 0

perm(3, 2) - 1 = 2

2-trito(3) = 4 + 2 = 6

perm(3,2) = (1,1,2), (1,2,1), (1,2,2);

perm(3, 1) = (1,1,1)

perm(3,3) = (3).

2-deutero(3): {3}, {{2}+1}, {{1+1}+1}, {1+1+1}

2-trito(3) : {3},

{{2}+1}, {1+{2}},

{{1+1}+1},{1 +{{1+1}},

{1+1+1}.

Sn2(4): 1,7,6,1.

2-Sn(4,4) and 2-Sn(4,1) have no second-level partitions.

Second-level partitions occur for 2-Sn2(4, 2) and 2-Sn2(4, 3).

Second-level types of addition and reflectional order of multi-level partitions

It should be reflected that a first-level and a second-level operation of addition based on first-order and second-order distinctions has to be differentiated. In a term like "{1 +{{1+1}}" there are 2 different kinds of additions involved: a first-level addition "$+_1$" and a second-level addition "$+_2$", hence the term is: "{1 $+_1${{1$+_2$1}}". Further more, a comparison of both levels of differentiation involves a *third* kind of additions. This third level might function as the level where the differences of the reflections are homogenized to the level 3 as level zero.

This distinction is eliminated by M. Klein with:

> "Either{{1+1}+{1+{2}}} or {{1+1}+{1+{1+1}}}. As long as there are more properties to test, we may still find that finally all balls are distinguishable. [...] We see every natural number as a superposition of its partitions. We take this research one step further and go beyond the partitions by using recursion." (M. Klein, Hilbert's Sixth Problem)

From the standpoint of a *theory of reflection* (Gunther) which takes into account the *difference* of the levels of reflection (distinction), it would be more interesting to study the reflectional properties of the *process* of "nivilation" of partitions towards a conglomerate of indistinguishable elements instead of the results of the elimination of the difference alone. Hence, the case of "{1 $+_1${1$+_2$1}}" implies two levels. Numbers like,

5 = {{1+1}+{1+{1+1}} becomes
{{1$+_2$1}$+_1${1$+_2${1$+_3$1}} and $Ord_{refl}(5) = (2,1,2,3)$ or
6 = {1 + 2 + 3} = {1 $+_1${1$+_2$1} $+_1${1 $+_2${2}}} = {1 $+_1${1$+_2$1} $+_1${1 $+_2${1 $+_3$1}}},
involves 3 levels of distinctions. And the order of the distinctions is ($+_1$,$+_2$,$+_1$,$+_2$,$+_3$) for number 6, i.e. $Ord_{refl}(6) = (1,2,1,2,3)$.
A multi-level partition of a number might have different results.

Reflectional property *Ord*$_{refl}$*(m)*

This intriguing property of numbers *Ord*$_{refl}$*(m)* is introducing a reflectional level of differentiation of numbers which is concerned only with the reflectional and differential character of numbers and is abandoning any references to counting or numerating objectivistic or mentalistic objects as legitimation and aim of the process of counting with numbers.

How are Klein's *"Number of sub-partitions"* related to the reflectional order of a number by $Ord_{refl}(m)$?

With that, an new step towards Gunther's project of "*philosophical*" numbers might be encouraged.

Kenogrammatic numbers, like trito-, deutero- and proto-numbers are based on kenogrammatic abstractions from the identity of signs. This just introduced *reflectional abstraction* is based on the reflectional levels of second-level partitions of ordinary natural numbers. That is, it counts the reflectional levels needed to sublimate (aufheben) all numerical differences of a natural number in a multi-level partition. As the *"Stirling Turn"* shows, kenogrammatic numbers, like proto-, deutero- and trito-numbers are based on *partitions* and *distinctions* and not on the action of counting objects. Hence, the are at least two different levels to deal with partitions, the first-level and the second-level partition as constitutions of kenogrammatic numbers.
In *"Number and Logos"*, Gunther has shown the mediating character of kenograms as mediating between numbers and concepts (logos).

Natural numbers in their linear order are the basic structure for the paradigm of Western mathematics. Distinction in the paradigm is reduced to a comparison of numbers as being directly comparable (equal, bigger smaller). No produced number in this framework is able to realize a differentiation onto itself.

An involvement of the reflectional order of a number invites, at first, to distinguish between *balanced*, *under*- and *over*-balanced reflectional configurations.
A *balanced* reflectional order is given if all the reflectional levels are of the same degree. Obviously, *under-balanced* order is given if the possible levels are not all implied. And an *over-balanced* situation is given if there are higher orders of distinctions involved as

the complexity of the isolated number on focus is able to realize. Such numbers are members of more complex numerical constellations where the neighbor number of higher reflectional order is intervening with the number under consideration. There might be much more interesting distinctions to be found.

Under-balanced situation:

5 = {{1+1}+{1+{1+1}} considered as $\{1 +_1\{1+_2 1\} +_1\{1 +_2\{2\}\}\}$, with $Ord_{rfl}(5)$ = (1,2,1,2)

5 = {{1+1}+{1+{1+1}} considered as {1+1+1+1+1}.

Balanced situation:

5 = {{1+1}+{1+{1+1}} considered as $\{1 +_1\{1+_2 1\} +_1\{1 +_2\{1 +_3 1\}\}\}$, with $Ord_{rfl}(5)$ = (1,2,1,2, 3)

5 = {{1+1}+{1+{1+1}} considered as $\{\{1+_3 1\}+_3\{1+_3\{1+_3 1\}\}$,

Over-balanced situation:

5 = {{1+1}+{1+{1+1}} considered as $\{1 +_4\{1+_2 1\} +_1\{1 +_2\{1 +_3 1\}\}\}$, with $Ord_{rfl}(5)$ = (4,2,1,2,3)

Therefore, additionally to the levels of partitions of a natural number, its reflectional order is to be considered as a further quality of numbers to towards the "natural" number's final philosophical elucidation.

> **Gunther's monomorphies**
>
> "In order to show the method in some detail we introduce two new concepts which we may call "*monomorphy*" and "*kenogrammatic equivalence*." A monomorphy is the set of all iterations of an individual kenogram. The boundary case of such monomorphy is a single kenogram.
> It is irrelevant whether a monomorphy is interrupted by one or more kenograms of different shape. It is only for the purpose of a simpler demonstration that we are going to write our monomorphies below in uninterrupted sequences."
> http://www.vordenker.de/ggphilosophy/gg_logic_structure.pdf

1.1.6. Mersenne calculi

Mersenne (occurence): partitions

For Mersenne calculi the interest is not in the occurence of the elements on an identity level. Two homogeneous occurrences of a string or number like [aa] and [bb] are considered as equal, (aa) $=_{Mers}$ (bb), while the differences of permutations of the occurrences of a string or number are of interest, and are therefore differentiated and distinguished, hence (ab)≠(ba). This points to the fact that Mersenne terms are not sequences of atomic signs and also no heaps but partitions of sequences of distinctions.

Two groups might be equal in complexity and complication, i.e. card(ind(2,2)) = card(occur(2,2)) = 3 but distinguished by their criteria of membership.
One membership is asking for different properties, the other for different constellations.
Different properties are defined by the indicational case: {aa, ab, bb}, while different constellations of the same properties are defined by the Mersenne constellations: {aa, ab, ba}.

For the Mersenne case, one might insist that two couples of the same property are the same. It doesn't matter if two different couples (aa) and (bb) are of the same property, i.e. (aa)$=_{Mers}$(bb). One occurrence of such a homogeneous couple is enough to be accepted. What counts are the differences of a couple, i.e. (ab) and (ba), i.e. (ab) $\neq_{Mers}$(ba).

In dual contrast, for the indicational case, one might insist that two couples of the same property are strictly different. It matters if two different couples (aa) and (bb) are of the same property or not. This defines the distinction. They have to be accepted as two different occurrences. What doesn't count as a distinction are the differences of a couple, i.e. (ab) and (ba). Both are identified and accepted as the same, (ab) $=_{Ind}$(ba).

Following the new investigations into second-level partitions by Moshe Klein it has to be questioned how those concepts are applicable for Mersenne differentiations too.

Generalization
Mersenne calculi might be generalized to: Mers(m, n) = m^n - (m - 1).
Example
Mers(3, 2) = 3^2 - (3-1) = 7
Mers(3, 2) = {aa, ab, ac, ba, bc, ca, cb}, with aa = bb = cc.

Interactions and consequences

Interaction

How are the two different semiotic types, Mersenne and indication, interacting?

It shouldn't be a serious problem to realize an interaction between an indicational and a Mersenne system if such an interaction is aware of its complementary properties. Hence, an implementation of both as interacting systems could enable new emerging properties of system dynamics on a fundamental semiotic level.
What has to be developed is thus a double calculus of interaction between Mersenne and Indicational formal systems.

Even on such a trivial level of reflection it is more than clear that the difference between Mersenne and Spencer Brown are definitive. On the same level of reflection it is more than clear too, that not only indication but also Mersenne, is incompatible with the 'semiotics' of two-valued logic or Boolean algebra. All three 'semiotics' are occupying different independent levels in the graphematic system of symbolization.

Logic(2,2)=(aa,ab,ba,bb).
trito(2, 2) = {aa, ab}.

[trito(2,2) is coinceding with deutro(2,2) and proto(2, 2)]

logic
↙ ↘
Mersenne Indication

Translation

$$\text{Mersenne} \quad : \text{model}(M) = \{(aa), (ab), (ba)\}$$

$$\text{Spencer-Brown}: \text{model}(B) = \{(aa), (ab), (bb)\}.$$

$$\text{model}(M): \quad (aa) \quad (ab) \quad (ba)$$

↕ ↙ ↘ ↖ ↗

$\text{model}(B): \quad (aa) \ (bb) \quad (ab)$

Consequences for system theory

There is a lot of argumentation for a indicational system theory in the sense of Spencer Brown and Niklas Luhmann.

System theoreticians are not considering the graphematic questions but are mesmerized by the recursive circularity of the re-entry form.

Ind: non(aa) = (bb) : the negation of the system is the enviroment,
non(ab) = (ab) : the negation of the system-environment is the system-environment.

Mers: non(aa) = (aa) : the negation of the system (environment) is the system (environment),
non(ab) = (ba) : the negation of the system-environment is the environment=system.

Blending and de-sedimentation

The intriguing situation of GSB's calculus of indication (CI) that is function as the hidden obstacle to its understanding is the fact of multiple coincidences with other calculi. For the basic beginnings of the CI the coincidence is given by the fact that the CI and the beginnings (axioms) Boolean algebra and semiotics are covering and blinding each other. A further and new coincidence appears with the Mersenne calculus that is understood as complementary to the CI. Further more, there is, at least for the beginnings, a close similarity to the trito- and deutero-structure of kenogrammatics.

Without a de-sedimentation and translocation of the different structures, an understanding of the CI (and the others too)is in fact impossible and leads to the well known misunderstandings, misconceptions and blinded defensive propaganda.

1.1.8. Paradoxes, reentry and recursivity

Reentry

What would a formal system theoretic application of Mersenne calculi look like?

Re-entry forms are not a privilege of the calculus of indication. Reentry for the calculus of indication is well studied. It runs similar like all other constructions too: a fixed point construction is achieved with the logical property of a contradiction, i.e. f(f) = f. That is, (aa) ≠ (bb) holds semiotically for the construction of the logical contradiction. This paradoxical situation then is conceptually 'resolved' with distributions in time or in space or both.

