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## Rudolf Kaehr

(1942-2016)

### Title

Ternary and Quaternary systems in morphoCAs  
Some tools for the analysis of number sequences in morphoCAs

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2015

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Morphograms, Cellular Automata, Semiotics

### Disciplines

Computer Science, Artificial Intelligence and Robotics, Logic and Foundations of Mathematics,  
Cybernetics, Theory of Science

### Abstract

Claviatures gives a glimpse into the usefulness of the sub-rule approach for all kind of cellular automata. The merits of the sub-rule approach becomes evident for highly complex automata where it is practically not achievable to manipulate all single rules of the automaton explicitly.

The topics that can be studied more directly are “number sequences” and “ternary and quaternay logics” related to morphoCAs.

Some preliminary results concerning number sequences for morphoCAs are sketched. The presented techniques might be used as tools for further elaborations.

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| K05 Interactivity                                 | K12 Cellular Automata                                |
| K06 Diamond Strategies                            | K13 RK and friends                                   |
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# Ternary and Quaternary systems in morphoCAs

Some tools for the analysis of number sequences in morphoCAs

Dr. phil Rudolf Kaehr

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ISSN 2041-4358

( work in progress, v. 0.2, August 2015 )

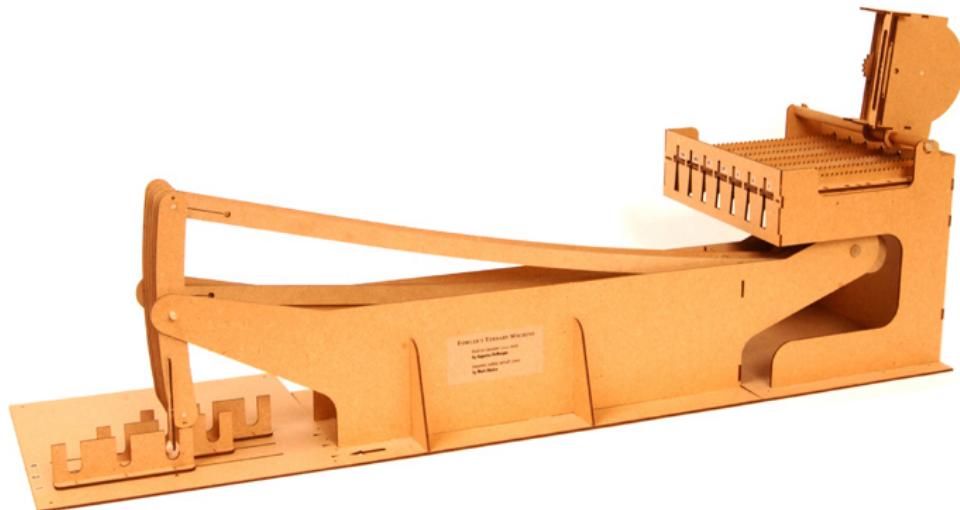
## Motivation

*Claviatures* gives a glimpse into the usefulness of the sub-rule approach for all kind of cellular automata. The merits of the sub-rule approach becomes evident for highly complex automata where it is practically not achievable to manipulate all single rules of the automaton explicitly.

The topics that can be studied more directly are “*number sequences*” and “*ternary and quaternary logics*” related to morphoCAs.

Some preliminary results concerning number sequences for morphoCAs are sketched.

The presented techniques might be used as tools for further elaborations.

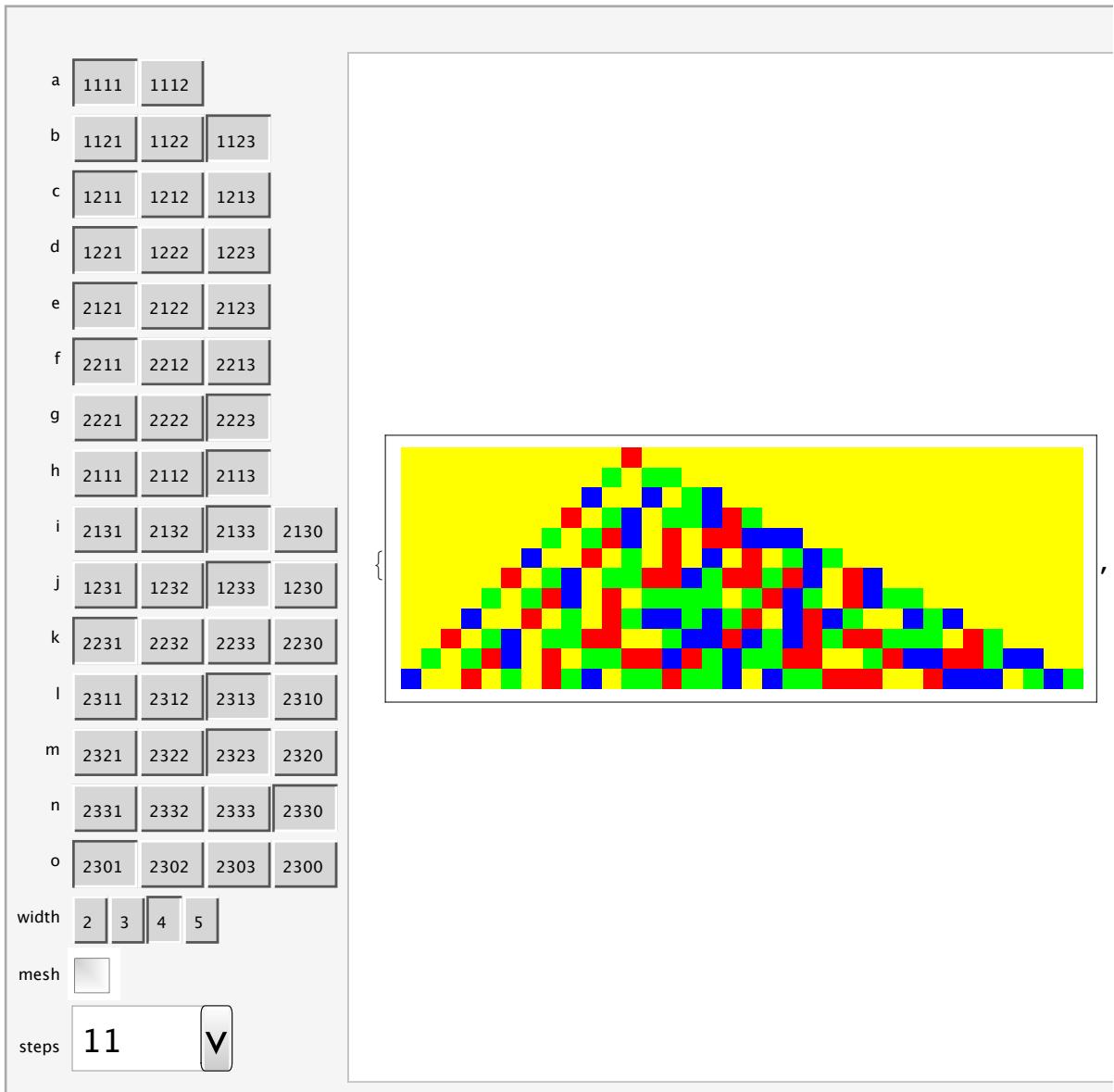


<http://www.mortati.com/glusker/fowler/> The ternary calculating machine of Thomas Fowler

## Initialization

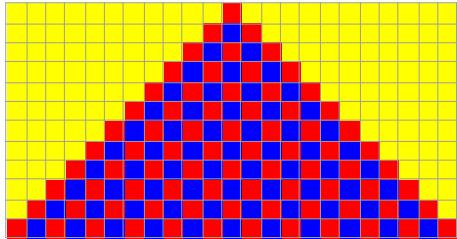
## Requisits

### Morphogram: ruleDCKV

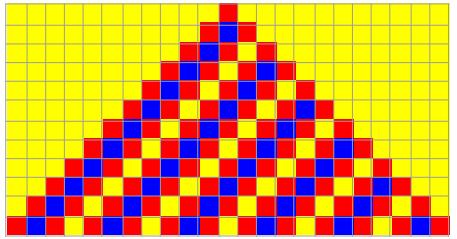




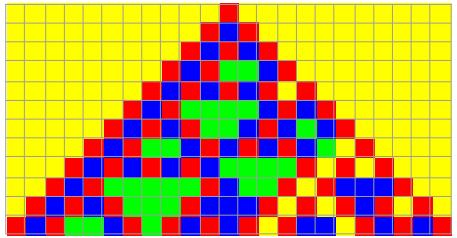
## Emphasis on the architectonic symmetry



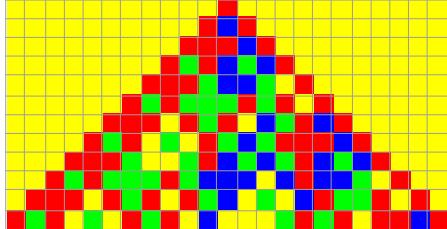
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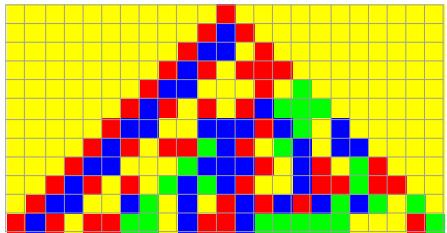
$$\left( \begin{array}{c} \{1\} \\ \{1, 2, 1\} \\ \{1, 2, 1, 0, 1\} \\ \{1, 2, 1, 0, 1, 2, 1\} \\ \{1, 2, 1, 0, 1, 2, 1, 0, 1\} \\ \{1, 2, 1, 0, 1, 2, 1, 0, 1, 2, 1\} \\ \{1, 2, 1, 0, 1, 2, 1, 0, 1, 2, 1, 0, 1\} \\ \{1, 2, 1, 0, 1, 2, 1, 0, 1, 2, 1, 0, 1, 2, 1\} \\ \{1, 2, 1, 0, 1, 2, 1, 0, 1, 2, 1, 0, 1, 2, 1, 0, 1\} \\ \{1, 2, 1, 0, 1, 2, 1, 0, 1, 2, 1, 0, 1, 2, 1, 0, 1, 2, 1\} \\ \{1, 2, 1, 0, 1, 2, 1, 0, 1, 2, 1, 0, 1, 2, 1, 0, 1, 2, 1, 0, 1\} \end{array} \right), \left( \begin{array}{c} 1 \\ 9 \\ 37 \\ 153 \\ 613 \\ 2457 \\ 9829 \\ 39321 \\ 157285 \\ 629145 \\ 2516581 \\ 10066329 \end{array} \right)$$



