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Rudolf Kaehr
(1942-2016)

Title

The Abacus of Universal Logics
Tabular Positionality and Tabular Morphogrammatics

Archive-Number / Categories

2_32 / K09

Publication Date

2007

Keywords

morphogrammatics, place-designator, polycontexturality, logic,

Disciplines

Cybernetics, Artificial Intelligence and Robotics, Logic and Foundations of Mathematics, Theory of Science, Philosophy of Science

Abstract

Forget about the tedious problems of combinatorial analysis of place-valued logics.

What is the real impact? And why is it so difficult to understand it?

It is very difficult to understand Günther’s approach because of endless confusions of it with other scientific trends, like many-valuedness, dialectics, deviant logics, etc.

The conceptual approach of place-valued logics is easy to understand, but nearly impossible to be accepted by mathematicians, philosophers and logicians.

In a subversive step of arithmetizing logic and logifying arithmetic, Günther revolutionized the old Chinese/Indian concept of Zero and positionality to a mechanism of distribution and mediation of logical systems, and later of formal systems in general.

Citation Information / How to cite

Rudolf Kaehr: "The Abacus of Universal Logics", www.vordenker.de (Sommer Edition, 2017) J. Paul (Ed.),
URL: http://www.vordenker.de/rk/rk_The-Abacus-of-Universal-Logics_2007.pdf

DERRIDA'S MACHINES PART IV

BYTES & PIECES

of

PolyLogics, m-Lambda Calculi, ConTeXtures, Morphogrammatics

The Abacus of Universal Logics

A. Tabular Positionality

B. Tabular Morphogrammatics

© by Rudolf Kaehr

ThinkArt Lab Glasgow 2007

"Interactivity is all there is to write about:

it is the paradox and
the horizon of realization."

The Abacus of Universal Logics

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A. Tabular Positionality

1 A revolution in logic?

Forget about the tedious problems of combinatorial analysis of place-valued logics. What is the real impact? And why is it so difficult to understand it?

It is very difficult to understand Gunther's approach because of endless confusions of it with other scientific trends, like many-valuedness, dialectics, deviant logics, etc.

The conceptual approach of place-valued logics is easy to understand, but nearly impossible to be accepted by mathematicians, philosophers and logicians.

In a subversive step of arithmetizing logic and logifying arithmetic, Gunther revolutionized the old Chinese / Indian concept of Zero and positionality to a mechanism of distribution and mediation of logical systems, and later of formal systems in general.

As we know, without the positionality system and its cipher Zero the whole Western science, technology and business wouldn't exist. On the other hand, without Western alphabetism the modern positional numeration (the zero and the place-value system) couldn't have such a historic impact on technology and society in general.

> "Therefore, albeit the Hindus perfected one of the greatest discoveries in human history -- the zero, they could not realize its cosmic function as a mathematical tool of science."

Gunther's approach is unseen subversivity! Never happend in the last 5000 years.

A concept, valuable inside a theory, i.e., in arithmetic is used / abused to place full logical theories in a distribution instead of numeral in a positional arithmetic. The part is treated as a whole and moved from arithmetic to the logical sphere.

Forget about the tedious problems of combinatorial analysis of place-valued logics and all the ambitious philosophical interpretations.

What is the real impact of Gunther's approach? And why is it so difficult to understand? What was the crazyness that Rowena Swanson was so much intrigued?

Linear positionality

gegenseitiger Abhängigkeit sich befinden. *Eine mehrwertige Logik beschreibt ein solches Abhängigkeitssystem der möglichen Stellenwerte, die die klassische Logik in dem Reflexionssystem unseres Bewußtseins einnehmen kann.*

Das soll an dem einfachen Beispiel des binarischen Zahlensystems erläutert werden. Die einzige dabei gebrauchte positive Ziffer „1" hat eine doppelte Bedeutung. Erstens als Einheit und zweitens als Quantität, je nach ihrem Stellenwert. Wenn wir also schreiben:

0	=	0	110	=	6
1	=	1	111	=	7
10	=	2	1 000	=	8
11	=	3	1 001	=	9
100	=	4	1 010	=	10
101	=	5	1 011	=	11

usw.

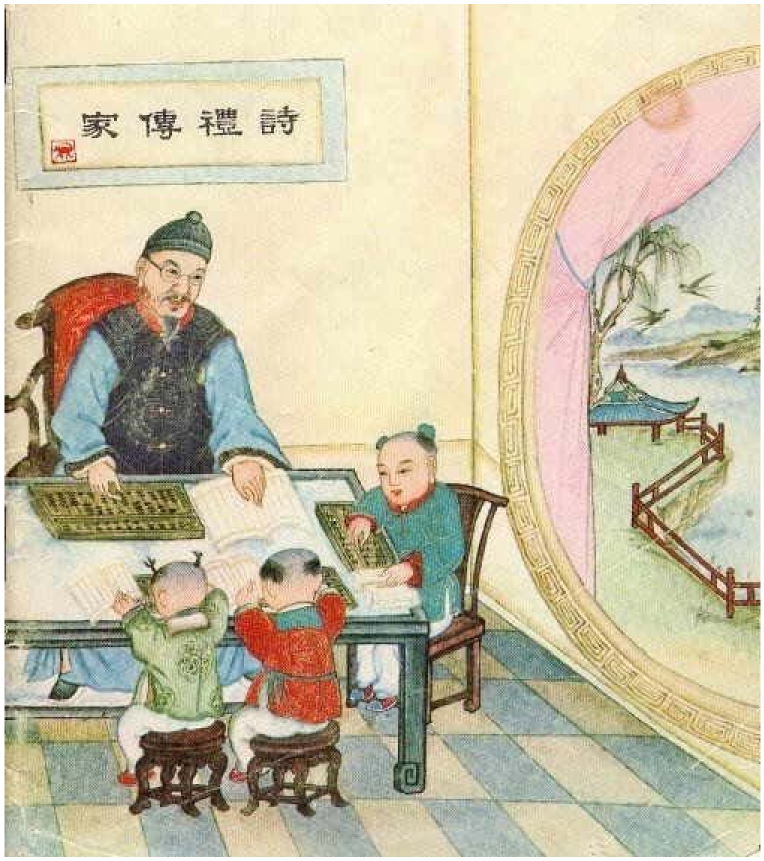
so ist es immer dieselbe identische „1", die sich in verschiedenen Stellen mit verschiedenen (quantitativen) Bedeutungen wiederholt. Eine „1" an der ersten Stelle bedeutet 1; an der vierten Stelle aber bedeutet dieselbe Ziffer 8.

Eine mehrwertige Logik ist nun nichts anderes als ein System, das uns erlaubt, unserer einzigen, „wirklichen" Logik verschiedene Stellenwerte im System des Bewußtseins derart zu geben, daß jeder Stellenwert mit einer verschiedenen semantischen Bedeutung des sich so wiederholenden zweiwertigen Kalküls verbunden ist. Ein solches mehrwertiges System erlaubt dann den strukturellen *Zusammenhang* der verschiedenen zweiwertigen Erlebnisstufen des Bewußtseins abzulesen.

In his paper *"Die Aristotelische Logik des Seins und die nicht-Aritotelische Logik der Reflexion"*, Gunther has given an exposition of the results of his research about a logic of reflection in such a concise and clear way that it is nearly impossible to no to understand his approach. But this exactly was the obstacle. How can we mix logic with the positional system of arithmetic? And how can we succed, later, from linearity to tabularity of a kenogrammatic positionality system?

Place-valued logics around Cybernetic Ontology, the BCL and AFOSR
http://www.thinkartlab.com/pkl/lola/AFOSR-Place-Valued-Logic.pdf

The concept of zero was conceived by the Chinese then improved on by Hindus



http://www.joernluetjens.de/sammlungen/abakus/abakus-en.htm

"They [the Chinese] then invented symbols for the content of each column to replace drawing a picture of the number of beads. Having developed symbols to express the content of each column, they had to invent a symbol for the numberless content of the empty column -- that symbol came to be known to the Hindus as "sunya", and sunya later became "sifr" in Arabic; "cifra" in Roman; and finally "cipher" in English.

Only an empty column of an abacus could possibly provide the human experience that called for the invention of the zero -- the symbol for "nothingness", and that discovery of the symbol for nothingness had an enormous significance upon subsequent humanity."
http://www.gupshop.com/print-view.php?t=2132&start=0&sid=1beb2b527b74aed4b36849721179c42b

A better understanding of Gunther's approach can be found in the fact of Gunther's early studies of Sanskrit and Chinese. I propose that it had a much more profound influence on his "unconscious" writing than anything consciously declared as Hegelian. Also an expert in German idealism, his interpretation of Hegel's Logik as a positional system of thought was in fact a departure from traditional Western philosophy.

As a consequence Gunther had to invent a new kind of zero, a proto-zero, able to distribute formal systems with their internal concept of zero and linearity over a tabular matrix opening up the way from the Abacus to trans-computation.

More at: http://www.thinkartlab.com/CCR/rudys-chinese-challenge.html

Algorist versus Abacist

http://library.thinkquest.org/22584/emh1100.htm

Are we not in a similar situation today? After the decline of the paradigm of algorithmic programming a new round has to be opened with "*interactionists*".

Interactionality, reflectionality and complexity of computation managed by the impotent and chaotic methods based on linear arithmetic and bivalent logic? The un-denied success of this paradigm is based on a self-destroying exploitation of natural and human resources. Not long ago, Medieval European scientists and mathematicians had been victims of their dysfunctional methods based on a Christian refutation of the Arabic positionality system.

Keith J Devlin:

"In the twenty-first century, biology and the human sciences will become the primary driving forces for the development and application of new mathematics. So far, we have seen some applications of mathematics in these fields, some quite substantial. But that has involved old mathematics, developed for other purposes. What we have not yet seen to any great extent are new mathematics and new branches of mathematics developed specifically in response to the needs of those disciplines. In my view, that is where we will see much of the mathematical action in the coming decades. I suspect that some of that new mathematics will look quite different from most of today's mathematics. But I really don't have much idea what it will look like."
http://www.spiked-online.com/index.php?/surveys/2024_article/1310/

2 Historical background of positionality and the Abacus

In Sanskrit (the scholarly language of the Hindus), the word for the zero is "sunya", meaning "void", and there is little doubt that the zero concept originated as the written symbol for the empty column of the abacus. The abacus had been used around the world since antiquity to provide a facile means of accumulating progressive products of multiplication by moving those products ever further leftward, column by column, as the operator filled the available bead spaces one by one and moved the excess over ten into the successive right-to-left-ward columns.

http://www.neo-tech.com/zero/part6.html

Some authorities believe that positional arithmetic began with the wide use of the abacus in China. The earliest written positional records seem to be tallies of abacus results in China around 400. In particular, zero was correctly described by Chinese mathematicians around 932, and seems to have originated as a circle of a place empty of beads.

In India, recognizably modern positional numeral systems passed to the Arabs probably along with astronomical tables, were brought to Baghdad by an Indian ambassador around 773.

http://en.wikipedia.org/wiki/Numeral_system

The first place-valued numerical system, in which both digit and position within the number determine value, and the abacus, which was the first actual calculating mechanism, are believed to have been invented by the Babylonians sometime between 3000 and 500 BC. Their number system is believed to have been developed based on astrological observations. It was a sexagesimal (base-60) system, which had the advantage of being wholly divisible by 2, 3, 4, 5, 6, 10, 15, 20 and 30. The first abacus was likely a stone covered with sand on which pebbles were moved across lines drawn in the sand. Later improvements were constructed from wood frames with either thin sticks or a tether material on which clay beads or pebbles were threaded.

Sometime between 200 BC the 14th century, the Chinese invented a more advanced abacus device. The typical Chinese swanpan (abacus) is approximately eight inches tall and of various widths and typically has more than seven rods, which hold beads usually made from hardwood.

This device works as a 5-2-5-2 based number system, which is similar to the decimal system. Advanced swanpan techniques are not limited to simple addition and subtraction. Multiplication, division, square roots and cube roots can be calculated very efficiently. A variation of this devise is still in use by shopkeepers in various Asian countries.

There is direct evidence that the Chinese were using a positional number system by 1300 BC and were using a zero valued digit by 800 AD.

http://astro.temple.edu/~joejupin/SynergyManual2.pdf

The void is empty but the emptiness is not void. That’s the difference!

For the first time since well 5000 years we have good reasons to state: the emptiness is not empty at all.

3 General positional matrix

The leading intuition behind the positional matrix is not in any sense based on linguistic, logical and cognitive science notions.

The position taken risks to think outside the paradigm of sentences, statements, propositions and also outside of notions, names, indentifiers.

Thus, logic of any kind is not leading the adventure.

The positional matrix is opting for positionality. It is believed that scientific rationality as codified by logic and math is occupying one and only one position in the world game.

The difficulty of introducing the positional matrix is not only its rejection of logos-based notions but also its insistence to reject topological concepts in the sense of mathematical topology and topos theory in the category theoretical and the philosophical sense. In fact, the positional matrix is not a mathematical concept.

So, nearly nothing is left to explain the positional matrix.

Obviously, there is no single PM and PM is not taking place in a field of possibilities.

A first step into the idea of positionality is given by Gotthard Gunther's theory of subjectivity. The distinction of an irreducible difference of I-subjectivity and Thou-subjectivity needs a structural space to distribute the domains of this difference. As a further consequence, Gunther introduced the idea and formalism of a *place-valued* logic. Here again, places, loci are fundamental. Further studies and attempts to formalize this idea of distributed rationality led him to discover kenogrammatics. Kenogrammatics are offering a grid for the distribution of a multitude of contextures.

Loci have to be understood in a most fundamental sense as empty. The voidness of their emptiness is beyond any logical, numeric and ontological notions of nothingness.

Philosophy, Occidental as well Oriental, knows emptiness only as an unstructured singularity. The West is identifying emptiness with nothingness. Thus, treating nothingness as an opposite to Being. The east is more radical in taking nothingness serious. But, like in the West (Parmenides about nothingness), there is nothing to say about the sunyata of emptiness. Happily enough, the Indian thinkers and mathematicians conceptualized sunyata as the arithmetical *zero*. Bramagupta (598-670) had the ingenious definition of zero: a number minus itself is zero.

But the concept zero is not identical with the concept of positionality. Both are in some sense complementary. The position system is a technique, a mechanism to organize numbers. The zero is a special "entity" to realize this organizational structure with great efficiency.

> *"In around 500AD Aryabhata devised a number system which has no zero yet was a positional system. He used the word "kha" for position and it would be used later as the name for zero."*
>
> http://turnbull.mcs.st-and.ac.uk/~history/HistTopics/Zero.html#s31

Even today there are all kinds of speculations and confusions about zero and positionality. At least since Alain Badiou's paper, the situation should be cleared.

I'm not aware about similar speculations about "zero" by Chinese thinkers. It is said that Chinese thinkers didn't develop abstractions in the Indic or Western sense. But the Chinese use of zero is highly technical: it is fundamental to the first hand-driven computer, the Abacus. Thus, the Chinese understanding of zero is in this context not speculative but mathematical, numeric and realized in a computational devise. Indian speculations are highly introspective, meditative and leading to inner insights not accessible to any mundane mechanism.

The positional matrix is only a part of the general theory of kenogrammatics but offers a more direct approach to a formal study of disseminated contextures, logical and semiotical.

Kenos (greek, empty) is empty but rejects any multitude. Kenograms are inscriptions of multitudes of empty places. Kenogrammatics is opening up the game of kenomic emptiness and the grammar of its inscription.

The decision for the positional matrix is linking this highly sophisticated speculations back or forwards to accessible formalisms.

Morphograms are beyond language

Polycontextural logics are logifications of morphogrammatics. This is not only be done by logical interpretations, say with truth-values of dialog-rules, but also by introducing sentential, i.e., propositional terms.

Technically, in the process of logification the morphograms get framed by propositional variables, connectors and interpreted by logical truth-values or equivalents.

Morphograms are linguistic-free; neither names nor sentences are involved.

Also morphograms are stripped off of any propositional attributes they are not something totally strange to anything logical. As Emil Lask would say, they perform the nakedness of logic.

Names vs. sentences

Chinese thinking is not sentence based. But it is also not name-based. What is a name if not in a sentence? Are there in a natural way sentence-free names? Names are part of sentences. What can be changed is the focus. We can emphasize names and treating sentences or propositions as names. Or we focus on sentences with their possible truth-values, i.e., their semantic and ontological function of mirroring or modeling reality. Mostly, all kind of sentences, commands, questions, imperatives, are modeled along the model of statements. To clear this kind confusions, analytic philosophy and the linguistic turn did some work, and produced some new confusions.

Rectifying names

> [...] that Chinese philosophical speculation tends to be guided more by considering of the *effect* of some doctrine on human behavior than on its empirical justification or "truth."
> [...] the role of language (and mind) to be predominantly *action guiding.*
> http://www.hku.hk/philodep/ch/Metaphysics%20of%20Dao%20doc.htm

ConTeXtures. Programming Dynamic Complexity
http://www.thinkartlab.com/pkl/lola/ConTeXtures.pdf

Operative or "elevator" terms used in morphogrammatics

Opposites:
statement, proposition, sentence, name, noun, notion
logical value, truth, consequence, deduction, proof
binary tree, hierarchy
number, zero

Elevators:
grid, matrix, locus, space, position,
place-designator
distribution, mediation
transformations

In a critical and reflexive treatment, elevator terms are not used dogmatically or unconsciously but are involved in explanations, constructions and even (circular) definitions. The matrix approach goes back to my paper *Deskriptive Morphogrammatik* (1974) and was then called O-Matrizen-Theorie in contrast to the Q-Matrizen-Theorie which was the genuine approach for morphogrammatics.

Reductions of complexity and complication

Reductions on the complexity of the positional matrix are possible and usual. Not all sub-system of the computing system has to have the ability to reflect all its neighbor systems into its own system. Neither is it necessary that all sub-systems are able to interact with all its neighbor systems. In a pragmatical context the system has to decide which properties are necessary.

Under the hegemony of 2-valued logic, every distinction is discrimination.

3.1 A numeric place-designator for numeric systems

A kenomic sequence like (000121121) can be seen as a doublet of a head and a body. The head, say (000), is marking the place where the body as a 0/1-sequence is located. It is called the *place-designator* (Gunther 1969). The relation between head and body is dynamic and depends on interpretation. Hence, the head of the sequence could also be (00) or (0001) instead of (000).

This understanding of the concept "place-designator' as constructed by the dynamics of the head/body difference of the numeric keno-sequence itself is not fully identical with the introduction of the place-designator by Gunther as the following citation may show. There, the natural numbers in trans-classic systems are localized by a place-designator and not their kenomic base.