For Mersenne systems with <aa> = <bb> there is no contradiction produced by self-application. A new kind of conflicts appears in a construction which claims <ab> = <ba>. This contradicts the 'axioms' of Mersenne systems in a complementary sense as the claim (aa) = (bb) contradicts the 'axioms' of the indicational system.

The term "(aa) = (bb)" is easily replaced by the Russell paradox "RR = Not(RR)". Obviously a statement like "(aa) = (ab)" is even a stronger contradiction and not easily to deduce if at all.

> **Kauffman**
> "The paradox occurs when we ask whether R can be a member of R.
> For if
>
> Rx = Not(xx)
>
> then, *substituting* R for x, we have
>
> **RR = Not(RR).**
>
> R is a member of R exactly when R is not a member of R." (L. Kauffman)

One trial more:

> "Suppose we have an operator or function F, and we define a new operator g by
>
> gx = F(xx).
>
> The operator g duplicates x, and applies F to the duplicate xx. Substituting g for x we have
>
> **gg = F(gg).**
>
> The operator F now has a fixed point gg, and we see that F(gg) is self-referential in that it "talks" about itself. This pattern is called the Church-Curry Fixed Point Theorem. The Fixed Point Theorem and Gödel's Theorem are but two sides to the same coin." (Kauffman, The Small Machine, p. 113)

Some Mersenne constellations

(1) No paradox occurs in a Mersenne constellation when we ask

whether R can be a member of R.
For if

$$Rx = Not(xx)$$

then, *substituting* R for x, we have

$$\mathbf{RR = Not(RR).}$$

R is a member of R exactly when R is not a member of R. But "RR = Not(RR)" is not a contradiction in Mersenne because the axiom "(aa) $=_{Mers}$ (bb)" holds.

Hence,

If $(aa) =_{Mers} (bb) \in$ MERS then $non(aa) =_{Mers} (aa) \in$ MERS

A logical model to the Mersenne constellation, "non(aa) $=_{Mers}$ (aa)", might be found in an application of *paraconsistent* logics.

(2) Suppose we have a Mersenne operator or a function F, and we define a new operator *g* by

$$gx = F(xx).$$

The operator g *replicates* x as xx, and applies F to the replicate xx. Substituting *g* for *x* we have

$$\mathbf{gg = F(gg).}$$

The operator F now has a fixed point gg, and we see that F(gg) is self-referential in that it "talks" to nobody. This pattern is called the Mersenne non-Church-Curry Fixed Point Theorem.

(3) A new kind of paradox occurs in a Mersenne constellation when we ask whether R can be a member of R.
For if

$$Rxy = perm(xy)$$

then, *substituting* R for xy, we have

$$\mathbf{RR = perm(RR).}$$

R is a member of R exactly when R is not a member of R. But "RR = perm(RR)" is a contradiction in Mersenne because the axiom "(ab) $\neq_{Mers}$ (ba)".

Hence:

If $(ab) =_{Mers} (ba) \notin$ MERS then $perm(ab) =_{Mers} (ab) \notin$ MERS

Slogan: *Contradiction in Mersenne is identity of permutation.*

In contrast, for the *calculus of indication*:

If $((ab) =_{Ind} (ba)) \in$ IND then $(perm(ab) =_{Ind} (ab)) \in$ IND.
But:
If $((aa) =_{Ind} (bb)) \notin$ IND then $(neg(aa) =_{Ind} (aa)) \notin$ IND.

Slogan: *Contradiction in the calculus of indication is identity of negation (difference).*

(4) Suppose we have a Mersenne operator or a function F, and we define a new operator *g* by

gx = F(xy).

The operator g *replicates* x as xy, and applies F to the replicate xy. Substituting *g* for *xy* we have

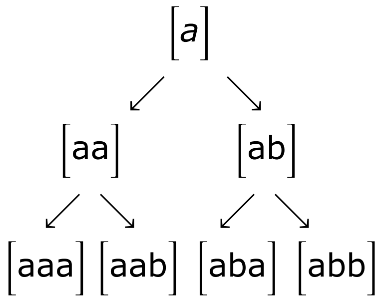
gh = F(gg).

But (ab) ≠ (aa) in Mersenne.

Retrograde recursivity?

Both types of repetition, Indication and Mersenne, don't have any retrograde feautures like retrograde recursivity in the sense of trito-systems. An aspect of retrogradeness might occur for the Browninan case with the equivalence of permutative terms and for the Mersenne case with the equivalence of homogeneous terms. Both aspects which are not anymore purely identitive, with $(aa) =_{Mers}(bb)$ and $(ab) =_{Ind}(ba)$, might be connected to the kenomic levels of deutero- and trito-structures and with that to aspects of retro-recursivity.

For just two kenograms for trito-systems, retrograde recursivity is very restricted.



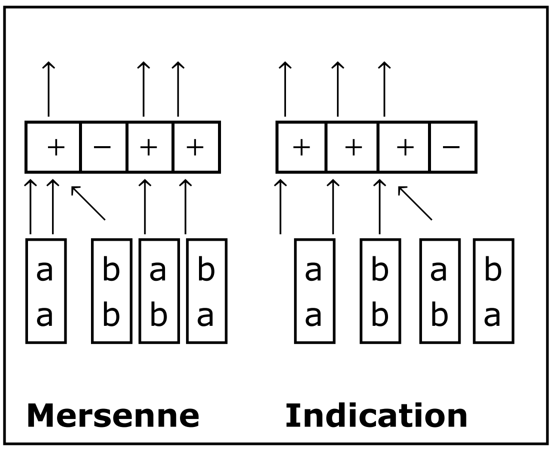
Retrogradness is hidden and covered by the minimal complexity of two kenograms. Nevertheless, the succession of [a] appears as iteration and as accretion determined by the preceding beginning [a]. This hints to the hidden retrogradness of the general case.
http://memristors.memristics.com/MorphoReflection/Morphogrammatics%20of%20Reflection.html

1.1.9. Possible applications

Classification systems, pattern recognition, automata theory, data types, complexity reduction, war propaganda.

Brownian mathematical noise-level reductor
"The consequences are of this arithmetical availability are sweeping." [...] Principia mathematica*. Allowing some 1500 symbols to the page, this represents a reduction of the mathematical noise-level by a factor of more than 40000."* (GSB, p. 117)

What's the corresponding complementary mathematical Mersenne noise-reduction?



Computation system for the simulation of the cerebral cortex, Bernhard Mitterauer
http://www.google.de/patents?id=J0EdAAAAEBAJ

Rhetorics in recent war propaganda
The left always was handicapped by the lack of appropriate analytical weapons.

> "The joint op-ed by Barack Obama, Nicolas Sarkozy and David Cameron published simultaneously in *The Times*, *The International Herald Tribune*, *Al-Hayat* and *Le Figaro* on 15 April 2011 stated that
>
> *"[Our goal]is not to remove Gaddafi by force. But it is impossible to imagine a future for Libya with Gaddafi in power."*
>
> This statement brings together two *contradictory* notions: Gaddafi is not the target of the military campaign against Libya, yet it is unthinkable that he should remain in power. Such a stance perfectly ties in with the *oxymoron* arising from the humanitarian war: the merging of *two mutually exclusive* terms. This procedure has the effect of *reversing* the meaning of each concept. War is peace and peace is war." (Jean-Claude Paye, Tülay Umay)
>
> http://www.voltairenet.org/Waging-war-in-the-name-of-the

Might it be possible that the mentioned war propaganda simply follows "unconsciously" the tricky features of the '*paraconsistency*' of Mersenne calculi? And on the other hand, the Brownian calculus which is accepting truth and false, albeit it is claimed that the truth of the calculus is "deeper than truth", but rejects the difference of the couples "truth/false" and "false/truth". Even if Indication is declared as *"deeper than truth",* the calculus of indication is strictly separating its terms from 'truth' and 'false'. In contrast, Mersenne calculi are playing with the perfect coincidence of 'truth' and false' at once.
Now, to fool people, the media and the almighty UN, and to be able - at once - to legitimate your statements (actions), you simply have to mix the logical systems of your argumentation according to the demands of the situation and jump between the gaps of your logical systems.

1.2. Calculus of mutual blending

1.2.1. Blending of Brownian and Mersenne semiotics

Indication calculi

Constellations

semiotics : aa ab ba bb

Mersenne : aa ab ba ..

Indication : aa ab .. bb

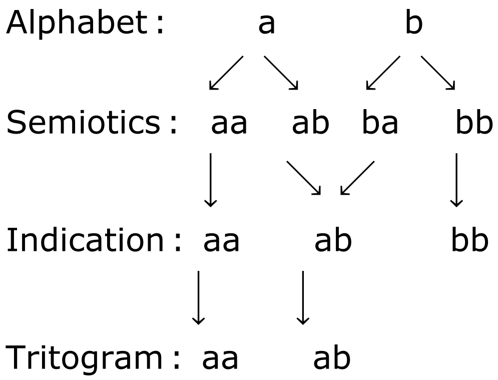
trito : aa ab .. ..

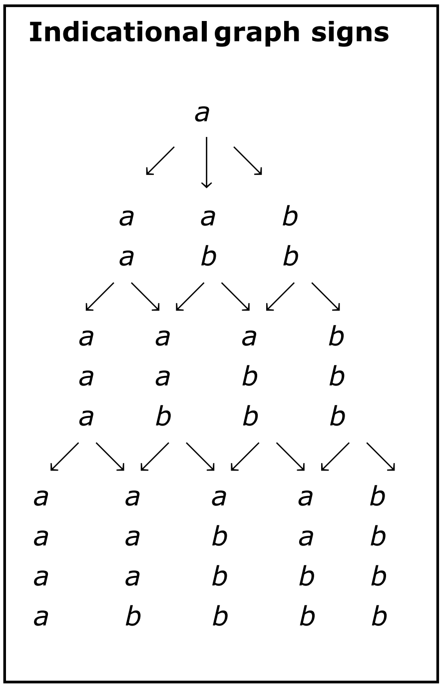
Logic

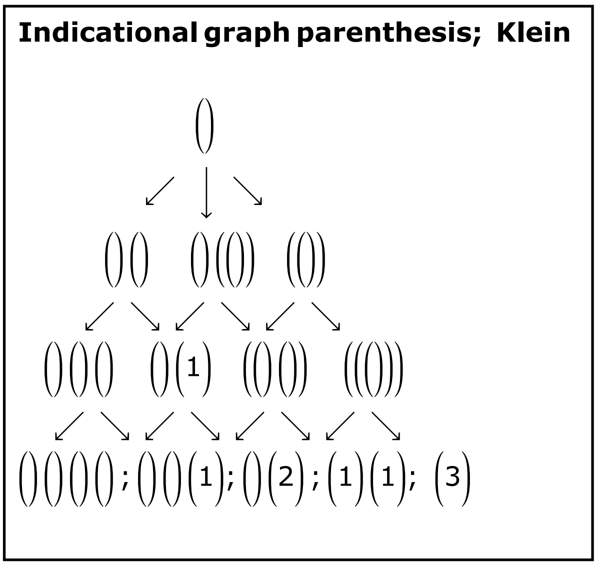
Indicational space $= \{aa,\ ab,\ bb\}$ corresponds to $\{tt,\ ft \equiv tf,\ ff\}$,

Mersenne space $= \{aa,\ ab,\ ba\}$ corresponds to $\{tt \equiv ff,\ tf,\ ft\}$.