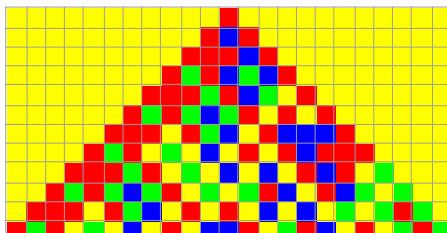
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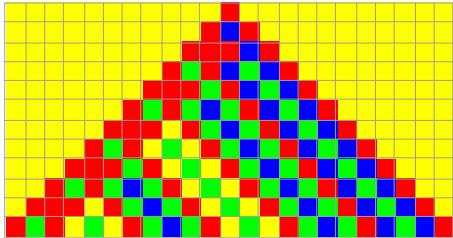
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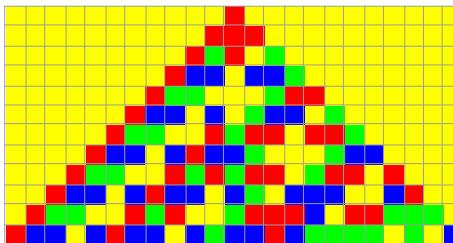
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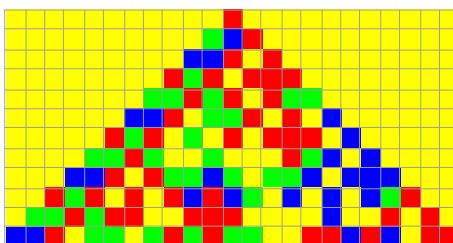
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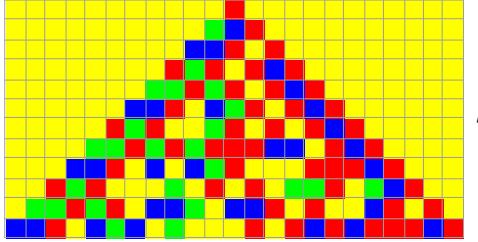
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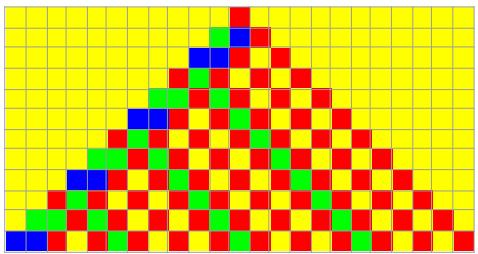
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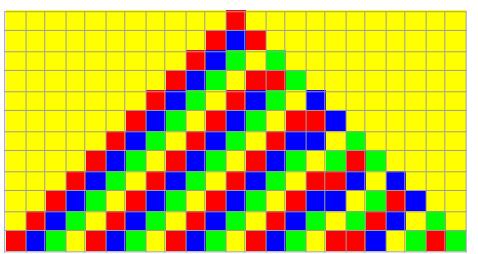
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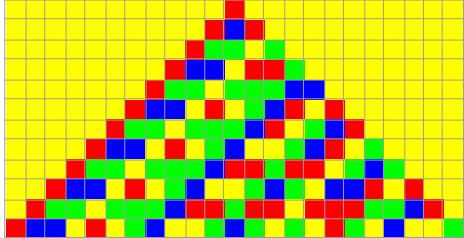
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$$\left( \begin{array}{c} \{1\} \\ \{3, 2, 1\} \\ \{2, 2, 1, 0, 1\} \\ \{1, 3, 1, 0, 1, 0, 1\} \\ \{3, 3, 1, 3, 1, 0, 1, 0, 1\} \\ \{2, 2, 1, 0, 1, 3, 1, 0, 1, 0, 1\} \\ \{1, 3, 1, 0, 1, 3, 1, 0, 1, 0, 1\} \\ \{3, 3, 1, 3, 1, 0, 1, 0, 1, 0, 1\} \\ \{2, 2, 1, 0, 1, 3, 1, 0, 1, 0, 1\} \\ \{1, 3, 1, 0, 1, 0, 1, 0, 1, 0, 1\} \\ \{3, 3, 1, 3, 1, 0, 1, 0, 1, 0, 1\} \\ \{2, 2, 1, 0, 1, 3, 1, 0, 1, 0, 1\} \end{array} \right), \left( \begin{array}{c} 1 \\ 17 \\ 53 \\ 181 \\ 1333 \\ 3509 \\ 11701 \\ 85429 \\ 224693 \\ 748981 \\ 5467573 \\ 14380469 \end{array} \right)$$

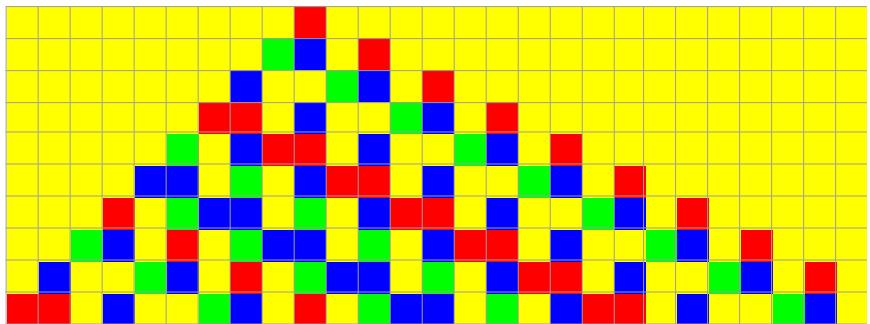


$$\left( \begin{array}{c} \{1\} \\ \{1, 2, 1\} \\ \{1, 2, 3, 0, 3\} \\ \{1, 2, 3, 0, 1, 1, 3\} \\ \{1, 2, 3, 0, 1, 2, 3, 0, 2\} \\ \{1, 2, 3, 0, 1, 2, 3, 0, 1, 1, 2\} \\ \{1, 2, 3, 0, 1, 2, 3, 0, 1, 2, 2, 0, 3\} \\ \{1, 2, 3, 0, 1, 2, 3, 0, 1, 2, 3, 0, 1, 1, 2, 0, 2\} \\ \{1, 2, 3, 0, 1, 2, 3, 0, 1, 2, 3, 0, 1, 2, 2, 0, 3, 1, 2\} \\ \{1, 2, 3, 0, 1, 2, 3, 0, 1, 2, 3, 0, 1, 2, 3, 0, 3, 1, 2, 0, 3\} \\ \{1, 2, 3, 0, 1, 2, 3, 0, 1, 2, 3, 0, 1, 2, 3, 0, 1, 2, 0, 3, 1, 3\} \end{array} \right), \left( \begin{array}{c} 1 \\ 9 \\ 47 \\ 185 \\ 750 \\ 3000 \\ 12011 \\ 48065 \\ 192226 \\ 768944 \\ 3075843 \\ 12303249 \end{array} \right)$$



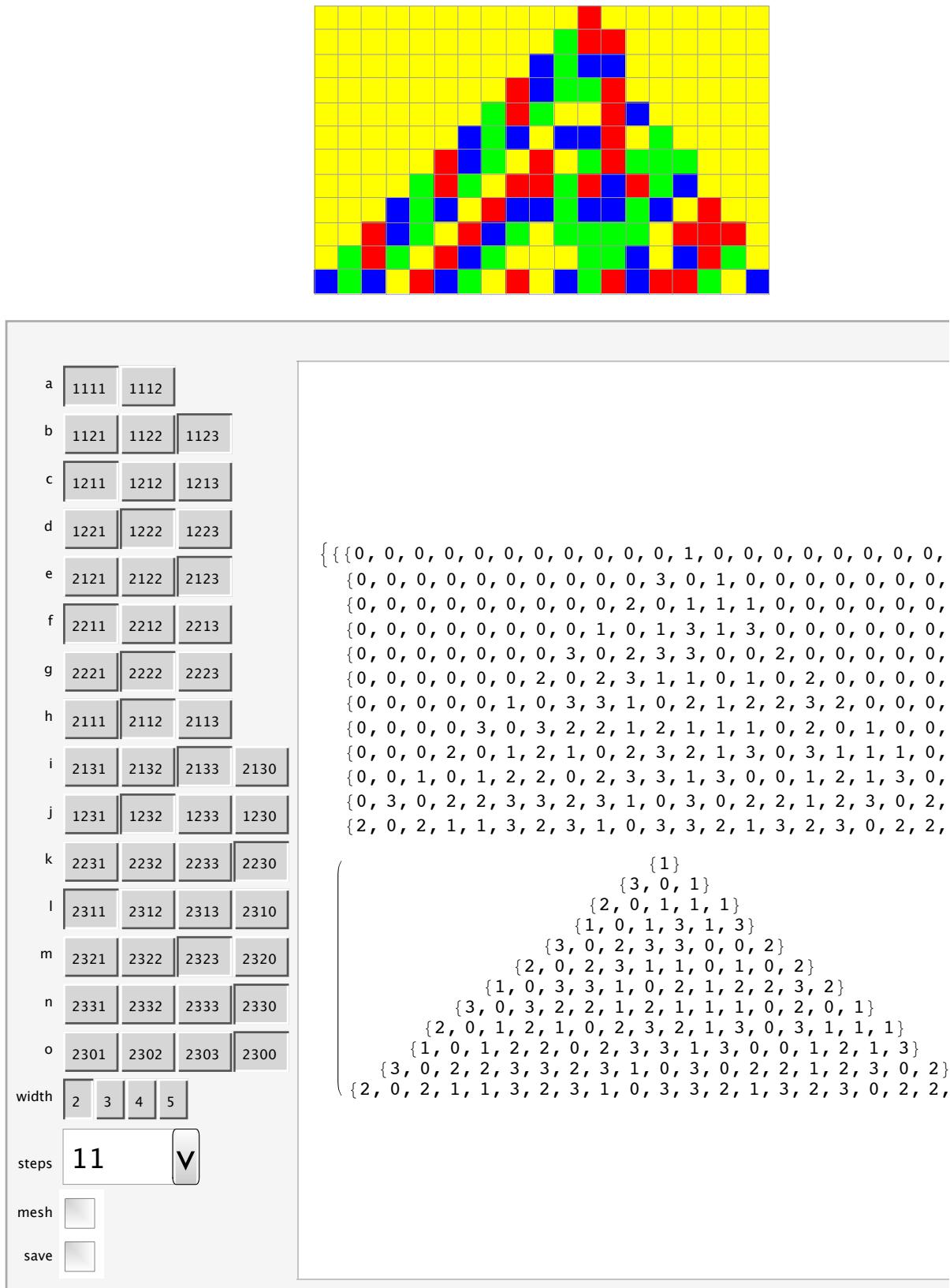
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Emphasis on the architectonic asymmetry of morphoCAs with radius = (r,2r)

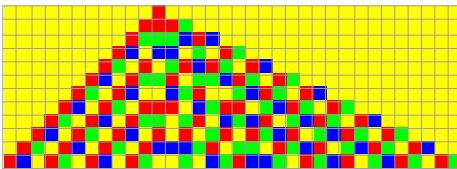
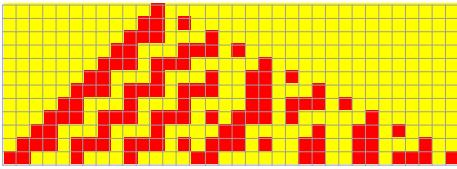


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c	1211	1212	1213	
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e	2121	2122	2123	
f	2211	2212	2213	
g	2221	2222	2223	
h	2111	2112	2113	
i	2131	2132	2133	2130
j	1231	1232	1233	1230
k	2231	2232	2233	2230
l	2311	2312	2313	2310
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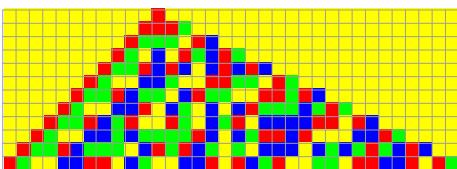
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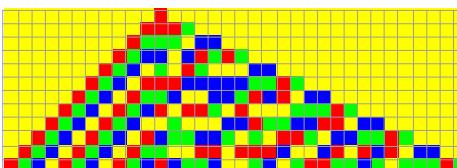
## Some selected configurations