> "The adding of a place-designator is not required in classic mathematics, because the natural numbers it employs are, logically speaking, always written against this backdrop of a potential infinity of zeros. In other words, the logical place of the traditional Peano numbers cannot change, since they appear only in one ontological locus.
>
> The situation is different in a trans-classic system. In this new dimension classic logic unfolds itself into an infinity of two-valued sub-systems, all claiming their own Peano sequences. It follows that natural numbers – running concurrently in many ontological loci – must then be written against an infinity of potential backdrops." Gunther, p. 21
>
> http://www.vordenker.de/ggphilosophy/gg_natural-numbers.pdf

> "In other words: a trito-number is no trito-number, unless it occupies a well defined place in a pattern of zeros." Gunther 1969

But things are much more dialectic than this. In the published work of Gunther only the very essentials are given and this in a highly abbreviated version. There is more to learn about the place-designator in the proposal from 1969 to Rowena Swanson from AFOSR. A "backdrop" may not only consist of a *"potential infinity of zeros"* but of all sorts of numeric constellations.

It seems to be clear that the place-designator is a further step in the realization of a new position system, not for numbers but for number systems. Hence, the positional function of the marker zero in a numeric position system is only a very first step to a fully developed reflectional and interactional position system.

To mark the place of the occurrence of a number in a kenomic texture, a place-designator has to be marked.

It follows, that the place-designator is also placing the place where a counting process might start.

Gaps, jumps (saltations) and place-designation (place-designator) connected with the successor operation are of fundamental relevance for polycontextural arithmetic.

Gaps, jumps (saltationss) and places are defining a system of constituents for the definition of the trans-classic concept of numbers.

The signature, i.e., the fundamental alphabet of polycontextural arithmetic consists of three very different categories of signs or marks: number signs (markers), empty signs (markers) and gap signs (markers) for each contexture.

4 Philosophical remarks about positionality and loci

The following philosophical remarks are in German language, writen in the early 90s. I don't see how I could translate them into a reasonable English. Polycontexturality also means the acceptance of different languages and the profound dis-contexturality between them.

4.1 Die Orte Ludwig Wittgensteins

Der Ort, bzw. der logische Ort, hat von jeher in der Logik eine Bedeutung gehabt und für eine gewisse Unruhe des Denkens gesorgt. Beim Aufbau der klassischen Logik, die wir zu verlassen versuchen, heißt es – chronologisch geordnet –:

„1.11.1914 Der Satz muß einen logischen Ort bestimmen.

7.11.1914 Der räumliche und der logische Ort stimmen darin überein, daß beide die Möglichkeit einer Existenz sind.

18.11.1914 Es handelt sich da immer nur um die Existenz des logischen Orts. Was – zum Teufel ist aber dieser 'logische Ort'!?". (Ludwig Wittgenstein)

Die logischen Orte Wittgensteins sind durch die Koordinaten der logischen Variablen und die Wahrheitswertverteilung bestimmt. Der logische Ort eines Satzes kann einen Punkt, einen Teilraum oder auch den ganzen logische Raum einnehmen. Die logischen Orte bilden die Möglichkeit für die Existenz von Welten. Die Orte sind nur bzgl. ihrer Indizes voneinander verschieden, sie sind die Orte eines und nur eines logischen Zusammenhanges. Ihre Logik begründende Extensionalität und Monokontexturalität unterscheidet sie entschieden von den qualitativen Orten der antiken Gedächtniskunst, der Mnemotechnik. Ausserhalb des logischen Raumes gilt keine Rationalität; einzig das Schweigen als lautloses Verstummen.

Die Orte Wittgensteins, heute noch Leitidee der KI–Forschung, insb. der logischen Programmierung, pflegen keine Verwandtschaft mit einer *„Architektur, die weder einschließt, noch aussperrt, weder abdichtet noch untersagt."* (Eva Meyer)

4.2 Orte und Polykontexturalität

Die Orte der Polykontexturaltät sind von denen Wittgensteins prinzipiell verschieden. Um das Bild des Koordinatensystems zu benutzen, in das nach Wittgenstein alle Elementarsätze der Welt und alle logischen Zusammenhänge zwischen ihnen, also die ganze Welt, abgebildet werden kann, wäre ein Ort einer Elementar–Kontextur der Nullpunkt des Koordinatensystems und die Polykontexturalität wäre über eine Vielheit solcher Nullpunkte und damit über einer Vielheit von Koordinatensystemen verteilt.

Wenn also die ganze (Leibniz–Wittgensteinsche) Welt in einem und nur einem Koordinatensystem abbildbar ist, so ist in ihrem Nullpunkt nichts abbildbar. Denn sowohl die Variablen für Elementarsätze, die Wahrheitswertverteilung, wie die Wahrheitswertfunktion des Zusammenhangs der Elementarsätze, fehlen an einem solchen Nullpunkt. Dieser Nullpunkt ist die Metapher für einen logisch–strukturellen Ort im Sinne der Kontexturalitätstheorie und der Kenogrammatik. Dieser Ort ist gewiss ohne Eigenschaften, ja er steht ausserhalb der Möglichkeit Eigenschaften zu haben und dennoch ist er der Ermöglichungsgrund aller möglicher logischer Sätze.

In der Kenogrammatik wird eine Vielheit von verschiedenen Orten dieser Eigenschaftslosigkeit unterscheiden. Aber hier endet der intra–metaphorische Gebrauch des Nullpunkts; ein Kenogramm markiert gewiß keinen Nullpunkt eines Systems von Koordinaten jedweder Art, sondern eher den Ort, den ein solcher Nullpunkt einnehmen könnte und verweist auf Emil Lask und das 'logisch Nackte'.

Die Polykontexturalitätstheorie wäre hier in Zusammenhang zu bringen mit der Vielheit der Wittgensteinschen Sprachspiele. Nur daß sie versucht, die Operativität des Kalküls in die Verstricktheit der Sprachspiele herüber zu retten. Was allerdings nur unter Hintergehung des common sense der Umgangssprache gelingen kann. Nicht die Unterscheidung Kalkül/Sprachspiel sollte hier leiten, sondern die Dekonstruktion der Hierarchien zwischen den beiden Konzeptionen bzw. Spielen und auch zwischen künstlichen und natürlichen Sprachen und Notationssystemen.

4.3 Genealogie, De-Sedimentierung und die Vier

Das Hauptproblem einer transklassischen und polykontexturalen Logik bzw. Kalkültheorie ist nicht so sehr die Einführung von neuen Werten oder neuartigen Funktionen, sondern die entschiedene Entledigung jeglicher Genealogie. Genealogie ist immer Herrschaft des Grundes über das Begründete, Verdekkung von Kalkül und Ermöglichungsgrund des Kalküls. Diese Sedimentierung ist es, die ent–deckt und entkoppelt werden muß.

Ohne diese Dekonstruktion des Grundes erklingt erneut das Lied von der nie versiegenden Quelle, diesmal von der 'Santa Cruz Triune': *„The void is the 'allowingness' prior to distinction; it can be viewed as the source from which forms arise, as well as the foundation within forms abide. To the extend that indicational space may be represented by a topological space, the void may be represented by an undifferentiated (homogeneus, isotropic and uniform) space that prevades all forms.“* (C. G. Berkowitz, 1988)

Eine Desedimentierung und Hineinnahme der begründendenden begrifflichen Instrumentarien in den Formalismus des Kalküls selbst, würde dem CI jene Operativität ermöglichen, die er für eine Kalkülisierung von doppelter Form, d.h. der Formation der Form, bzw. der Reflexionsform, benötigte. Dies würde aber die Simplizität sowohl seiner Grundannahmen wie auch seiner Architektur sprengen.

Erst wenn Grund und Begründetes als gleichursprüngliche Elemente eines komplexen Wechselspiels verstanden werden können, ist die Herrschaft des Grundes, die Genealogie, gebrochen und eine vom Grund losgelöste und damit autonome Realisation möglich. Eine solche Loslösung ist keine Negation des Grundes, sondern zieht den Prozeß des Negierens mit in die Loslösung ein.

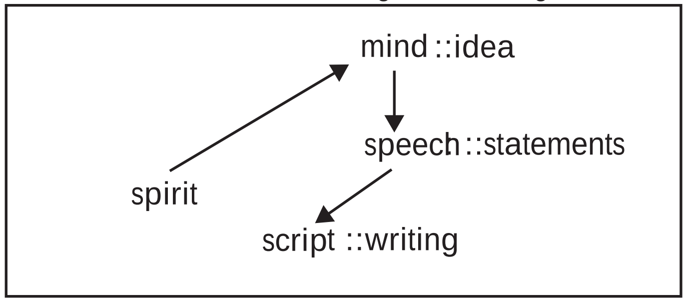
In einem von der Herrschaft der Genealogie befreiten Kalkül wie der Kenogrammatik gibt es jedoch keinen ausgezeichneten Ort der Begründung. Was Grund und was Begründetes ist, wird geregelt durch den Standort der Begründung. Der Wechsel des Standortes regelt den Umtausch von Grund und Begründetem. Jeder Ort der Begründung ist in diesem Fundierungsspiel Grund und Begründetes zugleich. Orte sind untereinander weder gleich noch verschieden; sie sind in ihrer Vielheit voneinander geschieden.

Die Ortschaft der Orte ist bar jeglicher Bestimmbarkeit. Orte eröffnen als eine Vierheit von Orten das Spiel der Begründung der Orte.

Kaehr, Disseminatorik: Zur Logik Der ’Second-Order Cybernetics’ in:
http://www.thinkartlab.com/pkl/media/DISSEM-final.pdf

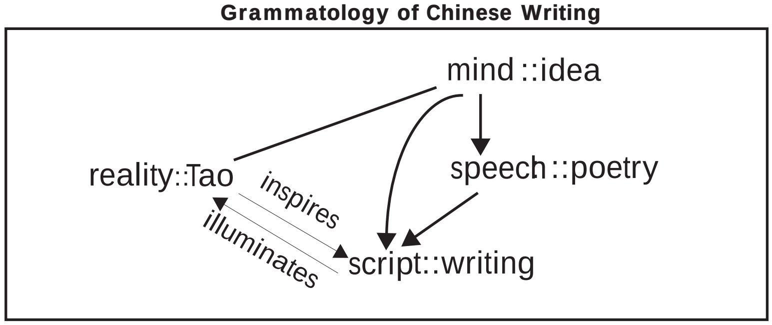
5 A little typology of interpretations of writing

Western Phonologism of Writing



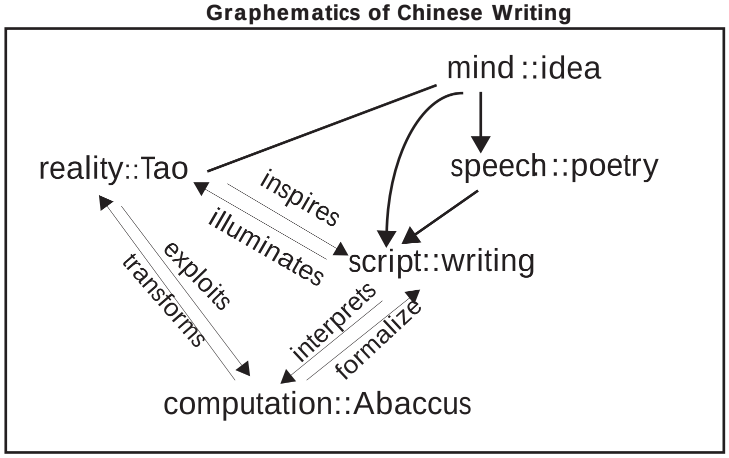
This is the scheme of a logocentric understanding of writing. It corresponds to the dominant tradition of Western philosophy and linguistics. But there are now surprises to observe that this scheme holds in a similar way in other cultures, too.

Grammatology of Chinese Writing



This scheme corresponds to the Chinese understanding of writing as it is exposed by Liu Hsieh. There are similarities in the pre-Aristotelian tradition of Plato and Pythagoras to find. It is not my aim to go into history, say of the Sumerian understanding of language as it is to discover in the Epic of Gilgamesh (2700 B.C.).

Graphematics of Chinese Writing



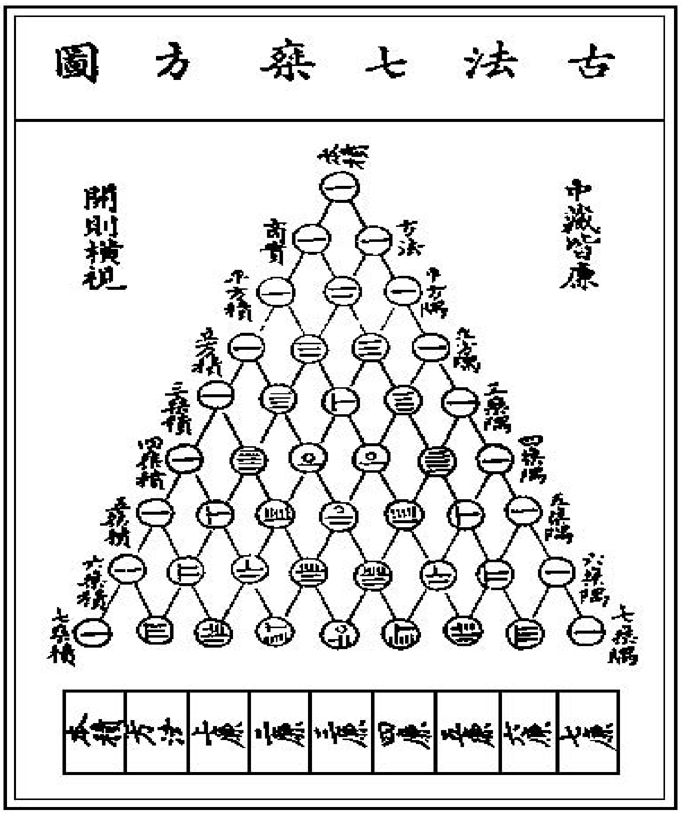
This scheme is considering the influence of technological and cultural practise on the paradigm of writing. The emphasis is on the influence of the usage of the Abacus on reality and on the concept of literal and algebraic writing. It is thought that the development of the concept of zero and the organizational system of positionality is an interpretation of the practice of the usage of the Abacus in calculations. Hence, techniques of computations have influenced the general structure of writing.

6 Reasoning beyond propositions and notions

Reasoning beyond apophansis and hierarchy (diairesis).
Semantic and ontological considerations about the new way of calculating.

Chad Hanson writes:

Chinese linguistic thought focused on names not sentences.
http://www.hku.hk/philodep/ch/lang.htm



Diairesis on Proto-Structures

Logic systems distributed over the proto-structure.

Linguistic and logical structure of diairesis: genus proximum/ differentia specifica.

Up and down; the same. (Diels)

But the *conceptual* use of the triangle is in strict conflict to the binary structure of the diaeresis.

Diairesis is applicable to both approaches, the sentence- and the notion-based.

http://www.vordenker.de/ggphilosophy/gg_life_as_polycontexturality.pdf

http://www.vordenker.de/ggphilosophy/gg_identity-neg-language_biling.pdf

Khu Shijiei triangle, depth 8, 1303.
http://www.roma.unisa.edu.au/07305/pascal.htm

Yang Hui (Pascal's) triangle, as depicted by the ancient Chinese using rod numerals.
Yang Hui (ókãP, c. 1238 - c. 1298)

http://people.bath.ac.uk/ma3mja/patterns.html

B. Tabular Morphogrammatics

1 Towards a tabular distribution of morphograms

Interactional and reflectional morphogrammatics as a kenomic abacus.

Morphograms are manipulated by operators and moved and transformed on the grid of the tabular kenomic abacus. Like with the abacus, the meaning of the morphograms is determined by their definition as elements and by their position in the positional grid.

A special type of morphogrammatics is introduced. It can be called a *quindecimal* positional morphogrammatic system because its basic elements are 15 morphograms – and not more. But these 15 morphograms can be distributed over arbitrary large grids of the positional system. Like the calculi (stones) or numbers, the meaning of the morphograms is realized by their positionality in the tabular positional system.

Cellular Automata can be seen in an analogy to the kenomic abacus. But CA is strictly remaining in the framework of identity of its signs and rules.

On the other hand, the classic Chinese Abacus is equivalent to a simple cellular automaton for numeric calculations.

The idea goes back to Gotthard Gunther's concept of *place-valued* logics and later, of *place-designator* for numeric systems.

To realize his place-valued logic he had to introduce a value-free dimension, called morphogrammatics, because it deals with the pure structure (Gestalt, morphe) of logical functions. Thus, the concept of positionality was moved from logical functions to their underlying morphograms.

This transitions from numeric to logical place-valued or positional systems has to be pushed further to a *tabular positional framework* for any kind formal systems (Church, Post, Thue, Markov, Smullyan).

poly-Lambda Calculus
Lambda Calculi in polycontextural Situations.
http://www.thinkartlab.com/pkl/lola/poly-Lambda_Calculus.pdf

1.1 From: Cybernetic Ontology

The pathos of quindecimality

Their full meaning still escapes us, but this much may be said now: no matter how comprehensive the logical systems we construct and no matter how many values we care to introduce, *these patterns and nothing else will be the eternally recurring structural units of trans-classic systems.*

Our values may change but these fifteen units will persist.

In order to stress the logical significance of these patterns, and to point out that they, and not their actual value occupancies, represent invariants in any logic we shall give them a special name. These patterns will be called "*morphograms*", since each of them represents an individual structure or Gestalt (äçèûÖ). And if we regard a logic not from the viewpoint of values but of morphograms we shall refer to it as a "*morphogrammatic*" system. p. 30

Subjectivity: A question of transjunctions

We propose as basis for a general consensus the following statement: if a cyberneticist states that an observed system shows the behavioral traits of subjectivity he does so with the strict understanding that he means only that the observed events show partly or wholly the logical structure of transjunction. p.34

de dicto/de re: radicalized

Everybody is familiar with these three aspects of subjectivity. The first is commonly called a thought; the second, an "objective" subject or person; the last, self-awareness or self-consciousness. These three distinctions correspond to the three varieties of rejection of a two-valued alternative which Table IV demonstrates:

a) partial rejection: morphograms [9] - [12] and [14]
b) total, undifferentiated, rejection: morphogram [13]
c) total, differentiated, rejection: morphogram [15]

A thought is always a thought of something. This always implies a partial refusal of identification of (subjective) form and (objective) content. This fact has been noted time and again in the history of philosophic logic, but the theory of logical calculi has so far neglected to make use of it.

Any content of a thought is, as such, strictly *objective*; it consequently obeys the laws of two-valued logic. It follows that for the content the classic alternative of two mutually exclusive values has to be accepted.

On the other hand, the form of a thought, relative to its content, is always *subjective*. It therefore rejects the alternative. In conformity with this situation the morphograms [9] - [12] and [14] always carry, in the second and third rows of Table IV, both an acceptance and a rejection value. Together, they represent all possible modes of acceptance and rejection.

Gunther, Cybernetic Ontology
http://www.vordenker.de/ggphilosophy/gg_cyb_ontology.pdf

After all, morphograms are neither *de dicto* nor *de re.* And *neither-nor* is still a language-dependent formulation of the morphogrammatic *action of rejection* by transjunctions. It is common to refer to the Sanskrit "neti/neti" in this case. Morphograms are enabling cognitive attitudes, they are the patterns of cognition but not themselves involved in *modeling* and representing subjective or objective world-data.