Indication graph




Indicational graph signs


Indicational graph parenthesis; Klein

Indicational graph numeric

$$\{1^1\}$$

↙ ↓ ↘

$$\{1^2\}, \{1^1 2^1\}, \{2^2\}$$

↙ ↘ ↙ ↘ ↙ ↘

$$\{1^3\}, \{1^2 2^1\}, \{1^1 2^2\}, \{2^3\}$$

↙ ↘ ↙ ↘ ↙ ↘ ↙ ↘

$$\{1^4\}, \{1^3 2^1\}, \{1^2 2^2\}, \{1^1 2^3\}, \{2^4\}$$

Numeric indicational rules

$$R1: \quad \Rightarrow \{1^1\}$$

$$R2: \{1^1\} \Rightarrow \{1^2\} \,|\, \{1^1 2^1\} \,|\, \{2^2\}$$

$$R3.1: \{1^n\} \Rightarrow \{1^{n+1}\} \,|\, \{1^n 2^1\}$$

$$R3.2: \{2^n\} \Rightarrow \{2^{n+1}\} \,|\, \{1^1 2^n\}$$

$$R3.3: \{1^n 2^n\} \Rightarrow \{1^{n+1} 2^n\} \,|\, \{1^n 2^{n+1}\}$$

Context-free language with the grammar: $S \longrightarrow SS|(S)|\lambda$ is generating the proper paranthesis for formal languages. Brown' s calculus of indication is abstracted from this context-free grammar by the abstraction: $(())() =_{Ind} ()(())$.

Indication graph $(2,\ 3)$

a   b   : Alphabet

↙ ↘ ↙ ↘

(aa) (ab) (ba) (bb)   : Semiotics$(2, 2)$

↙ ↘ ↙ ↘ ↙ ↘ ↙ ↘

aaa aab aba abb baa bab bba bbb   : Semiotics$(2, 3)$

↓ ↘ ↙ ↘ ↙ ↓

{aaa} {aab} {abb} {bbb} : Indication(2, 3)

↓ ↙ ↘ ↓

[aaa] [aab] [aba] [abb] : Tritogrammatics(2, 3)

↓ ↘ ↓ ↙

aaa aab : Deuterogrammatics(2, 3)

Mersenne calculi

Mersenne graph

a b : Alphabet

↙ ↘ ↙ ↘

aa ab ba bb : Semiotics

↓ ↓ ↓

aa ab ba : Mersenne

↓ ↘ ↙

aa ab : Tritogrammatics

Semiotic – Mersenne – Trito – Deutero graph (2, 3)

a b : Alphabet

↙ ↘ ↙ ↘

aa ab ba bb : Semiotics(2, 2)

↙ ↘ ↙ ↘ ↙ ↘ ↙ ↘

aaa aab aba abb baa bab bba bbb : Semiotics(2, 3)

↓ ↓ ↓ ↓ ↓ ↓ ↓

aaa aab aba abb baa bab bba : Mersenne(2, 3)

↓ ↓ ↓ ↓

aaa aab aba abb : Tritogrammatics(2, 3)

↓ ↘ ↓ ↙

aaa aab : Deuterogrammatics(2, 3)

Mersenne graph

a ↙ ↘ b : Alphabet, semiotic

aa ab ba bb : Semiotics, production

↓ ↓ ↓

aa ab ba : Mersenne$(2, 2)$, abstraction

↙ ↓ ↘ ↙ ↘ ↙ ↘

aaa bba aab aba abb baa bab : Mersenne$(2, 3)$, continuation

↓ ↘ ↙ ↓ ↓

aaa aab aba abb : Tritogrammatics$(2, 3)$, abstraction

↓ ↘ ↓ ↙

aaa aab : Deuterogrammatics$(2, 3)$, abstraction

Comment

Because $< aa > =_{Mers} < bb >$,
the prolongation of $< aa >$ for the Mersenne tree has to consider both cases :
$< aa > \rightarrow \{< aaa >, < aab >\}$ and $< aa > \rightarrow < bba >$.
The prolongation $< aa > \rightarrow <$
bbb $>$ is equivalent to the prolongation $< aa > \rightarrow < aaa >$,
because $< aa > =_{Mers} < bb >$, and therefore $< bbb > =_{Mers} < aaa >$.
This observation doesn't hold for compositions $< abb >$,
i.e. the part $< bb >$ of $< abb >$ is not equal $< aa >$.

Following the rules of Mersenne calculi the following tree is generated.
Again the rules have to accept the the strategies of accepting differences,
i.e. $[ab] \neq [ba]$ and abstracting from sameness, i.e. $[aa] = [bb]$.
The Mersenne specific step is given with the transition from $[a]$ to $[ba]$.
Because $[a]_{Mers}[b]$ the prolongation of $[a]$
has to accept this equality with the positionng of $[b -]$.
The full prolongation of $[a]$ then is iteratively $[ba]$.
An accretive prolongation
from $[a]$ to $[bb]$ would be erased by tha fact of $[aa] = [bb]$.
The final boundary of the Mersenne calculus is given by the fact of the
$2-$element semiotic alphabet $\{a, b\}$. A prolongation to,

say by [bc] is excluded by definition.
This principle of continuation is repeated for the following steps of recursion.

Mersenne tree : $2^n - 1$

a (1)

↙ ↓ ↘

a a **b**
a b a (3)

↙ ↓ ↘ ↙ ↘ ↙ ↘

a **b** a a a b b
a **b** a b b a a (7)
a a b a b a b

↙ ↘

a a **b** b b a a a a a a b b b b
a a **b** b b a a b b b b a a a a (15)
a a **b** a a b b a a b b a a b b
a b a a b a b a b a b a b a b

Arithmetic notation for Mersenne tree

Numeric Mersenne tree : $2^n - 1$

$\langle 1^3 \rangle$
$\langle \mathbf{2}^{\mathbf{2}} 1^1 \rangle$
$\boldsymbol{\langle 1^2 \rangle}$ $\langle 1^2 2^1 \rangle$

$\langle 1^1 2^1 1^1 \rangle$
$\langle 1^1 \rangle$ $\langle 1^1 2^1 \rangle$ $\langle 1^1 2^2 \rangle$

$\langle 2^1 1^2 \rangle$
$\langle 2^2 \rangle$ $\langle 2^1 1^1 2^1 \rangle$

Numeric Mersenne rules

$R1: \quad \Rightarrow \{1^1\}$

$R2.1: \{1^n\} \quad \Rightarrow \{1^{n+1}\} \,|\, \{2^n 1^1\} \,|\, \{1^n 2^1\}$

$R2.2: \{1^n 2^n\} \Rightarrow \{1^n 2^n 1^1\} \,|\, \{1^n 2^{n+1}\}$

$R2.3: \{2^n 1^n\} \Rightarrow \{2^n 1^{n+1}\} \,|\, \{2^n 1^n 2^1\}$

Mersenne numeric recursion

start $\quad : R1: \Rightarrow \{1^1\}$

Mersenne

$= \quad \neq$ $\nwarrow$ $\quad$: decision $\{\text{yes, no}\}$

$\swarrow \quad \searrow \; | \nu$ $\quad$: operation $\{R2.1,\ R2.2,\ R2.3\}$

stop $\quad$ [Mers] $\quad$: iteration $\quad n \Rightarrow n+1$

Mersenne 1 – partition 2 – partition

$< aaa > : \ < 1^3 >$

$< bba > : \ < 2^2,\ 1^1 > \quad \Rightarrow \ < \{2^1 2^1\},\ 1^1 >$

$< aab > : \ < 1^2,\ 2^1 > \quad \Rightarrow \ < \{1^1 1^1\},\ 2^1 >$

$< aba > : \ < 1^1 2^1 1^1 > \quad \Rightarrow \ < 1^1 2^1 1^1 >$

$< abb > : \ < 1^1 2^2 > \quad \Rightarrow \ < 1^1 \{2^1 2^1\} >$

$< baa > : \ < 2^1 1^2 > \quad \Rightarrow \ < 2^1 \{1^1 1^1\} >$

$< bab > : \ < 2^1 1^1 2^1 > \quad \Rightarrow \ < 2^1 1^1 2^1 > .$

First – and second – level partition tree

$< 1^2 >$

$\swarrow \quad \downarrow \quad \searrow$

$< 1^{3} > \quad < \mathbf{2}^{\mathbf{2}}\, 1^{1} > \qquad < 1^{2}\, 2^{1} >$

$| \qquad | < \left\{ 2^{1}\, 2^{1} \right\} 1^{1} > \; | < \left\{ 1^{1}\, 1^{1} \right\} 2^{1} >$

$\swarrow \downarrow \searrow \quad \downarrow \quad \downarrow \qquad \downarrow \quad \downarrow$

Mersenne prolongations

$\text{prolong}_{\text{Mers}} < 1^{1} > \quad = < 1^{2} >,\ < 1^{1}\, 2^{1} >,\ < 2^{1}\, 1^{1} >,$
with $< 1^{1} > = < 2^{1} >$

$\text{prolong}_{\text{Mers}} < 1^{2} > \quad = < 1^{2+1} >,\ < 2^{2}\, 1^{1} >,\ < 1^{2}\, 2^{1} >,$
with $< 1^{2} > = < 2^{2} >,$
$\text{prolong}_{\text{Mers}} < 1^{1}\, 2^{1} > \quad = < 1^{1}\, 2^{1}\, 1^{1} >,\ < 1^{1}\, 2^{1+1} >$
$\text{prolong}_{\text{Mers}} < 2^{1}\, 1^{1} > \quad = < 2^{1}\, 1^{1+1} >,\ < 2^{1}\, 1^{1}\, 2^{1} >.$

$\text{prolong}_{\text{Mers}} < 1^{3} > \quad = < 1^{3+1} >,\ < 1^{3}\, 2^{1} >,\ < 2^{3}\, 1^{1} >,$
with $< 1^{3} > = < 2^{3} >$
$\text{prolong}_{\text{Mers}} < 2^{2},\ 1^{1} > \quad = < 2^{2}\, 1^{1+1} >,\ < 2^{2}\, 1^{1}\, 2^{1} >,$
$\text{prolong}_{\text{Mers}} < 1^{2},\ 2^{1} > \quad = < 1^{2},\ 2^{1}\, 1^{1} >,\ < 1^{2},\ 2^{1+1} >,$
$\text{prolong}_{\text{Mers}} < 1^{1}\, 2^{1}\, 1^{1} > \; = < 1^{1}\, 2^{1}\, 1^{1+1} >,\ < 1^{1}\, 2^{1}\, 1^{1}\, 2^{1} >,$
$\text{prolong}_{\text{Mers}} < 1^{1}\, 2^{2} > \quad = < 1^{1}\, 2^{2}\, 1^{1} >,\ < 1^{1}\, 2^{2+1} >$
$\text{prolong}_{\text{Mers}} < 2^{1}\, 1^{2} > \quad = < 2^{1}\, 1^{2+1} >,\ < 2^{1}\, 1^{2}\, 2^{1} >$
$\text{prolong}_{\text{Mers}} < 2^{1}\, 1^{1}\, 2^{1} > \; = < 2^{1}\, 1^{1}\, 2^{1}\, 1^{1} >,\ < 2^{1}\, 1^{1}\, 2^{1+1} >.$