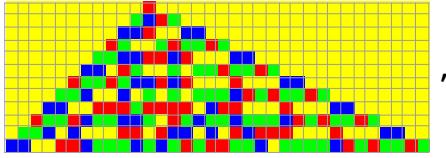
$\begin{pmatrix} \{1\} \\ \{1, 3, 3, 3, 2, 1, 2\} \\ \{1, 2, 0, 2, 2, 0, 3, 0, 1, 3\} \\ \{1, 3, 0, 1, 0, 3, 1, 2, 1, 3, 0, 1, 2\} \\ \{1, 2, 0, 1, 3, 3, 1, 0, 3, 3, 2, 1, 3, 0, 1, 3\} \\ \{1, 2, 0, 1, 2, 0, 2, 3, 1, 0, 0, 3, 2, 1, 3, 0, 1, 2\} \\ \{1, 2, 0, 1, 3, 0, 1, 1, 0, 2, 3, 1, 1, 0, 3, 2, 1, 3, 0, 1, 3\} \\ \{1, 2, 0, 1, 3, 0, 1, 2, 0, 1, 3, 0, 1, 0, 3, 2, 1, 3, 0, 1, 2\} \\ \{1, 2, 0, 1, 3, 0, 1, 2, 0, 1, 3, 0, 1, 0, 3, 2, 1, 3, 0, 1, 3\} \\ \{1, 2, 0, 1, 3, 0, 1, 2, 0, 1, 3, 0, 1, 0, 3, 2, 1, 3, 0, 1, 2\} \end{pmatrix}$	$\begin{pmatrix} 1 \\ 17 \\ 244 \\ 1245 \\ 11308 \\ 80045 \\ 728876 \\ 4915245 \\ 47055404 \\ 313946925 \\ 300139409 \\ 2017860589 \end{pmatrix}$
---	---



$\begin{pmatrix} 1 \\ 1, 1, 1, 1, 1, 3 \\ 1, 3, 3, 3, 3, 1, 0, 1, 2 \\ 1, 3, 0, 2, 0, 1, 3, 2, 1, 3 \\ 1, 3, 1, 2, 1, 2, 0, 2, 1, 2, 1, 2 \\ 1, 3, 0, 0, 2, 2, 3, 0, 0, 1, 1, 3, 3, 0, 1, 3 \\ 1, 3, 1, 2, 3, 3, 0, 2, 1, 3, 3, 0, 0, 1, 1, 3, 1, 2 \\ 1, 3, 0, 2, 1, 0, 2, 3, 0, 2, 0, 2, 1, 3, 3, 3, 2, 3, 1, 3 \\ 1, 3, 1, 2, 3, 1, 0, 2, 3, 1, 0, 2, 2, 2, 2, 1, 1, 0, 1, 1, 2 \\ 1, 3, 0, 2, 2, 3, 3, 3, 0, 2, 1, 3, 1, 2, 0, 2, 3, 1, 1, 2, 1, 0, 1, 2 \\ 1, 3, 0, 0, 2, 1, 0, 1, 2, 3, 0, 2, 0, 2, 1, 0, 1, 2, 3, 1, 3, 2, 1, 1, 2 \\ 1, 3, 0, 0, 2, 1, 0, 1, 2, 3, 0, 2, 0, 2, 1, 0, 1, 2, 3, 1, 3, 2, 1, 0, 1, 3 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 17 \\ 236 \\ 1461 \\ 14644 \\ 89773 \\ 958120 \\ 5700593 \\ 60653532 \\ 372129949 \\ 3941858452 \\ 23349097261 \end{pmatrix}$
--	---



$\begin{pmatrix} & & & \{1\} \\ & & & \{1, 1, 1, 1, 3\} \\ & & \{1, 3, 3, 3, 3, 0, 2, 2\} \\ & \{1, 3, 2, 2, 0, 2, 1, 3, 1, 3\} \\ & \{1, 3, 2, 0, 3, 0, 2, 1, 3, 0, 2, 2\} \\ & \{1, 3, 2, 0, 1, 3, 3, 2, 1, 2, 2, 3, 1, 2, 1, 0, 2, 2\} \\ & \{1, 3, 2, 0, 1, 3, 3, 2, 0, 1, 2, 2, 1, 3, 3, 2, 2, 1, 1, 0, 2, 2\} \\ & \{1, 3, 2, 0, 1, 3, 3, 2, 0, 1, 2, 2, 1, 3, 3, 2, 2, 1, 1, 0, 2, 2\} \\ & \{1, 3, 2, 0, 1, 3, 3, 2, 0, 1, 2, 2, 1, 3, 3, 2, 2, 1, 1, 0, 2, 2\} \\ & \{1, 3, 2, 0, 1, 3, 3, 2, 0, 1, 2, 2, 1, 3, 3, 2, 2, 1, 1, 0, 2, 2\} \\ & \{1, 3, 2, 0, 1, 3, 3, 2, 0, 1, 2, 2, 1, 3, 3, 2, 2, 1, 1, 0, 2, 2\} \\ & \{1, 3, 2, 0, 1, 3, 3, 2, 0, 1, 2, 2, 1, 3, 3, 2, 2, 1, 1, 0, 2, 2\} \\ & \{1, 3, 2, 0, 1, 3, 3, 2, 0, 1, 2, 2, 1, 3, 3, 2, 2, 1, 1, 0, 2, 2\} \end{pmatrix}$	$\begin{pmatrix} 1 \\ 17 \\ 238 \\ 1721 \\ 13222 \\ 102921 \\ 844814 \\ 6710617 \\ 53583710 \\ 429587721 \\ 3436413646 \\ 274948601 \end{pmatrix}$
--	--



$$\left( \begin{array}{c} \{1\} \\ \{3, 2, 1, 0, 3\} \\ \{2, 2, 0, 1, 3, 0, 3, 1, 3\} \\ \{1, 3, 0, 0, 3, 1, 3, 0, 2, 2\} \\ \{3, 3, 2, 2, 0, 1, 2, 1, 0, 3, 0, 2, 2\} \\ \{2, 2, 0, 1, 3, 0, 0, 3, 1, 3, 3, 1, 3\} \\ \{1, 3, 3, 1, 3, 2, 2, 1, 2, 1, 0, 0, 1, 0, 0, 0, 2, 2\} \\ \{3, 3, 2, 0, 0, 2, 0, 3, 0, 3, 0, 3, 3, 2, 1, 3, 1, 3, 1, 3\} \\ \{2, 2, 0, 0, 1, 1, 1, 3, 1, 1, 1, 0, 2, 1, 1, 0, 0, 0, 1, 0, 0, 2, 2\} \\ \{1, 3, 3, 1, 2, 3, 3, 2, 2, 3, 0, 1, 1, 3, 2, 3, 3, 2, 1, 3, 0, 1, 3, 1, 3\} \\ \{3, 3, 2, 0, 2, 0, 0, 0, 1, 1, 0, 1, 2, 2, 0, 2, 0, 1, 2, 1, 1, 0, 0, 1, 0, 0, 2, 2\} \\ \{2, 2, 0, 0, 1, 1, 2, 3, 3, 2, 3, 1, 1, 3, 2, 0, 3, 3, 2, 1, 3, 1, 3, 1, 3\} \end{array} \right), \left( \begin{array}{c} 1 \\ 37 \\ 214 \\ 1433 \\ 22054 \\ 109657 \\ 963110 \\ 10684121 \\ 52688422 \\ 492811993 \\ 5510614566 \\ 27234250457 \end{array} \right)$$

## Reduction example: morphoCA<sup>(5,4,5)</sup> $\Rightarrow$ morphoCA<sup>(5,2,5)</sup>

Reductions are an essential strategy for the study of morphoCAs. They are complementing the well known ECA techniques of *elimination*, *reversion* and *dualizations*. The proposed reductions by mapping the numeric presentation of the morphoCAs to a lower valued but isomorphic morphoCA is certainly just a quite simple first step. All reductive mappings that are preserving the structure of the output of the morphoCAs are natural candidates for reduction procedures.

Automata in comparison are: ECA, morphoDCM and morphoDM. The automaton morphoDCM is a restriction of morphoDM and is implementing the system morphoCA<sup>(3,3)</sup> that is restricted to the first 14 morphograms and functions with just 3 colors instead of 4 colors as for morphoDM are implemented. The system morphoCA<sup>(3,3)</sup> is to some parts a direct sub-system of morphoDM.

### Sub - rule ECA

k	<table border="1" style="border-collapse: collapse; width: 100%;"><tr><td>1</td><td>6</td></tr></table>	1	6
1	6		
l	<table border="1" style="border-collapse: collapse; width: 100%;"><tr><td>2</td><td>7</td></tr></table>	2	7
2	7		
m	<table border="1" style="border-collapse: collapse; width: 100%;"><tr><td>3</td><td>8</td></tr></table>	3	8
3	8		
n	<table border="1" style="border-collapse: collapse; width: 100%;"><tr><td>4</td><td>9</td></tr></table>	4	9
4	9		
o	<table border="1" style="border-collapse: collapse; width: 100%;"><tr><td>5</td><td>10</td></tr></table>	5	10
5	10		
p	<table border="1" style="border-collapse: collapse; width: 100%;"><tr><td>11</td><td>12</td></tr></table>	11	12
11	12		
q	<table border="1" style="border-collapse: collapse; width: 100%;"><tr><td>13</td><td>14</td></tr></table>	13	14
13	14		
r	<table border="1" style="border-collapse: collapse; width: 100%;"><tr><td>15</td><td>16</td></tr></table>	15	16
15	16		