What Gunther did is to interpret morphograms for cognitive attitudes. The morphograms [1] to [8] for the de-re-attitude, and the morphograms [9] to [12] for de-dicto-attitude, and the morphograms [13] to [15] for self-reflectional attitudes of cognitive systems.

1.2 Positionality in Dialogical Logics

Positionality in the sense of distribution of actors appears well also in dialogical logics (Lorenzen), game logics (Hintikka) and with great generality in ludics (Girard).

But this kind of positionality is not to confuse with the positionality of the numeric position system of arithmetic nor with anything kenogrammatic.

Distribution of Proponent and Opponent.
Reduction of m-actors to two actors: Abramsky

Superfluity of Lorenzen's criticism of Gunther's morphogrammatics.

Morphogrammatic abstractions of dialogue rules.
Morphograms of dialog rules.

$$morph(Opp, Prop, Rule) = [Rule]$$
$$morph(Opp, Prop, Rule) = morph(Prop, Opp, \overline{Rule})$$
$$morph(Opp, Prop, Rule_{(\wedge)}) = [\wedge]$$

The morph-abstraction is not depending on the existence of classical negation. It is not dealing with "values" but with the rules and the rules are depending on the opponent/proponent-dichotomy. Hence, the morphic abstraction is an abstraction from/of the opponent/proponent-frame. And is delivering the opponent/proponent-free inscription of the actions of the actors constituting logical operators.

Different morphic abstraction can be introduced. Gunther's classic abstraction is negation-based. Thus, his morphograms are negation-invariant patterns of logical operators. A stronger but still "symmetric" morphic abstraction is introduced by an abstraction based on duality. Thus, it includes an iterative application of negations, like in DeMorgan formulas. The result of such an abstraction could be called DeMorgan-independent or simply duality-independent patterns.

The timide positionality of dialogue logics or in general game logics has to be involved into the game of dissemination over the positional matrix to produce polylogic.

$$PolyLogics^{(m)} = (Opp^{(m)}, Prop^{(m)}, Rules^{(m)})$$
$$morph(Opp^{(m)}, Prop^{(m)}, Rules^{(m)}) = [Rules^{(m)}]$$
$$[Rules^{(m)}] = (morphograms^{(m)})$$
$$Morphogrammatics = (Operators, morphograms)$$

1.2.1 Correspondence between dialogical and tableaux rules for classical logic

(O)- *Cases*	(P)- *Cases*
$\dfrac{(\mathbf{O})A \wedge B}{\langle(\mathbf{P})_?\rangle(\mathbf{O})A \mid \langle(\mathbf{P})_?\rangle(\mathbf{O})B}$	$\dfrac{(\mathbf{P})A \wedge B}{\begin{array}{c}\langle(\mathbf{O})_?\rangle(\mathbf{P})A\\(\langle(\mathbf{O})_?\rangle(\mathbf{P})B)\end{array}}$
$\dfrac{(\mathbf{O})A \wedge B}{\begin{array}{c}\langle(\mathbf{P})_{?\mathrm{left}}\rangle(\mathbf{O})A\\(\langle(\mathbf{P})_{?\mathrm{right}}\rangle(\mathbf{O})B)\end{array}}$	$\dfrac{(\mathbf{P})A \wedge B}{\langle(\mathbf{O})_{?\mathrm{left}}\rangle(\mathbf{P})A \mid \langle(\mathbf{O})_{?\mathrm{right}}\rangle(\mathbf{P})B}$
$\dfrac{(\mathbf{O})A \rightarrow B}{(\mathbf{P})A\ldots \mid \langle(\mathbf{P})A\rangle(\mathbf{O})B}$	$\dfrac{(\mathbf{P})A \rightarrow B}{\begin{array}{c}(\mathbf{O})A\\(\mathbf{P})B\end{array}}$
$\dfrac{(\mathbf{O})\neg A}{(\mathbf{P})A, \otimes}$	$\dfrac{(\mathbf{P})\neg A}{(\mathbf{O})A, \otimes}$
$\dfrac{(\mathbf{O})\bigwedge_x A}{\begin{array}{c}\langle(\mathbf{P})_{?\tau}\rangle(\mathbf{O})A_{[\tau^*/x]}\\\tau \text{ has been labelled}\\\text{with a star before}\end{array}}$	$\dfrac{(\mathbf{P})\bigwedge_x A}{\begin{array}{c}\langle(\mathbf{O})_{?\tau^*}\rangle(\mathbf{P})A_{[\tau^*/x]\tau}\\\tau \text{ is new}\end{array}}$
$\dfrac{(\mathbf{O})\bigvee_x A}{\begin{array}{c}\langle(\mathbf{P})_?\rangle(\mathbf{O})A_{[\tau^*/x]}\\\tau \text{ is new}\end{array}}$	$\dfrac{(\mathbf{P})\bigvee_x A}{\begin{array}{c}\langle(\mathbf{O})_?\rangle(\mathbf{P})A_{[\tau/x]}\\\tau \text{ has been labelled}\\\text{with a star before}\end{array}}$

> "The philosophical point of dialogical logic is that this approach does not understand semantics as mapping names and relationships into the real world to obtain an abstract counterpart of it, but as *acting* upon them in a particular way." Rahman
> http://www.hf.uio.no/filosofi/njpl/vol3no1/symbexis/node2.html

Classical logic	*Intuitionistic logic*
Dialogical tableaux for negation	*Dialogical tableaux for negation*
$\dfrac{(O)\neg A}{(P)A,\otimes} \quad \dfrac{(P)\neg A}{(O)A,\otimes}$	$\dfrac{(O)\neg A}{(P)_{[O]}\,A,\otimes}\ ,\ \dfrac{(P)\neg A}{(O)_{[O]}\,A,\otimes}$
Morphogram of Dialogical negation	*Morphogram of Dialogical negation*
$morph\left(\dfrac{(O)\neg A}{(P)A,\otimes}, \dfrac{(P)\neg A}{(O)A,\otimes}\right) = (\bigcirc\blacktriangle)$	$morph\left(\dfrac{(O)\neg A}{(P)_{[O]}\,A,\otimes}, \dfrac{(P)\neg A}{(O)_{[O]}\,A,\otimes}\right) = (\bigcirc\blacktriangle)$

> "For the intuitionistic tableaux, the structural rule about the restriction on defences has to be considered. The idea is quite simple: the tableaux system allows all the possible defences (even the atomic ones) to be written down, but as soon as determinate formulae (negations, conditionals, universal quantifiers) of P are attacked all others will be deleted. Those formulae which compel the rest of P's formulae to be deleted will be indicated with the expression `` $O_{[O]}$'' (or ``$P_{[O]}$ ''), which reads *save O's formulae and delete all of P's formulae stated before*." Rahman

Because of the *proviso* [O], intuitionist negation rules are not as symmetric as the classic ones. Its symmetry is recovered on a structural level of opponent/proponent which coincides in the classic case with the symmetry of the rules.

Basic interpretations of morphograms

The operation *logification* is interpreting the morphograms *mg* as binary logical operators in the frame of the variables *p* and *q* of a m-valued polylogic.

Dialogification of the morphogram (ab) is producing the pair opponent/proponent, *valuation* of the morphogram is producing the truth-values (true, false).

$$dialog(\bigcirc\blacktriangle) = \begin{pmatrix} (Opponent, Proponent) \\ (Proponent, Opponent) \end{pmatrix}$$

$$val(\bigcirc\blacktriangle) = \begin{pmatrix} (true, false) \\ (false, rue) \end{pmatrix}$$

$$logification$$

$$logif([mg_1, ..., mg_s]) = p^{(m)}\ log_1, ...\ log_s\ q^{(m)}$$

Some wordings about the morphic abstraction of observer activities

"Durch den Durchgang durch alle strukturell möglichen 'subjektiven' Beschreibungen durch den Observer wird das Objekt der Beschreibung 'objektiv', d.h. observer-invariant 'als solches' bestimmt. Das Objekt ist also nicht bloß eine Konstruktion der Observation, sondern bestimmt selbst wiederum die Struktur der Subjektivität der Observation durch seine Objektivität bzw. Objektionalität. Der auf diesem Weg gewonnene Begriff der Sache entspricht dem Mechanismus des Begriffs der Sache und wird als solcher in der subjekt-unabhängigen Morphogrammatik inskribiert.

Kaehr, Diskontexturalitäten: Wozu neue Formen des Denkens? in:
http://www.thinkartlab.com/pkl/media/DISSEM-final.pdf

1.2.2 Multi-agent systems

"*Whither negation?* In 2-person Game Semantics, negation is interpreted as *role interchange*. This generalizes in the multi-agent setting to role *permutation*." Abramsky

http://www.illc.uva.nl/HPI/Draft_Information_Processes_and_Games.pdf

Hence, again, the morphic abstraction is independent of the number of agents involved. Simply, abstract over the permutations. That’s it. In this case, it doesn’t matter if n-person Game Semantics is reducible to 2-person games or not.

1.2.3 Loci in ludics

Only locations matters. Jean-Yves Girard

1.2.4 Tableaux based morphic abstractions for junctions

$$morph\left(\frac{T\neg X}{FX}\ ,\ \frac{F\neg X}{TX}\right) = morph\left(\frac{T\,\overline{X}}{FX}\ ,\ \frac{F\,\overline{X}}{TX}\right) = (\bigcirc\blacktriangle)$$

$$morph\left(\left(\frac{T\,(\vee)}{T\,|\,T}\ ,\ \frac{F\,(\vee)}{\begin{matrix}F\\F\end{matrix}}\right),\overline{\left(\frac{T\,(\vee)}{T\,|\,T}\ ,\ \frac{F\,(\vee)}{\begin{matrix}F\\F\end{matrix}}\right)}\right) = \begin{pmatrix}\bigcirc\bigcirc\\ \bigcirc\blacktriangle\end{pmatrix}$$

$$morph\left(\left(\frac{T\,(\supset)}{F\,|\,T}\ ,\ \frac{F\,(\supset)}{\begin{matrix}T\\F\end{matrix}}\right),\overline{\left(\frac{T\,(\supset)}{F\,|\,T}\ ,\ \frac{F\,(\supset)}{\begin{matrix}T\\F\end{matrix}}\right)},\right) = \begin{pmatrix}\bigcirc\blacktriangle\\ \bigcirc\bigcirc\end{pmatrix}$$

$$morph\left(\left(\frac{T\,(\subset)}{T\,|\,F}\ ,\ \frac{F\,(\subset)}{\begin{matrix}F\\T\end{matrix}}\right),\overline{\left(\frac{T\,(\subset)}{T\,|\,F}\ ,\ \frac{F\,(\subset)}{\begin{matrix}F\\T\end{matrix}}\right)},\right) = \begin{pmatrix}\bigcirc\bigcirc\\ \blacktriangle\bigcirc\end{pmatrix}$$

Generalized setting of the abstraction

$\forall i \in s(m):$

$$morph\left(\left(\frac{t_i\,(\vee)}{t_i\,|\,t_i}\ ,\ \frac{f_i\,(\vee)}{\begin{matrix}f_i\\f_i\end{matrix}}\right),\overline{\left(\frac{t_i\,(\vee)}{f_i\,|\,f_i}\ ,\ \frac{f_i\,(\vee)}{\begin{matrix}t_i\\t_i\end{matrix}}\right)}^{\,i}\right) = \begin{pmatrix}\bigcirc\bigcirc\\ \bigcirc\blacktriangle\end{pmatrix}$$

$\forall i \in s(m):$

$$morph\left(\overline{(Tabl(Op))}^{\,i}\right) = \left(MG_{Op}\right)$$

1.3 Smullyan's Unification as an abstraction

Another interesting abstraction is proved by Smullyan's unification method.

Smullyan's Unification

α^i	α_1^i	α_2^i	β^i	β_1^i	β_2^i
$f^i\ X \wedge Y$	$f^i\ X$	$f^i\ Y$	$t^i\ X \wedge Y$	$t^i\ X$	$t^i\ Y$
$t^i\ X \vee Y$	$t^i\ X$	$t^i\ Y$	$f^i\ X \vee Y$	$f^i\ X$	$f^i\ Y$
$t^i\ X \rightarrow Y$	$f^i\ X$	$t^i\ Y$	$f^i\ X \rightarrow Y$	$t^i\ X$	$f^i\ Y$
$t^i\ X \leftarrow Y$	$t^i\ X$	$f^i\ Y$	$f^i\ X \leftarrow Y$	$f^i\ X$	$t^i\ Y$

With the conjugational properties

$$
\begin{aligned}
&\textit{Conjugation } \rho \textit{ for } 2-\textit{logics}\\
&\rho(X) \neq X\\
&\rho(\rho(X)) = X\\
&\rho(\alpha) \Leftrightarrow (\beta)\\
&\rho(\alpha) \Leftrightarrow (\rho(\alpha_1) \textit{ et } \rho(\alpha_2))\\
&\rho(\beta) \Leftrightarrow (\rho(\beta_1) \textit{ vel } \rho(\beta_2))\\
&\rho(\alpha) \Leftrightarrow (\beta_1 \textit{ vel } \beta_2)\\
&\alpha = \rho(\beta) \Rightarrow (\alpha_1 = \rho(\beta_1) \textit{ et } \alpha_2 = \rho(\beta_2))
\end{aligned}
$$

$$
\begin{aligned}
&\textit{Junctional mediation}:\\
&\{\alpha, \beta\}_{/mediation} \in S^{(3)} \Leftrightarrow \{\alpha, \beta\} \in CF\\
&CF = \{(ccc), (cff), (fcf), (ffc), (fff)\}\\
&f = \{\wedge, \vee\}\\
&c = \{\rightarrow, \leftarrow, \leftrightarrow\}
\end{aligned}
$$

$$
\begin{aligned}
&\textit{Transjunctional mediation}:\\
&\{\alpha, \beta, \delta, \gamma\}_{/mediation} \in S^{(3)} \Leftrightarrow \{\alpha, \beta\ \delta, \gamma\} \in CF\\
&CF = \{(ccc), (cff), (fcf), (ffc), (fff)\}\\
&f = \{\wedge, \vee, <, >, <>, \triangleleft, \triangleright\}\\
&c = \{\rightarrow, \leftarrow, \leftrightarrow, \Uparrow, \Downarrow\}
\end{aligned}
$$

2 Notations for morphogrammatic compounds

Explicitness of information:

Logic-Tableaux (+ structure)
Structural Matrix
Logic-Matrix
Morphogram-Matrix.

There is no simple algorithm which is producing the logical tableaux out of the morphogrammatic matrix. Hence, the relationship between morphogrammatics and polylogics has to be specially considered in an own study.

Structure, meaning and relevance of morphograms.

Classic representation of morphogrammatic compounds as matrices

$[EEE]_1$	1	2	3
1	○	△	△
2	△	○	△
3	△	△	○

$[EEE]_2$	1	2	3
1	○	△	□
2	△	○	△
3	□	△	○

$[EEE]_3$	1	2	3
1	○	△	△
2	△	○	□
3	△	□	○

$[EEE]_4$	1	2	3
1	○	△	□
2	△	○	□
3	□	□	○

$[EEE]_5$	1	2	3
1	○	△	★
2	△	○	□
3	★	□	○

Patterns: [1, 1, 1] [1, 1, 3] [1, 3, 1] [1, 3, 3] [1, 3, 4]

Classic representation of morphogrammatic compounds as chains

$[EEE]_4$	1	2	3
1	○	–	○
2	△	–	–
3	–	–	□
4	△	–	–
5	○	○	–
6	–	□	–
7	–	–	□
8	–	□	–
9	–	○	○

$[EEE]_1$	1	2	3
1	○	–	–
2	△	–	–
3	△	–	–
4	△	–	–
5	○	–	–
6	△	–	–
7	△	–	–
8	△	–	–
9	○	–	–

$[EEE]_2$	1	2	3
1	○	–	○
2	△	–	–
3	–	–	□
4	△	–	–
5	○	–	–
6	△	–	–
7	–	–	□
8	△	–	–
9	○	–	○

$[EEE]_3$	1	2	3
1	○	–	–
2	△	–	–
3	△	–	–
4	△	–	–
5	○	○	–
6	–	□	–
7	△	–	–
8	–	□	–
9	○	○	–

$[EEE]_4$	1	2	3
1	○	–	○
2	△	–	–
3	–	–	□
4	△	–	–
5	○	–	○
6	–	–	□
7	–	–	□
8	–	–	□
9	–	–	○

New representation of morphograms onto positional matrix

$[EEE]_1$	$O1$	$O2$	$O3$
$M1$	S_1	$\varnothing$	$\varnothing$
$M2$	S_1	$\varnothing$	$\varnothing$
$M3$	S_1	$\varnothing$	$\varnothing$

$[EEE]_2$	$O1$	$O2$	$O3$
$M1$	S_1	$\varnothing$	$\varnothing$
$M2$	S_1	$\varnothing$	$\varnothing$
$M3$	$\varnothing$	$\varnothing$	S_3

$[EEE]_3$	$O1$	$O2$	$O3$
$M1$	S_1	$\varnothing$	$\varnothing$
$M2$	$\varnothing$	$\varnothing$	$\varnothing$
$M3$	S_2	S_3	$\varnothing$

$[EEE]_4$	$O1$	$O2$	$O3$
$M1$	S_1	$\varnothing$	$\varnothing$
$M2$	$\varnothing$	$\varnothing$	S_3
$M3$	$\varnothing$	$\varnothing$	S_3

2.1 Structure, Meaning and Relevance of Morphograms

Distribution and mediation of morphograms or logical operations is not yet considering the change or shift of meaning depending on the position of the morphograms or logical operations. It is presumed that their meaning is defined and stable and the distributed and mediated if possible.

The new emphasis on positionality is taking the fact serious that a morphogram is not only defined by its structure but also by the position it takes in the matrix.

The logical or contextual meaning of the morphogram E is defined by its place in the tabular positional system. Thus, a morphogram has two aspects of meaning: its definition as such and its place in the matrix. The definition of the morphogram determines its structural meaning and its place the contextual meaning which can be understood as the relevancy of the morphogram.

All occurrences of the morphogram E are morphogrammatically equivalent but they are placed at different loci in the matrix. Also they keep their structural meaning as the morphogram E their *relevance* is different at each place.

Some modi of relevance are given by the operators of distribution and replication, i.e., the way the position of a morphogram is defined. In a chain of morphograms, the morphogram is positioned by the distributor. In a reflectional or cloning situation the morphogram is positioned by the operator of replication. In general, the super-operator involved are defining the kind of positioning and thus, the relevance of a morphogram in a positional matrix.

The matrix system is in itself a composition and only for notational reasons written as a global matrix with global entries.