Mersenne recursivity

$< 1^{1} > \in \text{Sem} \Rightarrow \; < 1^{1} > \in \text{Mers}$

$< 1^{1} > \in \text{Mers} \Rightarrow \; \text{prolong}_{\text{Mers}}\left(< 1^{1} > \right) \in \text{Mers}$

Rule system for Mersenne

$\text{alph}\left(\text{Mers}\right) = \left\{ a,\ b \right\}$

$\text{Rule1}: \ \Rightarrow a$

$$\text{Rule2}: \ \text{Mers}\left(n\right) \Rightarrow \left(\begin{array}{l} \text{R2.1}: \text{Mers}\left(n\right) \wedge a,\ n = \left(\left\{ a_{i} \ldots b_{j} \right\} \right. \\ \text{R2.2}: \ \text{Mers}\left(n\right) \wedge b,\ n = \left(\left\{ a_{i} \ldots b_{j} \right\} \right. \\ \text{R2.3}: \ \text{Mers} - x \wedge a,\ n = \left(\left\{ x_{i} = x_{j} \right\} \right) \end{array} \right)$$

Example for Mers$\left(3\right)$:

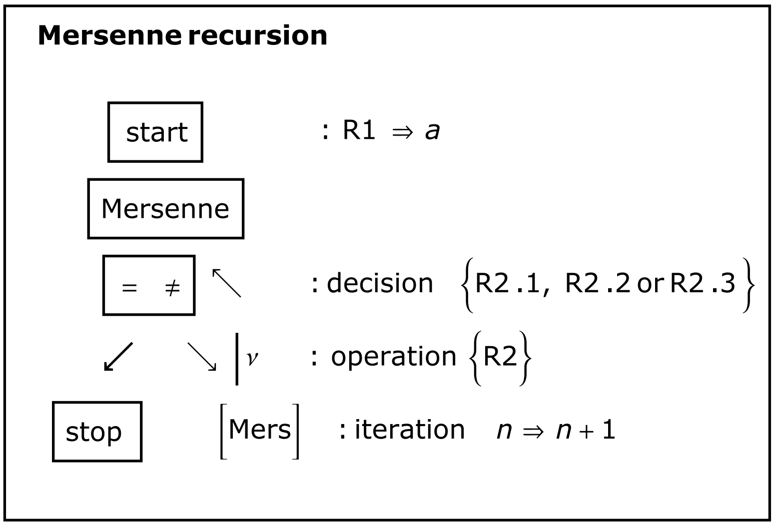
$$
\begin{aligned}
&\text{Rule1}: \ \Rightarrow a\\
&\text{Rule2}: \ \text{R2.1}: a \Rightarrow \text{aa}\\
&\qquad\quad \text{R2.2}: a \Rightarrow \text{ab}\\
&\qquad\quad \text{R2.3}: a \Rightarrow \text{ba.}
\end{aligned}
$$

$\text{Rule2}(\text{Rule2}):$

$$
\begin{aligned}
&\text{aa} \Rightarrow \text{aaa, aab} &&: \text{R2.1, R2.2}\\
&\text{aa} \Rightarrow \text{bba} &&: \text{R2.3}\\
&\text{ab} \Rightarrow \text{aba, abb} &&: \text{R2.1, R2.2}\\
&\text{ba} \Rightarrow \text{baa, bab} &&: \text{R2.3}
\end{aligned}
$$

$\text{Rule2}(\text{Rule2}(\text{Rule2})):$

$$
\begin{aligned}
&\text{aaa} \Rightarrow \text{aaaa, aaab}\\
&\text{aaa} \Rightarrow \text{bbba} \qquad : \text{R2.3}\\
&\text{aab} \Rightarrow \text{aaba, aabb}\\
&\text{bba} \Rightarrow \text{bbaa, bbab}\\
&\text{aba} \Rightarrow \text{abaa, abab}\\
&\text{abb} \Rightarrow \text{abba, abbb}\\
&\text{baa} \Rightarrow \text{baaa, baab}\\
&\text{bab} \Rightarrow \text{baba, babb.}
\end{aligned}
$$



1.2.2. Recursive arithmetics for Mersenne and Brownian calculi

Recursive arithmetics for Mersenne calculi

Mersenne calculi are semiotic calculi combined with a specific abstraction over homogeneous sign sequences. Therefore, the machinery of recursive word arithmetic (Goodstein, Vuckovic, Pogorzelski) is directly applicable. Based on the recursive Mersenne successor function, operations like addition, inversion, multiplication, etc.

are directly accessible to definition. The same holds for Brown calculi.

In contrast to semiotic and numeric recursivity, i.e. recursivity in the mode of identity, Mersenne and Brown recursivity has to introduce a *normal* form (standard notation) selection from the possible semiotic reperesentations of Mersenne and Brown "strings" or "numbers". Similar to the trito-normal form (tnf) for trito-kenogrammatic operations.

Recursion for Mersenne successor Succ

$a \in \text{Sign} \Rightarrow a \in \text{Mers}$
$x \in \text{Mers}_{hom},\ \text{Succ}(x) \Rightarrow xa,\ xb,\ \overline{x}a \in \text{Mers}$
$x \in \text{Mers}_{het},\ \text{Succ}(x) \Rightarrow xa,\ xb \in \text{Mers}$

Short :

$\text{Succ}(0) = 0 \quad : \text{R1}$
$\text{Succ}(x) = \{x\text{^}a,\ x\text{^}b,\ \overline{x}\text{^}a\} \ : \text{R2.1, R2.2, R2.3}$
$\text{Succ}(x) = \{xa,\ xb\} \quad : \text{R3.1, R3.3}$

$\overline{x} = (x_i \ldots x_j),\ i = j$
mnf(x) : Mersenne normal form of x.

Addition
$\text{Sum}(x,\ o) = x$
$\text{Sum}(x,\ \text{Succ}\, x) = \text{Succ}(\text{Sum}(x,\ y))$

Multiplication
$\text{Prod}(x,\ 0) = x$
$\text{Prod}(x,\ \text{Succ}(y)) = \text{Sum}(x,\ \text{Prod}(x,\ y))$

Examples for Mersenne calculi

Addition Sum

Sum(a, 0) = a
Sum(a, Succ 0) = Succ(Sum(0, a))
= Succ(a) = {aa, ab, ba}. : R2.x

Sum(a, Succ a) = Succ(Sum(a, a))
= Succ(aa, ab, ba) = {aaa, aab, bba; aba, abb; baa, bab}.

Sum(a, Succ aa) = Succ(Sum(a, aa))
= Succ(aaa, aab, bba),
= Succ(aaa) = {aaaa, aaab, bbba}, : R2.x

= Succ(aab) = {aaba, aabb}, : R2.1, R2.2
= Succ(bba) = {bbaa, bbab}. : R2.1. R2.2

Sum(a, Succ aaa) = Sum(a,(aaaa, aaab, bbba)
= {aaaaa, aaaab, bbbba; aaaba, aaabb; bbbaa, bbbab}.

Multiplication Prod
Prod(a, 0) = 0
Prod(a, Succ 0) = Sum(a, Prod(a, 0)) = Sum(a, 0)) = a
= Prod(a, a) = a
Prod(a, Succ a) = Sum(a, Prod(a; aa, ab, ba)) = Sum(a, (aa, ab, ba))
= {aaa, aab, bba; aba, abb; baa, bab}.

Recursive arithmetics for Brown calculi

Recursion for Brown successor Succ

$a \in \text{Sign} \Rightarrow a \in \text{Brown}$
$a \in \text{Brown}_{hom},\ \text{Succ}(a) \Rightarrow aa,\ ab,\ bb \in \text{Brown}$
$x \in \text{Brown}_{hom},\ \text{Succ}(x) \Rightarrow xa,\ xb \in \text{Brown}$
$x \in \text{Brown}_{perm},\ \text{Succ}(x) \Rightarrow \hat{x}a,\ \hat{x}b \in \text{Brown}$

Short :
$\text{Succ}(0) = 0$: R1
$\text{Succ}(a) = \{aa,\ ab,\ bb\}$: R2.1, R2.2, R2.3
$\text{Succ}(x) = \{xa,\ xb\}$: R3.1, R3.2
$\text{Succ}(x) = \{\hat{x}a,\ \hat{x}b\}$: R4.1, R4.2

$\hat{x} = (x_i, x_j),\ i \neq j$
bnf (x) : Brownian normal form of x.

Examples for Brown calculi

Addition Sum
Sum(a, 0) = a
Sum(a, Succ a) = Succ(Sum(a, a))
= Succ(aa, ab, bb) = {aaa, aab; abb; bbb} : R2.x
with {aba, bba} ∉ bnf
Sum(a, Succ aa) = Succ(Sum(a, aa))
= Succ(aaa, aab, bba, bbb)
= {aaaa, aaab, bbba; aaba, aabb; bbaa, bbab; bbbb}.
with {aaba, bbaa, bbab} ∉ bnf
Sum(a, Succ ab) = Succ(Sum(a, aa))
= Succ(aa, ab, bb) = {aaa, aab; abb; bbb}.

Sum(a, Succ bb) = Succ(Sum(a, aa))
= Succ(aa, ab, bb) = {aaa, aab; abb; bbb}.

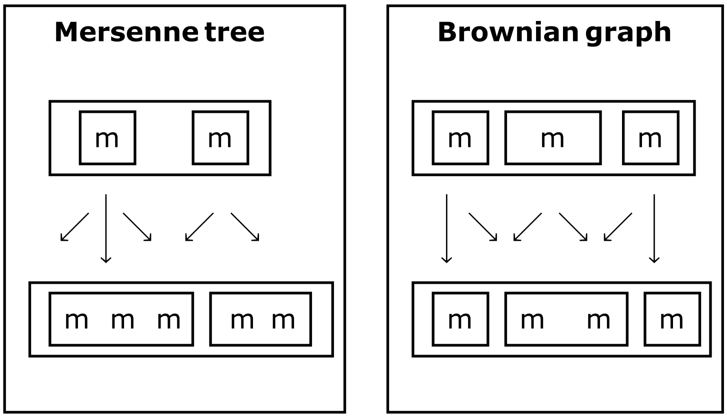
Multiplication Prod

Prod(a, 0) = 0
Prod(a, Succ 0) = Sum(a, Prod(a, 0)) = Sum(a, 0)) = a
= Prod(a, a) = a
Prod(a, Succ a) = Sum(a, Prod(a; aa, ab, bb)) = Sum(a, (aa, ab, bb))
= {aaa, aab; abb; bbb}.

Comparision

Prod(a, Succ a)
Brown: Sum(a, Prod(a; aa, ab, bb)) =
Sum(a, (aa, ab, bb)) =
{aaa, aab; abb; bbb}.
Mersenne: Sum(a, Prod(a; aa, ab, ba)) =
Sum(a, (aa, ab, ba)) =
{aaa, aab, bba; aba, abb; baa, bab}.