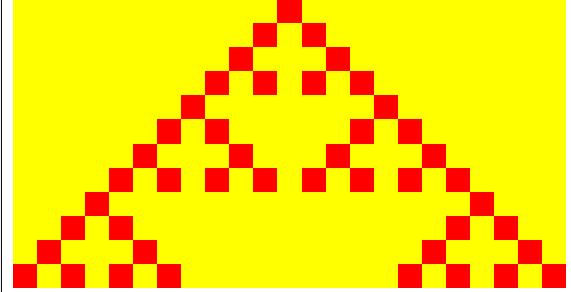
width	<table border="1" style="border-collapse: collapse; width: 100%;"><tr><td>2</td><td>3</td><td>4</td><td>5</td></tr></table>	2	3	4	5
2	3	4	5		

mesh	<input type="checkbox"/>
------	--------------------------

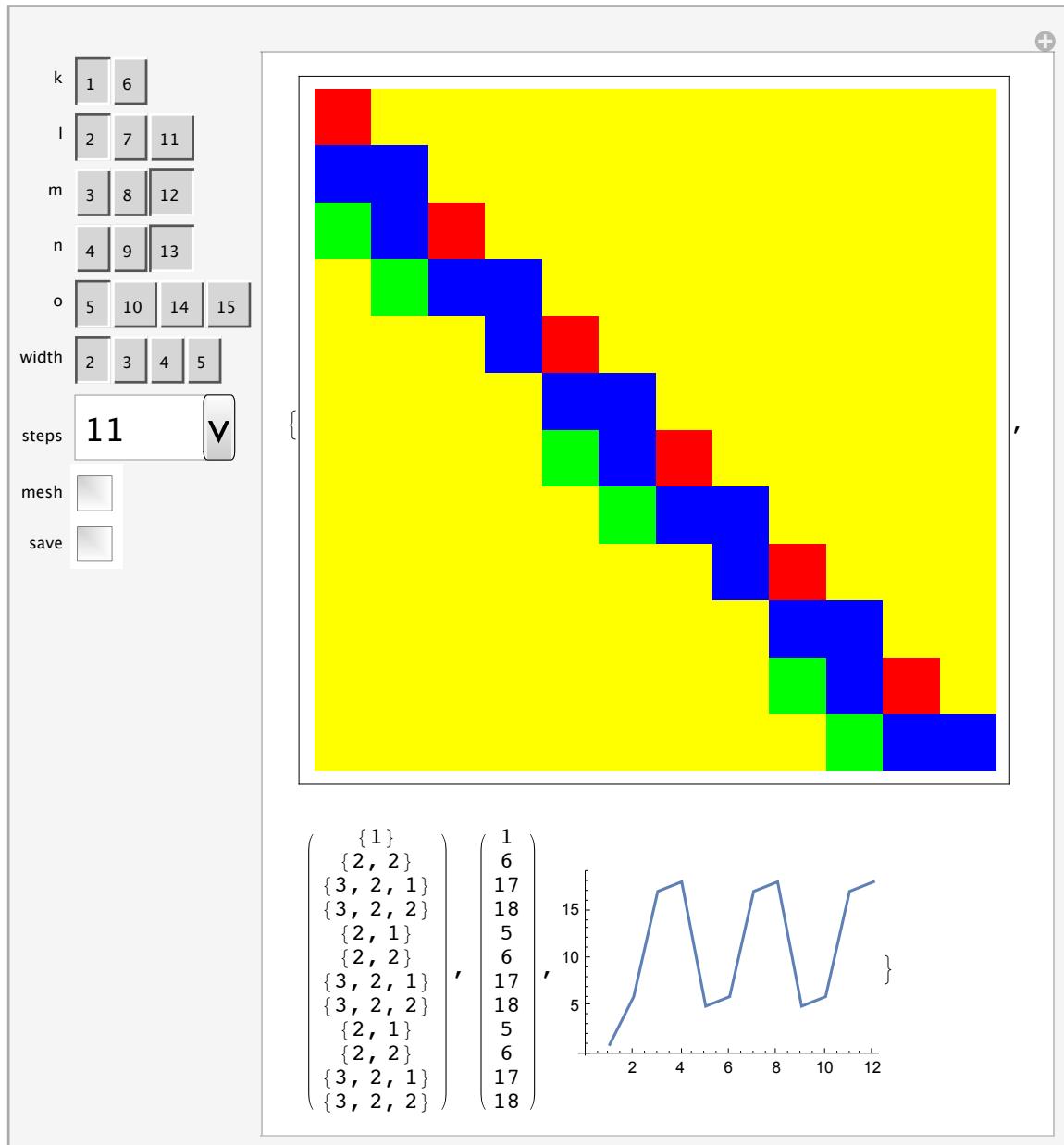
  

steps	11	<input type="button" value="▼"/>
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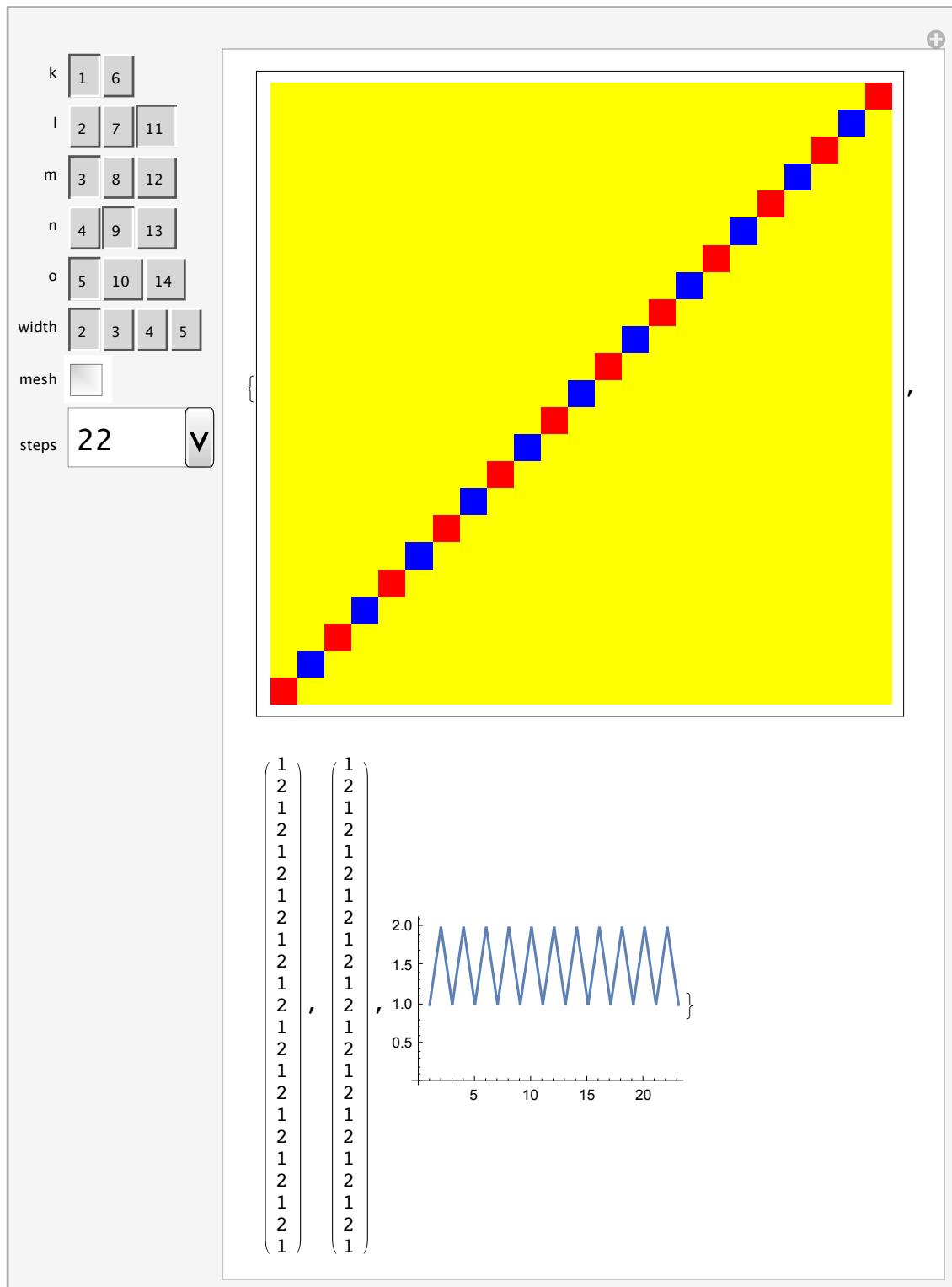


```
{
  {1},
  {1, 0, 1},
  {1, 0, 0, 0, 1},
  {1, 0, 1, 0, 1, 0, 1},
  {1, 0, 0, 0, 0, 0, 0, 0, 1},
  {1, 0, 1, 0, 0, 0, 0, 1, 0, 1},
  {1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1},
  {1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1},
  {1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1},
  {1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1},
  {1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1}
}
```

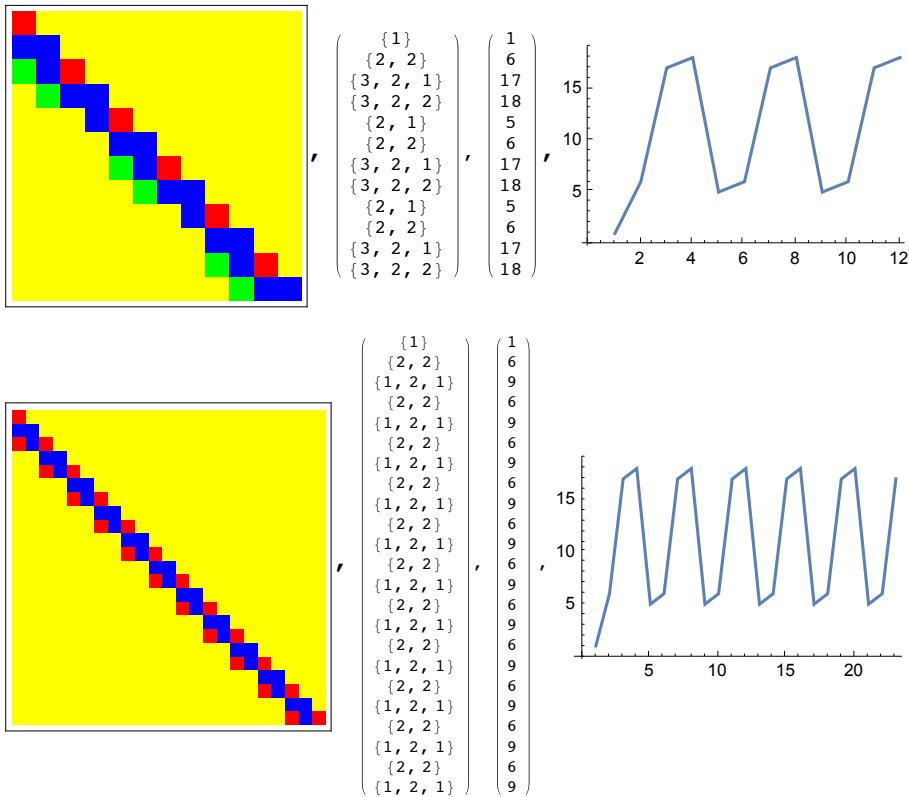
### Sub - rule morphoDM



**Sub – rule morphoDCM<sup>(3,3)</sup>**



A trivial difference between morphoDM and morphoDCM for rule[1, 2, 12, 13, 5]



Reduction example: morphoCA<sup>(5,3,4)</sup>  $\Rightarrow$  ECA<sup>(2,2)</sup>

```
ruleDM[{1, 11, 3, 13, 15}] =
  {{1}, {2, 0, 2}, {1, 0, 2, 0, 1}, {2, 0, 3, 0, 3, 0, 2}, {1, 0, 1, 0, 3, 0, 1, 0, 1}, {2, 0, 1, 0, 2, 0, 2, 0, 1, 0, 2}, {1, 0, 3, 0, 3, 0, 2, 0, 3, 0, 3, 0, 1}, {2, 0, 2, 0, 3, 0, 1, 0, 1, 0, 3, 0, 2, 0, 2}, {1, 0, 2, 0, 1, 0, 2, 0, 1, 0, 2, 0, 1, 0, 1}, {2, 0, 3, 0, 3, 0, 3, 0, 3, 0, 3, 0, 3, 0, 2}, {1, 0, 1, 0, 3, 0, 3, 0, 3, 0, 3, 0, 3, 0, 1, 0, 1}, {2, 0, 1, 0, 2, 0, 3, 0, 3, 0, 3, 0, 3, 0, 2, 0, 1, 0, 2}}]
  \Downarrow_{2,3\rightarrow1}
  {{1}, {1, 0, 1}, {1, 0, 1, 0, 1}, {1, 0, 1, 0, 1, 0, 1}, {1, 0, 1, 0, 1, 0, 1, 0, 1}, {1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1}, {1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1}, {1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1}}
= ruleECA[{1, 7, 3, 4, 5, 11, 13, 15}]
```