Chain representation onto positional matrix

$[EEE]_2$	S_1^1	S_2^1	S_3^1	S_1^2	S_2^2	S_3^2	S_1^3	S_2^3	S_3^3
1	○	–	–	–	–	–	–	–	○
2	△	–	–	–	–	–	–	–	–
3	–	–	–	–	–	–	–	–	□
4	△	–	–	–	–	–	–	–	–
5	○	○	–	–	–	–	–	–	–
6	–	△	–	–	–	–	–	–	–
7	–	–	–	–	–	–	–	–	□
8	–	△	–	–	–	–	–	–	–
9	–	○	–	–	–	–	–	–	○

$[EEE]_2$	$O1$	$O2$	$O3$
$M1$	E_1	$\varnothing$	$\varnothing$
$M2$	E_1	$\varnothing$	$\varnothing$
$M3$	$\varnothing$	$\varnothing$	E_3

$[EEE]_3$	S_1^1	S_2^1	S_3^1	S_1^2	S_2^2	S_3^2	S_1^3	S_2^3	S_3^3
1	○	–	–	–	–	○	–	–	–
2	△	–	–	–	–	–	–	–	–
3	–	–	–	–	–	□	–	–	–
4	△	–	–	–	–	–	–	–	–
5	○	–	○	–	–	–	–	–	–
6	–	–	△	–	–	–	–	–	–
7	–	–	–	–	–	□	–	–	–
8	–	–	△	–	–	–	–	–	–
9	–	–	○	–	–	○	–	–	–

$[EEE]_3$	$O1$	$O2$	$O3$
$M1$	E_1	$\varnothing$	$\varnothing$
$M2$	E_1	$\varnothing$	$\varnothing$
$M3$	$\varnothing$	E_2	$\varnothing$

$[EEE]_4$	S_1^1	S_2^1	S_3^1	S_1^2	S_2^2	S_3^2	S_1^3	S_2^3	S_3^3
1	○	–	–	–	–	–	–	–	○
2	△	–	–	–	–	–	–	–	–
3	–	–	–	–	–	–	–	–	□
4	△	–	–	–	–	–	–	–	–
5	○	–	–	–	–	–	–	○	–
6	–	–	–	–	–	–	–	□	–
7	–	–	–	–	–	–	–	–	□
8	–	–	–	–	–	–	–	□	–
9	–	–	–	–	–	–	–	○	○

$[EEE]_4$	$O1$	$O2$	$O3$
$M1$	E_1	$\varnothing$	$\varnothing$
$M2$	$\varnothing$	$\varnothing$	E_3
$M3$	$\varnothing$	$\varnothing$	E_3

In the example, polysemy of morphograms is interpreted as different distributions of the same morphogram over different loci of the tabular matrix. There is only one morphogram E but distributed over different places according to the super-operator *red* (reduction). The compounds of the distributed morphograms E are building, globally, different patterns of distributed Es.

The pattern of the distribution is: [Eøø, øøø, øEE].

Thus, polysemy is explained as the occurrence of the same morphogram in different contexts.

The introduction of the tabular setting may be only a systematic cleanup, without any changes in the combinatory results. The relevance of the tabular approach comes not only well into play with the representation of the morphograms for transjunctions but is significant for the management of the whole morphogrammatic calculus.

Matrix and brackets

The matrix and the bracket representations are neutral to morphogrammatics and logics. So, what is their meaning? The polycontextural matrix and its bracket representation are designing the framework of a general theory of structural interactionality and reflectionality for computational systems.

2.2 Pattern [id, id, red]: S_{123} –> S_{121}

$$\frac{t_1\ X \vee\supset\vee Y}{f_1\ X \mid f_1\ Y \parallel f_3\ X \mid f_3\ Y} \qquad \frac{f_1\ X \vee\supset\vee Y}{\begin{array}{c|c} f_1\ X & f_3\ X \\ f_1\ Y & f_3\ Y \end{array}}$$

$$\frac{t_2\ X \vee\supset\vee Y}{f_2\ X \mid t_2\ Y} \qquad \frac{f_2\ X \vee\supset\vee Y}{\begin{array}{c} t_2\ X \\ f_2\ Y \end{array}}$$

$$(O1O2O3)\ \left[\begin{array}{l} \left[\begin{array}{l} O1 \\ \left(\begin{array}{l} M1M2M3 \\ \quad (G101) \end{array}\right) \end{array}\right] \\ \left[\begin{array}{l} O2 \\ \left(\begin{array}{l} M1M2M3 \\ \quad (G020) \end{array}\right) \end{array}\right] \\ \left[\begin{array}{l} O3 \\ \left(\begin{array}{l} M1M2M3 \\ \quad (G000) \end{array}\right) \end{array}\right] \end{array}\right]$$

PM	$O1$	$O2$	$O3$
$M1$	$log1$	$\varnothing$	$\varnothing$
$M2$	$\varnothing$	$log2$	$\varnothing$
$M3$	$log1$	$\varnothing$	$\varnothing$

PM	$O1$	$O2$	$O3$
$M1$	or	$\varnothing$	$\varnothing$
$M2$	$\varnothing$	$impl$	$\varnothing$
$M3$	or	$\varnothing$	$\varnothing$

$$(\vee\supset\vee): L^{(3)} * L^{(3)} \longrightarrow L^{(3)} : [(L_1 \mid L_3), L_2, \varnothing]$$

$$\left[\begin{array}{l} Log_1 : L_1 * L_1 \xrightarrow[disjunction\ \vee]{} L_1 \mid L_3 \\ Log_2 : L_2 * L_2 \xrightarrow[imlication\ \rightarrow]{} L_2 \\ Log_3 : L_3 * L_3 \xrightarrow[disjunction\ \vee]{} L_1 \end{array}\right.$$

PM	$O1$	$O2$	$O3$
$M1$	S_1	$\varnothing$	$\varnothing$
$M2$	$\varnothing$	S_2	$\varnothing$
$M3$	S_1	$\varnothing$	$\varnothing$

$(\vee\supset\vee)$	S_1^1	S_2^1	S_3^1	S_1^2	S_2^2	S_3^2	S_1^3	S_2^3	S_3^3
1	T_1	–	T_1	–	–	–	–	–	–
2	T_1	–	–	–	–	–	–	–	–
3	–	–	T_1	–	–	–	–	–	–
4	T_1	–	–	–	–	–	–	–	–
5	F_1	–	–	–	T_2	–	–	–	–
6	–	–	–	–	F_2	–	–	–	–
7	–	–	T_1	–	–	–	–	–	–
8	–	–	–	–	T_2	–	–	–	–
9	–	–	F_1	–	T_2	–	–	–	–

$[\vee\supset\vee]$	$O1$	$O2$	$O3$
$M1$	$\vee_1$	$\varnothing$	$\varnothing$
$M2$	$\varnothing$	$\supset_2$	$\varnothing$
$M3$	$\vee_1$	$\varnothing$	$\varnothing$

$(\vee\supset\vee)$	1	2	3
1	$T_{1.1}$	T_1	T_1
2	T_1	F_1 / T_2	F_2
3	T_1	T_2	T_2 / F_1

PolyLogics
Towards a formalization of polycontextural Logics.
http://www.thinkartlab.com/pkl/lola/PolyLogics.pdf

$[\vee \supset \vee]$	S_1^1	S_2^1	S_3^1	S_1^2	S_2^2	S_3^2	S_1^3	S_2^3	S_3^3
1	○	–	○	–	–	–	–	–	–
2	○	–	–	–	–	–	–	–	–
3	–	–	○	–	–	–	–	–	–
4	○	–	–	–	–	–	–	–	–
5	△	–	–	–	△	–	–	–	–
6	–	–	–	–	□	–	–	–	–
7	–	–	○	–	–	–	–	–	–
8	–	–	–	–	△	–	–	–	–
9	–	–	△	–	△	–	–	–	–

$[\vee \supset \vee]$	1	2	3
1	○	○	○
2	○	△	□
3	○	△	△

$[\vee \supset \vee]$	$O1$	$O2$	$O3$
$M1$	$\vee_1$	$\varnothing$	$\varnothing$
$M2$	$\varnothing$	$\supset_2$	$\varnothing$
$M3$	$\vee_1$	$\varnothing$	$\varnothing$

Pattern: [bif, id, id] for transjunction

$[\oplus \vee \wedge]$	S_1^1	S_2^1	S_3^1	S_1^2	S_2^2	S_3^2	S_1^3	S_2^3	S_3^3
1	○	–	–	–	–	–	○	–	○
2	–	–	–	□	–	–	□	–	–
3	–	–	–	–	–	–	–	–	○
4	–	–	–	□	–	–	□	–	–
5	△	–	–	△	△	–	–	–	–
6	–	–	–	–	△	–	–	–	–
7	–	–	–	–	–	–	–	–	○
8	–	–	–	–	△	–	–	–	–
9	–	–	–	–	□	–	–	–	□

$[\oplus \vee \wedge]$	$O1$	$O2$	$O3$
$M1$	$[trans]_1$	$[trans]_1$	$[trans]_1$
$M2$	$\varnothing$	$[or]_2$	$\varnothing$
$M3$	$\varnothing$	$\varnothing$	$[and]_3$

$[\oplus \vee \wedge]$	1	2	3
1	○	□	□
2	□	△	△
3	□	△	□

$$\begin{array}{c} t_1\ X \oplus \vee \wedge Y \\ \hline t_1\ X \\ t_1\ Y \end{array} \qquad \begin{array}{c} f_1\ X \oplus \vee \wedge Y \\ \hline f_1\ X \\ f_1\ Y \end{array}$$

$$\begin{array}{c} t_2\ X \oplus \vee \wedge Y \\ \hline \begin{array}{c||c} t_2\ X \mid t_2\ Y & \begin{array}{c} f_1\ X \\ f_1\ Y \end{array} \end{array} \end{array} \qquad \begin{array}{c} f_2\ X \oplus \vee \wedge Y \\ \hline \begin{array}{c||c|c} \begin{array}{c} f_2\ X \\ f_2\ Y \end{array} & \begin{array}{c} f_1\ X \\ t_1\ Y \end{array} & \begin{array}{c} t_1\ X \\ f_1\ Y \end{array} \end{array} \end{array}$$

$$\begin{array}{c} t_3\ X \oplus \vee \wedge Y \\ \hline \begin{array}{c||c} \begin{array}{c} t_3\ X \\ t_3\ Y \end{array} & \begin{array}{c} t_1\ X \\ t_1\ Y \end{array} \end{array} \end{array} \qquad \begin{array}{c} f_3\ X \oplus \vee \wedge Y \\ \hline \begin{array}{c||c|c} f_3\ X \mid f_3\ Y & \begin{array}{c} f_1\ X \\ t_1\ Y \end{array} & \begin{array}{c} t_1\ X \\ f_1\ Y \end{array} \end{array} \end{array}$$

The double character of transjunctions as rejections (neti / neti) and re-positioning (imposing) the rejected situation in / on another system at another place is well documented by the different representations proposed. This insight in the double character of transjunctions was clear long ago (Materialien 1973). But only the different representations together given by tableaux, brackets and matrices, are inscribing the pattern and behavior of it adequately.

2.3 Surprises with the distribution of transjunctions?

How to explain this kind of distribution? What we learned in place-valued logics was that transjunctions are rejecting value-alternatives and marking this rejection with values not belonging to the sub-system from which the rejection happens. The frame values of the transjunction remain accepted. Thus, there is nothing mentioned which could justify this "wild" decomposition and distribution of parts of a transjunction over different sub-systems and being linked with a single core value to the guest sub-system. Again, the more mathematical settings of transjunctions by universal algebras and category theory have failed to give any further information usable for implementation.

Transjunctions are understood in the proposed setting as compositions of *partial* functions. Thus, the parts have to be mediated to build the whole function. Hence, a frame-element has to function as a mediation point, additional to the core elements as rejectional elements. Without such a partial mediation of the rejectional parts the partial function would be free floating in a neighbor system without a systematic reason. Hence, with this frame--element being mediated the partial function is fixed at it place in the neighbor system. On the other hand, if both frame-elements would be distributed there wouldn't be a transjunction but a replication of a transjunctional morphogram as such without a rejectional behavior.

This argumentation gets some justification in the context of polycontextural logics. Without the "additional" distribution of a frame-element the tableaux-based proof systems wouldn't work properly. This is based on *experiences* and not on proofs. There is still no general mathematical framework to produce reasonable proofs for transjunctional situations.

Such insights in the functioning of distributed transjunction becomes quite clear in the proposed notational order of the sub-systems by the tabular matrix of dissemination.

[⊕ v ⊕]	S_1^1	S_2^1	S_3^1	S_1^2	S_2^2	S_3^2	S_1^3	S_2^3	S_3^3
1	○	–	○	–	–	–	○	–	○
2	–	–	–	□	–	–	□	–	–
3	–	–	△	–	–	△	–	–	–
4	–	–	–	□	–	–	□	–	–
5	△	–	–	△	△	–	–	–	–
6	–	–	–	–	△	–	–	–	–
7	–	–	△	–	–	△	–	–	–
8	–	–	–	–	△	–	–	–	–
9	–	–	–	–	□	□	–	–	□

[⊕ v ⊕]	$O1$	$O2$	$O3$
$M1$	$[trans]$	$[trans]$	$[trans]$
$M2$	$\varnothing$	$[or]$	$\varnothing$
$M3$	$[trans]$	$[trans]$	$[trans]$

A full occupation

The desire to fill the matrix, and to involve it fully into the positional game, interactional operators, like total transjunctions, are disseminated over all main positions defined as the diagonal of the matrix. This full occupation, realized by 3 total transjunctions, is an example for the standard case of dissemination. That is, no replications are involved.

$[\oplus\oplus\oplus]$	S_1^1	S_2^1	S_3^1	S_1^2	S_2^2	S_3^2	S_1^3	S_2^3	S_3^3
1	○	–	○	–	–	–	○	–	○
2	–	–	–	□	–	–	□	–	–
3	–	–	△	–	–	△	–	–	–
4	–	–	–	□	–	–	□	–	–
5	△	–	–	△	△	–	–	△	–
6	–	○	–	–	–	–	–	○	–
7	–	–	△	–	–	△	–	–	–
8	–	○	–	–	–	–	–	○	–
9	–	□	–	–	□	□	–	–	□

$[\oplus\oplus\oplus]$	$O1$	$O2$	$O3$
$M1$	$[trans]$	$[trans]$	$[trans]$
$M2$	$[trans]$	$[trans]$	$[trans]$
$M3$	$[trans]$	$[trans]$	$[trans]$

Kenomic Abacus

$$keno-Abacus = \left[diss^{(m,\,n)},\ (mg_1, ..., mg_{15}),\ sop,\ \mathfrak{R}\right]$$

$$sop = \{id,\ perm,\ red,\ repl,\ bif\}$$

$$\mathfrak{R} = \{refl^1,\ ...,\ refl^s\}$$

2.4 Replications as Cloning

Additional to the super-operators based on mediation, i.e., identity, permutation, reduction, bifurcation, I introduced the operator replication. A morphogrammatic system with a complexity of 3 has a distribution of only 3 morphograms – and not more. But this is changing if we involve transjunctions and understand them as interactional operators.

Replications are understood in analogy to cloning. Cloned systems are not in the same sense mediated to their neighbors as the other sub-systems but they are nevertheless not arbitrarily added to the system as such. Thus, a morphogrammatic system with a complexity of 3 has a distribution of more than only 3 morphograms – all in all 9 morphograms can be involved – but not more on one level. With the concept of reflectional deepness more morphograms can be involved.

Morphograms are very flexible because they are not ruled by identity principles like their semiotic counter-parts. Hence, if we allow other mechanism of togetherness, then even more than 9 morphograms can be realized in a system of complexity 3. But this is working only with togetherness as fusion, overlapping etc., and not with "concatenation" or chaining.

The morphogrammatic modi of togetherness had been called in German: *Verkettung*, *Verknüpfung*, *Verschmelzung* (chaining, concatenation, fusion).

Replications and reductions can be, at a first glance, conflictive. Replications are augmenting the number of operators involved. Reductions are not changing the number of operators but reducing the number of different sub-systems in play.

$$
(O1O2O3)
\begin{bmatrix}
\begin{bmatrix} O1 \\ \begin{pmatrix} M1M2M3 \\ (G110) \end{pmatrix} \end{bmatrix} \\
\begin{bmatrix} O2 \\ \begin{pmatrix} M1M2M3 \\ (G020) \end{pmatrix} \end{bmatrix} \\
\begin{bmatrix} O3 \\ \begin{pmatrix} M1M2M3 \\ (G033) \end{pmatrix} \end{bmatrix}
\end{bmatrix}
$$

PM	$O1$	$O2$	$O3$
$M1$	S_1	$\varnothing$	$\varnothing$
$M2$	S_1	S_2	S_3
$M3$	$\varnothing$	$\varnothing$	S_3

What is the logical operator corresponding to this pattern and what is its morphogram compound?

The example shows clearly a reflectional, i.e., replicational situation for S_1 and S_3 and not a reduction of S_2.

Therefore, the example is not conventional to the common definitions of place-valued and polycontextural logic.

The new situation is using replication, thus this operator has to be justified.

Thus, the question is: Why do we need replicators?

From a purely formal point of view we have to take the chances given by the polycontextural matrix. Without replications the matrix is not fully interpreted. The matrix gives space to interpret the formal possibility as replication. Thus, we have to try it.

$$
\begin{array}{l}
(O1O2O3) \\
\left[\begin{array}{l}
\left[\begin{array}{l} O1 \\ \left(\begin{array}{r} M1M2M3 \\ (G110) \end{array}\right) \end{array}\right] \\
\left[\begin{array}{l} O2 \\ \left(\begin{array}{r} M1M2M3 \\ (G020) \end{array}\right) \end{array}\right] \\
\left[\begin{array}{l} O3 \\ \left(\begin{array}{r} M1M2M3 \\ (G033) \end{array}\right) \end{array}\right]
\end{array}\right]
\end{array}
$$

PM	$O1$	$O2$	$O3$
$M1$	S_1	$\varnothing$	$\varnothing$
$M2$	S_1	S_2	S_3
$M3$	$\varnothing$	$\varnothing$	S_3

$[JJ, J, JJ]$	$O1$	$O2$	$O3$
$M1$	$[junct]_1$	$\varnothing$	$\varnothing$
$M2$	$[junct]_1$	$[junct]_2$	$[junct]_3$
$M3$	$\varnothing$	$[\varnothing]$	$[junct]_3$

Partial transjunction

PM	$O1$	$O2$	$O3$
$M1$	S_1	$\varnothing$	$\varnothing$
$M2$	S_2	S_2	S_3
$M3$	$\varnothing$	$\varnothing$	S_3

$[J \oplus JJ]$	$O1$	$O2$	$O3$
$M1$	$[junct]_1$	$\varnothing$	$\varnothing$
$M2$	$[trans]_2$	$[trans]_2$	$[junct]_3$
$M3$	$\varnothing$	$[\varnothing]$	$[junct]_3$

Pattern: [id, bif, repl]

Iterative reflectionality

$$(O1O2O3)$$

$$\left[\begin{matrix}
\begin{bmatrix} O1 \\ \begin{pmatrix} M1M2M3 \\ (G110)\begin{pmatrix} M1 \\ (G010)\begin{pmatrix} M1 \\ (G010)\begin{pmatrix} M1 \\ (G010) \end{pmatrix} \end{pmatrix} \end{pmatrix} \end{pmatrix} \end{bmatrix} \\
\begin{bmatrix} O2 \\ \begin{pmatrix} M1M2M3 \\ (G222) \end{pmatrix} \end{bmatrix} \\
\begin{bmatrix} O3 \\ \begin{pmatrix} M1M2M3 \\ (G033) \end{pmatrix} \end{bmatrix}
\end{matrix}\right]$$

PM	$O1$	$O2$	$O3$
$M1$	S_1	S_2	$\varnothing$
$M2$	$S_{1.1.1.1}$	S_2	S_3
$M3$	$\varnothing$	S_2	S_3

Pattern: [$repl^4$, $repl^2$, $repl^1$]

Iterative reflectionality or self-reflection of system S_1, distributed twice for S_2 and once reflectionality for system S_3.