1.2.3. Systematic comparison



systems	properties	relation	repetition	□	□
semiotics	non – commutative	identity	iterative	□	□
Brown	commutative	difference	2 – recursive	□	□
Mersenne	non – commutative	similarity	2 – recursive	□	□
trito	non – commutative	bisimilarity	retrograde	□	□
deutero	commutative	□	□	□	□

Indication : **Mersenne :**

$J1 : \{\}\{\} = \{\}$ $M1 : \{\}\{\} = \varnothing$

$J2 : \{\{\}\} = \varnothing$ $M2 : \{\{\}\} = \{\}$.

$$M1: \{\ \}_{1.2}\{\ \}_{1.2} = \varnothing$$

$$M2: \{\{\ \}_{1}\}_{2} = \{\{\ \}_{2}\}_{1} = \{\{\ \}\}_{1.2} = \{\ \}$$

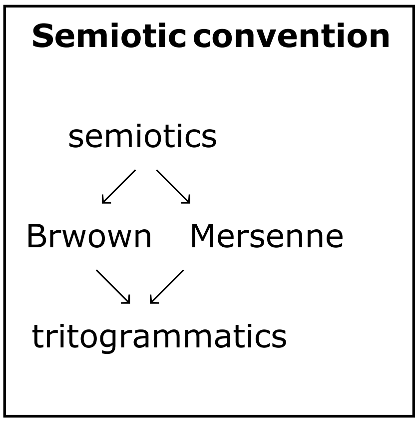
$$J1: \{\ \}_{1.2}\{\ \}_{1.2} = \{\ \}_{1.2} = \{\ \}$$

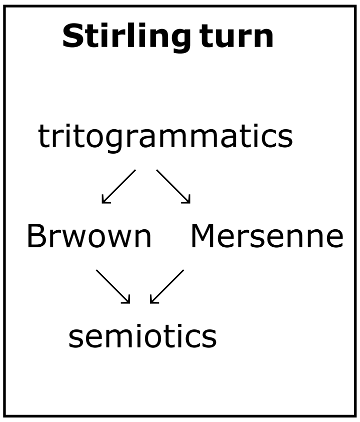
$$J2: \{\{\ \}_{1}\}_{2} = \{\{\ \}_{2}\}_{1} = \varnothing.$$

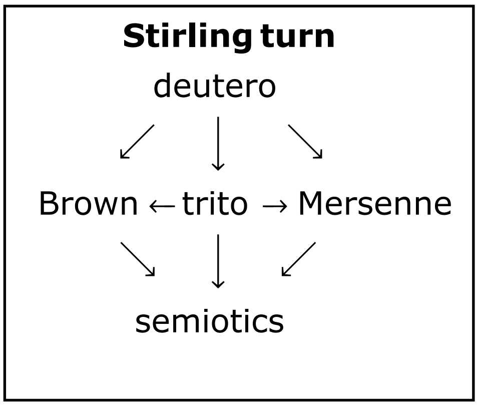
Mersennne and Brown calculi are based on complementary abstractions over a 2-element semiotics. They are 'unified' in their common tritogrammatic system :([aa], [ab]).

(aa) and (bb) are not different from the point-view of Mersenne-differences. Hence, (a)(a)=(b)(b)= ∅, i.e.,
M1: { }{ } = ∅.
But (ab) and (ba) are different, i.e. a(b) is Mersenne-different from b(a), i.e., a(b) = a and b(a) = b, thus by abstraction
M2: {{ }} = { }.

Graphematic system of minimal distinction







Definition

$M = \left(m, M_1, M_2\right)$

$B = \left(b, J_1, J_2\right)$

and $m = \left\{m_1, m_2\right\}$, $b = \left\{b_1, b_2\right\}$.

Theorem

M sim J iff M_1 sim J_2, M_2 sim J_1.

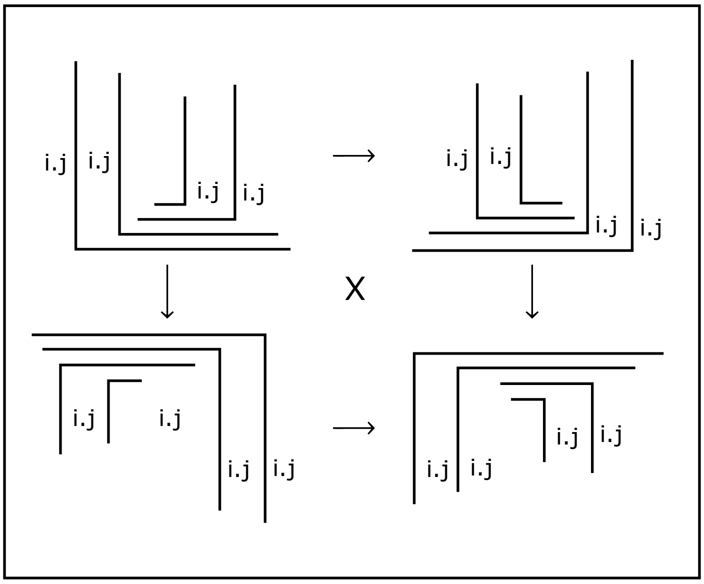
Proof

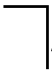
Axioms J and M are taken as operators. Sets of elements are *b* and *m*.

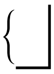
$J_1\left(m_2\right) = b_1,\ J_2\left(m_1\right) = b_2$

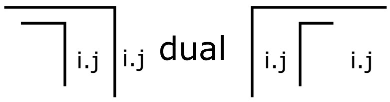
$M_1\left(b_2\right) = m_1,\ M_2\left(b_1\right) = m_2.$

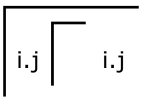
1.2.4. Interaction of Brown and Mersenne



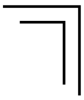
Spencer = { , ∅ }

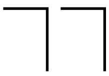
Mersenne = { , ∅ }.

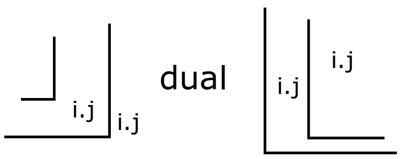
Spencer : 

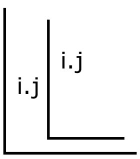
Dual – Spencer : 

 = ∅

∅(∅) = 

 = ∅.

Mersenne : 

Dual – Mersenne : 

Semiotics is a permutation-variant system, and is supporting the rules of negation in 2-valued logic.

Brown and Mersenne systems have different permutation invariant properties. Mersenne is partition-variant, Brown is partition-invariant under permutation.
Tritogrammatic systems are permutation invariant in respect of partition and distinction for m=n=2.

Deutero- and proto-systems are sedimated by trito-systems of complexity m=2. And therefore not accessible to reflection.
http://www.thinkartlab.com/pkl/lola/Quadralectic%20Diamonds/Quadralectic%20Diamonds.pdf

Semiotic models

$M = (m, M_1, M_2)$

$B = (b, J_1, J_2)$

and $m = \{m_1, m_2\}$, $b = \{b_1, b_2\}$.

$m, b = \{a, b\}$

$\text{model}(M) = \{(aa), (ab), (ba)\}$

$\text{model}(B) = \{(aa), (ab), (bb)\}$.

Translations

$\text{model}(M): \quad (aa) \quad (ab) \quad (ba)$

$\updownarrow \qquad \swarrow \; \searrow \quad \nwarrow \; \nearrow$

$\text{model}(B): \quad (aa) \quad (bb) \quad (ab)$

Null

1.2.5. Logical interpretations

Semiotics: propositional logic

Because of the identity of the signs, the semiotic constellation Sem(2, n) is well founding the semantics but also the syntactics of propositional logic.

Indicational semiotics: Brown

p eq non(p), p eq {{p}}
Indicational semiotics is acting as the deep-structure of the calculus of

indication of the Laws of Form.
Thus, the calculus of indication gets a proto-semantics in the form of its indicational semiotics. Both proto-semantics, the Mersenne and the Brownian, are based on graphematically independent systems that are in a complementary relationship.
Brown calculi are introduced be decisions based on a specific intuition. They are not reflecting explicitly their semiotic foundations.

Logically, it is declared that Brownian calculi are localized systematically *"deeper than truth"* (Varela).
This property of being logically "deeper than truth" is emphasized not so much on the level of the basic definition of the calculus of indication but on a *second*-order level which allows the construction self-referential forms of different kinds of reentries, with f(f) = f. A counter-argument might be used which says that a cross is at once an operator and an operand in the CI. Hence, a logical contradiction *per se*. Nevertheless, the CI as a calculus works with two operators on its operands: iteration and superposition, i.e.
{}{} = {} and {{}} = ∅.

As far as Mersenne calculi are studied it appears that the logically contradictory situation is directly covered by the basic axioms of Mersenne calculi, with $<aa> =_{Mers} <bb>$. This axiom is avoiding contradictions, paradoxes or antinomies of reentry forms at the very root of the calculus.

Mersenne semiotics: paraconsistent logics

p et non(p) eq p or non(p)
Because of the complementarity between indicational and Mersenne semiotics, the proto-semantics of Mersenne systems is well supported by the graphematics of Mersenne systems.

Despite the complementarity it seems that the pretension of being "*deeper than truth*" is more directly realized with the proto-semantics of Mersenne calculi.
It might be experimented with the logical interpretation of Mersenne calculi as a kind of proto-structural paraconsistent logic.
In this sense, indicational calculi have a logical model in a logic of contradictions and Mersenne calculi might have a logical model in the

proto-structure of paraconsistency.
http://plato.stanford.edu/entries/logic-paraconsistent/

For Mersenne calculi the equivalence <aa> $=_{\text{Mers}}$<bb> which represents logically a contraction holds, while the permutation {ab} $=_{\text{Ind}}${ba} which contradicts semiotic and logical superposition of an operand and an operator holds for indicational calculi. Hence, there are two different abstractions over semiotics involved to define Mersenne and Brown calculi.

Operator/operand-relationship
In operational terms of operator and operand, the difference is clear again. Mersenne calculi are accepting the operator/operand-difference, Brownian calculi are abstracting from it. Mersenne abstracts from the iterativity of operators and of operands: op(op) = rand(rand), while Brown accepts this difference. An operand of an operand is different from an operator of an operator.

Mersenne
op(op) = rand(rand), op(rand) ≠ rand(op), with the $base_{\text{Mers}}$ = {rand(rand), op(rand), rand(op)}

Brown
op(op) ≠ rand(rand), op(rand) = rand(op), with the $base_{\text{Brown}}$ = {op(op), op(rand), rand(rand)}.

To satisfy some *epistemological* desires, the differences might be put into a subject/object-scheme.
For Brown calculi, objective objectivity (oO) and subjective subjectivity (sS) are accepted as different. The difference between subjective objectivity (sO) and objective subjectivity (oS) is not recognized. Complementary, Mersenne calculi are accepting the sO/oS-difference but not the "type-free" oO and sS.

$$\textbf{Chiasm}\left(\textbf{op, rand, 1, 2}\right)$$
$$\begin{array}{lccc} S1: & op & \longrightarrow & rand \\ & \updownarrow & X & \updownarrow \\ S2: & rand & \longleftarrow & op \end{array}$$

Recall memristics
It should be recalled that a kenomic analysis of the possible memris-

tive behavior is covered by such epistemological cnsiderations as memristors are switching in their functionality between "memory" and "computing" with the 3 functional states of pure memory, chiastic switch between memory and computing and pure computing.