Reduction :

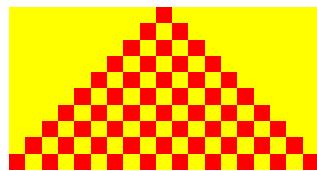
```

ruleDM : {[{1, 7, 3, 13, 5}], [{1, 7, 3, 13, 14}], [{1, 7, 3, 13, 15}],
           [{1, 11, 3, 4, 5}], [{1, 11, 3, 4, 14}], [{1, 11, 3, 4, 15}],
           [{1, 11, 3, 13, 5}], [{1, 11, 3, 13, 14}], [{1, 11, 3, 13, 15}],
           [{1, 7, 3, 4, x}]}

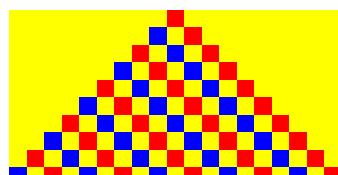
⇒2,3→1
ruleECA[{1, 7, 3, 4, 5, 11, 13, 15}]

```

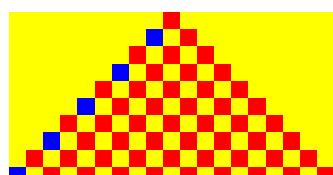
$$\text{ruleDM}[\{1, 7, 3, 4, x\}] = \left( \begin{array}{c} \{1\} \\ \{1, 0, 1\} \\ \{1, 0, 1, 0, 1\} \\ \{1, 0, 1, 0, 1, 0, 1\} \\ \{1, 0, 1, 0, 1, 0, 1, 0, 1\} \\ \{1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1\} \\ \{1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1\} \\ \{1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1\} \\ \{1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1\} \end{array} \right),$$



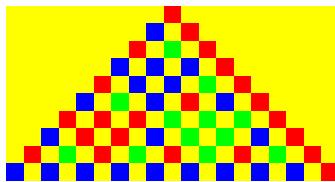
$$\text{ruleDM}[\{1, 11, 3, 4, 5\}] = \left( \begin{array}{c} \{1\} \\ \{2, 0, 1\} \\ \{1, 0, 2, 0, 1\} \\ \{2, 0, 1, 0, 2, 0, 1\} \\ \{1, 0, 2, 0, 1, 0, 2, 0, 1\} \\ \{2, 0, 1, 0, 2, 0, 1, 0, 2, 0, 1\} \\ \{1, 0, 2, 0, 1, 0, 2, 0, 1, 0, 2, 0, 1\} \\ \{2, 0, 1, 0, 2, 0, 1, 0, 2, 0, 1, 0, 2, 0, 1\} \\ \{1, 0, 2, 0, 1, 0, 2, 0, 1, 0, 2, 0, 1, 0, 2, 0, 1\} \\ \{2, 0, 1, 0, 2, 0, 1, 0, 2, 0, 1, 0, 2, 0, 1, 0, 2, 0, 1\} \end{array} \right),$$



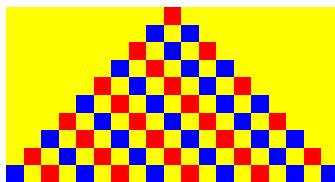
$$\text{ruleDM}[\{1, 11, 3, 4, 14\}] = \left( \begin{array}{c} \{1\} \\ \{2, 0, 1\} \\ \{1, 0, 1, 0, 1\} \\ \{2, 0, 1, 0, 1, 0, 1\} \\ \{1, 0, 1, 0, 1, 0, 1, 0, 1\} \\ \{2, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1\} \\ \{1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1\} \\ \{2, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1\} \\ \{1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1\} \\ \{2, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1\} \end{array} \right),$$



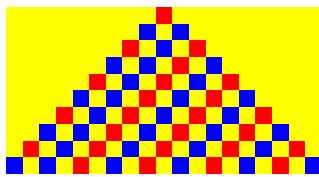
$$\text{ruleDM}[\{1, 11, 3, 4, 15\}] = \left( \begin{array}{c} \{1\} \\ \{2, 0, 1\} \\ \{1, 0, 3, 0, 1\} \\ \{2, 0, 2, 0, 2, 0, 1\} \\ \{1, 0, 2, 0, 2, 0, 3, 0, 1\} \\ \{2, 0, 3, 0, 2, 0, 1, 0, 2, 0, 1\} \\ \{1, 0, 1, 0, 1, 0, 3, 0, 3, 0, 3, 0, 1\} \\ \{2, 0, 1, 0, 1, 0, 2, 0, 3, 0, 3, 0, 2, 0, 1\} \\ \{1, 0, 3, 0, 1, 0, 3, 0, 1, 0, 3, 0, 1, 0, 3, 0, 1\} \\ \{2, 0, 2, 0, 2, 0, 2, 0, 2, 0, 2, 0, 2, 0, 2, 0, 1\} \end{array} \right),$$



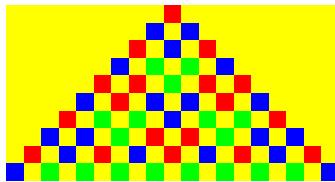
$$\text{ruleDM}[\{1, 11, 3, 13, 5\}] = \left( \begin{array}{c} \{1\} \\ \{2, 0, 2\} \\ \{1, 0, 2, 0, 1\} \\ \{2, 0, 1, 0, 2, 0, 2\} \\ \{1, 0, 2, 0, 1, 0, 2, 0, 1\} \\ \{2, 0, 1, 0, 2, 0, 1, 0, 2, 0, 2\} \\ \{1, 0, 2, 0, 1, 0, 2, 0, 1, 0, 2, 0, 1\} \\ \{2, 0, 1, 0, 2, 0, 1, 0, 2, 0, 1, 0, 2, 0, 2\} \\ \{1, 0, 2, 0, 1, 0, 2, 0, 1, 0, 2, 0, 1, 0, 2, 0, 1\} \\ \{2, 0, 1, 0, 2, 0, 1, 0, 2, 0, 1, 0, 2, 0, 1, 0, 2, 0, 2\} \end{array} \right),$$



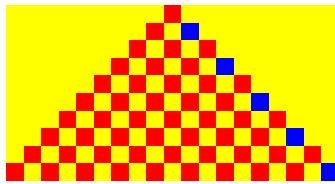
$$\text{ruleDM}[\{1, 11, 3, 13, 14\}] = \left( \begin{array}{c} \{1\} \\ \{2, 0, 2\} \\ \{1, 0, 2, 0, 1\} \\ \{2, 0, 2, 0, 1, 0, 2\} \\ \{1, 0, 2, 0, 1, 0, 2, 0, 1\} \\ \{2, 0, 2, 0, 1, 0, 2, 0, 1, 0, 2\} \\ \{1, 0, 2, 0, 1, 0, 2, 0, 1, 0, 2, 0, 1\} \\ \{2, 0, 2, 0, 1, 0, 2, 0, 1, 0, 2, 0, 1, 0, 2\} \\ \{1, 0, 2, 0, 1, 0, 2, 0, 1, 0, 2, 0, 1, 0, 2, 0, 1\} \\ \{2, 0, 2, 0, 1, 0, 2, 0, 1, 0, 2, 0, 1, 0, 2, 0, 1, 0, 2\} \end{array} \right),$$



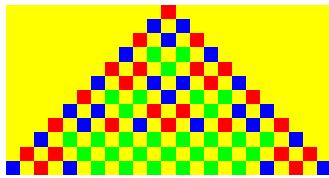
$$\text{ruleDM}[\{1, 11, 3, 13, 15\}] = \left( \begin{array}{c} \{1\} \\ \{2, 0, 2\} \\ \{1, 0, 2, 0, 1\} \\ \{2, 0, 3, 0, 3, 0, 2\} \\ \{1, 0, 1, 0, 3, 0, 1, 0, 1\} \\ \{2, 0, 1, 0, 2, 0, 2, 0, 1, 0, 2\} \\ \{1, 0, 3, 0, 3, 0, 2, 0, 3, 0, 3, 0, 1\} \\ \{2, 0, 2, 0, 3, 0, 1, 0, 1, 0, 3, 0, 2, 0, 2\} \\ \{1, 0, 2, 0, 1, 0, 2, 0, 1, 0, 2, 0, 1, 0, 2, 0, 1\} \\ \{2, 0, 3, 0, 3, 0, 3, 0, 3, 0, 3, 0, 3, 0, 2\} \end{array} \right),$$



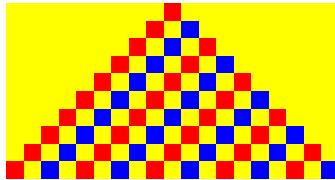
$$\text{ruleDM}[\{1, 7, 3, 13, 5\}] = \left( \begin{array}{c} \{1\} \\ \{1, 0, 2\} \\ \{1, 0, 1, 0, 1\} \\ \{1, 0, 1, 0, 1, 0, 2\} \\ \{1, 0, 1, 0, 1, 0, 1, 0, 1\} \\ \{1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 2\} \\ \{1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1\} \\ \{1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 2\} \\ \{1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1\} \\ \{1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1\} \end{array} \right),$$