Iterative reflectionality is producing a kind of a *deepness* which can be interpreted as the deepness of introspection. Deepness of reflection is not connected with the rank of a meta-language hierarchy.

The general matrix is giving the *broadness* of the interactional / reflectional system.

In another terminology, iterative reflectionality is opening up multi-dimensional matrices representing the layers of reflection.

The couple-terms deepness / broadness was used by Gunther in the 50s to describe the dimensions of his theory of reflection. In some sense, broadness was seen as European and deepness as Indian. It also mirrors well the basics of skiing and gliding. The complex action of proemiality, the reversal 'turn on one's skiis', is not yet involved.

Before you start skiing in the deepness and broadness of the new snow, drawing your loops and making your looping, you have to take position. Positioning your skis is the proemiality of any skiing.

2.5 Interpretation of the Polycontextural Matrix

I. Strictly mediated systems
a) accretive, by the diagonal
b) iterative, by reduction or replication (cloning)
II. Not-strictly mediated systems
a) by replication

There are for a 3-contextural system only 3 occupation of the PM by junctions. For transjunctions there are more than 3 occupations. Transjunction plus 2. There are more than 3 occupations of the PM for II. a)

Extensions of systems
a) by iteration (cloning, replication) augmenting the rows (complication)
b) accretive by mediation augmenting the columns

Extensions: iterative, accretive and mixed

$[E]$	O_1	O_2	O_3
M_1	E_1	$\varnothing$	$\varnothing$
M_2	E_1	$\varnothing$	E_3
M_3	E_1	$\varnothing$	E_3
M_4	E_1	$\varnothing$	$\varnothing$

$[E]$	O_1	O_2	O_3	O_4
M_1	E_1	$\varnothing$	$\varnothing$	$\varnothing$
M_2	E_1	$\varnothing$	E_3	E_4
M_3	E_1	E_4	E_3	E_4

$[E]$	O_1	O_2	O_3	O_4
M_1	E_1	$\varnothing$	$\varnothing$	$\varnothing$
M_2	E_1	$\varnothing$	E_3	E_4
M_3	E_1	E_4	E_3	E_4
M_4	E_1	$\varnothing$	E_1	E_4

A case of cloning E

$[EEE]_2$	S_1^1	S_2^1	S_3^1	S_1^2	S_2^2	S_3^2	S_1^3	S_2^3	S_3^3
1	○	–	○	–	–	–	–	–	○
2	△	–	△	–	–	–	–	–	–
3	–	–	–	–	–	–	–	–	□
4	△	–	–	–	–	–	–	–	–
5	○	○	–	–	–	–	–	–	–
6	–	△	–	–	–	–	–	–	–
7	–	–	–	–	–	–	–	–	□
8	–	△	△	–	–	–	–	–	–
9	–	○	○	–	–	–	–	–	○

$[EEE]_2$	1	2	3
1	○	△	□
2	△	○	△
3	□	△	○

$[EEE]_2$	$O1$	$O2$	$O3$
$M1$	E_1	$\varnothing$	$\varnothing$
$M2$	E_1	$\varnothing$	$\varnothing$
$M3$	E_1	$\varnothing$	E_3

Restricted mediated cloning.

2.5.1 Extended non-mediated cloning

Cloning can be justified by the replicator operator *rep*, and the replicans has not to be mediated with a neighbor system. This is not arbitrary and confusing but ruled by the system architecture and the replication operator.

If the tabular matrix makes any sense it has to have a reasonable interpretation on different levels of polycontextural logic and morphogrammatics.

Extended non-mediated cloning

	E	E	E					E	E
1	○	–	○	–	–	–	–	–	○
2	△		△						–
3	–		–						□
4	△		–						
5	○	○	–	–	–	–			
6		△	–						
7		–	–						□
8		△	△						–
9		○	○	–	–	–			○
10			–					○	–
11			–					□	–
12			–				–	–	–
13			–				–	□	–
14	–	–	–	–	–	–	–	○	–

$[EEE]_2$	$O1$	$O2$	$O3$
$M1$	E_1	$\varnothing$	$\varnothing$
$M2$	E_1	$\varnothing$	E_3
$M3$	E_1	$\varnothing$	E_3

3 An Abacus of Contextures

3.1 Elementary morphograms

To start somewhere, we say elementary contextures are the building blocks of compound contextures, i.e., of polycontextural objects. Elementary contextures are considered at first as bi-polar objects, bi-objects or dyads, they have the properties of duality, polarity, dichotomy, etc. But we could also start with triads or tetrads, in general with n-ads. It follows that the complexity of elementary contextures is not stable and reduced to bi-objects. Until know, we don't have a clear concept and apparatus for triads or n-ads in general. But we know quit well the definitions and behaviors of all kinds of dyads, numeric, semiotic, logical, etc. Thus, we decide to start our introduction of the theory of polycontexturality and morphogrammatics with dyads, i.e., bi-objects.

To start with triads could happen with the morphograms [1] to [5]. They would be the elementary structures for more complex morphogrammatic systems. The binary morphograms [A] and [B] would have to be understood as reductions of the triadic morphograms.

Such an elementary bi-object appears as an *iteration* [A] or as an *accretion* [B] of its position. Thus there are only 2 bi-objects in this interpretation of morphogrammatics: mg = {[A], [B]}. In fact, only [B] is a complete dyad and [A] is a monad.

.

mg	$[A]$	$[B]$
1	○	○
2	○	△

$MG^{(3)}$	$[1]$	$[2]$	$[3]$	$[4]$	$[5]$
1	○	○	○	○	○
2	○	○	△	△	△
3	○	△	○	△	□
$med^{(3)}$	$[AAA]$	$[ABB]$	$[BBA]$	$[BAB]$	$[BBB]$

Elementary bi-objects, like [A] and [B] have continuations, again in iterative and accretive form. The tree of this continuations is producing the set: $MG^{(3)}$= {[1], [2], [3], [4], [5]}. Both, mg and $MG^{(3)}$ are parts of the trito-structure of kenogrammatics.

From the point of view of logic we are dealing with the morphogrammatics of unary operations, thus avoiding complex considerations of the morphogrammatics of binary operations as introduced above. In Na's terms we are dealing the core-structure only.

Transjunctions between rejection and bifurcation

Transjunctions had been introduced by Gunther as binary functions with the *rejectional* property that a couple of different truth-values can be rejected and has not to be *accepted* as it happens with junctions in contrast to transjunctions. Thus, according to Gunther, a new "bi-valence" was introduced for morphogrammatic-based place-valued logics: *acception/rejection*.

Later I introduced the idea of transjunctions as logical *bifurcations*. The *splitting* into different logical systems, the holding at once at different places, was in focus. This too worked properly for binary functions. There was no need to think of transjunctions for *unary* operations. Simply because there is no value-pair to be rejected. But rejection is only one part of bifurcation. Transjunctions are *rejectional bifurcations*. But not all bifurcations are rejectional. The splitting functionality of bifurcation can happen without rejection. Thus, unary functions can be split. The idea of bifurcation is naturally applicable for unary functions. In short: unary transjunctions are bifurcations without rejection or rejection-free bifurcations. The term "bifurcation" has not to be reduced to a *"split (fork) into two"*, it can split into a multitude. It may be too much of terminology to use the term "multi-furcation" or n-furcation instead of bifurcation.

Guided by the idea of the splitting property of bifurcation unary function can by treated in a transjunctional manner. For morphograms, the splitting has nothing to do with a quantitative partition into parts. The morphogram splits in itself. Such a concept of splitting has some correspondence to two other important concepts of morphogrammatics, the concept of *cloning* and *replication*.

Replication is introduced as a cloning into itself. *Cloning* is a replication into others. In other words, replication is placed in the system it derives, like reflectional introspection. Cloning is replicating a morphogram at another place, outside of its derivation.

3.2 Mapping of bi-objects onto the polycontextural matrix

Based on the experiences we made with the morphogrammatics of binary functions, especially transjunctions, we are prepared to study the dissemination (distribution and mediation) of bi-objects. This dissemination is a mapping of bi-objects onto the polycontextural matrix, ruled by the super-operators, sops = {id, perm, red, repl, clon, bif}. Again, there is no information given by the common approach to morphograms and morphogrammatics to organize such mappings including replication, cloning and bifurcation. The following are examples for: identity, reduction, replication, cloning and bifurcation. Not including permutation.

As a consequence of the fact that the complexity of elementary contextures is flexible, decompositions in triads or general n-ads are possible. It is also of importance to see that mixed "based" systems, say dyads, triads, tetrads, etc. could hold at once inn morphogrammatic and as an interpretation in pluri-dimensional arithmetics. Depending on the complexity of domain under considerations the structural complexity can differ for different disseminated arithmetical systems. Obviously, this has nothing to do with the classic concept of n-ary representation of natural numbers by different numeric bases. As we know, all n-ary representations can be modeled and reduced to the dyadic representation of numbers.

Dyads in Triads

Standard mapping of morphogram $MG[5]$ *onto the matrix* $PM:[id, id, id]$

$[BBB]$	S_1^1	S_2^1	S_3^1	S_1^2	S_2^2	S_3^2	S_1^3	S_2^3	S_3^3
1	○	–	–	–	–	–	–	–	○
2	△	–	–	–	△	–	–	–	–
3	–	–	–	–	□	–	–	–	□

$[BBB]$	$O1$	$O2$	$O3$
$M1$	B_1	$\varnothing$	$\varnothing$
$M2$	$\varnothing$	B_2	$\varnothing$
$M3$	$\varnothing$	$\varnothing$	B_3

Standard mapping of $MG[1]$ *onto the matrix* $PM:[repl, \varnothing, id]$

	S_1^1	S_2^1	S_3^1	S_1^2	S_2^2	S_3^2	S_1^3	S_2^3	S_3^3
1	○	–	–	–	–	–	–	–	○
2	○	○	–	–	–	–	–	–	–
3	–	○	–	–	–	–	–	–	○

$[AA, \varnothing, A]$	$O1$	$O2$	$O3$
$M1$	A_1	$\varnothing$	$\varnothing$
$M2$	A_1	$\varnothing$	$\varnothing$
$M3$	$\varnothing$	$\varnothing$	A_3

$[repl, \varnothing, id]: (AAA) \longrightarrow (AAA, \varnothing\varnothing\varnothing, \varnothing\varnothing A)$

	S_1^1	S_2^1	S_3^1	S_1^2	S_2^2	S_3^2	S_1^3	S_2^3	S_3^3
1	○	–	○	–	–	–	–	–	○
2	○	○	–	–	–	–	–	–	–
3	–	○	○	–	–	–	–	–	○

$[AAA, \varnothing, A]$	$O1$	$O2$	$O3$
$M1$	A_1	$\varnothing$	$\varnothing$
$M2$	A_1	$\varnothing$	$\varnothing$
$M3$	A_1	$\varnothing$	A_3

$[id, id, red]: (BBB) \longrightarrow (B\varnothing B, \varnothing B\varnothing, \varnothing\varnothing\varnothing)$

	S_1^1	S_2^1	S_3^1	S_1^2	S_2^2	S_3^2	S_1^3	S_2^3	S_3^3
1	○	–	○	–	–	–	–	–	–
2	△	–	–	–	△	–	–	–	–
3	–	–	△	–	○	–	–	–	–

$[BB, B, \varnothing]$	$O1$	$O2$	$O3$
$M1$	B_1	$\varnothing$	$\varnothing$
$M2$	$\varnothing$	B_2	$\varnothing$
$M3$	B_3	$\varnothing$	$\varnothing$

$[bif, id, id]: (BBB) \longrightarrow (B\varnothing\varnothing, bB\varnothing, b\varnothing B)$

	S_1^1	S_2^1	S_3^1	S_1^2	S_2^2	S_3^2	S_1^3	S_2^3	S_3^3
1	○	–	–	○	–	–	–	–	○
2	△	–	–	–	△	–	△	–	–
3	–	–	–	–	□	–	–	–	□

$[B, bB, bB]$	$O1$	$O2$	$O3$
$M1$	B_1	b_1	b_1
$M2$	$\varnothing$	B_2	$\varnothing$
$M3$	$\varnothing$	$\varnothing$	B_3

$[id, clon, clon]:(BBA) \longrightarrow (BBA, \varnothing BA, \varnothing BA)$

	S_1^1	S_2^1	S_3^1	S_1^2	S_2^2	S_3^2	S_1^3	S_2^3	S_3^3
1	○	–	○	–	–	○	–	–	○
2	△	△	–	–	△	–	–	△	–
3	–	○	○	–	○	○	–	○	○

$[BBA, BA, BA]$	$O1$	$O2$	$O3$
$M1$	B_1	$\varnothing$	$\varnothing$
$M2$	B_2	B_2	B_2
$M3$	A_3	A_3	A_3

3.3 Dynamics of Dissemination

3.3.1 Dyads in Triads and Tetrads

[BBA]	[B]	[B]	[A]
1	○	–	○
2	△	△	–
3	–	○	○

	[B]	[B]	[B]	[A]	[A]	[B]
1	○	–	–	○	–	○
2	△	△	–	–	△	–
3	–	○	○	○	–	–
4	–	–	△	–	△	△

3.3.2 Triads in Tetrades and Quintads

$[i] \in MG^{(3)}$

	[2]	[3]	[3]	[1]
1	○	–	○	○
2	○	○	–	○
3	△	△	△	–
4	–	○	○	○

$[i] \in MG^{(3)}$

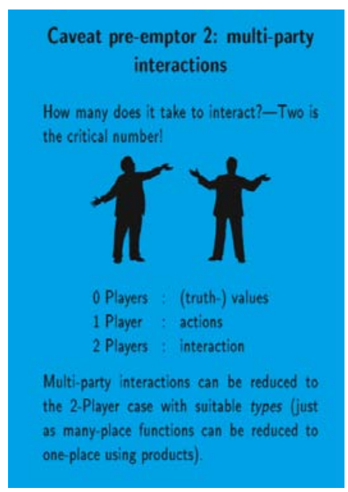
	[2]	[4]	[2]	[4]
1	○	–	○	○
2	○	–	○	–
3	△	△	–	–
4	–	□	–	□
5	–	□	□	□

It has to be mentioned again, that real triads, i.e., triadic-trichotomic objects, are not well understood today. Despite of Peirce's semiotics, category theory and others, all are based on dyads. Also Warren McCulloch's approach to triads didn't have much impact (Longyear). The same holds for Hegelian approaches. It is still an academic contract that n-ary relations, and whatsoever, can be reduced without loss to binarism. And to insist on a structural difference between dyads and triads is producing embarrassment for all parts.

3.3.3 n-ads in m-ads

	[2]	[B]	[A]	[1]
1	○	–	○	○
2	○	–	–	○
3	△	△	–	–
4	–	○	○	○

	[B]	[B]	[A]	[4]
1	○	–	○	○
2	△	△	–	–
3	–	○	○	–
4	–	–	–	□
5	–	–	–	□



"Perhaps you remember that Peirce has written Ernst Schröder that he likes his monograph about binary relations, but that he is *not in love* with binary relations at all. But since then we have all sorts of axiomatic set theories, and all are implementing the ingenious definitions of ordered pairs by Kuratowski and Wiener. With this definition of ordered pair you always can reduce a more complex situation to a binary one. The recent example can be found in the interesting paper of Abramsky about game theory in computer science."

SUSHI'S LOGICS

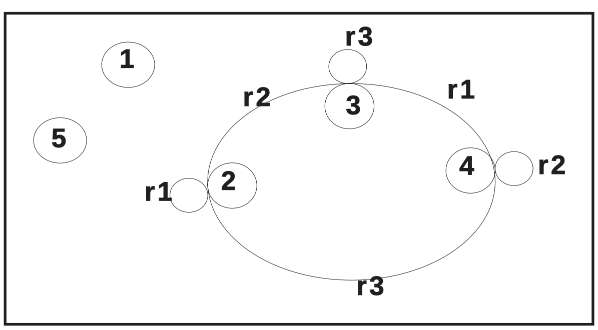
3.4 Operations on bi-objects

3.4.1 Reflectors

$$Reflector\,\Re \ on \ MG^{(3)}$$
$$self-dual: \begin{Bmatrix} \Re[1]=[1],\ \Re[3]=[3], \\ \Re[5]=[5] \end{Bmatrix}$$
$$dual: \{\Re[2]=[4],\ \Re[4]=[2]\}$$
$$\rho^2-dual=\{\rho^2[2]=[3],\ \rho^2[3]=[2]\}$$
$$\rho^2-self-dual=\{\rho^2[4]=[4]\}$$
$$\rho^3-dual=\{\rho^3[2]=[4],\ \rho^3[4]=[2]\}$$
$$\rho^3-self-dual=\{\rho^3[3]=[3]\}$$
$$\Re=\rho^1\rho^2\rho^3$$
$$\rho^1-self-dual=\begin{Bmatrix} \rho^1[1]=[1], \\ \rho^1[2]=[2], \\ \rho^1[5]=[5] \end{Bmatrix}$$
$$\rho^1-dual=\begin{Bmatrix} \rho^1[3]=[4], \\ \rho^1[4]=[3] \end{Bmatrix}$$
$$\rho^1[1]=\rho^2[1]=\rho^3[1]=[1]$$
$$\rho^1[5]=\rho^2[5]=\rho^3[5]=[5]$$
$$\rho^1(\rho^2[2])=\rho^2(\rho^1[3])=[4]$$
$$\rho^1(\rho^2[4])=\rho^2(\rho^1[2])=[3]$$
$$\rho^1(\rho^2[3])=\rho^2(\rho^1[4])=[2]$$

Again, such reflectional analysis, combined with several classification systems for compound structures, are well developed, documented and programmed in the book "*Morphogrammatik*" (Kaehr, Mahler). A lot of structural information and combiinatorics can be be gathered there, for free. What is missing in the 90s design is a free mapping onto the polycontextural matrix managed by the super-operators. Hence the job to do here is to bring these two trends together.