Obviously there is no *chiastic* interplay between operator and operand involved for both types of calculi.

Hence, both types of non-classical calculi are, at least partly, defined logically as "*deeper than truth*".
This property of being *"deeper than truth"* becomes more evident on the level of tritogrammatics. From a trito- and deutero-grammatic point of view, both, Mersenne and Brown, are introduced as specifications of the general graphematic structures.

1.3. Short survey

Graphematic system of minimal distinction

Stirling turn

deutero

↙ ↓ ↘

Brown ← trito → Mersenne

↘ ↓ ↙

semiotics

Semiotics

Sem $_{(m,n)} = m^n$

concatenation
identitive,
non – commutative,
associative,
linear

Brown

Ind $_{(m,n)} = \binom{n+m-1}{n}$

identive,
commutative,
associative,
linear – tabular

Ordered partition

Trito $_{(m,n)} = \sum^{M} S(n, k)$

Generalized Mersenne

Mers $_{(m,n)} = m^n - (m - 1)$

non – identitive,
non – commutative,
non – associative,
tabular

partial non – identitive,
non – commutative,
associative,
linear

Free partition

$$\textbf{Deutero}_{(\mathbf{m,n})} = \sum_{k=1}^{M} P(n, k)$$

non – identitive,
commutative,
associative,
tabular

2. Arithmetics of graphematic calculi

2.1. Arithmetics of the Calculus of Semiotics

2.1.1. Semiotics as theory of strings

Recursive Wordarithmetics is covering the domain of semiotics in the sense of a syntactical systems.

2.1.2. Semiotics as theory of signs

Max Bense and Alfred Toth studied the arithmetics of signs as triadic-trichotomic objects.

Toth presented a number theory of polycontextural semiotics.
http://www.mathematical-semiotics.com/pdf/signs%20and%20trito-numbers.pdf

Alfred Toth, Calculus semioticus: Was zählt die Semiotik?
http://www.mathematical-semiotics.com/pdf/Calculus%20semioticus.pdf

2.2. Arithmetics of Indicational Calculi

2.2.1. Brownian arithmetics

Spencer Brown, Kauffman, Bricken and others developed a Brownian arithmetic in the sense of modeling natural numbers in the framework

of the calculus of indication.
Nobody was able to realize the specific character of the numbers based on the calculus of indication and its "two-dimensional" structure defined by concatenation and crossing.
The aim was to reconstruct classical natural numbers by the means of the calculus of indiction, i.e. with the help of its 2 initial decisions (axioms).

Spencer Brown's late trick to allow an inequality between one mark and two repeated marks to define the order of natural numbers, {}{} ≠ {}, seems not only to be to a weak strategy and adhoc but more a kind of a desperation (Bricken is following bravely his master).

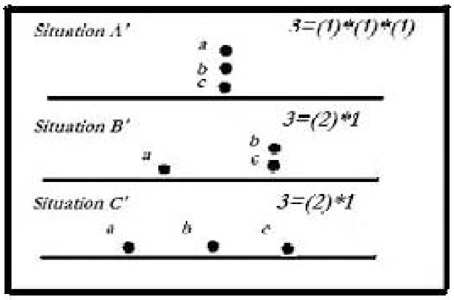
Policing the game of distinctions

> *"Interpreting the inner forms as numbers is* illegal*, so this particular problem can be defined away. However, stepping back prior to the distribution, we must introduce a restriction that blocks generating the illegal form."* William Bricken , Boundary Number Systems -- Spencer-Brown, January 2001
> http://www.wbricken.com/pdfs/01bm/06number/bnums-complete/04bnums-sb.pdf

2.2.2. Indicational arithmetic

Moshe Klein might be one of the few follower of GSB who was able to build the building blocks for a genuine indicational arithmetic. This happens with the distinction of "serial" and "parallel" numbers and their *intermediary* numbers as direct interpretations of the 'axioms' (initials) of the calculus of indication.

Unfortunately, Klein is not reflecting the specific *mathematical* character and relevancy of his numerical partitions for a general number theory but is offering an application or a model for the understanding of his first- and second-level partitions.



Hence, cardinality and ordinality of Indicational numbers are treated as different and the intermediary numbers are conceived as equivalent. Thus $\text{num}(\{\{\{\}\}\}) \neq_{\text{Ind}} \text{num}(\{\}\{\}\{\})$ and $\text{num}(\{\{\}\}\{\}) =_{\text{Ind}} \text{num}(\{\}\{\{\}\})$.

As was mentioned in the late 1960s by the logician Freytag-Loringhoff and the mathematician Hasse, Gunther's concept of *"Natural numbers in trans-Classic systems"* defines numbers simultaneously as a sequence of cardinal and a sequence of ordinal numbers and additionally as a system of intermediary numbers between the cardinality and ordinality of natural numbers. From a strictly mathematical point of view, based, say, on set-theory or recursive functions, this concept of tabular numbers was generally qualified as utter nonsense. Unfortunately, mathematicians are not aware of their semiotic frame in which they are working and which is in many senses just a cage for apologetic academics.

From a mathematical point of view the simultaneity of "*serial*" and "*parallel*" numbers is well modeled by the category-theoretic concepts of *bifunctoriality*. Intermediary numbers then occur as *internal* mixtures of both types of numbers, ordinal and cardinal, and are treated as well by bifunctoriality.

> *"Numbers can be represented as forms following either the original interpretation given by Spencer-Brown (1957), by adding further axioms and tokens not included in the original system (James 1993), or by relating form expressions to their corresponding Wolfram rule numbers (Schreiber 2004). This third approach is able to handle arbitrary integers or Boolean algebras of degree in general, and to reconstruct the 256 binary cellular automaton rules (Wolfram 1983, 2002) from 26 Spencer-Brown forms in particular. Large numbers can be repre-*

sented efficiently by constructing form expressions which specify only positions of ones."
Weisstein, Eric W. "Spencer-Brown Form." From MathWorld--A Wolfram Web Resource.
http://mathworld.wolfram.com/Spencer-BrownForm.html

Further information:

Jeffery M. James' approach

The term "*cardinaity*" has 87 occurences, the term "*ordinality*" zero in Jeffrey M. James' 1993 dissertation:
"A Calculus of Number Based on Spatial Forms".

Example for the J-multiplication 23 * 114

"Using these definitions, the multiplication 23 * 114 can be computed by making copies at each magnitude, collecting magnitudes, and doing a carry operation."

23 * 114

Given

{oo}ooo * {{o}o}oooo

Number Rewrite

([{oo}ooo][{{o}o}oooo])

Function Rewrite

([{oo}][{{o}o}oooo])([ooo][{{o}o}oooo])

Distribution

{([oo][{{o}o}oooo])}([ooo][{{o}o}oooo])

Promotion

{{{o}o}oooo{{o}o}oooo}{{o}o}oooo{{o}o}oooo{{o}o}oooo

Cardinality (2x)

{{{o}o}oooo{{o}o}oooo{o}o{o}o{o}o}oooooooooooo

Collection

{{{o}o}oooo{{o}o}oooo{o}o{o}o{o}o}boo

Replacement

{{{o}o}oooo{{o}o}oooo{o}o{o}o{o}oo}oo

Carry

{{{o}o{o}oooo}oooooooooooo}oo

Collection

{{{o}o{o}oooo}boo}oo
Replacement
{{{o}o{o}ooooo}oo}oo
Carry
{{{oo}oooooo}oo}oo
Collection
2622
Rewrite (JJames)
http://www.lawsofform.org/docs/jjames-thesis.txt

Bricken's Integers as Sets with ordinality

	Cardinality:	Ordinality:	Uniqueness:
0			
1	{ }	{ }	{ }
2	{ } { }	{ { } }	{{ }}
3	{ } { } { }	{ { { } } }	{{ },{{ }}}
4	{ } { } { } { }	{ { { { } } } }	{{ },{{ }},{{ },{{ }}}}
n	..n..	''n`	{1,..,n-1}

http://www.wbricken.com/pdfs/01bm/06number/bnums-complete/09boundary-numbers-all.pdf

A different numerical representation for the CI

num(Ind(i, j)):

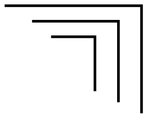
$\overline{\overline{\quad}|_{i}}|_{j} = \overline{\quad}|_{i}^{j}$: ord(ord(i) + ord(j)) : ord(i^j) : i^j = [i^j]

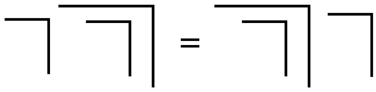
$\overline{\quad}|_{i}\,\overline{\quad}|_{j} = \overline{\quad}|_{i+j}$: card(card(i) + card(j)) : card(i+j) : i+j = [i,j]

$\overline{\overline{\quad}|_{i}}|_{j}\,\overline{\quad}|_{i}\,\overline{\quad}|_{j} = \overline{\quad}|_{i}^{j}\;\overline{\quad}|_{i+j}$: med((card(i+j), ord(i^j))): [[i,j], i^j].

Example m=3

card(3) : $\overline{\quad}|\;\overline{\quad}|\;\overline{\quad}|$: [1,1,1]

ord(3):  : [3]

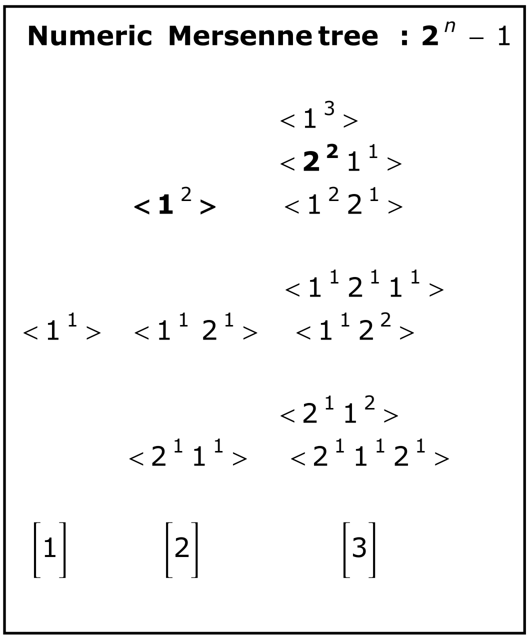
mix(3):  : [1, 2] = [2,1]

2.3. Arithmetics of Mersenne calculi

Mersenne systems are complementary to indicational systems and therefore their arithmetic is similarly complementary to the indicational arithmetic.
Hence, cardinality and ordinality of Mersenne numbers are treated as equivalences and the intermediary numbers are conceived as different.

Example for number 3:

Mersenne	**Brown**
[3] : <1+1+1> eq 1(1(1))	[3]: {1+1+1}
<2 +1> eq 1(1)+1	{3}
<1+2> eq 1+1(1)	{1+2} eq {2 + 1}



Numeric Mersenne rules

$$\text{R1}: \quad \Rightarrow \left\{1^{1}\right\}$$

$$\text{R2.1}: \left\{1^{n}\right\} \Rightarrow \left\{1^{n+1}\right\} \mid \left\{2^{n}1^{1}\right\} \mid \left\{1^{n}2^{1}\right\}$$

$$\text{R2.2}: \left\{1^{n}2^{n}\right\} \Rightarrow \left\{1^{n}2^{n}1^{1}\right\} \mid \left\{1^{n}2^{n+1}\right\}$$

$$\text{R2.3}: \left\{2^{n}1^{n}\right\} \Rightarrow \left\{2^{n}1^{n+1}\right\} \mid \left\{2^{n}1^{n}2^{1}\right\}$$

2.4. Arithmetics of Tritogrammatics

In contrast to the deutero-numbers and the indicational numbers, trito-arithmetics has to consider the order of the partitions.