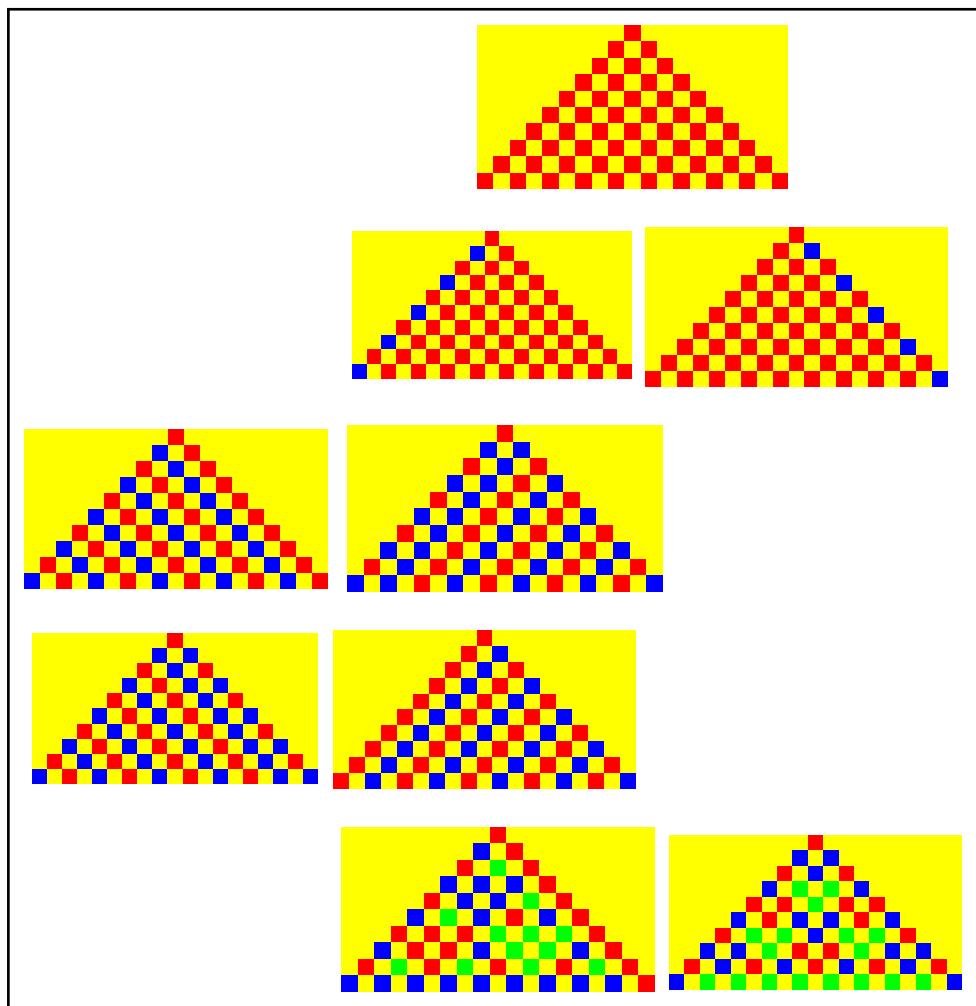
$$\text{ruleDM}[\{1, 7, 3, 13, 15\}] = \left( \begin{array}{c} \{1\} \\ \{1, 0, 2\} \\ \{1, 0, 3, 0, 1\} \\ \{1, 0, 2, 0, 2, 0, 2\} \\ \{1, 0, 3, 0, 2, 0, 2, 0, 1\} \\ \{1, 0, 2, 0, 1, 0, 2, 0, 3, 0, 2\} \\ \{1, 0, 3, 0, 3, 0, 3, 0, 1, 0, 1, 0, 1\} \\ \{1, 0, 2, 0, 3, 0, 3, 0, 2, 0, 1, 0, 1, 0, 2\} \\ \{1, 0, 3, 0, 1, 0, 3, 0, 1, 0, 3, 0, 1, 0, 3, 0, 1\} \\ \{1, 0, 2, 0, 2, 0, 2, 0, 2, 0, 2, 0, 2, 0, 2, 0, 2\} \end{array} \right),$$



$$\text{ruleDM}[\{1, 7, 3, 13, 14\}] = \left( \begin{array}{c} \{1\} \\ \{1, 0, 2\} \\ \{1, 0, 2, 0, 1\} \\ \{1, 0, 2, 0, 1, 0, 2\} \\ \{1, 0, 2, 0, 1, 0, 2, 0, 1\} \\ \{1, 0, 2, 0, 1, 0, 2, 0, 1, 0, 2\} \\ \{1, 0, 2, 0, 1, 0, 2, 0, 1, 0, 2, 0, 1\} \\ \{1, 0, 2, 0, 1, 0, 2, 0, 1, 0, 2, 0, 1, 0, 2\} \\ \{1, 0, 2, 0, 1, 0, 2, 0, 1, 0, 2, 0, 1, 0, 2, 0, 1\} \end{array} \right),$$

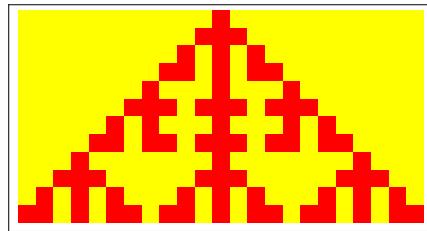


**Alternating system of differentiation and reduction**

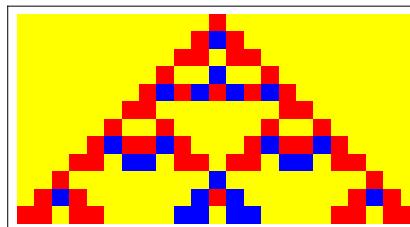


## Comparisons

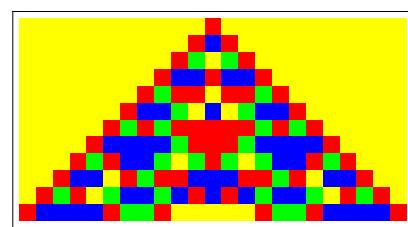
`ruleDM[{1, 7, 8, 4, x}], ECA150`



`ruleDM[{1, 7, 12, 4, 10}]`

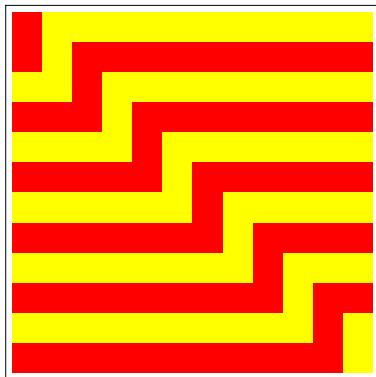


`ruleDM[{1, 7, 12, 4, 15}]`

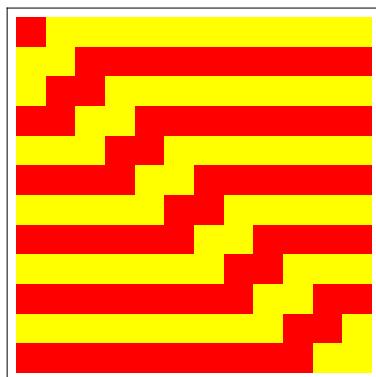


`ruleDM[{6, 7, 8, 9, 15}]`

`ruleDM[{6, 7, 3, 9, 15}]`

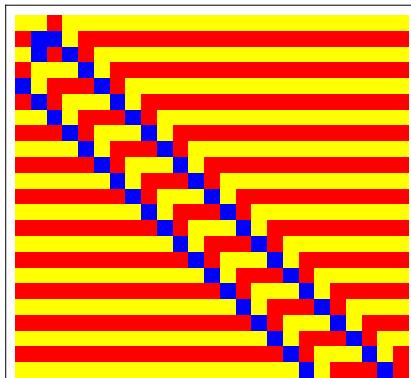


ruleDM[{6, 11, 12, 9, 5}]

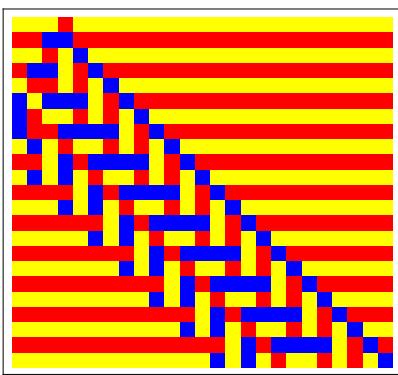


ruleDM[{6, 11, 12, 9, 10}]

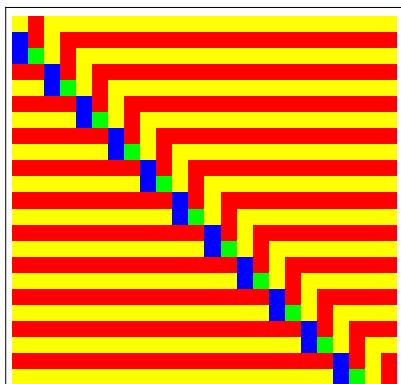
ruleDM[{6, 11, 12, 4, 10}]



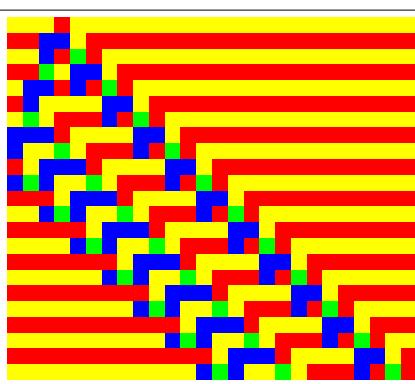
ruleDM[{6, 11, 8, 9, 15}]



ruleDM[{6, 11, 12, 9, 15}]



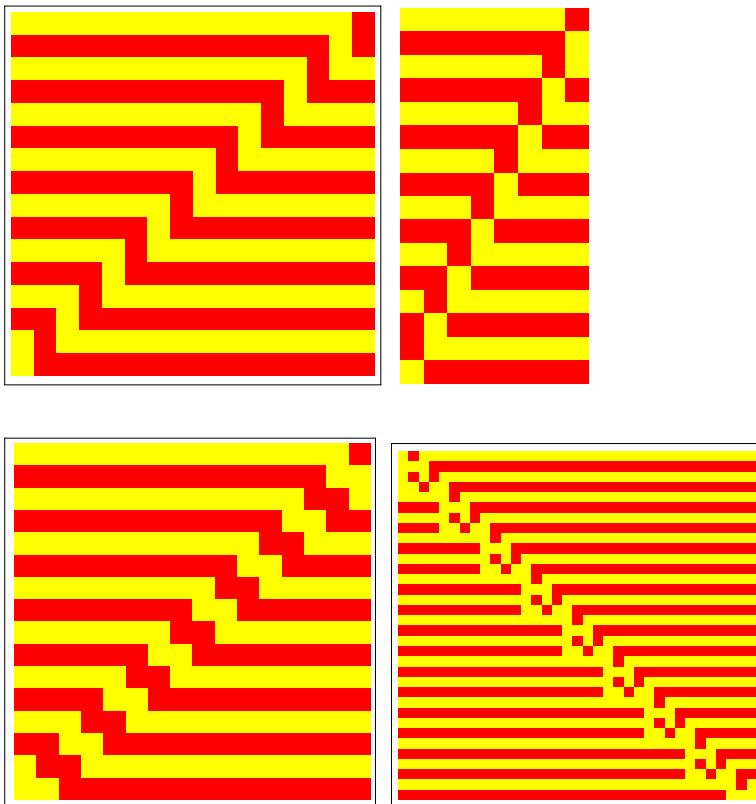
ECA 170



ECA 202

ECA 138

ECA 148



### Analogs to code 420, NKS p.63

*"In detail, some of the patterns are definitely more complicated than those seen in elementary rules. But at the level of overall behavior, there are no fundamental differences. [...]*

*The only new structure not seen in elementary rules is the one in code 420 - but this occurs only quite rarely."* (NKS, p. 65)

Interestingly, the difference of code 420 to ECA rule is an internal difference of the produced patterns and is not considering the underlying architectonic design.