Cycles in [MG$^{(3)}$, r]



In a binary arithmetic system with X={0, 1}, the only structure is a 2-valued permutation, corresponding to a 2-valued negation: neg(0)=1 and neg(1) = 0, thus (neg(neg(X))=X.

Hence, the negational cycle of binary arithmetic is the shortest possible non-self-cycle.

3.5 Position system for contextures

To map bi-objects as the morphograms [A] and [B] on the polycontextural matrix PM means to use the PM as a positional system for morphograms.

In contrast to the linearity of the numeric position system, the positional matrix is at least tabular. This tabularity, suggesting 2-dimensionality, con be repeated and augmented by iterative operations, like introspection.

Until now, the position matrix PM was restricted to a regular standard form of (3, 3x3)-entries, thus $PM^{(3,9)}$. The elements used had been the elementary bi-objects (morphograms) [A] and [B]. Both together, the PM and the bi-objects, are determining a kind of a "binary" arithmetic of morphograms distributed over a tabular organization scheme. And restricted, because of the complexity m=3, to a counting of only 2 steps involving a third "mediating" step between the first and the second step.

On the base of this restriction interesting behaviors of dissemination ruled by the super-operators can be studied.

But now, we would like to ”count’ further in this game of tabularity.

A new important difference to classic position systems has to be considered. That is, the *identity* of the elements of distribution. In other words, in a binary arithmetic, the elements 0 and 1 are always the same. Their numeric meaning is changing in respect to their position but their identity as markers, 0 and 1, for the numeric value is definitively identical.

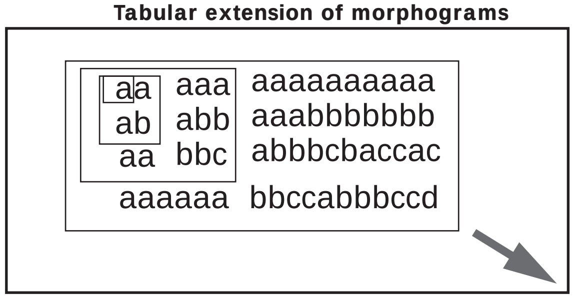
Also our small morphogrammatic system with only [A] and [B] is repeating these elements over different positions there identity is changing in the process of occupying a position.

This was the case at the very beginning from [A] and [B] to, say [B, B, A] where the first and the second occurrence of [B] is marked differently.

Another obvious difference to a classic position system is the fact that in the morphogrammatic case we are dealing with *differences* and not with atomic terms. We are always adding in a succession the full difference, represented by [A] and [B], to the existing configuration. This can be seen as if we would always add 0 and 1 at once if we want to add something, that is a unit. In other words, to augment a configuration by one bi-object, one part of the bi-partition is glued to the existing element of the configuration and the other part represents the augmentation.

3.5.1 Continuation operators

Tabular extension of morphograms



3.5.2 Composition operators

Without being involved with the positional matrix, the rules of such successions are the rules of trito-kenogrammatic successors. There are many kinds of successions, concatenating, gluing melting which shouldn't be confused.

Gluing :

$$[\bigcirc\bigcirc\triangle] \,\&\, [\bigcirc\triangle] = \{[\bigcirc\bigcirc\triangle\bigcirc], [\bigcirc\bigcirc\triangle\square]\}$$

$$\Rightarrow$$

$$[ABB] \,\&\, [B] = \{[ABBBAA], [ABBBB]\}$$

Fusion :

$$[\bigcirc\bigcirc\triangle]\$[\bigcirc\triangle] = \{[\bigcirc\bigcirc\triangle]\}$$

De – Fusion :

$$[\bigcirc\bigcirc\triangle] \Rightarrow \{[\bigcirc\bigcirc\triangle], [\bigcirc\triangle]\}$$

If necessary, this gluing process can be interpreted as a coalition building procedure or as a fusion. Where the environment of the parts are adopting each other and a new domain is added to the new whole. This can happen, in this case, in only two ways. One is to support a domain of the partner system by replicating it in the domain of the cooperating part, or by adding a new domain to the coalition system. Support and novelity are the two mody of such a cooperation.

$$[\bigcirc\bigcirc\triangle]@[\bigcirc] = \{[\bigcirc\bigcirc\triangle\bigcirc], [\bigcirc\bigcirc\triangle\triangle], [\bigcirc\bigcirc\triangle\square]\}$$

Concatenation

$$[\bigcirc\bigcirc\triangle]@[\bigcirc\triangle] = \left\{\begin{matrix} [\bigcirc\bigcirc\triangle\bigcirc\triangle], [\bigcirc\bigcirc\triangle\triangle\bigcirc], [\bigcirc\bigcirc\triangle\bigcirc\square], [\bigcirc\bigcirc\triangle\triangle\square], \\ [\bigcirc\bigcirc\triangle\square\bigcirc], [\bigcirc\bigcirc\triangle\square\triangle], [\bigcirc\bigcirc\triangle\square\blacktriangle] \end{matrix}\right\}$$

$$[ABB]@[B] = \left\{\begin{matrix} [ABBBAABABB], [ABBABBBBAA], [ABBBAABBBB]. \\ [ABBABBBBBB], [ABBBBBBBAA], \\ [ABBBBBBABB], [ABBBBBBBBB] \end{matrix}\right\}$$

Morphogrammatik (PDF)
http://www.thinkartlab.com/pkl/media/mg-book.pdf

[○○△○□]	[○○△△□]	[○○△□○]	[○○△□△]	[○○△□▲]
A B B B	*A B A B*	*A B B B*	*A B B B*	*A B B B*
B A B	*B B B*	*B B B*	*B B A*	*B B B*
A B	*B B*	*B A*	*B B*	*B B*
B	*B*	*A*	*B*	*B*
[*ABBBAABBBB*]	[*ABBABBBBBBB*]	[*ABBBBBBBAA*]	[*ABBBBBBABB*]	[*ABBBBBBBBB*]

The whole tedious *intricateness* is inherited by the insistence on the simple idea to distribute morphic dyads [A] and [B] over a tabular position systems.

The decision for kenomic dyads is nothing ultimate. Once the game is understood, we can start with flexible "bases", triads, tetrads, etc.

It is not the place to go into philosophical reflections about the dyads as they occur in Platonism or the I Ching. The only thing to recognize is that these dyads are not part of any economy of logical, ontological and semiotical identity and its dualism.

These dyads are functioning as the kenomic realizations of dynamic sameness and difference in the calculations of the Abacus.

To determine the behavior of the kenomic Abacus the only ”thing" we need are the dyads disseminated over the positional matrix.

Arithmetical representations in the sense of kenomic or dialectical numbers are then introduced as special interpretations of the morphogrammatic structure.

And again, natural numbers in their uniqueness are naturally obtained by freezing the whole kenomic game into linearity and atomic identity.

But there is no need to freeze kenomic behaviors to get connected with numbers. It is possible to interpret kenomic configurations as distributed binary number systems.

A kenomic Abacus is involved in computing, interactional and reflectional modeling of computations.

Binary arithmetic is computing with 0 and 1. This can be interpreted as yes/no or on/off, etc. decisions which are defining the *states* of a system. Kenomic computing is dealing with sameness and difference. This is corresponding to *behaviors*, behaviors are distinguished as the *same* or *different*, not as identifiable and separable entities or states, identical or non-identical, but as observable actions or behaviors. Hence, morphograms are not representing the states but the processuality of the switch from one state to the other. Independently of the state "0" or "1" in the switch form (01) to (10) and from (10) to (01), the structure of the switch is the same and is represented by the morphogram [B]. If nothing happens to "0" or to "1", i.e., an identity holds for (00) and (11), the structure or pattern is the same for both behaviors, and the corresponding morphogram is [A].

3.5.3 Multiplication operators

3.5.4 Decomposition and Monomorphy

3.5.5 Kenomic Bisimulation

3.6 Remembering the epsilon/nu-structure

In the context of the book *Morphogrammatik, p. 66,* the mapping of the [A] and [B] elements into kenomic sequences, not yet into the positional matrix, was called the *epsilon/nu-structure,* and as usual it is well documented and programmmed.

The possibility of specific isomorphisms between different presentations is not denying the legitimacy of the fact that some presentations are opening up other developments than others.

Als eine andere Veranschaulichungsweise der Quotientenstruktur läßt sich die ϵ/ν-*Darstellung* verwenden. Die ϵ/ν-Schreibweise betont stärker den Aspekt der Struktur von Gleichheit und Verschiedenheit *zwischen* den Leerstellen[18], der *Tritostruktur*[19]. Auch sie ist isomorph zur Quotientenmenge und damit auch zu der entsprechenden Kenogrammsequenz.

Definition 3.15 **(ϵ/ν-Tripel)** *Die Funktion $\delta((i,j),z)$, mit $z = [b_1, \ldots, b_n]$, $b_k \in B$, ordne jedem Paar von Positionen (i,j) das Tripel (i,j,ϵ) zu falls $b_i = b_j$, sonst (i,j,ν).*

Die vierte Position, $j = 4, b_4 = c$ muß mit ihren drei Vorgängern verglichen werden:

$$\begin{array}{ccccccc} & & & \boxed{(1,4,\nu)} & & & \\ & & (1,3,\epsilon) & & \boxed{(2,4,\nu)} & & \\ & (1,2,\nu) & & (2,3,\nu) & & \boxed{(3,4,\nu)} & \\ \hline a & & b & & a & & c \end{array}$$

Offensichtlich sind die Positionen der Tripel in dieser Darstellung eindeutig bestimmt, so daß auf die Positionsangabe (i,j) verzichtet werden kann:

$$\begin{array}{ccccccc} & & & \nu & & & \\ & & \epsilon & & \nu & & \\ & \nu & & \nu & & \nu & \\ \hline a & & b & & a & & c \end{array}$$

Die Struktur dieser ϵ/ν-Tripel einer n-stelligen Sequenz kann somit als Liste von n Listen mit jeweils $(j-1)$ Tripeln dargestellt werden:

$$[[], [(1,2,\epsilon/\nu)], [(1,3,\epsilon/\nu), (2,3,\epsilon/\nu)], \ldots, [(1,n,\epsilon/\nu), \ldots, (n-1,n,\epsilon/\nu)]]$$

Definition 3.16 **(ϵ/ν-Struktur)** *Die ϵ/ν-Struktur einer n-stelligen Sequenz $z = [b_1, \ldots, b_j, \ldots, b_n]$ ist die Liste der n Listen von jeweils $(j-1)$ ϵ/ν-Tripel von z:*

$$[[], [(1,2,\epsilon/\nu)], \ldots, [(1,j,\epsilon/\nu), \ldots, (j-1,j,\epsilon/\nu)], \ldots, [(1,n,\epsilon/\nu), \ldots, (n-1,n,\epsilon/\nu)]]$$

Die ϵ/ν-Struktur einer Sequenz wird durch die ML-Funktion `ENstructure` berechnet:

3.7 Numeric interpretation of dyads

$$pluri-Dyads^{(m)}:\{0,1\}^{(m)}\longrightarrow MG^{(m)}$$

$$Tcontexture(3)=$$
$$\big((\bigcirc),((\bigcirc\bigcirc),(\bigcirc\triangle)),((\bigcirc\bigcirc\bigcirc),(\bigcirc\bigcirc\triangle),(\bigcirc\triangle\bigcirc),(\bigcirc\triangle\triangle),(\bigcirc\triangle\square))\big)$$

$$numb(Tcontexture(3))=$$
$$\big((0),((00),(01)),((000),(001),(010),(011),(012))\big)$$

$$dyads(numb(Tcontexture(3)))=$$
$$\left((0),((00),(01)),\left(\begin{pmatrix}00-\\-0\ 0\\0-0\end{pmatrix},\begin{pmatrix}00-\\-0\ 1\\0-0\end{pmatrix},\begin{pmatrix}01-\\-1\ 0\\0-0\end{pmatrix},\begin{pmatrix}01-\\-1\ 1\\0-1\end{pmatrix},\begin{pmatrix}01-\\-1\ 2\\0-2\end{pmatrix}\right)\right)$$

$$dyads(Tcontexture(3))=$$
$$\left((d0),((d0),(d1)),\left(\begin{pmatrix}d0\\d0\\d0\end{pmatrix},\begin{pmatrix}d0\\d1\\d0\end{pmatrix},\begin{pmatrix}d1\\\overline{d1}\\d0\end{pmatrix},\begin{pmatrix}d1\\d0\\d1\end{pmatrix},\begin{pmatrix}d1\\d1\\d1\end{pmatrix}\right)\right)$$

Dyads are the numeric base system of binary numbers. There are two kinds of base systems, complete and incomplete bases. Dyads with only one element for two places, monads, are incomplete numeric bases. They are the base systems for purely iterative systems, they may have nil-markers but are without a positional system. Dyads with two elements are complete binary bases for binary positional number systems. Thus, for a system with 3 contextures we have 5 compositions of a distribution and mediation of monads and dyads.

$$Tree(dyads(Tcoontexture(3)))=$$
$$\begin{pmatrix}d0\\d0\\d0\end{pmatrix}=\begin{bmatrix}UnaryTree\\UnaryTree\\UnaryTree\end{bmatrix},\ \begin{pmatrix}d0\\d1\\d0\end{pmatrix}=\begin{bmatrix}UnaryTree\\BinaryTree\\UnaryTree\end{bmatrix},\ \begin{pmatrix}d1\\\overline{d1}\\d0\end{pmatrix}=\begin{bmatrix}BinaryTree\\\overline{BinaryTree}\\UnaryTree\end{bmatrix},\ \begin{pmatrix}d1\\d0\\d1\end{pmatrix}=\begin{bmatrix}BinaryTree\\UnaryTree\\BinaryTree\end{bmatrix},\ \begin{pmatrix}d1\\d1\\d1\end{pmatrix}=\begin{bmatrix}BinaryTree\\BinaryTree\\BinaryTree\end{bmatrix}$$

$$\begin{pmatrix}d0\\d0\\d0\end{pmatrix}=\begin{bmatrix}UnaryTree\\UnaryTree\\UnaryTree\end{bmatrix}=\begin{bmatrix}\rightarrow\rightarrow\cdots\rightarrow\\\rightarrow\rightarrow\cdots\rightarrow\\\rightarrow\rightarrow\cdots\rightarrow\end{bmatrix}\qquad\begin{pmatrix}d0\\d1\\d0\end{pmatrix}=\begin{bmatrix}UnaryTree\\BinaryTree\\UnaryTree\end{bmatrix}=\begin{bmatrix}\rightarrow\rightarrow\cdots\rightarrow\\\nearrow\cdots\\\searrow\nearrow\cdots\\\searrow\cdots\\\rightarrow\rightarrow\cdots\rightarrow\end{bmatrix}$$

3.7.1 Unary tree & the abstractness of computation

> "We can, in principle, make do [auskommen] with an alphabet which contains only a single letter, e.g. the letter |. The words of this alphabet are (apart from the empty word): |, ||, |||, etc.These words are in a trivial way be identified with the natural numbers 0, 1, 2, ...[...] The use of an alphabet consisting one element only does not imply an essential limitation." Hans Hermes (1961), after: Epstein, Carnielli, p.67

(Extense studies about such a "stroke calculus", see Lorenzen 1962.)

Also this is very obvious, it is not as familiar as the binary introduction of natural numbers. Unary systems don't have a position system for their words, binary systems can be used as the prototype for numeric position systems. The statement that an unary alphabet is not putting any limitation on a theory of computability is well accepted. What is not mentioned within this statement is the fact that such a conception of computability is independent or neutral to the concept of positionality. Thus, computationality needs not to be positioned, it doesn't take a place and is therefore, again, a purely abstract system. This abstractness is based in the abstractness or ideality of sign systems.

3.7.2 Mixed, unary and binary systems

$$\begin{pmatrix} d1 \\ \overline{d1} \\ d0 \end{pmatrix} = \begin{bmatrix} BinaryTree \\ \overline{BinaryTree} \\ UnaryTree \end{bmatrix}$$

This case of distributing binary tree is of special interest because it demands for a mediation of a tree and its *dual* form. Interpreted as the representation "0" (or "1") and "1" it has to be understood that "0" (or "1") in the first system has the value of "1" (or "0") in the second system.

$$\begin{pmatrix} d1 \\ d1 \\ d1 \end{pmatrix} = \begin{bmatrix} BinaryTree \\ BinaryTree \\ BinaryTree \end{bmatrix}$$

This case of distribution corresponds to the common situation that all 3 sub-systems are structurally equivalent and are representing 3 complete binary number systems if interpreted as numeric.

$$\begin{pmatrix} d1 \\ d0 \\ d1 \end{pmatrix} \begin{pmatrix} d0 \\ d1 \\ d0 \end{pmatrix} \begin{pmatrix} d0 \\ d0 \\ d0 \end{pmatrix}$$

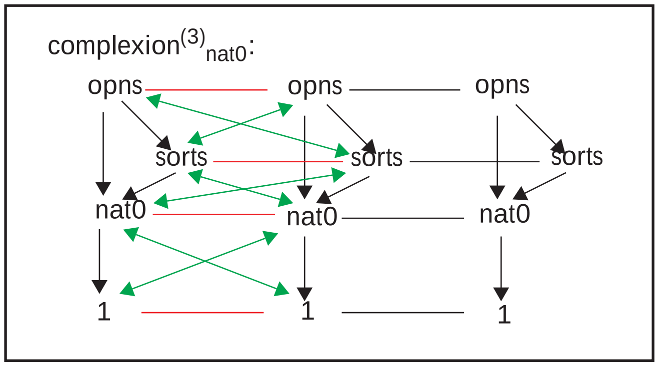
This are the mixed cases with unary and dyadic structures and the fully reduced case where all dyadic subsystems are reduced to the unary form.

All these cases above are basic forms without reflectionality and interactionality involved. An example for a more complex constellation is given below.

$$Tree\left(dyads\left([id,\ clon,\ clon]\right)\right) : (BBA) \longrightarrow (BBA,\ \varnothing BA,\ \varnothing BA)$$

	S_1^1	S_2^1	S_3^1	S_1^2	S_2^2	S_3^2	S_1^3	S_2^3	S_3^3
1	○	–	○	–	–	○	–	–	○
2	▵	▵	–	–	▵	–	–	▵	–
3	–	○	○	–	○	○	–	○	○

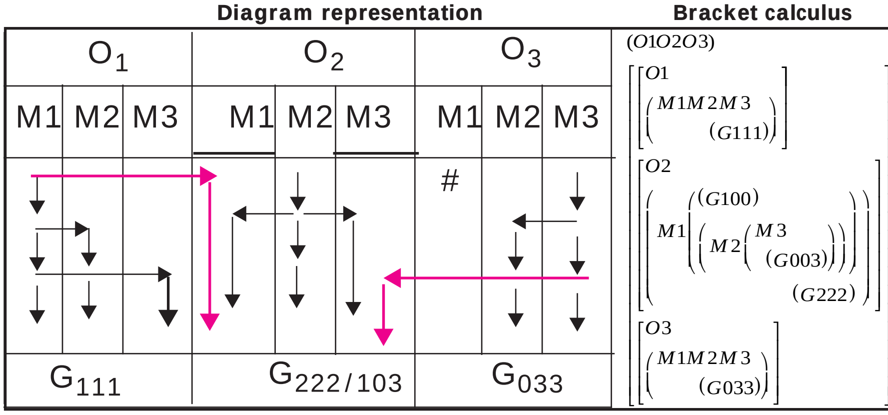
$[BBA, BA, BA]$	$O1$	$O2$	$O3$
$M1$	$BinTree_1$	$\varnothing$	$\varnothing$
$M2$	$BinTree_2$	$BinTree_2$	$BinTree_2$
$M3$	$UnTree_3$	$UnTree_3$	$UnTree_3$



Because this analysis is focussed on unary and binary *trees* and their data it could be called a *data-oriented* approach. There are other interpretations of the dyads, too. A dyad could be interpreted as an operator/operand-pair of a formal operation. To the unary dyads only operators or operand would correspond, and to the binary dyads the dichotomy of operator and operand.