2.5. Arithmetics of Deuterogrammatics

Deutero-arithmetics is covered, again, by Gunther/Schadach and elaborated in “Morphogrammatik” (1993).

2.6. Arithmetics of Protogrammatics

Proto-arithmetics is covered, again, by Gunther/Schadach and elaborated in “Morphogrammatik” (1993).

3. Logical interpretations

3.1. Classical logical interpretation of the CI

"I want to canclued by emphazising once again, that the calculi of indication are not a subtle form of logic. They really intent something quite different ..." (Varela, 1979)

Recalling Varela:

CI: Calculus of Indication,
PC: Propositional Calculus,
Variables: A, B, ... $\in$ CI, PC.
Procedure: Π.

Definition B.1

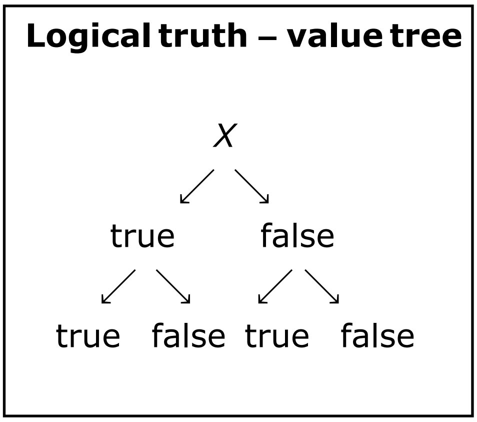
If A is ¬ B, write $\overline{\mathrm{B}}|$ for A in CI;
If A is B ∨ C, write BC for A in CI;
If ⊢ A in PC, write $\Pi(A) = \overline{\;\;}|$ in CI;
If ⊢ ¬A in PC, write $\Pi(\overline{A}) = \overline{\;\;}|$ in CI.

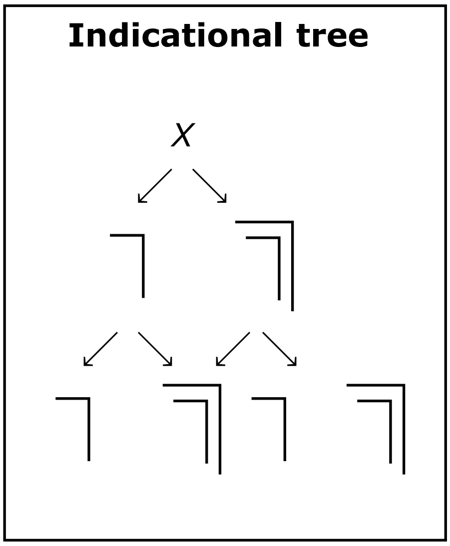
Lemma B.2
To every expression in PC there is a corresponds an indicational form.

Lemma B.3
Every demonstrable expression in PC is equivalent to the cross, $\overline{\;\;}|$, in CI.
(Varela, Principles of Biological Autonomy, 1979, p.285)

$$\text{Boolean domain}: B = \{\text{true, false}\}.$$

$$\text{Brownian domain}: CI = \{\overline{\;\;}|,\ \overline{\overline{\;\;}|}|\}.$$





3.2. Graphematical interpretation

3.2.1. Meta-semantics of the calculi

This classical semantic modeling with $\overline{\;\;}|$ or { } for *true* and $\overline{\overline{\;\;}|}|$ or {{ }} for *false* is generally accepted by the followers of GSB - obviously, he introduced it himself -, and I don't intent to criticise this possibility at all.

But there is another approach possible that seems, at least for my taste, to be more close to the *intentions* of the calculus of indications, as far as I understand them, and as far I can see a motivation to spent some interest on working on it.
Earlier work (1980) at: http://www.vordenker.de/ggphilosophy/rk_meta.pdf

A new kind of inter-relationship between propositional and indicational calculi is achieved with a change of abstraction. Not a mapping from logical truth-values onto indicational crosses on an atomic level but a mapping between the meta-semantical properties of PC is introduced. Hence, the meta-semantic properties of *tautology, saturation* and *contradiction* are mapped onto the graphematic properties of the calculus of indication, i.e. (aa, ab, bb).

Meta-semantics

logic : aa ab ba bb 1 2 3 4 taut sat1 sat1 contr
Brown: aa ab –– bb 1 2 – 4 taut sat1 *gap* contr
Mersenne: aa ab ba –– 1 2 3 – taut sat1 sat2 *gap*

$sem(X)$	neg_{Brown}	neg_{Mers}	neg_{logic}	–
taut	contr	taut	contr	hom
sat1	sat1	sat2	sat2	–
sat2	gap	sat1	sat1	–
contr	taut	gap	taut	het

For classical propositional logical systems, PC, there is a clear distinction between tautology and contradiction as well as between saturation in "b", (tf), and saturation in "a", (ft). The (meta-)semantics of PC is gap-free.

For *Brownian* calculi, CI, there is a negation between the properties of tautology (tt) and contradiction (ff) on the "background" of saturation ({tf}). The (meta-)semantics of CI has a (ft)-gap.

For *Mersenne* calculi, MC, there is a negation between the properties of saturations {(tf), (ft)} on the ”background” of tautology (tt). The (meta-)semantics of MC has a (ff)-gap.

The “backgrounds” of the meta-semantics are negation-invariant for Mersenne and Brownian calculi.
Brown: neg(ab) = ab,
Mersenne: neg(aa) = aa.

Kenogrammatics is *negation-free*. For the example with m=2, n=1, there are two configurations, a homogeneous, hom = (xx), and a heterogeneous, het =(xy). An analogon to negation is reflection, with the reflector-operator *refl*: refl(hom) = hom and refl(het) = het.

Gaps

From the point of view of the meta-semantics or graphematics of PC, 2 different kinds of gaps appear for CI and MC. A *gap* is an empty valuation, hence a **“**neutral” or “non-valent” semantic state. Such gaps of MC and CI are not to be mixed with the semantic gaps of the *“Logic of “Fiction”* where the existence *designators* relates to an ontologically non-existing entity. MC and CI are not related to ontology but to *graphematics* as the general framework of inscription.

Non-accessibility of gaps

Gaps in CI and MC are not accessible by the means of their systems. There is no procedure to produce a formula filling the gaps inside CI or inside MC.

Inter-relation between MC and CI

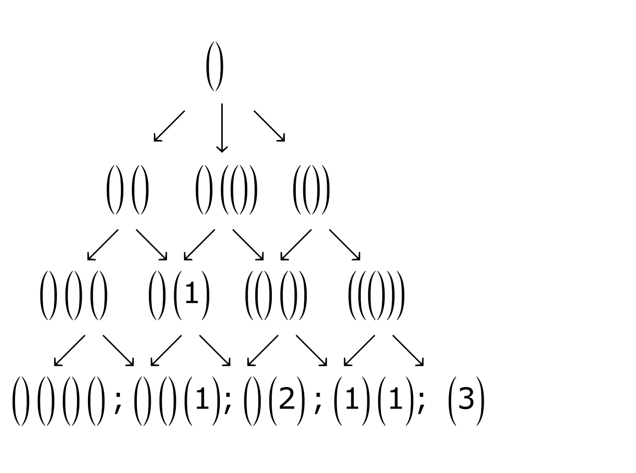
An inter-relation between the two 1-gap calculi, MC and CI, enables a mapping between gaps and “values” of different calculi.
This kind of interaction gets two different ways of enfolding: *one* from the gap-free logic PC and *one* from the gap-free kenogrammatics KG. Kenogrammatic systems are collecting PC, MC and CI into an identity- and gap-free calculus of {(aa), {ab)}.

3.2.2. Calculus of Indication

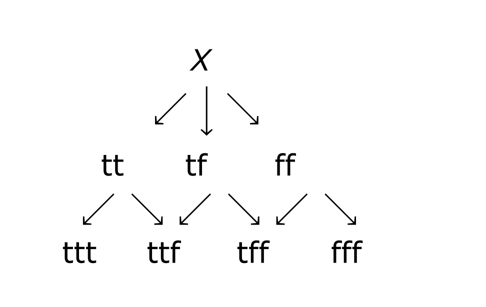
The *indicational* domain of the CI is trichotomic: *Brown* = {tt, tf, ff}. This takes into consideration that the indicational space is not properly characterized by atomical terms, like a cross and a blank alone. Therefore, terms like *true* and *false,* in the sense of classical propostional logic, are not adequete to mark the differences. What counts from a graphematic point of view is the behavior, expressed by the difference between marks, i.e. the rules of the action between 'elements' of the CI. This domain of action is defined for the CI by the graphematic constellation (aa, ab, bb) with (ab) $=_{Brown}$ (ba) and (aa) $\neq_{Brown}$ (bb).

Hence, it is argued that the elements of the CI are not properly defined simply by {{}} and {} and its indicational tree. The Brownian system starts wit 3 elements: {}{}, {}{{}}, {{{}}} and develops into a graph and not into a tree.

Indicational graph parenthesis; Klein



Truth – value graph for CI



Semantics of the indicational domain

val({aa, ab, bb}) = {tt, tf, ff}

val(aa) = (tt)

val(ab) = (tf)
val(bb) = (ff).

Negation
non(tt) = ff
non(tf) = tf, because (ab) $=_{\text{Ind}}$(ba)

Negation in Brown is inversion (negation)

Numerical truth-values
num(tt) = (1)
num(tf) = (2)
num(ff) = (3).

non(1, 2, 3) = (3, 2, 1)

Conjunction
(tt) (tt) $\longrightarrow$ (tt)
(tt) (tf) $\longrightarrow$ (tf)
(tt) (ff) $\longrightarrow$ (ff)

(tf) (tt) $\longrightarrow$ (tf)
(tf) (tf) $\longrightarrow$ (tf)
(tf) (ff) $\longrightarrow$ (ff)

(ff) (tt) $\longrightarrow$ (ff)
(ff) (tf) $\longrightarrow$ (ff)
(ff) (ff) $\longrightarrow$ (ff)

Truth-tables:

conj	tt	tf	ff
tt	tt	tf	ff
tf	tf	tf	ff
ff	ff	ff	ff

conj	1	2	3
1	1	2	3
2	2	2	3
3	3	3	3

Comparision of truth-tables

$conj_{Brown}$	1	2	–	4
1	1	2	–	4
2	2	2	–	4
–	–	–	–	–
4	4	4	–	4

$conj_{Mers}$	1	2	3	–
1	1	2	3	–
2	2	2	3	–
3	3	3	3	–
–	–	–	–	–

standardized:

$conj_{Brown}$	1	2	3
1	1	2	3
2	2	2	3
3	3	3	3

$conj_{Mers}$	1	2	3
1	1	2	3
2	2	2	3
3	3	3	3

3.2.3. Mersenne calculus

The *distinctional* domain of the Mersenne calculus is trichotomic, too:
Mers = {tt, tf, ft}.

non(tt) = tt
non(tf) = ft

Negation in Mers is permutation.

num({tt, tf, ft}) = (1,2,3)

non(1,2,3) = (1, 3, 2)

conj	1	2	3
1	1	2	3
2	2	2	3
3	3	3	3

conj	tt	tf	ft
tt	tt	tf	ft
tf	tf	tf	ft
ft	ft	ft	ft

3.2.4. DeMorgan for Brownian and Mersenne calculi

A separation and interaction of both calculi, the Brownian and Mersenne, might be managed by the category-theoretic methods of bifunctoriality.
Bifunctoriality and its extension to a general concept of interchangeability offers the methods to study the interactions of paradigmatically different calculi without the need to subordinate to a unifying general paradigm.