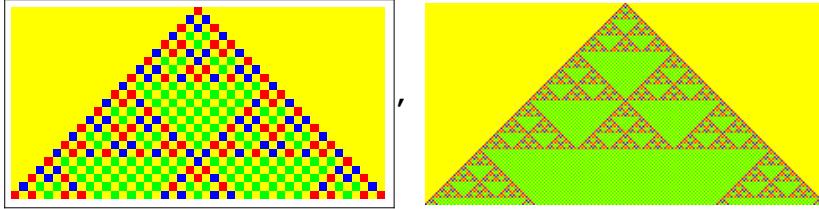
Therefore, many examples of morphoCA<sup>(3,4)</sup> rules are different from ECA patterns albeit they are still defined in the classical architecture.

This holds for `ruleDM[{1, 11, 3, 13, 15}]`, `ruleDM[{1, 7, 3, 13, 15}]` and the rules `ruleDCM[{1,11,12,13,14}]`, `ruleDCM[{1,2,12,13,14}]` that are analogons of *code 420*.

Hence, there are at least 4 deviant forms in morphoCA. It turns out that, in this context, most patterns of morphic CAs are non-conventional.

CAs defined by the DCKV-rules are set in a different architecture and are thus producing internally and architectonically different patterns.

**`ruleDM[{1, 11, 3, 13, 15}]`**

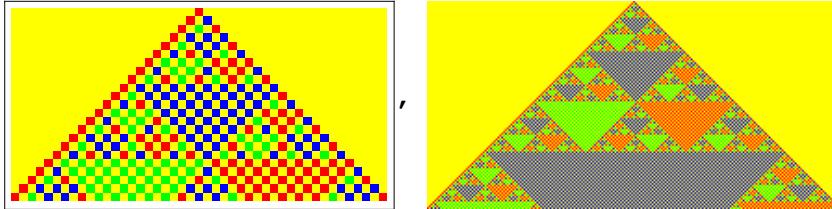


```

    {1}
    {2, 0, 2}
    {1, 0, 2, 0, 1}
    {2, 0, 3, 0, 3, 0, 2}
    {1, 0, 1, 0, 3, 0, 1, 0, 1}
    {2, 0, 1, 0, 2, 0, 2, 0, 1, 0, 2}
    {1, 0, 3, 0, 3, 0, 2, 0, 3, 0, 3, 0, 1}
    {2, 0, 2, 0, 3, 0, 1, 0, 1, 0, 3, 0, 2, 0, 2}
    {1, 0, 2, 0, 1, 0, 2, 0, 1, 0, 2, 0, 1, 0, 2, 0, 1}
    {2, 0, 3, 0, 3, 0, 3, 0, 3, 0, 3, 0, 3, 0, 3, 0, 2}
    {1, 0, 1, 0, 3, 0, 3, 0, 3, 0, 3, 0, 3, 0, 3, 0, 1, 0, 1}
    {2, 0, 1, 0, 2, 0, 3, 0, 3, 0, 3, 0, 3, 0, 3, 0, 2, 0, 1, 0, 2}
    {1, 0, 3, 0, 3, 0, 1, 0, 3, 0, 3, 0, 1, 0, 3, 0, 3, 0, 1, 0, 1}
    {2, 0, 2, 0, 3, 0, 2, 0, 2, 0, 3, 0, 3, 0, 3, 0, 2, 0, 2, 0, 3, 0, 2}
    {1, 0, 2, 0, 1, 0, 1, 0, 2, 0, 1, 0, 3, 0, 3, 0, 3, 0, 1, 0, 2, 0, 1, 0, 1}
    {2, 0, 3, 0, 3, 0, 1, 0, 3, 0, 2, 0, 3, 0, 3, 0, 2, 0, 3, 0, 3, 0, 2}
    {1, 0, 1, 0, 3, 0, 2, 0, 2, 0, 1, 0, 1, 0, 3, 0, 1, 0, 2, 0, 1, 0, 1, 0, 2, 0, 1}
    {2, 0, 1, 0, 2, 0, 1, 0, 2, 0, 1, 0, 1, 0, 3, 0, 1, 0, 2, 0, 2, 0, 1, 0, 1, 0, 1}
    {2, 0, 1, 0, 2, 0, 1, 0, 2, 0, 1, 0, 1, 0, 3, 0, 1, 0, 2, 0, 2, 0, 1, 0, 2, 0, 1}
    {1, 0, 3, 0, 3, 0, 3, 0, 3, 0, 3, 0, 3, 0, 3, 0, 3, 0, 3, 0, 3, 0, 1}
    {2, 0, 2, 0, 3, 0, 3, 0, 3, 0, 3, 0, 3, 0, 3, 0, 3, 0, 3, 0, 3, 0, 2}
    {1, 0, 2, 0, 1, 0, 3, 0, 3, 0, 3, 0, 3, 0, 3, 0, 3, 0, 3, 0, 3, 0, 1, 0, 2, 0, 1, 0, 1}
    {2, 0, 3, 0, 3, 0, 2, 0, 3, 0, 3, 0, 3, 0, 3, 0, 3, 0, 3, 0, 3, 0, 2, 0, 3, 0, 3, 0, 2}
    {1, 0, 1, 0, 3, 0, 1, 0, 3, 0, 3, 0, 3, 0, 3, 0, 3, 0, 3, 0, 3, 0, 1, 0, 1, 0, 3, 0, 1, 0, 1}

```

**ruleDM[{1, 7, 3, 13, 15}]**

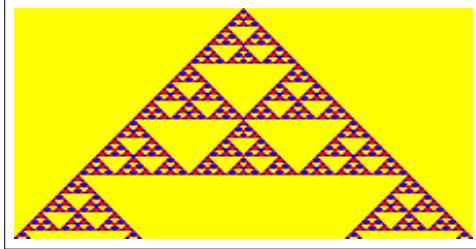


```

    {1}
    {1, 0, 2}
    {1, 0, 3, 0, 1}
    {1, 0, 2, 0, 2, 0, 2}
    {1, 0, 3, 0, 2, 0, 2, 0, 1}
    {1, 0, 2, 0, 1, 0, 2, 0, 3, 0, 2}
    {1, 0, 3, 0, 3, 0, 3, 0, 1, 0, 1, 0, 1}
    {1, 0, 2, 0, 3, 0, 3, 0, 2, 0, 1, 0, 1, 0, 2}
    {1, 0, 3, 0, 1, 0, 3, 0, 1, 0, 3, 0, 1, 0, 3, 0, 1}
    {1, 0, 2, 0, 2, 0, 2, 0, 2, 0, 2, 0, 2, 0, 2, 0, 2}
    {1, 0, 3, 0, 2, 0, 2, 0, 2, 0, 2, 0, 2, 0, 2, 0, 1}
    {1, 0, 2, 0, 1, 0, 2, 0, 2, 0, 2, 0, 2, 0, 2, 0, 2}
    {1, 0, 3, 0, 3, 0, 3, 0, 2, 0, 2, 0, 2, 0, 2, 0, 1, 0, 1}
    {1, 0, 2, 0, 3, 0, 3, 0, 1, 0, 2, 0, 2, 0, 2, 0, 2, 0, 1, 0, 2}
    {1, 0, 3, 0, 1, 0, 3, 0, 2, 0, 3, 0, 2, 0, 2, 0, 1, 0, 2, 0, 1}
    {1, 0, 2, 0, 2, 0, 1, 0, 1, 0, 1, 0, 2, 0, 2, 0, 2, 0, 3, 0, 2}
    {1, 0, 3, 0, 2, 0, 2, 0, 3, 0, 2, 0, 2, 0, 2, 0, 1, 0, 3, 0, 2, 0, 1}
    {1, 0, 2, 0, 1, 0, 2, 0, 2, 0, 1, 0, 2, 0, 3, 0, 2, 0, 3, 0, 2, 0, 2}
    {1, 0, 3, 0, 3, 0, 3, 0, 1, 0, 2, 0, 2, 0, 2, 0, 2, 0, 3, 0, 1, 0, 1, 0, 2}
    {1, 0, 2, 0, 3, 0, 3, 0, 3, 0, 3, 0, 3, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1}
    {1, 0, 3, 0, 1, 0, 3, 0, 3, 0, 3, 0, 3, 0, 3, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1}
    {1, 0, 2, 0, 2, 0, 2, 0, 3, 0, 3, 0, 3, 0, 3, 0, 3, 0, 2, 0, 2, 0, 2, 0, 1, 0, 1, 0, 1, 0, 1, 0, 2}
    {1, 0, 3, 0, 2, 0, 2, 0, 1, 0, 3, 0, 3, 0, 3, 0, 3, 0, 3, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1}

```

**morphoCA**<sup>(3,3)</sup> ruleDCM[{1, 11, 12, 13, 14}])



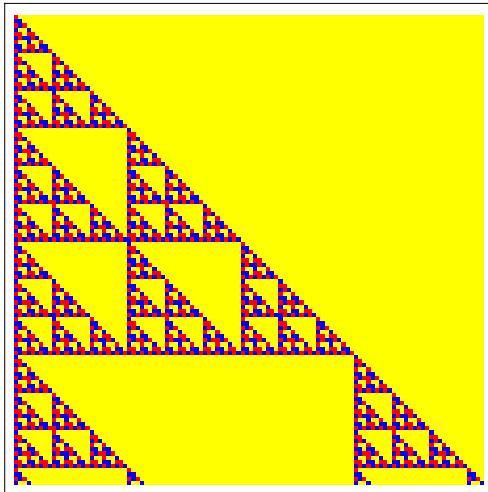
```

{1}
{2, 2, 2}
{1, 1, 2, 1, 1}
{2, 2, 0, 0, 2, 2}
{1, 1, 1, 0, 1, 1, 1}
{2, 2, 1, 1, 2, 2, 1, 1, 2, 2}
{1, 1, 0, 0, 0, 2, 0, 0, 0, 0, 1, 1}
{2, 2, 2, 0, 0, 1, 1, 0, 0, 0, 2, 2, 2}
{1, 1, 2, 2, 1, 1, 2, 1, 2, 2, 1, 1, 2, 2, 1, 1}
{2, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 2}
{1, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1}
{2, 2, 1, 1, 2, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 1, 1, 2, 2}
{1, 1, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 1, 1}
{2, 2, 2, 0, 0, 2, 2, 2, 0, 0, 0, 0, 0, 0, 0, 0, 2, 2, 2, 0, 0, 2, 2, 2}
{1, 1, 2, 2, 1, 1, 1, 1, 2, 1, 1, 0, 0, 0, 0, 1, 1, 2, 2, 1, 1, 1, 2, 2, 1, 1}
{2, 2, 0, 0, 0, 1, 1, 0, 0, 0, 0, 2, 2, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 2, 2}
{1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1}
{2, 2, 1, 1, 2, 2, 1, 1, 2, 2, 1, 1, 2, 2, 1, 1, 2, 2, 1, 1, 2, 2, 1, 1, 2, 2}
{1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1}
{2, 2, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 2}
{1, 1, 2, 2, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 2, 2, 1, 1}
{2, 2, 0, 0, 0, 0, 2, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 2, 0, 0, 0, 0, 2, 2}
{1, 1, 1, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 1, 1, 1, 1}

```

1
14
35
198
495
3698
6275
61 918
157 387
786 438
1 966 095
14 942 322
25 559 235
255 592 350
638 980 875
3 247 202 502
8 306 751 375
61 847 581 490
103 079 739 395
1 030 793 986 078
2 576 995 582 027
13 400 323 916 166
33 501 004 432 335