3.8 More complex situations

Diagram representation **Bracket calculus** **Matrix**



PM	$O1$	$O2$	$O3$
$M1$	S_1	$S_{2.1}$	$\varnothing$
$M2$	S_1	$S_{2.0}$	S_3
$M3$	S_1	$S_{2.3}$	S_3

4 Calculating with the new Abacus

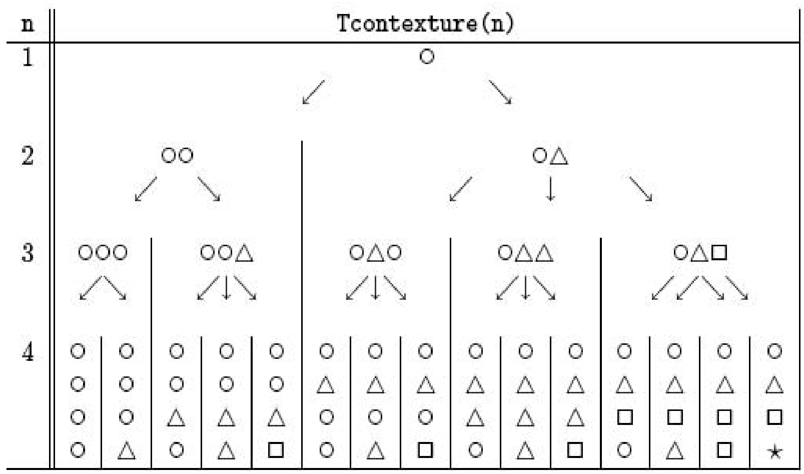
4.1 Pluri-dimensional binary arithmetic

Morphogrammatics as sketched until now is not telling how to calculate with numbers in an analogues way as we know it from the linear position system.

Morphogrammatics are the deep-structure of trans-computation.

Disseminated binary arithmetic systems are the fields of pluri-dimensional computation. Each dimension is realizing a binary number system. Other representations are possible. Each has its advantages and disadvantages for the purpose of an introduction of transclassic number systems.

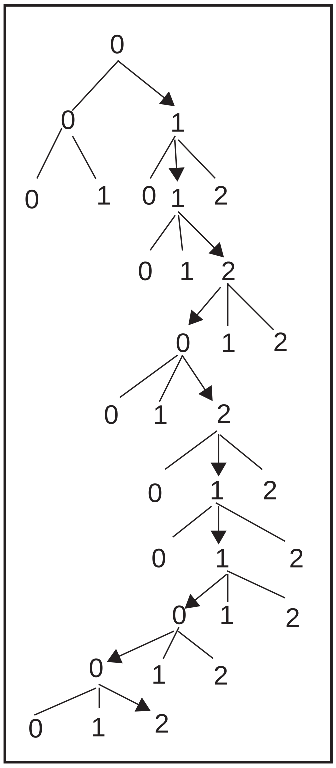
$$pluri - Dyads^{(m)} : \{0, 1\}^{(m)} \longrightarrow KG^{(m)}$$



$$\left[\begin{array}{l} tree \\ \left(\begin{array}{l} sops \\ \left(\begin{array}{l} dyads \\ (pattern) \end{array}\right) \end{array}\right) \end{array}\right]$$

4.1.1 Decompositions

TZ= (01120211002)-tree



The chain of events interpreted as a tissue of 3 binary systems S_1, S_2 and S_3 with 3 elements {0, 1, 2}. Each 2 elements are defining a binary system.

TZ=(01120211002)

this chain is having at least 2 numeric interpretations:

a) [011/12/202/211/100/02]
with the chain of sub-systems: $S_1S_2S_3S_2S_1S_3$
and
b) [011/112/202/211/1100/002]
with the chain of sub-systems: $S_1S_2S_3S_2S_1S_3$

In this case the chain of syb-systems of a) and b) are equal but the resolutions are of different length.

As in binary systems each number has a well defined position. In binary systems the possible positions of numbers are calculated by 2^n, trito-numbers in kenomic systems are calculated by their Stirling numbers of the 2. kind. The unspecified graphic representation of the number TZ as a tree is specified by the following presentation. Again, only an abbreviation can be given because of their complexity. The scheme is sketching the Stirling development. The indices of the trito-numbers in focus, in red, are indicated and marked by their place-number.

TZ= (01120211002)-sequence

$$T^{(3)} - number\ \ sequence\ (01120211002)$$

$$\left(\begin{array}{l}
1: \ (0)_1 \\
2: \ ((0), (1)_2) \\
3: \ (((0), (1)), ((0), (1)_4, (2))) \\
4: \ (((0), (1)), ((0), (1), (2)), ((0), (1), (2)), ((0), (1), (2)_{11}), ((0), (1), (2))) \\
5: \ (((0), (1)), ((0), (1), (2)), ((0), (1), (2)), ..., ((0)_{32}, (1), (2)), ..., ((0), (1), (2))) \\
6: \ (((0), (1)), ((0), (1), (2)), ((0), (1), (2)), ..., ((0), (1), ((2))_{95})), ..., ((0), (1), (2)) \\
7: \ (((0), (1)), ((0), (1), (2)), ((0), (1), (2)), ..., ((0), (1)_{284}, (2))), ..., ((0), (1), (2)) \\
8: \ (((0), (1)), ((0), (1), (2)), ((0), (1), (2)), ..., ((0), (1)_{851}, (2))), ..., ((0), (1), (2)) \\
9: \ (((0), (1)), ((0), (1), (2)), ((0), (1), (2)), ..., ((0)_{2552}, (1), (2))), ..., ((0), (1), (2)) \\
10: \ (((0), (1)), ((0), (1), (2)), ((0), (1), (2)), ..., ((0)_{7655}, (1), (2))), ..., ((0), (1), (2)) \\
11: (((0), (1)), ((0), (1), (2)), ((0), (1), (2)), ..., ((0), (1), (2)_{23988})), ..., ((0), (1), (2))
\end{array}\right)$$

TZ= (01120211002)-decompositions

$$dec(01120211002)$$

$$\begin{bmatrix} (011)_{1.2.4} \\ \quad (12)_{4.11} \\ \quad\quad (202)_{11.32.93} \\ \quad\quad\quad (211)_{93.284.851} \\ \quad\quad\quad\quad (100)_{851.2552.7655} \\ \quad\quad\quad\quad\quad (02)_{7655.23988} \end{bmatrix} \begin{bmatrix} (011)_{1.2.4} \\ \quad (112)_{2.4.11} \\ \quad\quad (202)_{11.32.93} \\ \quad\quad\quad (211)_{93.284.851} \\ \quad\quad\quad\quad (1100)_{284.851.2552.7655} \\ \quad\quad\quad\quad\quad (002)_{2552.7655.23988} \end{bmatrix}$$

$$T^{(3)} - number\ \ sequence$$

$$(0112000211002)$$

$$\begin{pmatrix} (0) \\ ((0),(1)) \\ (((0),(1)),((0),(1),(2))) \\ (...,((0),(1),(2)),...) \\ (...,((0),(1),(2)),...) \\ (...,((0),(1),(2)),...) \\ (...,((0),(1),(2)),...) \\ (...,((0),(1),(2)),...) \\ (...,((0),(1),(2)),...) \\ (...,((0),(1),(2)),...) \\ (...,((0),(1),(2)),...) \\ (...,((0),(1),(2)),...) \\ (...,((0),(1),(2)),...) \end{pmatrix}$$

Another example

This trito-number TZ= (0112000211002) has interpretations with different chains of sub-systems and different length of resolutions. The length of the chains of sub-systems c) is l=8 and the length of d) is l=6. The 4. resolutions of c) and d) are of different length, c) is 3 with (000) and d) is 6 with (200002).

c) [01/12/20/000/02/211/100/02] with

$S_1S_2S_3S_1S_3S_2S_1S_3$, l=8, r4=3

d) [01/12/200002/211/100/02] with

$S_1S_2S_3S_2S_1S_3$, l=6, r4=6

4.1.2 Cracks and gaps

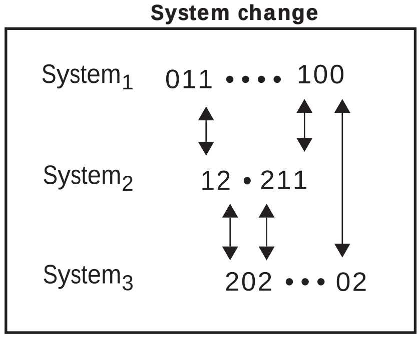
"The law which we applied was the principle of numerical induction; and although nobody has ever counted up to 10^{1000}, or ever will, we know perfectly well that it would be the height of absurdity to assume that our law will stop being valid at the quoted number and start working again at 10^{10000}.

We know this with absolute certainity because we are aware of the fact that the principle of induction is nothing but an expression of the reflective procedure our consciousness employs in order to become aware of a sequence of numbers. The breaking down of the law even for one single number out of the infinity would mean there is no numerical consciousness at all!" Gotthard Gunther, Cybernetic Ontology, p. 360

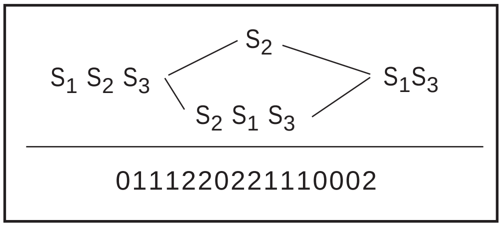
4.1.3 Leaps and saltations

4.2 Interpretations

System change

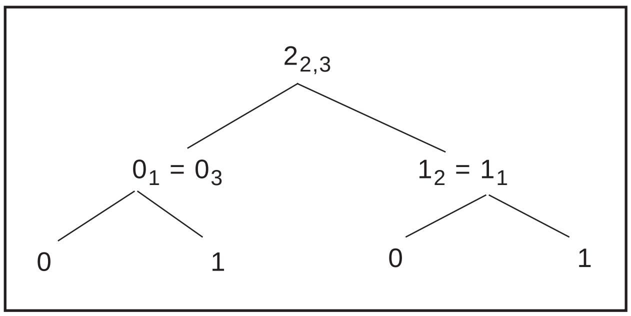


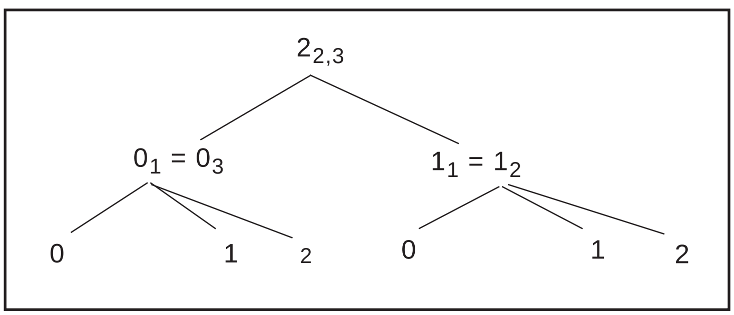
4.2.1 Negotations about interpretations

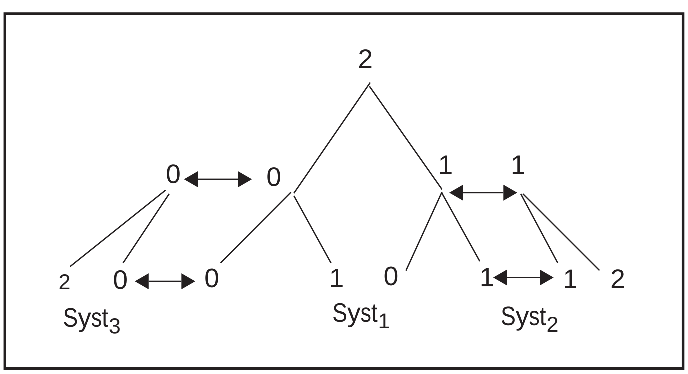


S1 = (0111) ==
S2 = (12) ==
S3 = (2022) ==
S2 = (2111) :: S2=(211), S1=(110), S3=(00)
S1 = (1000) =≠
S3 = (02) ==

The possibility to interpret a sequence in different ways enables an asymmetry between the construction and the destruction of the sequence. The way down has not to be the way up. Asymmetric inversions are possible. And obviously, a separation and reunion of the path of the sequence is accessible, too.







system change with bifurcation

4.3 Bisimulations vs. equivalence and equality

Bisimulation – the Basic Case

We first give the definition for the basic modal language.

Let M = (W, R, V) and M´= (W´, R‘, V‘) be two models.

A non-empty binary relation Z WxW‘is called bisimulation between M and M‘if the following conditions are satisfied:

(i) If wZw‘then w and w‘satisfify the same letters.

(ii) If wZw‘and Rwv, then there exists v‘(in M‘) such that vZv‘and R‘w‘v´ (the *forth condition*).

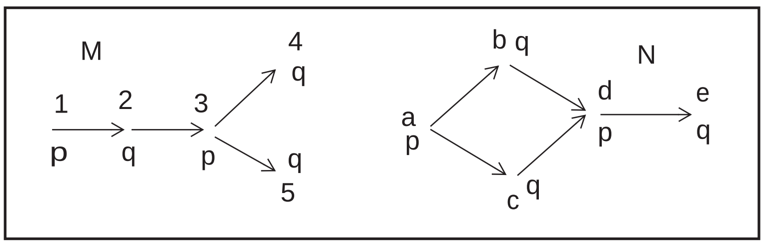
(iii) The converse of (ii): if wZw´and R‘w‘v‘. then there exists v (in M) such that vZv‘and Rwv (the *back condition*).

Example:

Models M and N are bisimilar under the relation Z.

Z = {(1,a), (2,b), (2,c), (3,d), (4,e), (5,e)}

Bisimilar Models



The two models M and N have the same behavior in respect to the relation Z. To each transition in M there is a corresponding transition in N which is fulfilling the states of the knots p and q. Hence, the models M and N are bisimilar.

"Quite simply, a bisimulation is a relation between two models in which related states have identical atomic information and matching possibilities."

Modal Logic (Blackburn et al.)

Bisimulation, Locality, and Computation

"Evaluating a modal formula amounts to running an automaton: we place it at some state inside a structure and let it search for information. The automaton is only permitted to explore by making transitions to neighboring states; that is, it works locally.

Suppose such an automaton is standing at a state w in a model M, and we pick it up and place it at state w´in a different model M´; would it notice the switch? If w and w´are bisimilar, no. Our atomaton cares only about the information at the current state and the information accessible by making a transition – it is indifferent to everything else. (...)" p. 68

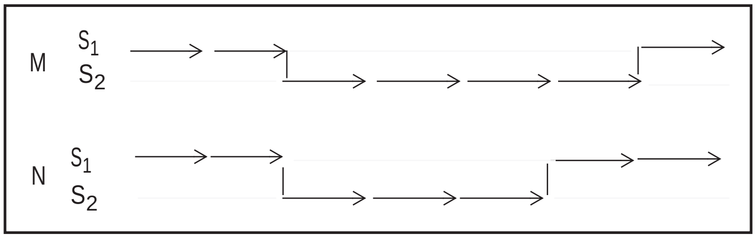
Morphogramms and Bisimulation

A morphogram MG = (aabcbcbaa) can be interpreted as a trito-number TZ = (001212100). The behavior of this trito-number can be observed only by its actions in accessible sub-systems which are here the binary components. The trito-number TZ is showing two different behaviors M and N which are represented by the two different developments of binary systems.

M = ($S_{1122221}$) and

N = ($S_{1122211}$).

M and N are different at the second last position in respect to S_1 and S_2.



In contrast, the two trito-numbers TZ_1= (001212) with a sub-system development S_{11222} and TZ_2 = (001012) with a sub-system development S_{11112} are not bisimilar because the states at the position 4 of both differs with "2" for TZ_1 and "0" for TZ_2.

5 Computation and Iterability

5.1 Turing, Zuse and Gurevich

"The basic idea is very simple, at least in the sequential case, when time is sequential (the algorithm starts in some initial state S_0 and goes through states S_1, S_2, etc.) and only a bounded amount of work is done each step.

Each state can be represented by a first-order structure: a set with relations and functions. (...) Thus, the run can be seen as a succession of first-order structures, but this isn´t a very fruitful way to see the process.

How do we get from a state S_i to the next state S_{i+1}? Following the algorithm, we perform a bounded number of transition rules of very simple form." Gurevich, p. 5

"A *computation* of *R* consists of a finite or infinite sequence of states $M_0 \ldots M_n \ldots$, such that for each a 0 M_n arises from M_{n-1} by one application of some rule in *R.*"

In short: "IF b, THEN U_1 .....U_k".

„What Turing did was to show that calculation can be broken down into the iteration (controlled by a ´program´) of extremely simple concrete operations; ...“ Gandy, in: Herken, p. 101

Konrad Zuse writes: *"Rechnen heisst: Aus gegebenen Angaben nach einer Vorschrift neue Angaben bilden."* (Plankalkül)

[Computation means: Producing new information from given information according to a rule.]

In all those descriptions of computation as iterations there is no proviso mentioned that restricts computation to linearity. Obviously, it is, also not declared, the conditio sine qua non of any computation. Computation, as we know it, is restricted to linearity. This is not in conflict with parallel and concurrent computation as we can learn from Zuse’s Plankalkül.

Because computation has a very abstract model it is also independent of any positionality. A stroke calculus is doing the job of defining the realm of *computability*.

Hence, computations don’t take place. They simply happen as physical events, i.e., in space and time. But space is not a structural place, locus, position, like in a positional system.

5.2 TransComputation as Accretion

Iterability is not reduced to iteration it also includes alteration in the sense of accretion.

6 Abaci in an interactional/reflectional game

The following descriptions of reflectional interactions are not necessarily different to the models known by Second-Order Cybernetics, especially Gordon Pask, Paul Pangaro and Vladimir Levebvre. The main difference consists in the fact that the polycontextural modeling and design is intended on the level of computation and not in an applicative way. Thus, it is postulated that mathematics as such should be designed as interactional and reflectional in all its basic constituents beginning with its morphograms.