The bifunctorial system [Brown, Mersenne, Semiotics] is opening up the framework to study the inter-relationship between the Mersenne and the Brown calculi from the background of semiotics.

DeMorgan for Mersenne

$\text{non}\left(\text{conj}\left(\text{non X, nonY}\right)\right)$:

$\text{conj}\left(\text{non } X, \text{non } Y\right)$:

$\text{conj}\left(\bar{X}, \bar{Y}\right)$	1	3	2
1	1	3	2
3	3	3	3
2	2	3	2

$\Rightarrow$

$\text{non}\left(-\right)$	1	3	2
1	1	2	3
3	2	2	2
2	3	2	3

non – aeq

$\text{disj}\left(X, Y\right)$	1	2	3
1	1	1	1
2	1	2	2
3	1	2	3

$\text{non}\left(\text{conj}\left(\text{non X, nonY}\right)\right)$ *non –aeq* $\text{disj}\left(\text{X, Y}\right)$.

$\text{conj}\left(\text{X, nonX}\right) \in \text{Satisfaction}\left(\notin \text{Contradiction}\right)$

$X, \text{non}\left(X\right)$	$\text{conj}\left(X, \text{non } X\right)$
1 1	1
2 3	3
3 2	3

DeMorgan for Brown

$\text{non}\left(\text{conj}\left(\text{non X, nonY}\right)\right)$:

$\text{conj}\left(\bar{X}, \bar{Y}\right)$	3	2	1
3	3	3	3
2	3	2	2
1	3	2	1

$\Rightarrow$

$\text{non}\left(--\right)$	3	2	1
3	1	1	1
2	1	2	2
1	1	2	3

aeq

$\text{disj}\left(X, Y\right)$	1	2	3
1	1	1	1
2	1	2	2
3	1	2	3

$\text{non}\left(\text{conj}\left(\text{non}\,X, \text{non}Y\right)\right)$ *aeq dis j* $\left(X,\ Y\right)$.

$\overline{\overline{X}\rceil\ \overline{Y}\rceil}\rceil = XY$.

$\text{conj}\left(X, \text{non}X\right)\right) \in \text{contradiction}$

$\overline{X}\rceil = X$

X, $\text{non}\left(X\right)$	$\text{conj}\left(X, \text{non}\,X\right)$
1 3	3
2 2	2
3 1	3

3.2.5. Varela's ECI and self-referentiality

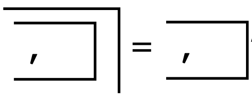
Funnily enough, the meta-semantic interpretation of the CI and its representation by truth-tables reminds strongly at Varela's introduction and formalization of an Extended Calculus of Identification, ECI, with an additional "value" for a third state of re-entry, $\overline{\ ,\quad}\rceil$, the "self-cross", "self-naming", "autonomous" value, with the property of *constancy.*

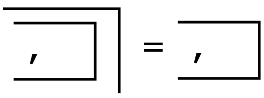
> *"Beyond these considerations, let us look more in detail the* autonomous *value as a paradigm for self-reference. As it now stands in $\mathcal{L}(\mathcal{E})$ it is only a third value which can deal with self-*

> *reference in a very loose way, namely, insofar self-referring statements require a value which is identical to its negation."* (Varela, The Extended Calculus of Indications Interpreted as a Three-valued Logic, in: Notre Dame Journal of Formal Logic, Vol. XX, No. 1, Jan. 1979)
> http://projecteuclid.org/DPubS/Repository/1.0/Disseminate?view=body&id=pdf_1&handle=euclid.ndjfl/1093882412

Although Varela generously "*allows*" in his ECI self-referring forms and in its logical interpretation as an *autonomous*, self-referring value, there is no intrinsic need by the calculus CI as such to ask for a domestication of self-reference. Albeit the topic of self-reference was, and still is, virulent, the extension of the CI towards an ECI comes without systematic motivation. It is, and remains, in fact, an *ad hoc* construction.

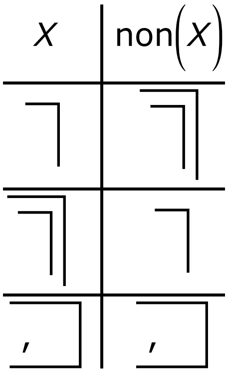
> *"It can be showed that, although all self-referring forms are allowed in **ECI**, their diversity can essentially be reduced to the atomic case of the autonomous value."* (Varela, ibd)

In contrast, the understanding of the CI as based on the pattern (aa, ab, bb), involves self-referentiality from the very beginning. The graphematic situation: (ab) $=_{Ind}$(ba) is, considered from an external viewpoint and not reflecting its chiastic immanent structure, logically equivalent to non(tf) = (tf). Changing the wording and symbolism from "non(tf) = (tf)" to "  " we get Varela's adhoc implementation as a necessity of the concept of indication.

Varela's *Constancy*:  .

The formulas says that the distinction of a self-distinction is indicationally equal the self-distinction. Hence, a distinction of a self-distinction doesn't make a distinction.

Logified, *constancy* corresponds to the truth-table for negation in a special 3-valued logic (Kleene).



numerically equal

to:

X	$non(X)$
1	2
2	1
3	3

or alternatively

X	$non(X)$
1	3
2	2
3	1

Varela, Principles of Biological Autonomy, 1979, p.127

Reduction of complexity

> *"The point of view of indication greatly simplifies the discussion of self-referential situations, by simply having an expression indicate itself. Expressions where self-indication is allowed, are called Boolean expressions of higher degree by Spencer Brown, [...]"*

This intended complexity reduction for the implementation of self-referentiality can be strengthened by the graphematic approach to indication. On the other hand, the graphematic approach gives foundation for self-referential complexity higher than it is possible with a single solitaire autonomous value.

Blending together

$conj_{Brown/Mers}$	1	2	3	4
1	1	2	3	4
2	2	2	3	4
3	3	3	3	–
4	4	4	–	4

red: Brown $\frown$ Mersenne

blue: Mersenne \ {red, green}
green: Brown \ {red, blue}
Semiotics: Browm ⌣ Mersenne

3.3. Proof-theory and tableaux calculi

For Mersenne calculi a branch of the tableau terminates if the same formaulas contains the signatures (tf) and (ft).
For Brown calculi a branch of the tableau terminates if the same formula contains the signatures (tt) and (ff).

3.4. Bifunctoriality of Brownian and Mersennian calculi

From the point of view of a theory of polycontexturality as proposed by Gotthard Gunther and elaborated in many ways by my own studies it is natural to model an interplay between the 4 different graphematic systems, semiotic (logic), Brownian and Mersennian calculi together with the kenogrammatics systems (proto-, deutero- and trito-structures) with the techniques of generalized category-theoretic bifunctoriality.

BIFUNCTORIALITY OF MERSENNE, BROWN and SEMIOTICS

$$\begin{bmatrix} g_{\text{Mers}} & - & g_{\text{Sem}} \\ f_{\text{Mers}} & g_{\text{Brown}} & - \\ - & f_{\text{Brown}} & f_{\text{Sem}} \end{bmatrix}:$$

$$\left(\begin{matrix} \left(\begin{matrix} \left(f_{\text{Mers}} \circ^{1.0\quad .0} g_{\text{Mers}}\right) \\ \amalg_{1.2\quad .0} \\ \left(f_{\text{Brown}} \circ^{0.2\quad .0} g_{\text{Brown}}\right) \end{matrix}\right) \\ \amalg_{1.2\quad .3} \\ \left(f_{\text{Sem}} \circ^{0.0\quad .3} g_{\text{Sem}}\right) \end{matrix}\right) =$$

$$\left(\begin{matrix} \left(\begin{matrix} f_{\text{Mers}} \\ \amalg_{1.2\quad .0} \\ f_{\text{Brown}} \end{matrix}\right) \\ \amalg_{1.2\quad .3} \\ f_{\text{Sem}} \end{matrix}\right) \circ_{1.2\quad .3} \left(\begin{matrix} \left(\begin{matrix} g_{\text{Mers}} \\ \amalg_{1.2\quad .0} \\ g_{\text{Brown}} \end{matrix}\right) \\ \amalg_{1.2\quad .3} \\ g_{\text{Sem}} \end{matrix}\right)$$

REDUCED BIFUNCTORIALITY OF SEMIOTICS, MERSENNE, BROWN AND TRITO – SYSTEM

$$\left[\begin{array}{cccccc} g_{Mers} & - & g_{Sem} & - & - & g \\ f_{Mers} & g_{Brown} & - & - & g & - \\ - & f_{Brown} & f_{Sem} & g_{Trito} & - & - \\ - & - & - & f_{Trito} & f & f \end{array}\right]:$$

$$\left(\begin{array}{c} \left(\begin{array}{c} \left(f_{Mers} \circ^{1.0\ .0\ .0} g_{Mers}\right) \\ \text{Ш}_{1.2\ .0\ .0} \\ \left(f_{Brown} \circ^{0.2\ .0} g_{Brown}\right) \end{array}\right) \\ \text{Ш}_{1.2\ .3\ .0} \\ \left(\begin{array}{c} \left(f_{Sem} \circ^{0.0\ .3\ .0} g_{Sem}\right) \\ \text{Ш}_{1.2\ .3\ .4} \\ \left(f_{Trito} \circ^{0.0\ .0\ .4} g_{Trito}\right) \end{array}\right) \end{array}\right) =$$

$$\left(\begin{array}{c} \left(\begin{array}{c} f_{Mers} \\ \text{Ш}_{1.2\ .0\ .0} \\ f_{Brown} \end{array}\right) \\ \text{Ш}_{1.2\ .3 \times .0} \\ \left(\begin{array}{c} f_{Sem} \\ \text{Ш}_{1.2\ .3\ .4} \\ f_{Trito} \end{array}\right) \end{array}\right) \circ_{1.2\ .3\ .4} \left(\begin{array}{c} \left(\begin{array}{c} g_{Mers} \\ \text{Ш}_{1.2\ .0\ .0} \\ g_{Brown} \end{array}\right) \\ \text{Ш}_{1.2\ .3\ .0} \\ \left(\begin{array}{c} g_{Sem} \\ \text{Ш}_{1.2\ .3\ .4} \\ g_{Trito} \end{array}\right) \end{array}\right)$$

Again,*"In a nutshell, stateful logic means that the 'state' of the memristor acts as both the computer and the memory. That's a pretty big change from current computers, which typically load data from memory, perform operations on it, and then send it back*" [Nature]
http://blogs.nature.com/news/2010/04/memristance_is_not_futile.html