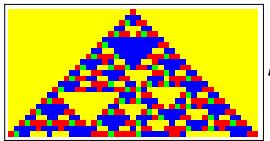
**morphoCA**<sup>(3,3)</sup> **ruleDCM**[{1, 2, 12, 13, 14}]



{1}	1
{2, 2}	6
{1, 2, 1}	9
{2, 0, 0, 2}	18
{1, 1, 0, 1, 1}	27
{2, 1, 2, 2, 1, 2}	108
{1, 0, 0, 2, 0, 0, 1}	81
{2, 2, 0, 1, 1, 0, 2, 2}	414
{1, 2, 1, 2, 1, 2, 1, 2, 1}	681
{2, 0, 0, 0, 0, 0, 0, 0, 2}	1026
{1, 1, 0, 0, 0, 0, 0, 0, 0, 1}	1539
{2, 1, 2, 0, 0, 0, 0, 0, 0, 2, 1, 2}	6156
{1, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 1}	4617
{2, 2, 0, 2, 2, 0, 0, 0, 0, 2, 0, 2, 0, 2}	27702
{1, 2, 1, 1, 2, 1, 0, 0, 0, 1, 2, 1, 1, 2, 1}	41553
{2, 0, 0, 1, 0, 0, 2, 0, 0, 2, 0, 0, 1, 0, 0, 2}	70794
{1, 1, 0, 2, 2, 0, 1, 1, 0, 1, 1, 0, 2, 2, 0, 1, 1}	124659
{2, 1, 2, 1, 2, 1, 2, 1, 2, 2, 1, 2, 1, 2, 1, 2, 1, 2}	437076
{1, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 1}	263169
{2, 2, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 2, 2}	1574406
{1, 2, 1, 0, 0, 0, 0, 0, 0, 2, 1, 2, 0, 0, 0, 0, 0, 1, 2, 1}	2365449
{2, 0, 0, 2, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 2, 0, 0, 2}	4723218
{1, 1, 0, 1, 1, 0, 0, 0, 0, 2, 2, 0, 2, 2, 0, 0, 0, 1, 1, 0, 1, 1}	7105563

## A further slightly deviant analog to code 420

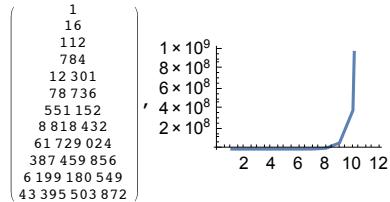
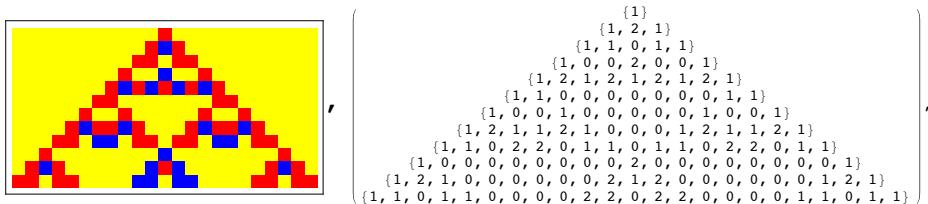
```
ruleDM[{1, 11, 12, 13, 15}]]
```



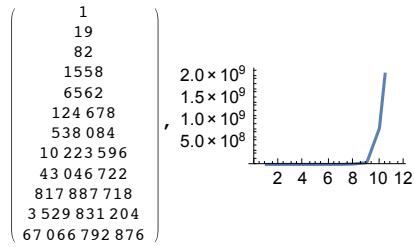
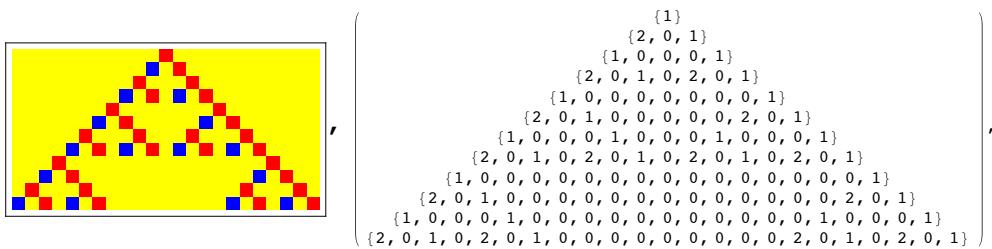
```
morphoCA(3,3) ruleDM[{1, 11, 12, 13, 14}]]
```

## Examples

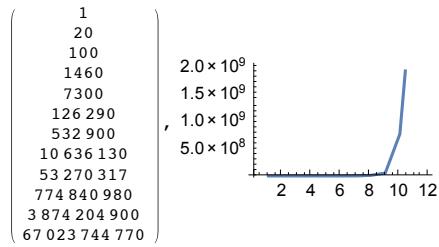
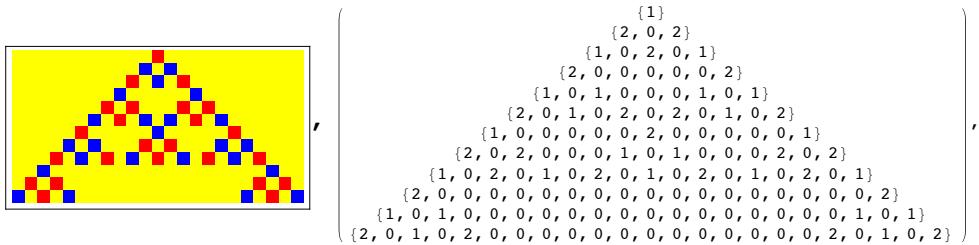
```
ruleDM[{1, 7, 12, 4, 10}]
```



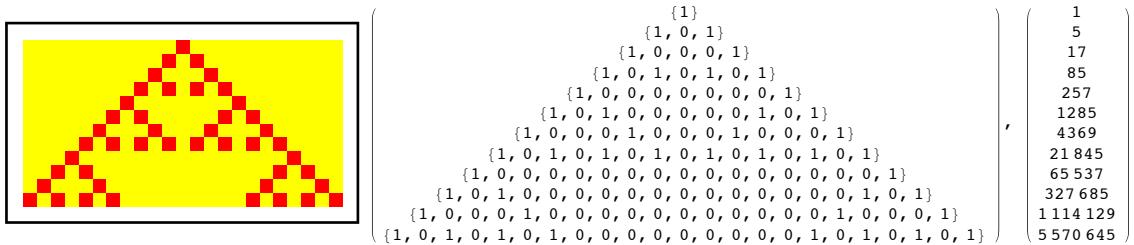
ruleDM[{1, 11, 3, 4, 10}]



ruleDM[{1, 11, 3, 13, 10}]



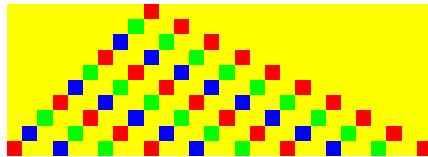
Contrast ECA rule 90



### Reduction in morphoDCKV

```
ruleDCKV[{1111, 1123, ...}] :
```

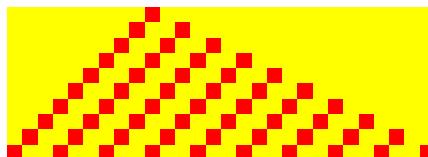
$$\left( \begin{array}{c} \{1\} \\ \{3, 0, 0, 1\} \\ \{2, 0, 0, 3, 0, 0, 1\} \\ \{1, 0, 0, 2, 0, 0, 3, 0, 0, 1\} \\ \{3, 0, 0, 1, 0, 0, 2, 0, 0, 3, 0, 0, 1\} \\ \{2, 0, 0, 3, 0, 0, 1, 0, 0, 2, 0, 0, 3, 0, 0, 1\} \\ \{1, 0, 0, 2, 0, 0, 0, 1, 0, 0, 2, 0, 0, 3, 0, 0, 1\} \\ \{3, 0, 0, 1, 0, 0, 2, 0, 0, 3, 0, 0, 1, 0, 0, 2, 0, 0, 3, 0, 0, 1\} \\ \{2, 0, 0, 3, 0, 0, 1, 0, 0, 2, 0, 0, 3, 0, 0, 1, 0, 0, 2, 0, 0, 3, 0, 0, 1\} \\ \{1, 0, 0, 2, 0, 0, 3, 0, 0, 1, 0, 0, 2, 0, 0, 3, 0, 0, 1, 0, 0, 2, 0, 0, 3, 0, 0, 1\} \end{array} \right),$$



$\Downarrow_{2,3 \rightarrow 1}$

```
ruleDCKV[{1111, 1122, ...}] :
```

$$\left( \begin{array}{c} \{1\} \\ \{1, 0, 0, 1\} \\ \{1, 0, 0, 1, 0, 0, 1\} \\ \{1, 0, 0, 1, 0, 0, 1, 0, 0, 1\} \\ \{1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1\} \\ \{1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1\} \\ \{1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1\} \\ \{1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1\} \\ \{1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1\} \\ \{1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1\} \end{array} \right),$$



### Examples for morphoCA<sup>(5,5,5)</sup>

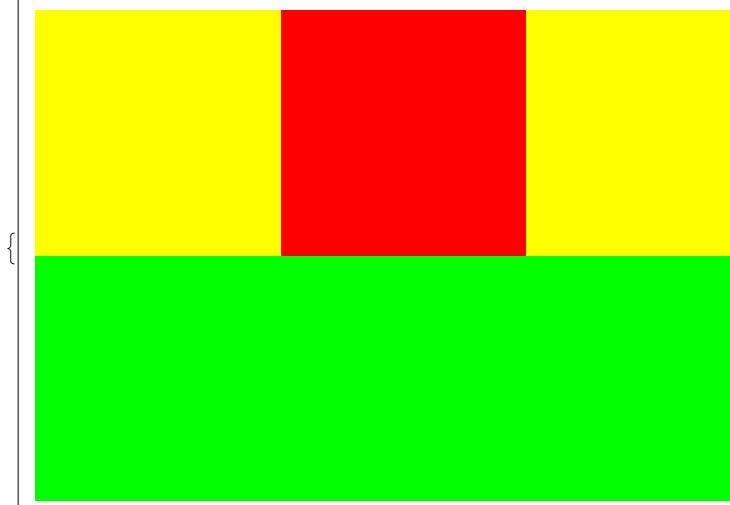
#### Requisites

#### Procedures morphoDCKV

#### Rules

(Debug) Out[3582]=

a	1111	1112			
b	1120	1121	1122	1123	
c	1210	1211	1212	1213	
d	1220	1221	1222	1223	
e	2120	2121	2122	2123	
f	2210	2211	2212	2213	
g	2220	2221	2222	2223	
h	2110	2111	2112	2113	
i	2130	2131	2132	2133	2134
j	1230	1231	1232	1233	1234
k	2230	2231	2232	2233	2234
l	2310	2311	2312	2313	2314
m	2320	2321	2322	2323	2324
n	2330	2331	2332	2333	2334
o	2340	2341	2342	2343	2344
width	2	3	4	5	
mesh	<input type="checkbox"/>				
steps	1	<input type="button" value="▼"/>			



## Examples