$[\varnothing B \varnothing]$	S_1^2	S_2^2	S_3^2
1	–	–	–
2	–	△	–
3	–	○	–

$[BB, B, BA]$	$O1$	$O2$	$O3$
$M1$	B_1	$\varnothing$	$\varnothing$
$M2$	B_2	B_2	B_2
$M3$	$\varnothing$	$\varnothing$	A_3

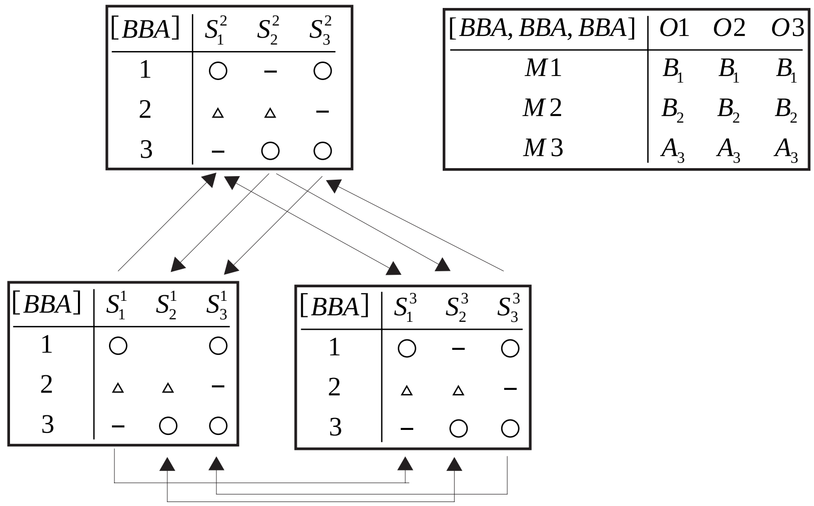
$[BB\varnothing]$	S_1^1	S_2^1	S_3^1
1	○		–
2	△	△	–
3	–	○	–

$[\varnothing BA]$	S_1^3	S_2^3	S_3^3
1	–	–	○
2	–	△	–
3	–	○	○

$$[(id, clon, \varnothing), (\varnothing, id, \varnothing), (\varnothing, clon, id)] : (BBA) \longrightarrow (BB\varnothing, \varnothing B\varnothing, \varnothing BA)$$

System S1 has a model of the inner structure of system S2 and is placing this model in its interactional space S1.2. System S2 is not involved in any reflection or interaction, i.e., S2 is not modeling its environment consisting of the systems S1 and S3. Also system S3 itself is of reduced structure, it has a model of system S2, too. Because system S3 is mediating between the systems S1 and S2, system S1 and system S3 can communicate about system S2. But this will happen without a structural representation as an interactional/reflectional mode between S1 and S3 because S1 has no representation of S3 and S3 neither from S1.

The interaction of S2 with S1 and S3 is not sending information but the structural frame in which information can be set. The structural frame is a structured place-holder for information but not itself information in the sense of a message.



$$\left[(id, clon, clon), (clon, id, clon), (clon, clon, id)\right] : (BBA) \longrightarrow (BBA, BBA, BBA)$$

In this case a structural representation happens as an interactional but with no reflectional mode between S1.1 and S2.2 because both have a representation of each other as S1.2 and S2.2 and are mediated by their representations of S3 as S1.3 and S2.2. Additionally, S3.3 has representations from S1 and S2 as S3.1 and S3.2. Interactional behaviors are realized by the cloning operator "*clon*".

The same morphogrammatic pattern may have a different realization including a reflectional action represented by the replication operator "*repl*".

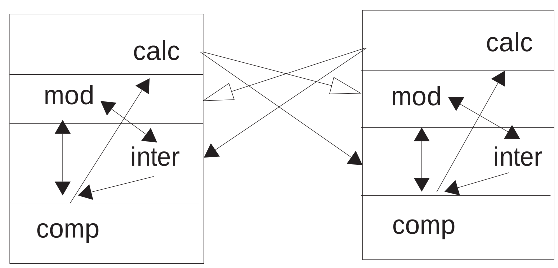
In fact cloning is also a bi-directional action because the cloned "object" has to be accepted by the neighbor systems. It has to be offered a structural place to set the cloned object. Thus, the interaction happens as a double action of duplicating (cloning) and acceptance of the duplicate at place in a neighbor system involved into the interaction.

$[BBA, BBA, BBA]$	$O1$	$O2$	$O3$
$M1$	B_1	B_1	B_1
$M2$	$\overline{B_1}$	B_2	B_3
$M3$	A_3	A_3	A_3

$$\left[(id, repl, clon), (clon, id, clon), (clon, clon, id)\right] : (BBA) \longrightarrow (BBA, BBA, BBA)$$

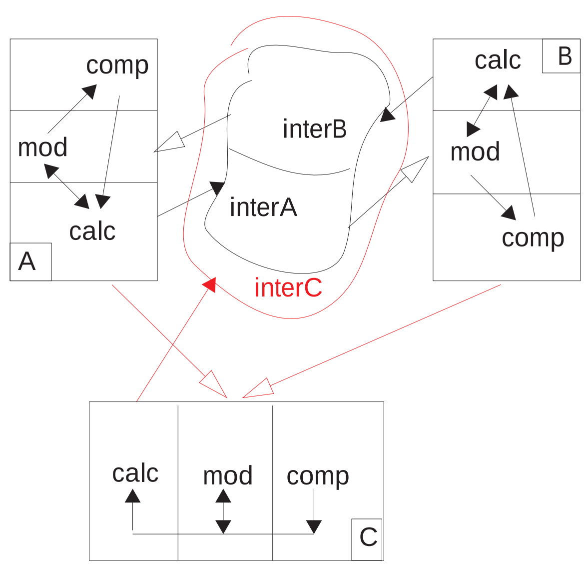
6.1 A model of a reflectional/interactional 3-agent system

Model of a 2-agent system



$[inter/refl - Sys]$	$O1$	$O2$	$O3$
$M1$	$calc$	$\varnothing$	$\varnothing$
$M2$	mod	$inter$	$\varnothing$
$M3$	$comp$	$\varnothing$	$\varnothing$

Model of a 3-agent system



	S_1^1	S_2^1	S_3^1	S_1^2	S_2^2	S_3^2	S_1^3	S_2^3	S_3^3
1	$calc_1$	–	–	–	$comp_2$	–	–	–	$comp_3$
2	mod_1	$inter_2$	$inter_3$	$inter_1$	$calc_2$	$inter_3$	$inter_1$	$inter_2$	mod_3
3	$comp_1$	–	–	–	mod_2	–	–	–	$calc_3$

6.2 An interpretation of the model

6.2.1 How does it work?

System S3 (C) is giving place to reflect / interact for System1 (A) and systemS2 (B) about their common goals and rules. Thus, system C is playing the part of a super-visor enabling S1 and S2 to realize a kind of self-reflection about their common actions. Without S3, the goals and rules would be *implicit* for S1 and S2 and pre-given for their game. And thus, not changeable *during* the game. If they would like to change the game, they would have to stop, to change and then to restart a new game. Start and end of an interactional / reflectional game between system1 and system2 is placed in system3. The negotiation about the goals and the rules and the decision or even the contract to accept the situation is outside the actual actions between A and B and is therefore localized, systematically, at the place C.

6.2.2 Metaphor of an application

My	Your	Our
calculation interaction reflection comparision	calculation interaction reflection comparision	calculation interaction reflection comparision

calculation (goal)
reflection (modeling)
interaction (action, realization)
comparison (correction).
Comparision can be seen as a computation.

Calculations
My calculations are my-calculations,
Your calculations are your-calculations,
Our calculations are our-calculations.

Interaction
My interactions are accepted by your-modeling as my-interactions,
Your interactions are accepted by my-modeling as your-interactions,
Our interactions are accepted by our-modeling as our-interactions.

Modeling
I am reflecting / modeling your calculation in my-reflection,
You are reflecting / modeling my-calculations in your-reflection,
We are reflecting / modeling our-calculations in our-reflection.

Comparision
I am comparing (reflecting, modeling)
your-interaction with my-reflection on your-calculation in my-comparison.
You are comparing
my-interaction with your-reflection on my-calculation in your-comparison.
We are comparing (super-vision)
our-interactions with our-reflections on our-calculations in our-comparison.

Leibniz-Monads
Each agent is able to give structural space to himself and to the neighbor agents to *model* all his neighbor agents' *interactions*; *comparising* and correcting his model

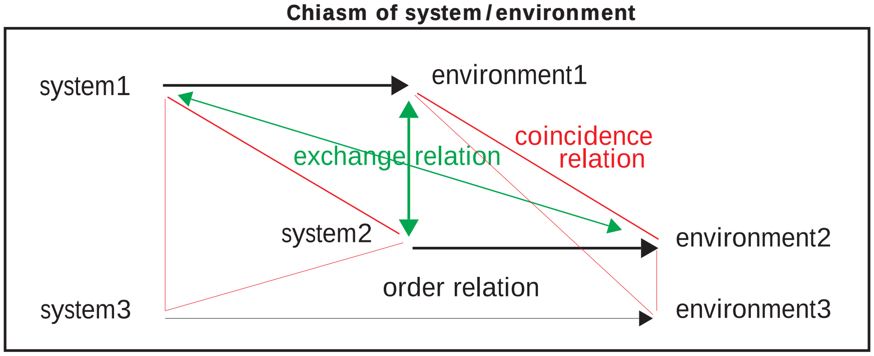
about the others *calculations* and *interactions*, and being able to be interacted by all his neighbor agents.

This is the case of a *harmonized* agent system, called the *Leibniz-Monads*.

6.2.3 System environment distinction

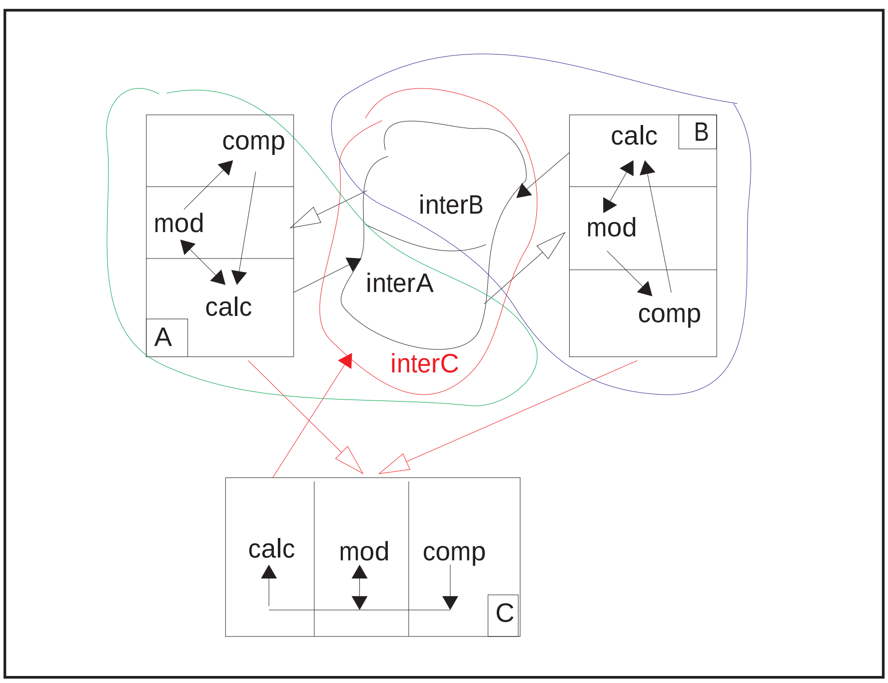
What's my environment is your system,
What's your environment is my system,
What's our environments and our systems is the environment of our-system.

Chiasm of system/environment



Chiastic interdependency

Interactions are based on computations and reflections.
Computations are based on interactions and reflections.
Reflections are based on interactions and computations.
Comparisions are based on reflections and interactions.



From Dialogues to Polylogues
http://www.thinkartlab.com/pkl/lola/Games-short.pdf

6.2.4 To calculate means to take part in the culture of calculation

What are we doing if we are using an Abacus?

The common answer is: Buy an Abacus, follow the instructions and then use it for your business calculations. What you are doing while using an Abacus is to calculate with the physical devise Abacus according to the rules you learned from your buckled. You have not to understand that your Abacus is based on a positional system to organize your calculations.

This might not be totally wrong. But this explanation is presupposing a lot more.

Even in the solitaire use of an Abacus the complexity of the game always happens. Even if my-calculation are my-calculation, they are not reasonable in isolation. I learned the rules from a teacher. He represents your-calculation. And our-calculation happens as a result of my-calculation and your-calculation, that is, if my-calculations correspond to the calculation I learned from your-calculation. This gives my-calculation the guarantee that my-calculations are correct. The correctness of my-calculations are represented in our-calculations, i.e., in the accordance with the general rules. Thus, there is never something like my-calculation in a solitaire isolation.

To know about these intricate relationships is a first step to implement them in a physical mechanism, i.e., to *objectify* the mental processes of learning and using an Abacus.

Now we can leave the metaphor of the Abacus and turn to better funded research programs for cognitive systems in robotics and game development.

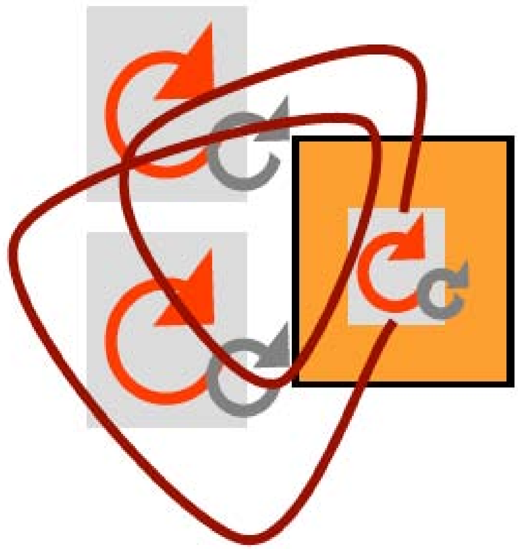
And all the rest is the work to be done by a plumper. But as we know, there are no plumpers left.

Other wordings

Interaction is based on *inquiries* and not on calls (send, receive). Inquiries can be *rejected* or *accepted*. The inquiring forms an internal model of the inquirer, only if this succeeds, can it step into a communication process. In the communication model, defined through (process, send, receive, buffer), additionally to the non-interactive structure of the algorithms, this basic encounter structure of the agents must be pre-given by the designer.

FIBONACCI in ConTeXtures
http://www.thinkartlab.com/pkl/lola/FIBONACCI.pdf

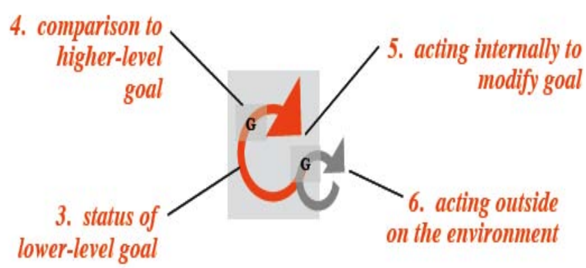
Other reflectional interaction models (Pangaro, Levebvre, Pask)



A nicer design is given for similar situations of *participative interaction* by Paul Pangaro.

danm.ucsc.edu/courses/2004-05/spring/204/lectures/paul_pangaro Paul_Pangaro_lecture

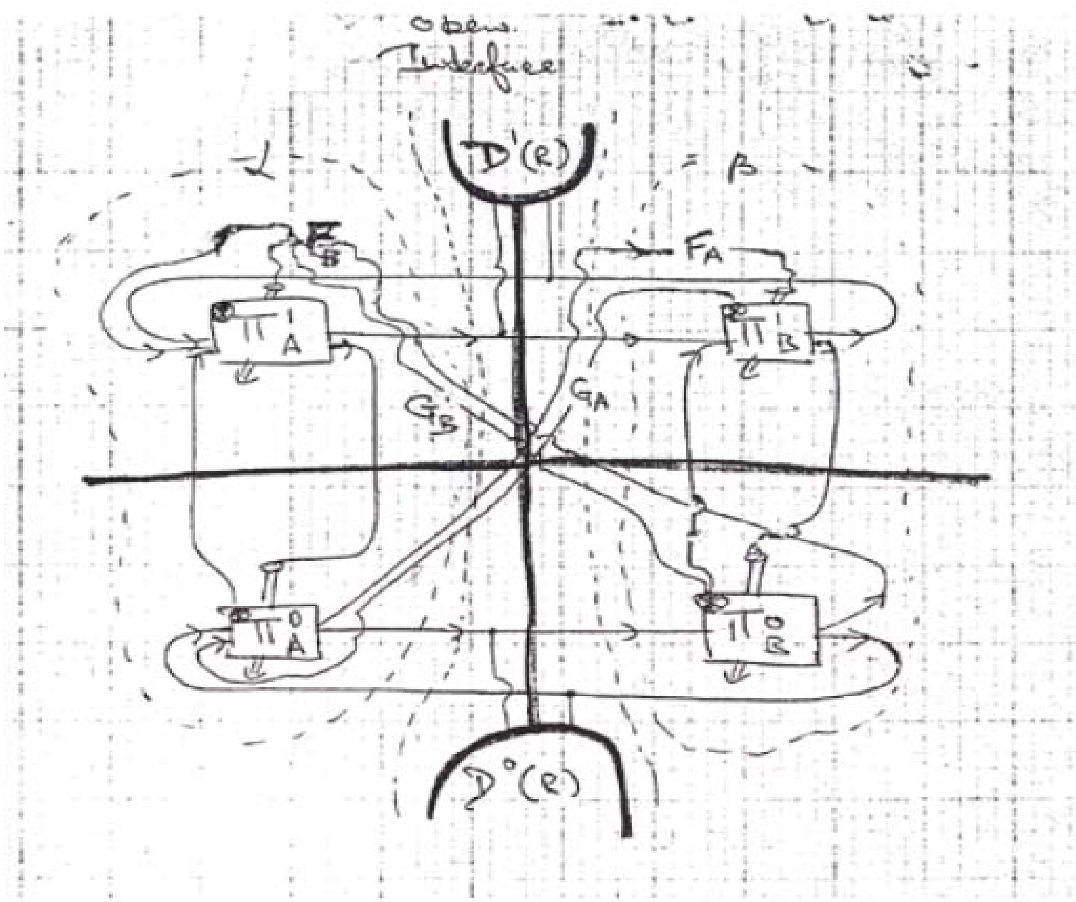
www.pangaro.com



Vladimir Levebvre

http://www.c4ads.org/files/day.1.1300.1400.vladimir.lefebvre.pdf?PHPSESSID=928087361390dbc3005b4b1e16ba6448

Gordon Pask's Interface



An interface is a "Schnitt und Naht"-Stelle

Chiasms vs. circles